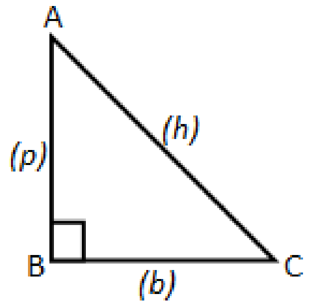
TRIGONOMETRY



- $sin\theta = \frac{p}{h}$; $cosec\theta = \frac{h}{p}$
- $cos\theta = \frac{b}{h}$; $sec\theta = \frac{h}{b}$
- $ten\theta = \frac{p}{b}$; $cot\theta = \frac{b}{p}$

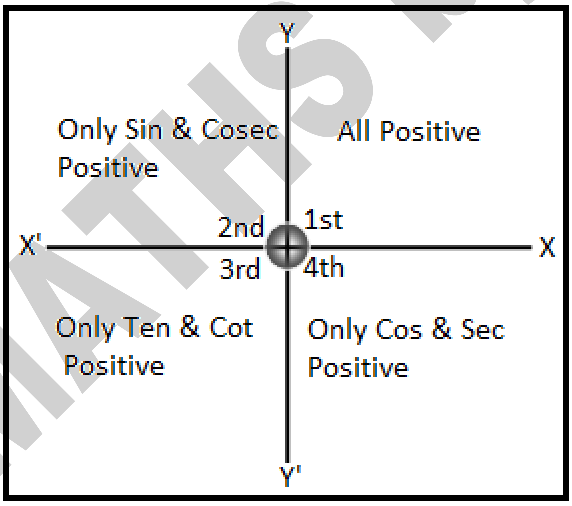
- $-1 \leq sin\theta \text{ or } cos\theta \leq +1$
- $-\infty \leq tan\theta \text{ or } cot\theta \leq +\infty$
- $+1 \leq sec\theta \text{ or } cosec\theta \leq -1$

a) $sin\theta cosec\theta = 1$

b) $cos\theta sec\theta = 1$

c) $ten\theta cot\theta = 1$

- ✓ $ten\theta = \frac{sin\theta}{cos\theta} = \frac{1}{cot\theta}$
- ✓ $cot\theta = \frac{cos\theta}{sin\theta} = \frac{1}{ten\theta}$
- ✓ $sec\theta = \frac{1}{cos\theta}$
- ✓ $cosec\theta = \frac{1}{sin\theta}$



- 1st= 0° to 90° ; 2nd= (90°+) to 180°
- 3rd=(180°+) to270° ; 4th=(270°+) to 360°

❖ Changes between Angles:-

➢ $sin\theta \leftrightarrow cos\theta$ // $ten\theta \leftrightarrow cot\theta$// $sec\theta \leftrightarrow cosec\theta$

- $sin(90° - \theta) = cos\theta$
- $cos(90° - \theta) = sin\theta$
- $ten(90° - \theta) = cot\theta$
- $cot(90° - \theta) = ten\theta$
- $sec(90° - \theta) = cosec\theta$
- $cosec(90° - \theta) = sec\theta$
- $sin(90° + \theta) = cos\theta$
- $cos(90° + \theta) = -sin\theta$
- $ten(90° + \theta) = -cot\theta$
- $cot(90° + \theta) = -ten\theta$
- $sec(90° + \theta) = -cosec\theta$
- $cosec(90° + \theta) = sec\theta$
- $sin(180° - \theta) = sin\theta$
- $cos(180° - \theta) = -cos\theta$
- $ten(180° - \theta) = -ten\theta$
- $cot(180° - \theta) = -cot\theta$
- $sec(180° - \theta) = -sec\theta$
- $cosec(180° - \theta) = cossec\theta$
- $sin(180° + \theta) = -sin\theta$
- $cos(180° + \theta) = -cos\theta$
- $ten(180° + \theta) = ten\theta$
- $cot(180° + \theta) = cot\theta$
- $sec(180° + \theta) = -sec\theta$
- $cosec(180° + \theta) = -cosec\theta$
- $sin(270° - \theta) = -cos\theta$
- $cos(270° - \theta) = -sin\theta$
- $ten(270° - \theta) = cot\theta$
- $cot(270° - \theta) = ten\theta$
- $sec(270° - \theta) = -cosec\theta$
- $cosec(270° - \theta) = -sec\theta$
- $sin(270° + \theta) = -cos\theta$
- $cos(270° + \theta) = sin\theta$
- $ten(270° + \theta) = -cot\theta$
- $cot(270° + \theta) = -ten\theta$
- $sec(270° + \theta) = cosec\theta$
- $cosec(270° + \theta) = -sec\theta$
- $sin(360° - \theta) = -sin\theta$
- $cos(360° - \theta) = cos\theta$
- $ten(360° - \theta) = -ten\theta$
- $cot(360° - \theta) = -cot\theta$
- $sec(360° - \theta) = sec\theta$
- $cosec(360° - \theta) = -cosec\theta$
- $sin(360° + \theta) = sin\theta$
- $cos(360° + \theta) = cos\theta$
- $ten(360° + \theta) = ten\theta$
- $cot(360° + \theta) = cot\theta$
- $sec(360° + \theta) = sec\theta$
- $cosec(360° + \theta) = cosec\theta$

TRIGONOMETRY

❖ SOME IMPORTANT FACTS:-

- $sec\theta + tan\theta = 1/(sec\theta - tan\theta)$
- $cosec\theta + cot\theta = 1/(cosec\theta - cot\theta)$
- $sin^2\theta + cos^2\theta = 1$
- $sec^2\theta - tan^2\theta = 1$
- $cosec^2\theta - cot^2\theta = 1$
- $sin2\theta = 2\ sin\theta\ cos\theta$
- $sin2\theta = 2\ tan\theta/(1 + tan^2\theta)$
- $cos2\theta = cos^2\theta - sin^2\theta$
- $cos2\theta = 2cos^2\theta - 1$
- $cos2\theta = 1 - 2sin^2\theta$
- $cos2\theta = (1 - tan^2\theta)/(1 + tan^2\theta)$
- $tan2\theta = 2tan\theta/(1 - tan^2\theta)$
- $sin3\theta = 3sin\theta - 4sin^3\theta$
- $cos3\theta = 4cos^3\theta - 3cos\theta$
- $tan3\theta = (3tan\theta - tan^3\theta)/(1 - 3tan^2\theta)$
- $tan^2\theta - sin^2\theta = tan^2\theta\ sin^2\theta$
- $cot^2\theta - cos^2\theta = cot^2\theta\ cos^2\theta$
- $cosec^2\theta + sec^2\theta = cosec^2\theta\ sec^2\theta$
- $sin^4\theta + cos^4\theta = 1 - 2sin^2\theta\ cos^2\theta$
- $sin^6\theta + cos^6\theta = 1 - 3sin^2\theta\ cos^2\theta$
- $sin\theta * sin2\theta * sin4\theta = \frac{1}{4}sin3\theta$
- $cos\theta * cos2\theta * cos4\theta = \frac{1}{4}cos3\theta$
- $tan\theta * tan2\theta * tan4\theta = tan3\theta$
- $tan^{-1}x + tan^{-1}y = tan^{-1}\frac{(x+y)}{(1-xy)}$
- $tan^{-1}x - tan^{-1}y = tan^{-1}\frac{(x-y)}{(1+xy)}$
- $\frac{a}{sinA} = \frac{b}{sinB} = \frac{c}{sinC} = 2R$
- $cosA = (b^2 + c^2 - a^2)/2bc$
- $cosB = (a^2 + c^2 - b^2)/2ac$
- $cosC = (a^2 + b^2 - c^2)/2ab$
- $sin15^0 = (\frac{\sqrt{3}-1}{2\sqrt{2}}) = cos75^0$
- $cos15^0 = (\frac{\sqrt{3}+1}{2\sqrt{2}}) = sin75^0$
- $tan15^0 = (2 - \sqrt{3}) = cot75^0$
- $cot15^0 = (2 + \sqrt{3}) = tan75^0$

❖ If $(x + y) = 90^0$: –

- $sinx - cosy = 0$
- $cosx - siny = 0$
- $tanx - coty = 0$
- $cotx - tany = 0$
- $cosecx - secy = 0$
- $secx - cosecy = 0$
- $tanx * tany = 1$
- $cotx * coty = 1$
- $sinx/cosy = 1$
- $cosx/siny = 1$
- $tanx/coty = 1$
- $cotx/tany = 1$
- $cosecx/secy = 1$
- $secx/cosecy = 1$
- $sin^2x + sin^2y = 1$
- $cos^2x + cos^2y = 1$
- $sec^2x - cot^2y = 1$
- $cosec^2y - tan^2x = 1$
- $cosec^2x - tan^2y = 1$
- $sec^2y - cot^2x = 1$

TRIGONOMETRY

❖If $(A+B+C)=180^0$: −

- $sinA + sinB + sinC = 4\,cos\frac{A}{2}\,cos\frac{B}{2}\,cos\frac{C}{2}$
- $sin2A + sin2B + sin2C = 4\,sinA\,sinB\,sinC$
- $tanA + tanB + tanC = tanA\,tanB\,tanC$
- $cotA\,cotB + cotB\,cotC + cotC\,cotA = 1$

TRIGONOMETRY

❖ Some Special Formulas:-

- $\sin(A+B) = sinA\ cosB + cosA\ sinB$
- $\sin(A-B) = sinA\ cosB - cosA\ sinB$
- $\cos(\mathrm{A}+\mathrm{B}) = cosA\ cosB - sinA\ sinB$
- $\sin(A-B) = cosA\ cosB + sinA\ sinB$
- $\tan(A+B) = {(tanA + tanB)}\big/{(1 - tanA\ tanB)}$
- $\tan(A-B) = {(tanA - tanB)}\big/{(1 + tanA\ tanB)}$
- $\cot(A+B) = {(cotA\ cotB - 1)}\big/{(cotA + cotB)}$
- $\cot(A-B) = {(cotA\ cotB + 1)}\big/{(cotB - cotA)}$
- $\tan(A+B+C) = \frac{tanA+tanB+tanC-tanA\ tanB\ tanC}{1-tanA\ tanB-tanB\ tanC-tanC\ tanA}$
- $\sin(A+B) + \sin(A-B) = 2\ sinA\ cosB$
- $\sin(A+B)$ - $\sin(A-B) = 2\ cosA\ sinB$
- $\cos(A+B) + \cos(A-B) = 2\ cosA\ cosB$
- $\cos(A-B)$ - $\cos(A+B) = 2\ sinA\ sinB$
- $\sin(A+B)*\sin(A-B) = sin^2A - sin^2B = cos^2B - cos^2A$
- $\cos(A+B)*\cos(A-B) = cos^2A - sin^2B = cos^2B - sin^2A$
- $sinC + sinD = 2\sin\left(\frac{C+D}{2}\right)\ \cos\left(\frac{C-D}{2}\right)$
- $sinC - sinD = 2\cos\left(\frac{C+D}{2}\right)\ \sin\left(\frac{C-D}{2}\right)$
- $cosC + cosD = 2\cos\left(\frac{C+D}{2}\right)\ \cos\left(\frac{C-D}{2}\right)$
- $cosC - cosD = 2\sin\left(\frac{C+D}{2}\right)\ \sin\left(\frac{D-C}{2}\right)$
- $if\ sec\theta + tan\theta = x,\ \ then :\gg\ \ sec\theta = \frac{x^2+1}{2x}\ ; tan\theta = \frac{x^2-1}{2x}\ ; sin\theta = \frac{x^2-1}{x^2+1}$
- $if\ cosec\theta + cot\theta = x\ \ then :\gg\ \ cosec\theta = \frac{x^2+1}{2x}\ ; cot\theta = \frac{x^2-1}{2x}\ ; cos\theta = \frac{x^2-1}{x^2+1}$
- $if\ (sin\theta + cosec\theta) = 2,\ \ then :\gg\ \ (sin^n\theta + cosec^n\theta\ ) = 2$ $\quad [\because (sin\theta + cosec\theta) \geq 2]$
- $if\ (cos\theta + sec\theta) = 2,\ \ then :\gg\ \ (cos^n\theta + sec^n\theta\ ) = 2$ $\quad [\because\ (cos\theta + sec\theta) \geq 2]$
- $if\ (tan\theta + cot\theta) = 2,\ \ then :\gg\ \ (tan^n\theta + cot^n\theta\ ) = 2$ $\quad [\because\ (tan\theta + cot\theta) \geq 2]$
- $if\ (a\ sin\theta \pm b\ cos\theta) = c,\ \ then :\gg\ \ (a\ cos\theta\ \mp b\ sin\theta) = \pm\sqrt{a^2+b^2-c^2}$
- $if\ (a\ cos\theta \pm b\ sin\theta) = c,\ \ then :\gg\ \ (a\ sin\theta\ \mp b\ cos\theta) = \pm\sqrt{a^2+b^2-c^2}$

TRIGONOMETRY

- $if\ (sin\theta \pm cos\theta) = x,\ then :\gg\ (sin\theta \mp cos\theta) = \sqrt{2 - x^2}$
- $if\ (2\ cos\theta) = x + \frac{1}{x},\ then :\gg\ (2\ cos3\theta) = x^3 + \frac{1}{x^3}$
- $if\ (sin\theta + sin^2\theta) = 1,\ then :\gg\ (cos^2\theta + cos^4\theta) = 1\ \&\ (tan^2\theta + tan^4\theta) = 1$
- $if\ (cos\theta + cos^2\theta) = 1,\ then :\gg\ (sin^2\theta + sin^4\theta) = 1\ \&\ (cot^2\theta + cot^4\theta) = 1$
- $if\ (A\ sin\theta + B\ cos\theta) = C, then :\gg sin\theta = \frac{A}{C}; cos\theta = \frac{B}{C}; tan\theta = \frac{A}{B}\quad [Where\ (C^2 = A^2 + B^2)]$
- $if\ (A^2sin^2\theta + B^2cos^2\theta) = C^2, then :\gg sin\theta = \sqrt{\frac{B^2-C^2}{B^2-A^2}};\ cos\theta = \sqrt{\frac{C^2-A^2}{B^2-A^2}};\ tan\theta = \sqrt{\frac{B^2-C^2}{C^2-A^2}}$
- $sin12^0 * sin24^0 * sin48^0 * sin84^0 = 1/16$

❖ Maximum & Minimum Values :-

- $Minimum\ value\ of\ (a\ sin^2\theta + b\ cosec^2\theta) = 2\sqrt{a * b}$
- $Minimum\ value\ of\ (a\ cos^2\theta + b\ sec^2\theta) = 2\sqrt{a * b}$
- $Minimum\ value\ of\ (a\ tan^2\theta + b\ cot^2\theta) = 2\sqrt{a * b}$
- $Maximum\ value\ of\ (a\ sin\theta + b\ cos\theta) = \sqrt{a^2 + b^2}$
- $Minimum\ value\ of\ (a\ sin\theta + b\ cos\theta) = -\sqrt{a^2 + b^2}$
- $if\ A = k\ sin^m\theta + k\ cos^n\theta\ [m\ \&\ n\ are\ even];\ to\ find\ the\ Minimum\ value\ of\ A, put\ \theta = 45^0\ \&$ $\&\ put\ \theta = 0^0\ or\ 90^0\ to\ find\ Maximum\ value\ of\ A.$
- $Maximum\ value\ of\ (sin\theta * cos\theta)^n = (1/2)^n$

❖ Trigonometric values:-

	0^0	30^0	45^0	60^0	90^0
$sin\theta$	0	1/2	$1/\sqrt{2}$	$\sqrt{3}/2$	1
$cos\theta$	1	$\sqrt{3}/2$	$1/\sqrt{2}$	1/2	0
$tan\theta$	0	$1/\sqrt{3}$	1	$\sqrt{3}$	∞
$cot\theta$	∞	$\sqrt{3}$	1	$1/\sqrt{3}$	0
$sec\theta$	1	$2/\sqrt{3}$	$\sqrt{2}$	2	∞
$cosec\theta$	∞	2	$\sqrt{2}$	$2/\sqrt{3}$	1