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→ MENSURATION
- Area
- Volume

→ TRIGNOMETRY

→ GEOMETRY

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$$\frac{1}{\operatorname{cosec}\theta} + \frac{1}{\sec\theta} = \frac{13}{2}$$

$$\frac{\operatorname{cosec}\theta + \sec\theta}{\ldots}$$

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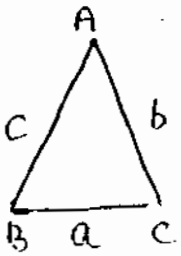
MENSURATION

Meansuration
- 2D
  - Area
  - Perimeter
- 3D
  - Area
  - Volume

Area :→ Area is the Whole space Covered. of any Diagram

Perimeter :→ Perimeter is the Total Length of any Diagram.



जो Angle जितना बड़ा होगा उसके सामने की भुजा का Size उतना ही बड़ा होगा। और जो Angle छोटा होगा उसके सामने की भुजा उतनी ही छोटी होगी।

Sum of any two side of a traingle is always greater than 3rd side $a+b>c$

Difference of any two side of a traingle is always less than 3rd side $a-b<c$

e.g 8 5 4 → This is a Triangle.

$8+5>4$, $5+4>4$, $8+4>5$, $5+4>8$ $\therefore a+b>c$

$8-5<4$, or $5-4<8$, $8-4<5$ $\therefore a-b<c$

→ Two Side of a Triangle is 6, 8. find the Rangle of third Side.

We know for a Triangle $a+b>c$, $a-b<c$

$\therefore 6+8>c$ $\quad$ $8-6<c$

$14>c$ $\quad$ $2<c$

$\therefore 2<c<14$. [Range of Third Side]

→ Sum of All three Angles of a Triangle is 180°

→ The Ratio of angle of a Triangle is 1:2:3 find the Smallest angle

Total Value of Angle $= 1+2+3=6$

Let Smallest angle $= a = \frac{1}{6}\times 180^\circ = 30^\circ$ Ans.

$b = 2\times 30^\circ = 60^\circ$ $\quad$ $c = 3\times 30^\circ = 90^\circ$ Ans

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⇒ If Each Angle of a Triangle is Decreased By 10° then the Ratio of the angle Become 1:2:3. Find the each Angle of a Triangle:

We know that $\angle a + \angle b + \angle c = 180°$

Now Every Angle is Decreased By 10°.

$\therefore \angle a - 10° + \angle b - 10° + \angle c - 10° = 180° - 30°$

$\angle a - 10° + \angle b - 10° + \angle c - 10° = 150°$

$\therefore a : b : c = 1 : 2 : 3 \Rightarrow 6$

$\therefore a = \frac{1}{6} \times 150 = 25°$ $\quad b = 2 \times 25° = 50°$ $\quad c = 3 \times 25° = 75°$

अब जो Decrease किया था उसको Add भी करेंगे

$\therefore \angle a = 25° + 10° = 35°$ $\quad \angle b = 50° + 10° = 60°$ $\quad \angle c = 75° + 10° = 85°$

⇒ If Each Angle of a Triangle is Decreased By 20° then the Ratio of Angle Becomes 2:3:5. find the Each Angle of a Triangle.

$\angle a + \angle b + \angle c = 180°$

$\angle a - 20° + \angle b - 20° + \angle c - 20° = 180° - 60 = 120°$

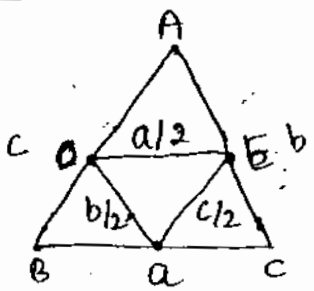
$\therefore a : b : c = 2 : 3 : 5 = 10$

$\angle a = \frac{2}{10} \times 120° = 12 \times 2 = 24°$ $\quad \angle b = \frac{3}{10} \times 120° = 36°$ $\quad \angle c = \frac{5}{10} \times 120 = 60°$

Ab अब 20° Add भी करेंगे

$\angle a = 24° + 20 = 44°$ $\quad \angle b = 36° + 20° = 56°$ $\quad \angle c = 60° + 20° = 80°$

Next



अगर किसी दो भुजाओं के mid Point को मिलाते हैं तो वह भुजा तीसरी भुजा के Parallel होती है

$DE \parallel BC$

और वह भुजा तीसरी भुजा से आधी होती है।

$DE = \frac{1}{2} \times BC$

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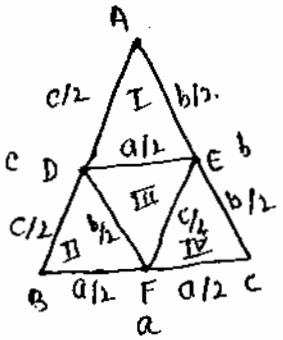


Triangle की सभी भुजाओं के mid point को मिलाते हैं तो उस Triangle के अंदर 4 Triangle बनते हैं और इन सभी का Area Equal होता है। सभी की Side भी बराबर होती हैं और Perimeter भी बराबर होता है।

$\therefore$ Area of ΔADE = Area of ΔDBF = Area of ΔDEF = Area of ΔEFC

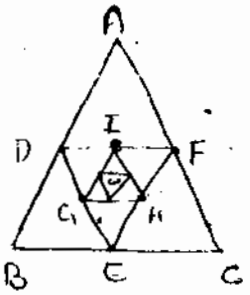
और इन सभी Traingle का Area पूरे Triangle के Area का $\frac{1}{4}$th होता

$\therefore$ Area of ΔADE = Area of ΔDEF = Area of ΔDBF = Area of ΔEFC

$= \frac{1}{4} \times$ Area of ΔABC

→ एक त्रिभुज की भुजाओं के मध्य बिंदु को मिलाकर एक त्रिभुज बनाया जाता है और उस त्रिभुज के मध्य बिन्दु को मिलाकर एक और त्रिभुज बनाया जाता है और यह प्रक्रिया अनंत तक दोहराई जाती है। find the sum of area of All Triangle formed.

$\therefore$ Let Area of Triangle ABC = $\underline{A}$



$\therefore$ Sum of Area of all Triangle

$= A + A \times \frac{1}{4} + A \times \frac{1}{4} \times \frac{1}{4} + \cdots \infty$

$= A\left[1 + \frac{1}{4} + \left(\frac{1}{4}\right)^2 + \cdots \infty\right]$ becomes an G.P

$= A\left[\frac{1}{1-\frac{1}{4}}\right]$ $\left[\because S_\infty = \frac{a}{1-r}\right]$

$A\left[\frac{4}{3}\right] = \frac{4}{3}A$

Area of $\Delta DEF = \frac{1}{4} \times \Delta A$
$\therefore$ Area of ΔGHE
$= \frac{1}{4} \times \Delta DEF$
$= \frac{1}{4} \times \frac{1}{4} \times \Delta ABC$
Continuos

$\therefore$ Sum of area of All Triangle formed = $\frac{4}{3}A$

OR

Sum of area of All Traingle formed = $\frac{4}{3} \times$ Area of ΔABC

→ समबाहु त्रिभुज (Equilateral Triangle) के प्रत्येक भुजा की लंबाई 12 cm है। इस त्रिभुज की भुजाओं के मध्य बिंदु को मिलाकर एक त्रिभुज बनाया जाता है, और इस प्रकार से बने त्रिभुज के मध्य बिंदु को मिलाकर एक त्रिभुज और बनाया जाता है और यह प्रक्रिया अनंत तक दोहराई जाती है। इस प्रकार से बने त्रिभुज के क्षेत्रफल का योग क्या होगा?

Area of Equilateral Triangle = $\frac{\sqrt{3}}{4}(\text{side})^2$

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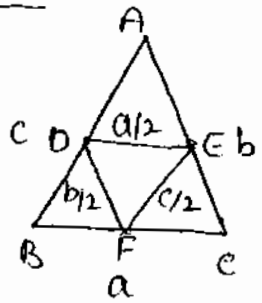
$\therefore$ Area $= \frac{\sqrt{3}}{4} \times 12 \times 12 = 36\sqrt{3}$

$\therefore$ Sum of Area of whole Triangle $= \frac{4}{3} \times 36\sqrt{3} = 48\sqrt{3}$

Next :-



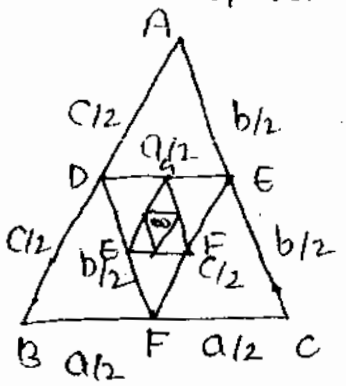
$\therefore$ Perimeter of $\Delta ABC = a+b+c$

$\therefore$ Perimeter of $\Delta DEF = \frac{a}{2} + \frac{b}{2} + \frac{c}{2} = \frac{a+b+c}{2}$

$\therefore$ Perimeter of $\Delta DEF = \frac{1}{2} \times$ Perimeter of ΔABC

Just

एक त्रिभुज की भुजाओं के मध्य बिंदु को मिलाकर एक त्रिभुज बनाया जाता है और इस प्रकार से बने त्रिभुज के मध्य बिंदु को मिलाकर एक और त्रिभुज बनाया जाता है और यह प्रक्रिया अनंत तक दोहराई जाती है। find the sum of Perimeter of triangle formed.



Let Perimeter of $\Delta ABC = P$

$\therefore$ Sum of Perimeter of whole Triangle formed

$= P + \frac{1}{2} \times P + \frac{1}{2} \times \frac{1}{2} \times P + \cdots \infty$

$= P\left[1 + \frac{1}{2} + \left(\frac{1}{2}\right)^2 + \cdots \infty\right]$

$= P\left[\frac{1}{1-\frac{1}{2}}\right] = 2P$

[$\because$ Perimeter of $DEF = \frac{1}{2}\Delta ABC$
Perimeter of $\Delta EFG = \frac{1}{2}\Delta DEF$
$= \frac{1}{2} \times \frac{1}{2} \times \Delta ABC$
continuslly]

$\therefore$ Sum of Perimeter of whole Traingle
$= 2 \times$ Perimeter of ΔABC

$\Rightarrow$ एक त्रिभुज की भुजाओं के मध्य बिंदु को मिलाकर एक त्रिभुज बनाया जाता है इस प्रकार से बने त्रिभुज की भुजाओं के मध्य बिन्दु को मिलाकर एक अन्य त्रिभुज बनाया जाता है और यह प्रक्रिया अनंत तक दोहराई जाती है इस प्रकार से बने त्रिभुजो के परिमाप का योग क्या होगा यदि बड़े त्रिभुज की भुजाएं 5, 6, 7 cm हो।

Perimeter of Triangle $= 5+6+7 = 18$ cm.

$\therefore$ Sum of Perimeter of whole Triangle $= 2 \times P = 2 \times 18 = 36$ cm.

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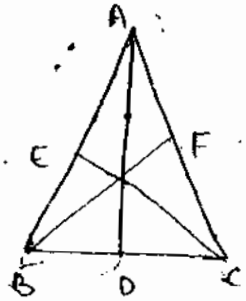
Next Type :→

Median (माध्यिका) :→ (Vertex (शीर्ष-बिंदु) से आने वाली वह रेखा जो किसी भी Triangle को समान भागों में बांट देती है, भुजाओं को दो समान भागों में और Area को भी दो समान भागों में बांट देती है।



Area of ΔADB = Area of ΔADC

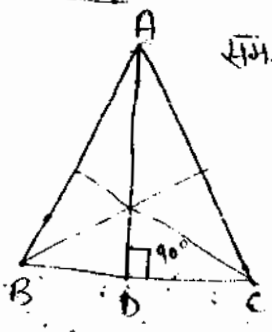
Similarly Area of ΔBFC = Area of ΔAFB

Area of ΔAEC = Area of ΔBEC

OR $BD = DC$, $AF = FC$, $AE = EB$

यह जरूरी नहीं कि माध्यिका किसी भी Angle को दो बराबर भागों में बांटती है और किसी भी Side पर माध्यिका डाले तो वह 90° का Angle बनाए।

Next Type :→



समद्विबाहु त्रिभुज

If $AB = BC$ then अगर हम AD Median डालते हैं BC पर तो AD BC को Bisect करती है और 90° का Angle बनाती है।

$\therefore BD = DC$

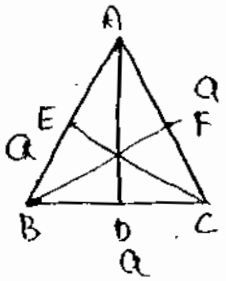
or $\angle ADB = \angle ADC = 90°$

पर यह जरूरी नहीं कि B और C से Median डाले तो वह भी 90° का Angle बनाए।

if $AB = BC$ then Median AD, Angle $\angle A$ का Bisector भी होगा।

$\therefore \angle BAD = \angle CAD$

Next Type :→



Equilateral Triangle (समबाहु त्रिभुज)

if $AB = BC = CA$

then $AD \perp BC$, $BF \perp AC$ or $CE \perp AB$

$\angle BAD = \angle CAD$, $\angle BCE = \angle ACE$, $\angle CBF = \angle ABF$

Median जिस Point पर एक दूसरे को काटती है उसे केन्द्रक (Centroid) कहते हैं। वह Point Median को 2:1 में काटती है।

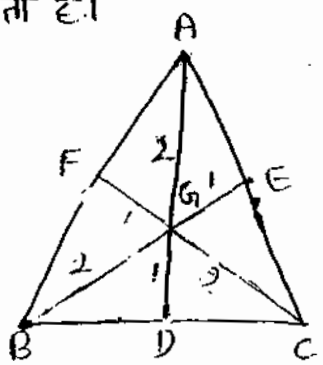
$AD \neq BE \neq CF$

$AG : GD = 2:1$

$BG : GE = 2:1$

$CG : GF = 2:1$

यहां केवल Ratio बराबर होगा, Length may be Various.



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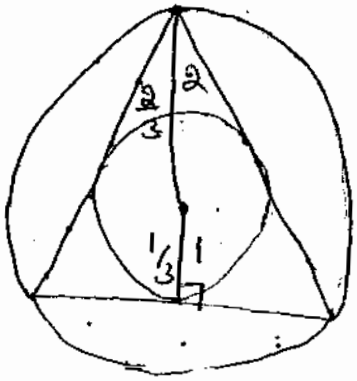
समबाहु त्रिभुज में अंत वृत्त (Incentre) का केन्द्र ~~होगा~~ वही होगा जो Circumcentre (परिकेन्द्र) का होगा।

Incentre की जो Radius होगी वो $\frac{1}{3}$ होगी और Circumcentre की जो Radius होगी वो $\frac{2}{3}$ होगी



Radius of incentre : Radius of Circumcentre

$= \frac{1}{3} : \frac{2}{3}$

$= 1 : 2$

Diameter $d_1 : d_2 = 1 : 2$

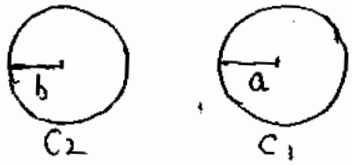
Circumcentre $C_1 : C_2 = 1 : 2$

Area $A_1 : A_2 = 1 : 2$

जब दो वृत्त दिए हो उनकी Radius का Ratio वही होगा जो उनके Diameter और परिधी का होगा।

Let $r_1 : r_2 = a : b$

$d_1 : d_2 = 2a : 2b = a : b$



(Circum) $C_1 : C_2 = 2\pi a : 2\pi b = a : b$

⇒ दो Circle में उनके Area का Ratio उनके परिधि व त्रिज्या के Ratio का Square के Equal होता है।

Let $r_1 : r_2 = a : b$ Or $C_1 : C_2 = a : b$

$\therefore A_1 : A_2 = \pi a^2 : \pi b^2$

$= a^2 : b^2$

⇒ जब दो वृत्तों के क्षेत्रफल का अनुपात दिया गया हो तो उनके त्रिज्या/परिधी का अनुपात क्षेत्रफल के अनुपात के Square Root के बराबर होता है।

e.g यदि दो वृत्त के क्षेत्रफल का अनुपात 25 : 16 है। तब परिधि/त्रिज्या का अनुपात 5 : 4 होगा।

⇒ जब दो square दिए हो तब उनकी Side का Ratio जो होगा, वही उनके Perimeter व Diagnol का Ratio होगा।

Side $S_1 : S_2 = a : b$

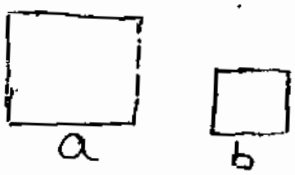
Perimeter $P_1 : P_2 = 4a : 4b$

$= a : b$

Diagnol $d_1 : d_2 = \sqrt{2}a : \sqrt{2}b$

$= a : b$

Area $A_1 : A_2 = a^2 : b^2$



अगर Area का Ratio $a^2 : b^2$ है तो

$$\boxed{\text{Ratio of Side} = \sqrt{\text{Ratio of Area}}}$$

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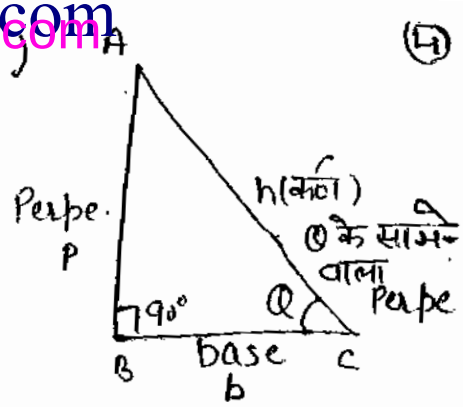
⇒ In Right Angle Triangle (समकोण त्रिभुज में)

$\angle ABC = 90^\circ$

According To Pythagorus Theorem

$h^2 = P^2 + b^2$ $[\because AC^2 = AB^2 + BC^2]$

$\Rightarrow \boxed{h = \sqrt{P^2 + b^2}}$



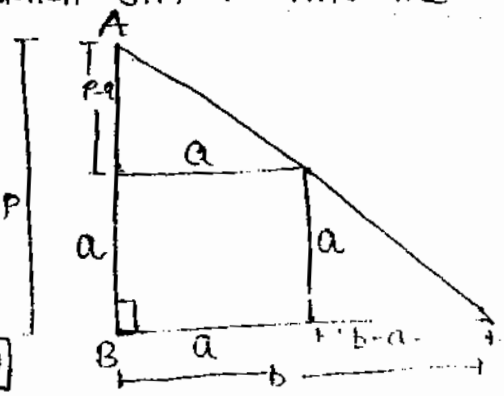
$\boxed{\text{Area} = \frac{1}{2}[\text{Base} \times \text{Height}]}$

$\boxed{\text{परिमाप} = h + P + b \text{ [सभी भुजाओं का योग]}}$

समकोण त्रिभुज में बड़े से बड़े आकार का square बनाया जाए तो find the Length of Side of Square.

Let length of Each Side of square = a

Area of Square = Area of whole ΔABC − Area of Two Small Triangle



$a^2 = \frac{1}{2} \times P \times b - \left[\frac{1}{2} \times a \times (b-a) + \frac{1}{2} \times a \times (P-a)\right]$

$2a^2 = Pb - [ab - a^2 + ab - a^2]$

$= Pb - [ab + aP - 2a^2]$

$2a^2 = Pb - ab - aP + 2a^2$

$0 = Pb - ab - aP$

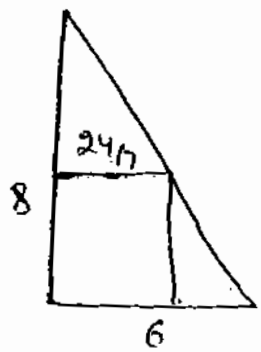
$Pb = ab + aP \Rightarrow a(b+P)$

$\boxed{a = \frac{Pb}{b+P}}$ where P और b is the Side of Triangle, a is the Side of square.

⇒ एक समकोण त्रिभुज के समकोण बनाने वाली भुजा की लंबाई 8 व 6 cm है। इस त्रिभुज के अन्दर बड़े से बड़े आकार का वर्ग बनाया जाता है तब वर्ग की भुजा व क्षेत्रफल ज्ञात कीजिए।

Using $a = \frac{Pb}{P+b}$ (a = Side of square)

$a = \frac{8 \times 6}{8+6} = \frac{48}{14} = \frac{24}{7}$ cm.



Area $= \left(\frac{24}{7}\right)^2 = \frac{576}{49}$ cm²

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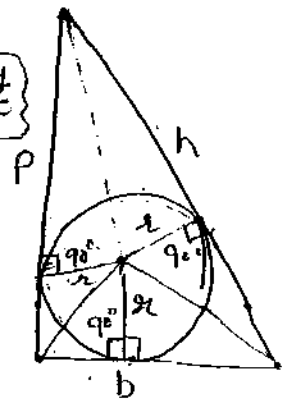
एक समकोण त्रिभुज के अन्दर बड़े से बड़े आकार का Circle बनाया जाता है।



Radius हमेशा स्पर्श रेखा पर 90° का Angle बनाती है

$$\frac{1}{2}\times P\times b = \frac{1}{2}\times \underset{\text{Base}}{P}\times r + \frac{1}{2}\times \underset{\text{Base}}{b}\times r + \frac{1}{2}\times \underset{\text{Base}}{h}\times r \ (\text{Height})$$

$$\frac{1}{2}\times P\times b = \frac{1}{2} r[P+b+h]$$

$$Pb = r[P+b+h]$$

$$\boxed{r = \frac{Pb}{P+b+h}} = \frac{Pb}{(P+b)+h}\times\frac{(P+b)-h}{(P+b)-h} = \frac{Pb(P+b-h)}{(P+b)^2-h^2}$$

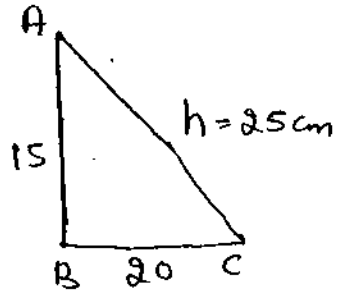
$$= \frac{Pb(P+b-h)}{P^2+b^2+2Pb-h^2}$$

$$= \frac{Pb(P+b-h)}{h^2+2Pb-h^2} \quad [\because P^2+b^2=h^2]$$

$$= \frac{Pb(P+b-h)}{2Pb} = \frac{P+b-h}{2}$$

$$\boxed{r = \frac{P+b-h}{2}}$$

⇒ एक समकोण त्रिभुज में समकोण बनाने वाली भुजा की लंबाई 15cm, 20cm है find the Radius of Incircle.



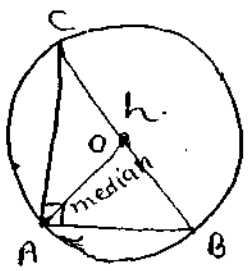
Here ⇒ $P = 15cm$, $b = 20cm$

$$\therefore h = \sqrt{P^2+b^2} = \sqrt{(20)^2+(15)^2} = \sqrt{625} = 25cm$$

$$\therefore \text{Radius of incircle} = \frac{15+20-25}{2} = \frac{10}{2} = 5cm$$

$$\left[\text{using } r = \frac{P+b-h}{2}\right]$$

समकोण त्रिभुज (में परिवृत (Circumcentre) का centre कर्ण का मध्य बिंदु होता है।



Diameter = h = BC

radius = (OB = OC) = $h/2$

केंद्र से परिधि तक सभी त्रिज्या बराबर होती है।

$\therefore OC = OB = OA = h/2$, AO → Median भी है।

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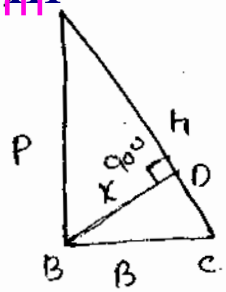
31/8/15 Given BD ⊥ [illegible]



Area of Triangle ABC $= \frac{1}{2} \times P \times b$ $\left[\because \frac{1}{2} \times \text{base} \times \text{Height}\right]$ —(1)

Let $x = BD =$ Perpendicular Height

Then $h \rightarrow$ Base

$\therefore$ Area of $\Delta ABC = \frac{1}{2} \times x \times h$ — II

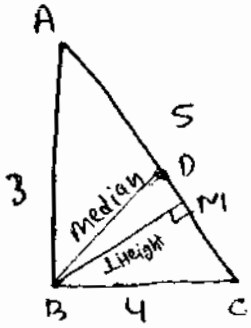
from I and II

$$\frac{1}{2} \times P \times b = \frac{1}{2} \times x \times h \Rightarrow \boxed{x = \frac{Pb}{h}}$$

Next :→ Here Perp. (P) = 3cm, Base (b) = 4cm

Hypo (h) = 5 Cm

Median BD $= \frac{1}{2} \times$ Hypotaneous $= \frac{1}{2} \times 5 = \frac{5}{2}$ cm

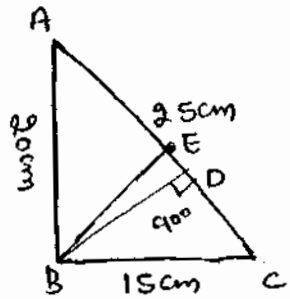


⊥ Height BM $= \frac{P \times b}{h} = \frac{3 \times 4}{5} = \frac{12}{5} = 2.4$ cm

{Median की लंबाई कर्ण की लंबाई का आधा होती है।}

Next :→ {Median व Perpendicular बराबर भी हो सकते हैं जब Right angle side equal हो।}

⇒ एक समकोण त्रिभुज के समकोण बनाने वाली भुजा की लंबाई 15cm और 20cm है। Find the Length of Median and Longest Side ⊥ Height.



$$AC = \sqrt{20^2 + 15^2} = \sqrt{400 + 225} = \sqrt{625} = 25 \text{ cm}$$

Perpendicular Height

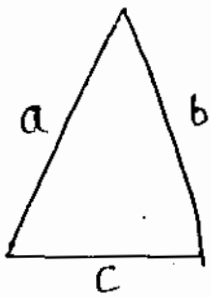
$$= \frac{P \times b}{h} = \frac{\overset{4}{\cancel{20}} \times \overset{3}{\cancel{15}}}{\underset{5}{\cancel{25}}} = 12 \text{ cm}$$

Median (BE) $= \frac{1}{2} \times h = \frac{1}{2} \times 25 = 12.5$ cm

Next Type :→ Scalane Triangle [विषमबाहु त्रिभुज]

in Scalane Triangle

Perimeter $= \boxed{a + b + c}$



By Heron's Formula

$$S = \frac{a + b + c}{2} \quad \text{[Semiperimeter]}$$

$$\boxed{\text{Area} = \sqrt{S(S-a)(S-b)(S-c)}}$$ or $\frac{1}{2} \times$ Base $\times$ Height जब ⊥ Height दी गई हो

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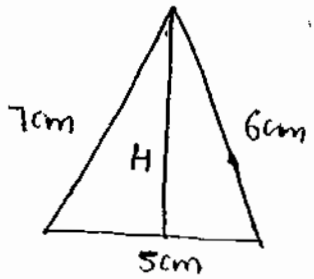
⇒ If the Side of a Scalane Triangle is 5, 6, 7 cm. find the length of Perpendicular drawn on smallest side.



$$S = \frac{a+b+c}{2} = \frac{5+6+7}{2} = 9\text{ cm}.$$

$$\text{Area} = \sqrt{S(S-a)(S-b)(S-c)} = \frac{1}{2} \times \text{Base} \times \text{Height}$$

$$\Rightarrow \sqrt{9(9-7)(9-6)(9-5)} = \frac{1}{2} \times 5 \times H$$

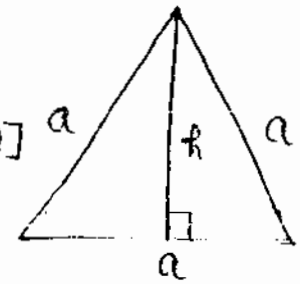
$$\sqrt{9 \cdot 2 \cdot 3 \cdot 4} = \frac{1}{2} \times 5 \times H \Rightarrow \frac{1}{2} \times 5 \times H = 3 \times 2\sqrt{6}$$

$$H = \frac{2 \times 3 \times 2\sqrt{6}}{5} = \frac{12}{5}\sqrt{6}$$

$$= 2.4\sqrt{6}\text{ cm}$$

Next:→

Equilateral Triangle (समबाहु त्रिभुज):→



$$\text{Perimeter} = a + a + a = 3a$$

[All sides are equal]

$$\text{Area} = \frac{\sqrt{3}}{4} a^2 \quad \text{--- I}$$

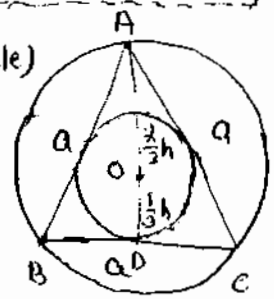
और जब ⊥ Height दी हो. ∴ $\text{Area} = \frac{1}{2} \times \text{Base} \times \text{Height}$

$$= \frac{1}{2} \times a \times h \quad \text{--- II}$$

from I and II

$$\frac{\sqrt{3}}{4} a^2 = \frac{1}{2} \times a \times h, \quad \boxed{h = \frac{\sqrt{3}}{\cancel{4}_2} a^{\cancel{2}} \times \frac{\cancel{2}}{\cancel{a}} = \frac{\sqrt{3}}{2} a}$$

Next:→ अगर किसी भी समबाहु त्रिभुज में अंतवृत्त (Incircle) और Circumcircle (बाह्यवृत्त) दिए गए हो



then Radius of Circumcircle

$$= \frac{2}{3} \times h = \frac{2}{3} \times \frac{\sqrt{3}}{2} a \quad \left[\because h = \frac{\sqrt{3}}{2} a\right]$$

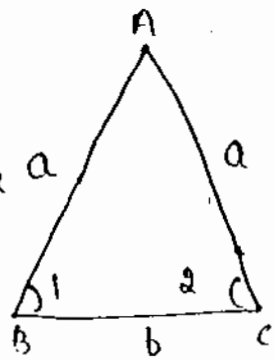
$$= \frac{1}{\sqrt{3}} a$$

Radius of Incircle

$$= \frac{1}{3} \times h = \frac{1}{3} \times \frac{\sqrt{3}}{2} a = \frac{1}{2\sqrt{3}} a$$

Next:→ Isoceles Triangle (समद्विबाहु त्रिभुज):→

समबाहु त्रिभुज में दो भुजाएँ बराबर होती हैं और जो side equal होती हैं उनके opposite वाला Angle बराबर होता है।



$$AB = AC$$

$$\therefore \angle 1 = \angle 2$$

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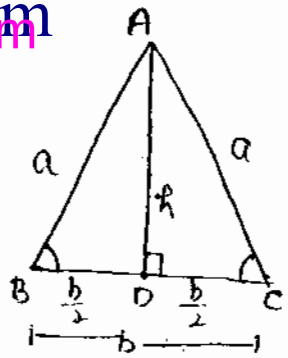
In Isoceles Δ ABC



Here $AB = AC$, $\angle B = \angle C$

Perimeter $= a + a + b = 2a + b$

Area $= \sqrt{S(S-a)(S-b)(S-c)}$

$= \sqrt{S(S-a)(S-a)(S-b)}$

$= (S-a)\sqrt{S(S-b)}$ where $S \Rightarrow$ Semiperimeter

In Diagram AD दो Equal side के बीच का Perpendicular है तो Median है और BC को 2 Equal parts में Divide करता है।

$\therefore BD = DC = b/2$

Now In ΔABD

$$h^2 = a^2 - \frac{b^2}{4} = \frac{4a^2 - b^2}{4}$$

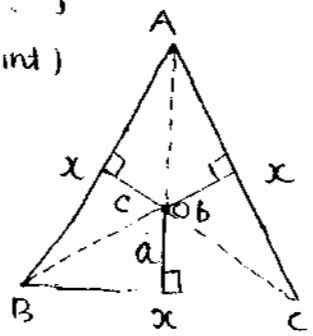
$$h = \sqrt{\frac{4a^2-b^2}{4}} = \frac{\sqrt{4a^2-b^2}}{2}$$

$$\text{Area} = \frac{1}{2} \times \text{Base} \times \text{Height} = \frac{1}{2} \times b \times \frac{\sqrt{4a^2-b^2}}{2} = \frac{1}{4} b\sqrt{4a^2-b^2}$$

$\therefore$ Area of Isoceles Triangle $= \frac{1}{4} b\sqrt{4a^2-b^2}$

Next :-

समबाहु त्रिभुज में किसी आंतरिक बिन्दु (Interior Point) से इसके तीनों भुजाओं पर डाले गए लंब की लंबाई a, b तथा c है और त्रिभुज की प्रत्येक भुजा x है



$\therefore$ Area of $\Delta AOB = \frac{1}{2} \times c \times x$

Area of $\Delta BOC = \frac{1}{2} \times a \times x$

Area of $\Delta AOC = \frac{1}{2} \times b \times x$ $\left[\because \text{Area} = \frac{1}{2} \times \text{Base} \times \text{Height}\right]$

Sum of Area

$$= \frac{1}{2} a \times x + \frac{1}{2} b \times x + \frac{1}{2} c \times x = \text{Area of } \Delta ABC$$

$$\frac{1}{2} x(a+b+c) = \frac{\sqrt{3}}{4} x^2 \qquad \left[\because \text{Area of } \underset{(\text{समबाहु})}{\Delta ABC} = \frac{\sqrt{3}}{4} a^2\right]$$

$$a+b+c = \frac{\sqrt{3}}{2} x \Rightarrow x = \frac{2}{\sqrt{3}}(a+b+c)$$

$\therefore$ Side of Triangle $= \frac{2}{\sqrt{3}}(a+b+c)$

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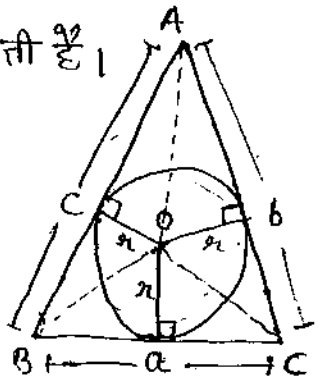
Next:→ विषमबाहु त्रिभुज की सभी भुजाएं अलग-अलग होती हैं।



$OA \perp BC$ [∵ BC is tangent]

Now Area of ΔABC
= Area of ΔAOB + Area of ΔBOC + Area of ΔAOC

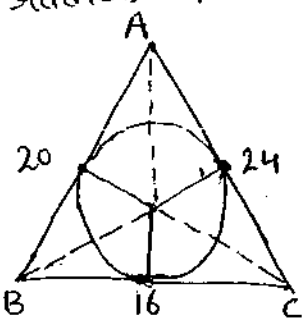
$$\Rightarrow \frac{1}{2} \times c \times r + \frac{1}{2} \times a \times r + \frac{1}{2} \times b \times r = \underbrace{\sqrt{S(S-a)(S-b)(S-c)}}_{\text{Area of } \Delta ABC}$$

$$\Rightarrow \frac{1}{2} r(a+b+c) = \text{Area of } \Delta ABC$$

$$r = \frac{\text{Area of } \Delta ABC \times 2}{a+b+c} \Rightarrow \frac{\text{Area of } \Delta ABC}{S} \quad \left[\because S = \frac{a+b+c}{2}\right]$$

$OA = OB = OC$ $\boxed{\therefore \text{Radius of Incircle} = \frac{\text{Area of } \Delta ABC}{S}}$ Very Important.

eg एक त्रिभुज की भुजाएं 20cm, 16cm, 24cm हैं, Find the radius of Incircle.



$$\text{Semiperimeter } (S) = \frac{20+16+24}{2} = \frac{60}{2} = 30 \text{ cm}$$

$$\text{Area of } \Delta ABC = \sqrt{30(30-20)(30-16)(30-24)}$$
$$= \sqrt{30 \times 10 \times 14 \times 6}$$
$$= \sqrt{3 \times 10 \times 10 \times 7 \times 2 \times 3 \times 2}$$
$$= 3 \times 10 \times 2\sqrt{7} = 60\sqrt{7} \text{ cm.}$$

Radius of Incircle

$$= \frac{\text{Area of } \Delta ABC}{S} = \frac{60\sqrt{7}}{30} = 2\sqrt{7} \text{ cm.}$$

⇒ एक त्रिभुज के अंतवृत्त की त्रिज्या 5cm है, और उसका परिमाप 60cm है त्रिभुज का क्षेत्रफल ज्ञात कीजिए।

$$\text{Radius of Incircle} = \frac{\text{Area of Triangle}}{S \rightarrow \text{semiperimeter}}$$

$$\text{Semiperimeter} = \frac{60}{2} = 30 \text{ cm.}$$

$$\text{Area of Triange} = 30 \times 5 = 150 \text{ cm}^2$$

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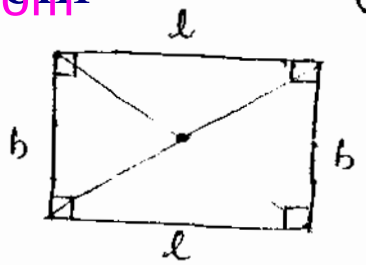
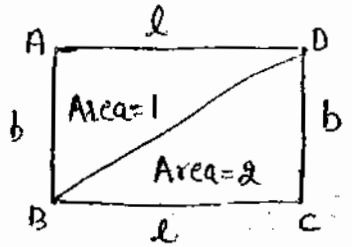
Next :→ 'Rectangle' (आयत)

* आयत के आमने-सामने की भुजाएं बराबर होती हैं।
* दोनों विकर्ण बराबर होते हैं।
* Diagnol (विकर्ण) जिस Point पर काटते हैं वहां 90° का Angle नहीं बनाते।



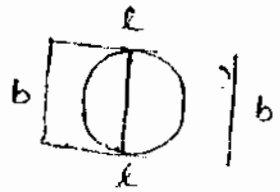
Perimeter = $2(l+b)$
Area = length × Breadth = $l \times b$

Area of ΔABD = Area of ΔBCD
Length of Diagnol = $\sqrt{l^2+b^2}$



⇒ एक आयत के अन्दर बड़े से बड़े आकार का वर्त्त का व्यास आयत के Breadth के बराबर होगा।

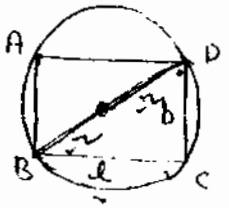
∴ Diameter of Circle = b
Radius of Circle = $b/2$



⇒ एक आयत के बाहर बड़े से बड़े आकार का वर्त्त का व्यास आयत के Diagnol के बराबर होगा।

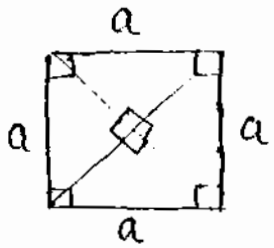
Diagnol = $\sqrt{l^2+b^2}$

∴ Diameter of Circumcircle = $\sqrt{l^2+b^2}$
Radius of Circumcircle = $\frac{1}{2}\sqrt{l^2+b^2}$



Next :→ Square (वर्ग)

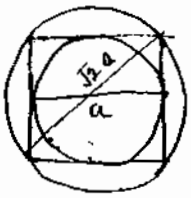
* वर्ग की सभी भुजाएं बराबर होती हैं।
* वर्ग के दोनों Diagnol बराबर होते हैं।
* विकर्ण जिस Point पर काटते हैं, वहां 90° का Angle बनाते हैं



Perimeter = $4a$
Area = $(\text{Side})^2 = a^2$
Diagnol = $\sqrt{a^2+a^2} = \sqrt{2}\,a$

* एक Square के अंदर बड़े से बड़े Circle का व्यास Square की Side के Equal होगा।

Diameter = a



* एक Square के बाहर बड़े से बड़े Circle का व्यास Square के Diagnol के Equal होगा।

Diameter = $\sqrt{2}\,a$

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In a square

$$\text{Diameter}_{\text{incircle}} : \text{Diameter}_{\text{circum}} = a : \sqrt{2}a$$
$$= 1 : \sqrt{2}$$

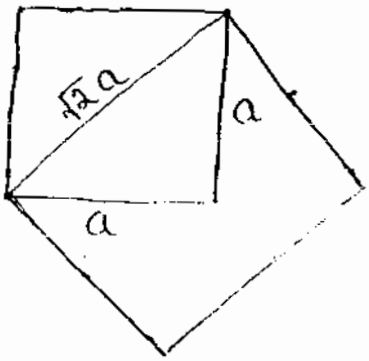
Similarly $R_{\text{incircle}} : R_{\text{circum}} = \frac{1}{2} : \frac{\sqrt{2}}{2} = 1 : \sqrt{2}$

$C_{\text{incircle}} : C_{\text{circum}} = 1 : \sqrt{2}$ [where C = Circumference]

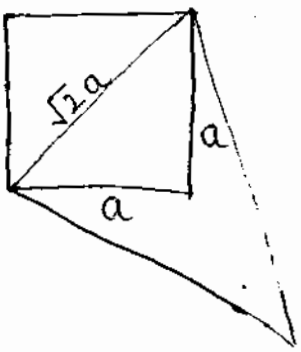
$$A_{\text{incircle}} : A_{\text{circum}} = 1^2 : \sqrt{2}^2$$
$$= 1 : 2 \quad \text{[where A = Area]}$$

Next Type :→

एक square के विकर्ण पर यदि एक और square बनाया जाता है।
दोनों square की side का Ration
$= a : \sqrt{2}a = 1 : \sqrt{2}$ होगा।
Ratio of Area
$= 1^2 : \sqrt{2}^2 \Rightarrow 1 : 2$ Ans.



Next Type :→ किसी square के विकर्ण पर एक समबाहु त्रिभुज बनाया जाता है तो उसकी side का Ratio $1 : \sqrt{2}$ होगा। Area का Ratio $2 : \sqrt{3}$ होगा।



Side of square : Side of Equilateral

$$\Rightarrow S_{\square} : S_{\Delta} = a : \sqrt{2}a$$
$$= 1 : \sqrt{2}$$

Area of square : Area of Equilateral

$$A_{\square} : A_{\Delta} = a^2 : \frac{\sqrt{3}}{4}a^2 \quad \left[\because \text{Area of Equilateral } \Delta = \frac{\sqrt{3}}{4}a^2\right]$$

$$= a^2 : \frac{\sqrt{3}}{4}(\sqrt{2}a)^2 \quad [\because \text{Side of Equilateral} = \sqrt{2}a]$$

$$= a^2 : \frac{\sqrt{3}}{\cancel{4}_2} \times \cancel{2}a^2$$

$$= 1 : \frac{\sqrt{3}}{2} \Rightarrow 2 : \sqrt{3}$$

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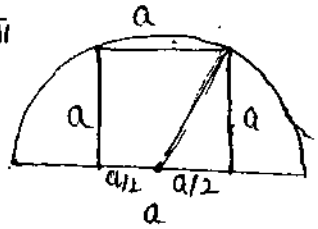
Next Type :→ यदि किसी semicircle में बड़े से बड़ा वृत्त बनाया जाता है तो



$$\text{Radius of circle (semi)} = r = \sqrt{a^2+\left(\frac{a}{2}\right)^2}$$

$$r = \sqrt{a^2+\frac{a^2}{4}} = \sqrt{\frac{5a^2}{4}}$$

$$r = \frac{\sqrt{5}a}{2} \Rightarrow \frac{r}{a} = \frac{\sqrt{5}}{2}$$

$$\boxed{r : a = \sqrt{5} : 2}$$

eg. एक अर्धवृत्त (semicircle) की त्रिज्या 20cm है, इसके अन्दर बने बड़े से बड़े square का क्षेत्रफल क्या होगा।

Using $r^2 = a^2 + \frac{a^2}{4}$

$\Rightarrow 20^2 = \frac{5a^2}{4}$

$400 \times 4 = 5a^2$

$80 \frac{400 \times 4}{5} = a^2$

$\boxed{a^2 = 320\ cm^2}$

OR

$r : a = \sqrt{5} : 2$

$\sqrt{5} \rightarrow 20$

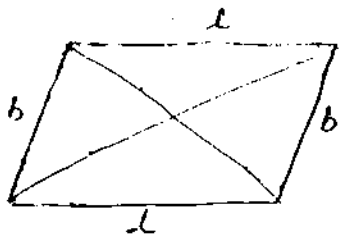
$2 \rightarrow \frac{20}{\sqrt{5}} \times 2$

$a \rightarrow 8\sqrt{5}$

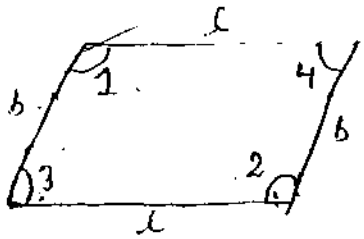
$a^2 = (8\sqrt{5})^2$

$= 320 cm^2$

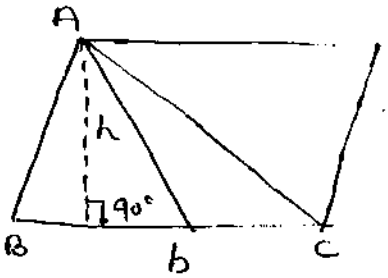
Next Type :→ Parallelogram (समांतर चतुर्भुज)



* समांतर चतुर्भुज में आमने सामने की भुजा बराबर होती है।
* Diagnol की लंबाई अलग-अलग होती है।
* $\angle 1 = \angle 2$ & $\angle 3 = \angle 4$
* $\angle 1 + \angle 3 = \angle 3 + \angle 2 = \angle 2 + \angle 4 = \angle 4 + \angle 1 = 180^\circ$



$$\boxed{\begin{array}{l}\text{Perimeter} = 2(l+b) \\ \text{Area} = \text{Base} \times \text{Height}\end{array}}$$



Area of $\Delta ABC = \frac{1}{2} \times \text{Base} \times \text{Height}$

$\therefore$ Area of whole Parallogram

$= 2 \times \frac{1}{2} \times \text{Base} \times \text{Height}$

$= \text{Base} \times \text{Height}$

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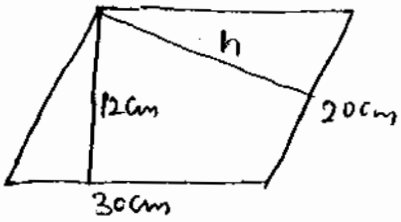
एक समांतर चतुर्भुज के आसन्न भुजा (Adjecent side) की लंबाई 30cm और 20cm है अगर बड़ी भुजा के बीच की लंबवत दूरी 12cm है तो इनके छोटी भुजा के बीच की लंबवत दूरी क्या होगी।



when Base = 30cm, Height = 12cm

Area = $30 \times 12\ cm^2$ [H×B] —I

when Base = 20cm, let Height = h cm

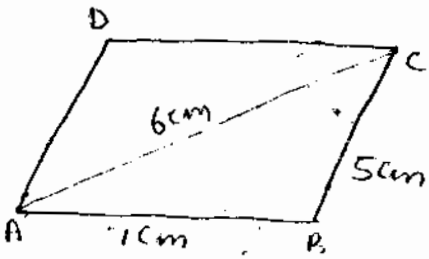
Area = $20 \times h\ cm^2$ —II

from I and II

$30 \times 12 = 20 \times h$

$h = \frac{30 \times 12}{20} = 18$ cm Ans.

⇒ एक समांतर चतुर्भुज के आसन्न भुजा की लंबाई 7cm और 5cm है अगर एक विकर्ण की लंबाई 6cm है। find Area.



ΔABC is a Scalane (विषमबाहु) Triangle

$\therefore S = \frac{a+b+c}{2} = \frac{5+6+7}{2} = 9cm$

Area = $\sqrt{S(S-a)(S-b)(S-c)}$

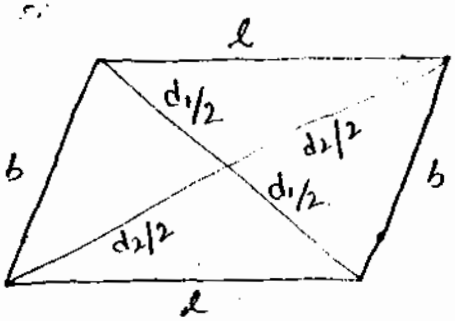
$= \sqrt{9(9-7)(9-6)(9-5)} = \sqrt{9 \cdot 2 \cdot 3 \cdot 4}$

$= 6\sqrt{6}\ cm^2$

Area of Parallelogram = 2 × Area of Δ ABC

$= 2 \times 6\sqrt{6} = 12\sqrt{6}\ cm^2$

⇒ Nom



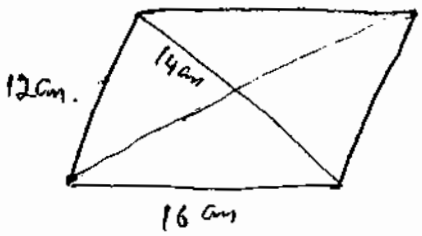
$l^2 + b^2 = 2\left(\frac{d_1}{2}\right)^2 + 2\left(\frac{d_2}{2}\right)^2$

$= 2 \times \frac{d_1^2}{4} + 2 \times \frac{d_2^2}{4}$

$\Rightarrow \frac{d_1^2}{2} + \frac{d_2^2}{2}$

$$\boxed{d_1^2 + d_2^2 = 2(l^2 + b^2)}$$

⇒ एक समांतर चतुर्भुज के आसन्न भुजा की लंबाई 16cm और 12cm है और एक विकर्ण की लंबाई 14cm है तो दूसरे विकर्ण की लंबाई क्या होगी।



Using $d_1^2 + d_2^2 = 2(l^2 + b^2)$

$\Rightarrow (14)^2 + d_2^2 = 2(12^2 + 16^2)$

$= 2(144 + 256)$

$d_2^2 = \frac{2 \times 400}{\cancel{196}} - 196$ $\quad 800 - 196 = 604$

$d_2 = \sqrt{604}$ cm.

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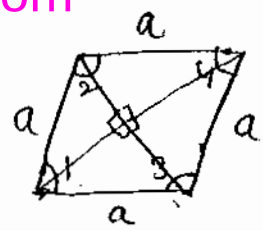
Next Type :→ Rhombus (समचतुर्भुज)

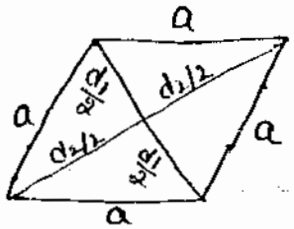


* समचतुर्भुज की सभी भुजाएं बराबर होती है।
* दोनों विकर्णों की लंबाई अलग-अलग होती है।
* दोनों विकर्ण एक दूसरे को जिस Point पर काटते हैं 90° का Angle बनाते हैं।
* $\angle 1 = \angle 4$ & $\angle 2 = \angle 3$
* $\angle 1 + \angle 2 = \angle 2 + \angle 4 = \angle 4 + \angle 3 = \angle 3 + \angle 1 = 180°$

Perimeter = 4a

Area = $\frac{1}{2} \times d_1 \times d_2$ = Base × Height

Side (a) = $\sqrt{\left(\frac{d_1}{2}\right)^2 + \left(\frac{d_2}{2}\right)^2} = \frac{\sqrt{d_1^2 + d_2^2}}{2}$



⇒ किसी समचतुर्भुज के दोनों विकर्णों की लंबाई 16cm और 12 cm है। Find the Area, Perimeter and Height.

$d_1 = 16cm$ $d_2 = 12cm$

∴ Side = a = $\frac{\sqrt{d_1^2 + d_2^2}}{2} = \frac{\sqrt{(16)^2 + (12)^2}}{2} = \frac{\sqrt{256 + 144}}{2} = \frac{20}{2} = \underline{10cm}$

Area = $\frac{1}{2} \times d_1 \times d_2 = \frac{1}{\cancel{2}} \times \overset{8}{\cancel{16}} \times 12 = 96\ cm^2$

Perimeter = $4 \times a = 4 \times 10 = 40cm$.

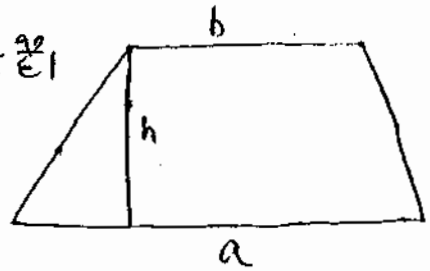
Now Area = $\frac{1}{2} \times d_1 \times d_2$ = Base × Height

⇒ 96 = 10 × Height

Height = 9.6 cm.

Next Type :→ Trapizium (समलंब चतुर्भुज)

* समलंब चतुर्भुज में दो भुजाएं Parallel होती है।

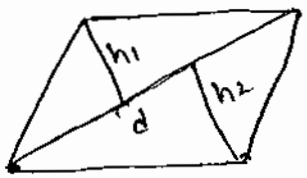


Area = $\frac{1}{2}(a+b) \times h$

= $\frac{1}{2}$(Sum of ∥ side) × h

Next Type :→ Quadilateral (चतुर्भुज)

अगर किसी भी चतुर्भुज के विकर्ण पर दो लंब डालते हैं।



Then Area = $\frac{1}{2} d h_1 + \frac{1}{2} d h_2$

= $\frac{1}{2}(h_1 + h_2) \times d$

∴ Area = $\frac{1}{2}$ (Sum of ⊥ on Diagnol) × Diagnol

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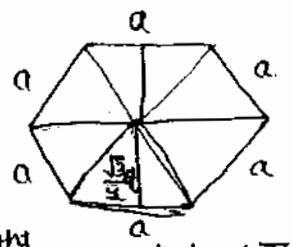
Next :→ Regular Hexagonal (सम षट्भुज)



* Regular Hexagonal में सभी भुजाएं बराबर होती हैं।

Perimeter = 6a

Height of Hexagonal = $2 \times \frac{\sqrt{3}}{2} a$ [∵ Height ~~Area~~ of Equilateral Triangle = $\frac{\sqrt{3}}{2}a$]

= $\sqrt{3}a$ [It is the ⊥ Distance Betn Two Parallal Side]

Regular Hexagonal के अंदर 6 Equilateral Triangle बनते हैं।

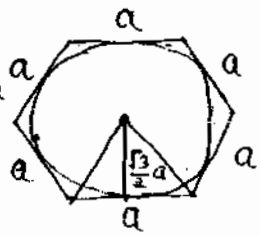
∴ Area of 1 Equilateral Traingle = $\frac{\sqrt{3}}{4}a^2$

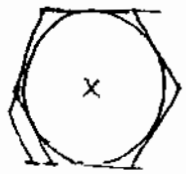
∴ = Area of Regular Hexagonal = 6 × Area of Equilateral Triangle

$= 6 \times \frac{\sqrt{3}}{4} a^2 \quad \frac{3\sqrt{3}}{2}a^2$

⇒ अगर Hexagonal के अन्दर Circle हो तो उसकी Radius वही होगी जो Equilateral Triangle की Height होगी।

$r = \frac{\sqrt{3}}{2}a$





⇒ अगर Hexagonal के बाहर कोई भी circle हो तो उसकी Radius वही होगी जो Hexagonal की Side की लंबाई होगी।

$r = a$

Now

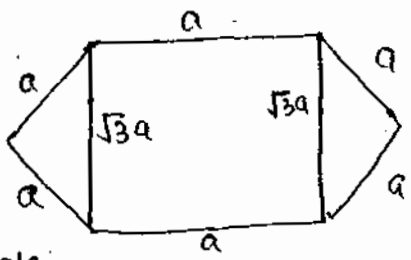
$r_{incircle} : r_{circum} = \frac{\sqrt{3}}{2}a : a$

$= \sqrt{3} : 2$

$Area_{incircle} : Area_{circum} = \pi(\sqrt{3})^2 : \pi(2)^2$

$3 : 4$

⇒ किसी भी आयत Hexagonal के अंदर A आयत Draw करते हैं। तो आयत का Area क्या होगा।



Side of Hexagonal = Length of Reactangle

= a

Height of Hexagonal = Bredth of Reactangle

= $\sqrt{3}a$

∴ Area of Reactangle = $\sqrt{3}a \times a = \sqrt{3}a^2$

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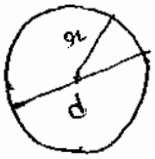
Next Type :→ Circle (वृत्त)



* Radius of Circle = r
* Diameter of Circle = d

Diameter = 2 × Radius
$d = 2r$
Circumference (परिधि) $= 2\pi r$
Area $= \pi r^2$

Where π is Irrational Number.
$\pi = \frac{22}{7}$ or 3.14

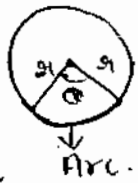
Ques :→ Choose the Irrational Number
(i) π (ii) $22/7$ (iii) 3.14 (iv) All of above
∵ π की कोई Specific Value नहीं है।

Next :→ Sector of a Circle (त्रिज्या खंड) :→ किसी भी Circle का छोटा सा Area Sector कहलाता है।

∴ जब Angle 360° होता है तब Area $= \pi r^2$
जब Angle θ° होगा तब Area $= \frac{\pi r^2}{360^\circ} \times \theta$

$$\therefore \text{Area of Sector} = \frac{\theta}{360} \times \pi r^2$$

Next :→ Arc of a Circle (चाप) :→ किसी भी Circle की छोटी सी परिधि (Circumference) Arc कहलाती है।



∴ जब Angle 360° होता है तब परिधि $= 2\pi r$
जब Angle θ° होगा तब परिधि $= \frac{2\pi r}{360} \times \theta = \frac{\theta}{180} \times \pi r$

$$\therefore \text{Length of Arc} = \frac{\theta}{180} \times \pi r$$

⇒ Relation Between Length of Arc & Area of Sector :→

$$\text{Area of Sector} = \frac{\theta}{360} \times \pi r^2 = \frac{1}{2} \times \underbrace{\frac{\theta}{180} \times \pi r}_{} \times r$$

$$= \frac{1}{2} \times \text{Length of Arc} \times r$$

$$\therefore \text{Area of Sector} = \frac{1}{2} \times \text{Length of Arc} \times r$$

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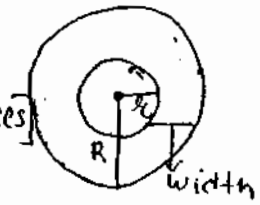
Next :→ Concentric Circle (ऐसे circle जिनका एक ही centre हो)

Find the Length of Width



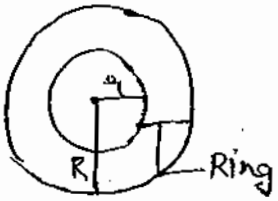
Now $2\pi R - 2\pi r = C_1 - C_2$ [C_1, C_2 are Circumferences]

$2\pi(R-r) = C_1 - C_2$

Width ⇒ $\boxed{R - r = \frac{C_1 - C_2}{2\pi}}$

⇒ Area of Ring

Area of Ring $= \pi R^2 - \pi r^2$
$= \pi(R^2 - r^2)$
$= \pi(R+r)(R-r)$



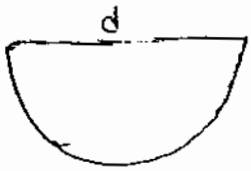
∴ $\boxed{\text{Area of Ring} = \pi(R+r)(R-r)}$

⇒ Perimeter

Perimeter = Total Length
$= 2\pi r + 2\pi R$
$= 2\pi(r+R)$

∴ $\boxed{\text{Perimeter} = 2\pi(r+R)}$

Next Type :→ Semi-Circle (अर्धवृत्त)

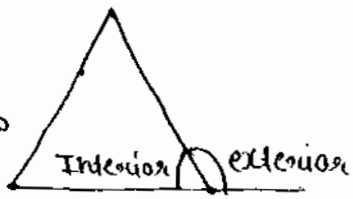


Area of Semicircle $= \frac{\pi r^2}{2}$

Perimeter $= \pi r + d = \pi r + 2r$
$= (\pi + 2) r$

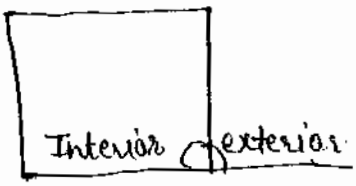
Next Type :→ Polygen (बहुभुज) :→ दो से अधिक भुजाओं की बंद आकृति Polygen कहलाती है।

In a Polygen



∠Interior Angle + ∠Exterior Angle = 180°

(आंतरिक कोण + बाह्य कोण) = 180°



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⇒ एक समबहुभुज (Polygon) के अंतःकोण (Interior Angle) उसके बाह्य कोण (Exterior Angle) का $\frac{3}{2}$ है। Find Each Interior Angle.

$$\text{Interior} : \text{Exterior} = \frac{3}{2} : 1$$
$$= 3 : 2$$

$$\text{Now} \angle \text{Interior} + \angle \text{Exterior} = 180^\circ$$
$$\therefore 3+2 \rightarrow 180^\circ$$
$$5 \rightarrow 180^\circ$$
$$\therefore 1 \rightarrow \frac{180}{5} = 36^\circ$$

$\therefore$ Interior Angle $= 3 \times 36^\circ = 108^\circ$, Exterior Angle $= 180^\circ - 108^\circ = 72^\circ$

⇒ एक समबहुभुज के प्रत्येक अंतःकोण उसके प्रत्येक बाह्य कोण के 2 गुणे से 60 अधिक है। प्रत्येक अंतःकोण का माप क्या होगा।

Let Each Exterior Angle $= x$
Each Interior Angle $= 2x+60$

$$\therefore x + 2x + 60 = 180^\circ$$
$$3x + 60 = 180^\circ$$
$$3x = 120^\circ$$
$$x = 40^\circ \text{ (Each Exterior Angle)}$$

$\therefore$ Each Interior Angle $= 2 \times 40 + 60^\circ = 140^\circ$

⇒ Sum of All Exterior Angle of a Polygen $= 360^\circ$

Each exterior angle of a Regular Polygen $= \frac{360^\circ}{n}$

$n \rightarrow$ No of Sides

No. of Side of a Regular Polygen $= \frac{360^\circ}{\text{Each Exterior Angle}}$

Sum of all Interior Angle of a polygen $= (n-2) \times 180^\circ$

Each Interior Angle of a Regular Polygen $= \frac{(n-2) \times 180^\circ}{n}$

$n \rightarrow$ No of Side

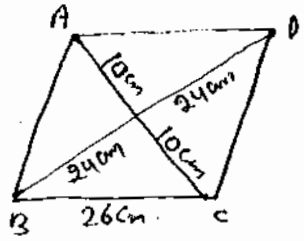
No. of sides of a Regular Polygen $= \frac{\text{Sum of all Interior Angle}}{\text{Each Interior Angle}}$

⇒ एक समचतुर्भुज के दो विकर्णों की लंबाई 48cm और 20cm है। Find the Area, Perimeter and Height.

Given $d_1 = 48\text{cm}$, $d_2 = 20\text{cm}$

$$\text{Area} = \frac{1}{2} \times \overset{24}{\cancel{48}} \times 20 = 480\text{cm}^2 \quad \left[\text{using Area} = \frac{1}{2} \times d_1 \times d_2\right]$$

$$\text{Side} = a = \sqrt{\left(\frac{d_1}{2}\right)^2 + \left(\frac{d_2}{2}\right)^2} = \sqrt{\left(\frac{20}{2}\right)^2 + \left(\frac{48}{2}\right)^2}$$
$$= \sqrt{(10)^2 + (24)^2} = \sqrt{676}$$
$$= 26\text{cm}$$



$\therefore$ Perimeter $= 4a = 4 \times 26 = 104\text{cm}$

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Now Using

Area ⇒ Base × Height = $\frac{1}{2} \times d_1 \times d_2$

$$26 \times H = 480$$

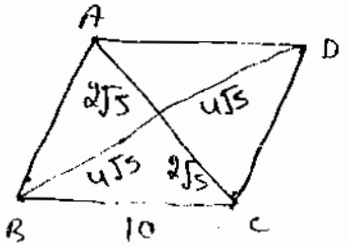
$$H = \frac{\cancel{480}\ 240}{\cancel{26}\ 13} = \frac{240}{13}\ cm.$$

⇒ एक समचतुर्भुज (Rhombus) का क्षेत्रफल $80 cm^2$ है। एक विकर्ण की लंबाई दूसरे विकर्ण की लंबाई का दो गुना है, find the Height of Rhombus.

~~Let Height of 1st~~

$$d_1 : d_2 = 2 : 1$$
$$= 2x \text{ or } x$$

[Let $d_2 = x$
then $d_1 = 2x$]



∴ Area = $\frac{1}{2} \times d_1 \times d_2 = 80$

$$\frac{1}{\cancel{2}} \times \cancel{2}x \times x = 80$$

$$x^2 = 80 \qquad x = 4\sqrt{5}\ cm.$$

∴ $d_1 = 8\sqrt{5} cm$ or $d_2 = 4\sqrt{5} cm$.

$$\text{Base} = a = \sqrt{\left(\frac{d_1}{2}\right)^2 + \left(\frac{d_2}{2}\right)^2} = \sqrt{\left(\frac{8\sqrt{5}}{2}\right)^2 + \left(\frac{4\sqrt{5}}{2}\right)^2}$$

$$= \sqrt{80 + 20} = \sqrt{100} = 10 cm.$$

Now Using

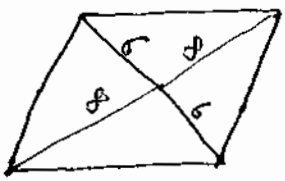
Base × Height = Area

$$10 \times H = 80$$

$$H = 8 cm$$

⇒ एक समचतुर्भुज का क्षेत्रफल $96 cm^2$ है। एक विकर्ण की लंबाई दूसरे विकर्ण की लंबाई का $\frac{4}{3}$ गुना है। find the Perimeter of Rhombus.

$$d_1 : d_2 = \frac{4}{3} : 1$$
$$\Rightarrow 4 : 3$$
$$\Rightarrow 4x, 3x$$



Area = $\frac{1}{2} \times d_1 \times d_2 = 96$

$$\frac{1}{\cancel{2}} \times \cancel{4}x \times 3x = 96$$

$$x^2 = 16$$

$$x = 4 cm$$

∴ Side = $a = \sqrt{8^2 + 6^2}$

$$= \sqrt{100}$$

$$= 10$$

∴ Length of first Diagnol = $4 \times 4 = 16 cm$
Length of 2nd Diagnol = $4 \times 3 = 12 cm$.

Perimeter = $4a$
$= 4 \times 10$
$= 40 cm$.

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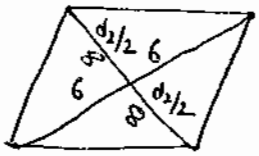
⇒ एक समचतुर्भुज का परिमाप 40cm है। उसके एक विकर्ण की लंबाई 12cm है। Find the Area. (12)

Perimeter = 4a

∴ 4a = 40

a = 10cm [a → Side of Rhombus]



Using $a^2 = \left(\frac{d_1}{2}\right)^2 + \left(\frac{d_2}{2}\right)^2$

$100 = \left(\frac{12}{2}\right)^2 + \left(\frac{d_2}{2}\right)^2$

$\left(\frac{d_2}{2}\right)^2 = 100 - 36 = 64$

∴ $\frac{d_2}{2} = 8$

$d_2 = 16cm$

OR

$\frac{d_2}{2} = \sqrt{(10)^2 - (6)^2}$

$= \sqrt{100 - 36} = \sqrt{64}$

$\frac{d_2}{2} = 8$

$d_2 = 16cm$.

∴ Area $= \frac{1}{2} \times d_1 \times d_2 = \frac{1}{2} \times 12 \times 16 = 96 cm^2$

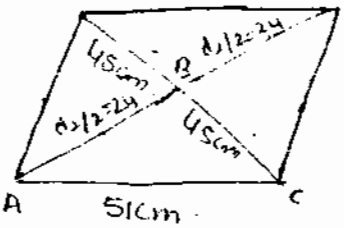
⇒ एक समचतुर्भुज का परिमाप 204cm है। एक विकर्ण की लंबाई 90cm है। Find the Height of Rhombus.

Perimeter = 4a

4a = 204

a = 51cm.

∴ $d_1 = 90cm$, $d_2 = 48cm$.



Using

Base × Height $= \frac{1}{2} \times d_1 \times d_2$

$51 \times H = \frac{1}{2} \times 90 \times 48$

$H = \frac{90 \times 48}{2 \times 51} = \frac{720}{17} cm$.

$\left(\frac{d_2}{2}\right)^2 = 51^2 - 45^2$

$\frac{d_2}{2} = \sqrt{2601 - 2025}$

$= \sqrt{576} = 24$

$d_2 = 24 \times 2 = 48cm$

⇒ एक समचतुर्भुज के दोनों विकर्णों की लंबाई x और y cm है। find the Height of Rhombus.

Using Side $= a = \frac{1}{2}\sqrt{d_1^2 + d_2^2} = \frac{1}{2}\sqrt{x^2 + y^2}$

Now Using

Base × Height $= \frac{1}{2} \times d_1 \times d_2$

$\Rightarrow \frac{1}{2}\sqrt{x^2+y^2} \times H = \frac{1}{2} \times x \times y$

$H = \frac{xy}{\sqrt{x^2+y^2}}$

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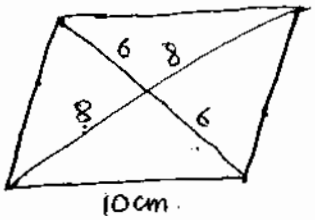
⇒ एक समचतुर्भुज का परिमाप 40cm है। और क्षेत्रफल 96cm² है, Find its Diagnol.



Detail :→ Using Area $= \frac{1}{2} \times d_1 \times d_2$

$4a = 40$

$a = 10cm$

$96 = \frac{1}{2} \times d_1 \times d_2$

$d_1 \times d_2 = 192$ — I

Side $= a = \frac{1}{2}\sqrt{d_1^2 + d_2^2}$

$10 = \frac{1}{2}\sqrt{d_1^2 + d_2^2}$

$\sqrt{d_1^2 + d_2^2} = 20$

$d_1^2 + d_2^2 = 400$ — II

$d_1 + d_2 = \sqrt{d_1^2 + d_2^2 + 2d_1d_2}$

$= \sqrt{400 + 2(192)} = \sqrt{400 + 384}$

$= \sqrt{784} = 28$ — III

Short Trick :→
We know that
$(10)^2 = 8^2 + 6^2$
$\because \frac{d_1}{2} = 8, \frac{d_2}{2} = 6$
$d_1 = 16cm$ $d_2 = 12cm$

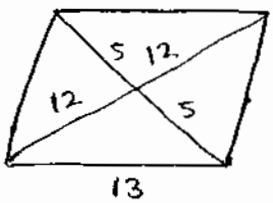
$d_1 - d_2 = \sqrt{d_1^2 + d_2^2 - 2d_1d_2} = \sqrt{400 - 384} = \sqrt{16} = 4$ — IV

Solving III and IV

$d_1 = 16$, $d_2 = 12$

Sum of Two Diagnol = 28cm Subtract of Two Diagnol = 4cm.

⇒ एक समचतुर्भुज का परिमाप 52cm है और क्षेत्रफल = 120cm² है। Find the Length of Longest Diagnol.



$4a = 52$

$a = \frac{52}{4} = 13cm$

Detail is Same as in Previous Example

$13^2 = 12^2 + 5^2$

$\therefore \frac{d_1}{2} = 12$, $\frac{d_2}{2} = 5$

$d_1 = 12 \times 2 = 24cm$ $d_2 = 5 \times 2 = 10cm$.

∴ Length of Longest Diagnol = ~~10~~ 24cm.

⇒ एक समचतुर्भुज का परिमाप 2a cm है। और क्षेत्रफल b cm है। find the sum of two Diagnol.

परिमाप = 2a cm

$\therefore$ Side (A) $= \frac{2a}{4} = \frac{a}{2}$ cm.

or $\frac{1}{2} \times d_1 \times d_2 = b$

$d_1 \times d_2 = 2b$

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Now Side of Rhombus

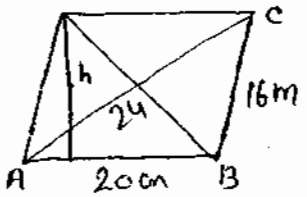
$$\frac{a}{2} = \frac{1}{2}\sqrt{d_1^2+d_2^2}$$

$$d_1^2+d_2^2 = a^2$$

Using

$$d_1+d_2 = \sqrt{d_1^2+d_2^2+2d_1d_2} = \sqrt{a^2+2\cdot 2b} = \boxed{\sqrt{a^2+4b}}$$

⇒ एक समांतर चतुर्भुज (Parallelogram) की आसन्न भुजा क्रमशः 20 और 16cm है। एक विकर्ण की लंबाई 24 cm है। Find the Area and ⊥ Distance Between the Longest side.



ΔABC is a Scalane Triangle

$$S = \frac{a+b+c}{2} = \frac{20+24+16}{2} = 30 \text{ cm}.$$

$$\text{Area of } \Delta ABC = \sqrt{S(S-a)(S-b)(S-c)}$$

$$= \sqrt{30(30-20)(30-24)(30-16)}$$

$$= 60\sqrt{7} \text{ cm}^2$$

∴ Area of Parallelogram $= 2\times 60\sqrt{7} = 120\sqrt{7} \text{ cm}^2$

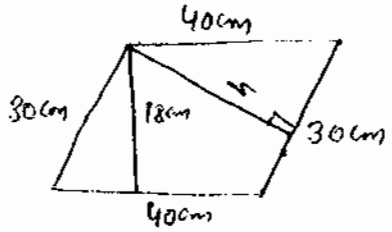
Now Base × Height = Area

$$20 \times H = 120\sqrt{7}$$

$$H = 6\sqrt{7} \text{ cm}.$$

⊥ Distance Betn Two Longest Side $= 6\sqrt{7}$cm.

⇒ एक समांतर चतुर्भुज की आसन्न भुजा (Adjecent) की लंबाई 40 और 30cm है। अगर इसके बड़ी भुजा के बीच की लंबवत दूरी 18cm है। Find the ⊥ Distance Between smallest side.



We know that Area = Base × Height

$$\therefore 18\times 40 = 30\times h$$

$$h = \frac{18\times 40}{30} = 24\text{cm}$$

∴ ⊥ Distance Between smallest side = 24cm.

⇒ एक समांतर चतुर्भुज की आसन्न भुजा की लंबाई का अनुपात 8:9 है। इसकी बड़ी भुजा और संगत ऊंचाई का अनुपात 3:2 है यदि इसकी छोटी भुजा की लंबाई 40 cm है। Find Area.

Give $l : b = 8 : 9$ [Now b is greater than l] —I

$\therefore b : h = 3 : 2$ —II

छोटी भुजा की लंबाई = 40cm

$\therefore 8 \rightarrow 40$

$9 \rightarrow \frac{40}{8}\times 9 = 45\text{cm}$ [बड़ी भुजा की लंबाई]

Acc. to II cond^n Now $3 \Rightarrow 45\text{cm}$ $2 \Rightarrow \frac{45}{3}\times 2 = 30\text{cm}$

∴ Length of Big Side (Base) = 45cm, Height = 30cm.

Area = Base × Height = $[45\times 30] = 1350 \text{ cm}^2$

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⇒ एक समांतर चतुर्भुज की आसन्न भुजा की लंबाई का अनुपात 3:2 है। इसकी बड़ी भुजा और संगत ऊँचाई का अनुपात 5:4 है यदि इसकी छोटी भुजा की लंबाई 30 cm है. find Area

$l : b = 3 : 2$ [$l > b$]

$\therefore l : h = 5 : 4$

Now 2(छोटी भुजा) → 30

$l = \because 3 \rightarrow \frac{30}{2} \times 3 = 45cm$

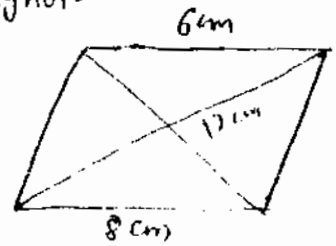
Now $l : h = 5 : 4$

$5 = 45 \quad \therefore 4 \rightarrow \frac{45}{5} \times 4 = 36cm$

$\therefore l = 45cm, \quad h = 36cm$

$Area = l \times h = 45 \times 36 = 1620cm^2$.

⇒ एक समांतर चतुर्भुज के आसन्न भुजा की लंबाई 8 और 6 cm है। एक विकर्ण की लंबाई 12 cm है find the Length of Other Diagnol.



Using $d_1^2 + d_2^2 = 2(l^2 + b^2)$

$\therefore 12^2 + d_2^2 = 2(8^2 + 6^2)$

$144 + d_2^2 = 200$

$d_2^2 = 200 - 144 = 56$

$d_2 = \sqrt{56} = 2\sqrt{14}$ cm. [Length of other Diagnol]

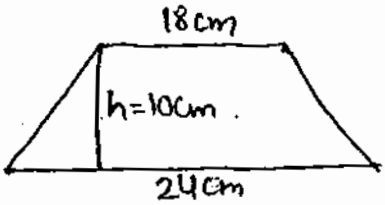
Next Type:→

⇒ एक समलंब चतुर्भुज के समांतर भुजाओं (∥side) की लंबाई 18 और 24 cm है। समलंब चतुर्भुज (Trapizium) का क्षेत्रफल 210 cm^2 है. इनके बीच की लंबाई लंबवत दूरी (Height) क्या होगी।

Using

Area of Trapizium

$= \frac{1}{2}$(sum of ∥ side) × Height



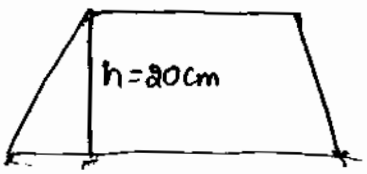
$\therefore 210 = \frac{1}{2}(18 + 24) \times h \Rightarrow 210 = \frac{42}{2} \times h$

$\Rightarrow h = 10$ cm.

∴ Length of Perpendicular Height = 10 cm

एक समलंब चतुर्भुज का क्षेत्रफल 270 cm^2 है। अगर ⊥ Distance 20 cm हो। Ratio of Length of ∥ side 4:5. Then Find the Length of Unparallel Side.

Length of ∥ side = $4x$ & $5x$



Using Area = $\frac{1}{2}$(Sum of ∥ side) × h

$270 = \frac{1}{2}(4x + 5x) \times 20$

$x = \frac{270 \times 2}{20} = \frac{270 \times 2}{4 \times 20} = 3$

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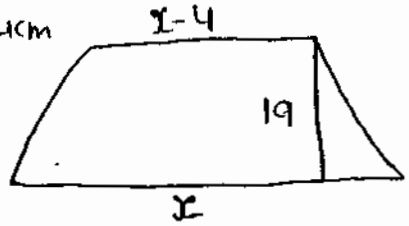
∴ Length of ll side = Resultant height = 5×3 = 15cm
or 5×3 = 15cm.

→ एक समलंब चतुर्भुज का क्षेत्रफल 475cm² है। Length of ⊥ Side (height) is 19c Difference Between Length of ll Side is 4cm. Find the Length of ll Side. Let Length of one Side = x cm, ∴ Second Side = x−4cm



Using Area

$= \frac{1}{2}(\text{sum of ll side}) \times h$

$475 = \frac{1}{2}(x + x - 4) \times 19$

$2x - 4 = \frac{475 \times 2}{19}$ ⇒ $2x - 4 = 50$

$x = 54/2 = 27cm$

∴ Length of one Side = 27cm. Second Side = 27−4 = 23cm

$475 = \frac{1}{2}(A+B) \times 19$

$A + B = \frac{475 \times 2}{19}$

$A + B = 50$

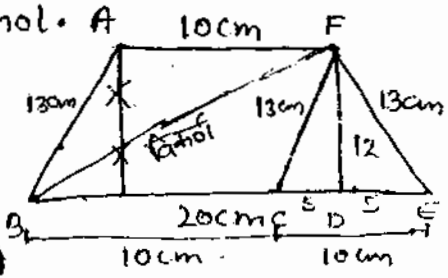
II→ $A - B = 4$

$2A = 54$ $A = 27$

∴ $B = A - 4 = 23$

NEXT TYPE :→

एक समलंब चतुर्भुज की समांतर भुजाओं की लंबाई क्रमशः 10cm और 20cm है। अन्य भुजाओं की प्रत्येक की लंबाई 13cm है, Find The Area and Length of Diagnol.



[जब किसी समलंब चतुर्भुज की असमांतर भुजाओं की लंबाई बराबर दी हो तो हम एक Side के ll side Drawn करते हैं जिससे एक समद्विबाहु त्रिभुज (Isoceles Triangle) बनाते हैं। समद्विबाहु त्रिभुज के ऊपर एक Median Drawn करते हैं तो वह तीसरी भुजा को दो समान भागों में Divide करती है।

$\text{Height} = \sqrt{13^2 - 5^2} = \sqrt{169 - 25} = \sqrt{144} = 12cm.$

∴ $\text{Area} = \frac{1}{2}(20 + 10) \times 12 = 15 \times 12 = 180cm^2$

(BF) Diagnol $= \sqrt{(BD)^2 + (ED)^2} = \sqrt{15^2 + 12^2}$ [∵ BD = BC + CD = 10 + 5 = 15cm]

$= \sqrt{225 + 144} = \sqrt{369}$ cm. [जब दोनों side (Unparallel) Equal हो तो दोनों विकर्ण भी बराबर होंगे]

OR

Using

$\text{Area} = \frac{a+b}{K}\sqrt{S(S-K)(S-C)(S-D)}$ where $K = a - b$

C, D → Unparellel side
a, b → Parellel side

$\text{Height} = \frac{2}{K}\sqrt{S(S-K)(S-C)(S-D)}$, where $S = \frac{K + C + d}{2}$

$\text{Area} = \frac{20+10}{20-10}\sqrt{18(18-10)(18-13)(18-13)}$

[∵ $S = \frac{20 - 10 + 13 + 13}{2} = 18$
$K = 20 - 10 = 10cm$]

$= \frac{30}{10}\sqrt{18 \times 8 \times 5 \times 5}$

$= 3\sqrt{2 \times 9 \times 2 \times 4 \times 5 \times 5} = 3 \times 2 \times 3 \times 2 \times 5 = 180cm^2$

$\text{Height} = \frac{2}{10}\sqrt{18 \times 8 \times 5 \times 5} = \frac{2}{10} \times 12 \times 5 = 12cm.$

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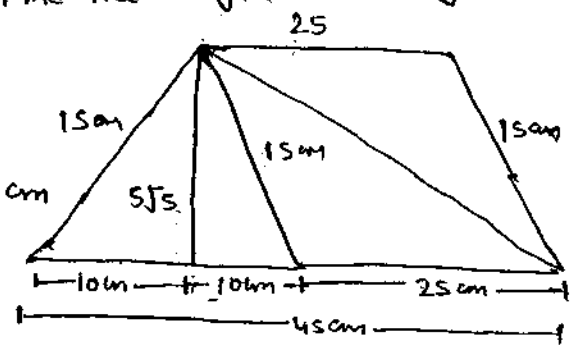
एक समलंब चतुर्भुज की समांतर भुजाओं की लम्बाई क्रमशः 25 और 45 cm है। अन्य असमांतर भुजाओं की लम्बाई प्रत्येक की 15 cm है। Find the Length and Diagnol of and Area of side.

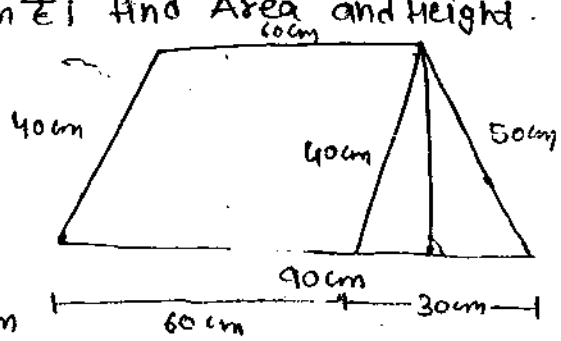


$$\text{Height} = \sqrt{15^2 - 10^2}$$
$$= \sqrt{225-100} = \sqrt{125} = 5\sqrt{5}\text{ cm}$$

$$\text{Area} = \frac{1}{2}(25+45) \times 5\sqrt{5}$$
$$= \frac{70}{2} \times 5\sqrt{5} = 175\sqrt{5}\text{ cm}^2$$

$$\text{Diagnol} = \sqrt{(25+10)^2 + (5\sqrt{5})^2} = \sqrt{35^2+125} = \sqrt{1225+125} = \sqrt{1350}\text{ cm}$$
$$= 15\sqrt{6}\text{ cm}$$

एक समलंब चतुर्भुज की समांतर भुजाओं की लंबाई क्रमशः 60cm और 90cm है। और असमांतर भुजाओं की लंबाई 40 और 50 cm है। find Area and Height.



Using Area $= \frac{a+b}{K}\sqrt{S(S-K)(S-c)(S-d)}$

$S = \frac{\overset{K}{a-b}+c+d}{2}$ [K = a−b (a/b are ∥ side) c/d are unparallel]

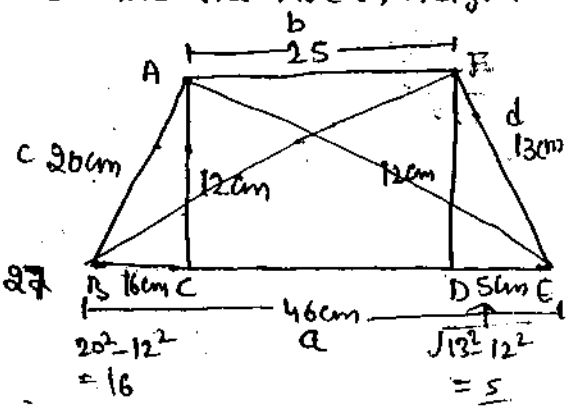
$K = 90-60 = 30\text{cm}$ $\quad S = \frac{30+40+50}{2} = 60\text{cm}$

$$\text{Area} = \frac{90+60}{30}\sqrt{60(60-30)(60-40)(60-50)}$$
$$= \frac{150}{30}\sqrt{60\times30\times20\times10} = \frac{150}{\cancel{30}} \times \overset{20}{\cancel{600}} = 3000\text{cm}^2$$

$$\text{Height} = \frac{2}{K}\sqrt{S(S-K)(S-c)(S-D)} = \frac{2}{30}\sqrt{60\times30\times20\times10} = \frac{2}{\cancel{30}} \times \overset{20}{\cancel{600}} = 40\text{cm}.$$

OR

⇒ ~~एक समांतर~~ एक समलंब चतुर्भुज के समांतर भुजा की लंबाई 25 cm और 46 cm है, और असमांतर भुजाओं की लंबाई 20 और 13 cm है find the Area, Height and Diagnol of Trapizium.



$$\text{Area} = \frac{a+b}{K}\sqrt{S(S-K)(S-b)(S-d)}$$

$K = a-b \qquad S = \frac{K+c+d}{2}$

$K = 46-25 = 21\text{cm} \qquad S = \frac{21+20+13}{2} = 27$

$$= \frac{46+25}{21}\sqrt{27(27-21)(27-20)(27-13)}$$
$$= \frac{71}{21}\times\sqrt{3\times9\times6\times7\times14} = \frac{71}{21}\times\sqrt{3\times3\times3\times3\times2\times7\times2\times7} = \frac{71}{\cancel{21}}\times3\times\cancel{3}\times2\times\cancel{7}$$
$$= 426\text{ cm}^2$$

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Height $= \frac{2}{K}\sqrt{s(s-a)(s-b)(s-c)}$

$= \frac{2}{21} \times \sqrt{27(27-21)(27-20)(27-13)}$

$= \frac{2}{21} \times 3\times3\times2\times7 = 12cm$.

(1) $\therefore DE = \sqrt{13^2-12^2} = \sqrt{169-144} = \sqrt{25} = 5cm$

$\therefore BD = BE - DE = 46 - 5 = 41 cm$.

In ΔBDF

$BF^2 = BD^2 + DF^2$

$\therefore$ Diagnol $BF = \sqrt{(41)^2+(12)^2} = \sqrt{1681+144} = \sqrt{1725} = 25\sqrt{69} cm$.

(2) $BC = \sqrt{20^2-12^2} = \sqrt{400-144} = \sqrt{256} = 16cm$.

$\because CE = BE - BC = 46 - 16 = 30 cm$.

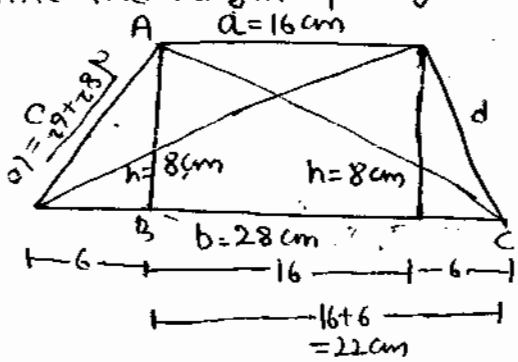
ΔACE $\therefore$ Diagnol $AE = \sqrt{AC^2+CE^2} = \sqrt{12^2+30^2} = \sqrt{144+900} = \sqrt{1044} = 6\sqrt{29} cm$

(C) एक समलंब चतुर्भुज का क्षेत्रफल $176 cm^2$ है इसमें असमांतर भुजा में से प्रत्येक की लंबाई बराबर है। Height of Trapizium is $\frac{2}{11}$ times of sum of parallel side और समांतर भुजाओं की लंबाई का अनुपात 4:7 है। find the length of Diagnol.

$a : b = 4 : 7$

$\therefore a = 4x$ or $b = 7x$

$h = \frac{2}{11}(4x+7x) = \frac{2}{11} \times 11x = 2x$



Area $= \frac{1}{2}$ (Sum of ∥ Side) $\times h$

$176 = \frac{1}{2}(4x+7x) \times 2x$

$11x^2 = 176$ $x^2 = 176/11 = 16$

$x = 4 cm$.

$\therefore a \Rightarrow 4\times4 = 16cm$ $b \Rightarrow 7\times4 = 28cm$, $h = 2\times4 = 8cm$

Now $c = d = \sqrt{8^2+6^2} = \sqrt{64+36} = 10cm$.

$\therefore$ In ΔABC

$AC^2 = AB^2 + BC^2$

Diagnol $\therefore AC = \sqrt{8^2+22^2} = \sqrt{64+484} = \sqrt{548}$ [दोनों विकर्ण बराबर होंगे।

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NEXT TYPE :→ एक सम षट्भुज (Regular Hexagonal) के प्रत्येक भुजा की लंबाई 8cm है। Find Area and ⊥ Distance Between two side.

Using Area $= \frac{\sqrt{3}}{4} a^2 \times 6 = \frac{\sqrt{3}}{4} \times 8^2 \times 6 = \frac{\sqrt{3}}{4} \times 8 \times 8 \times 6 = 96\sqrt{3}\ cm^2$

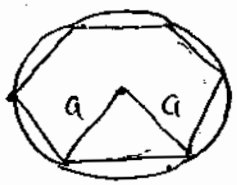
⊥ Distance (Height) $= \sqrt{3}\, a = \sqrt{3} \times 8 = 8\sqrt{3}$ cm.

⇒ एक समषट्भुज (Regular Hexagonal) की लंबाई 14 cm है find an Area of Incircle and Circumcircle.

Radius of Incircle $= \frac{\sqrt{3}}{2} a = \frac{\sqrt{3}}{2} \times 14 = 7\sqrt{3}$ cm.

Area of Incircle $= \pi r^2 = \frac{22}{7} \times 7\sqrt{3} \times 7\sqrt{3}$

$154 \times 3 = 462\ cm^2$.



Circum

Radius of CircumCircle = Side of Hexagonal

$= 14$ cm.

∴ Area of Circumcircle $= \frac{22}{7} \times 14 \times 14$

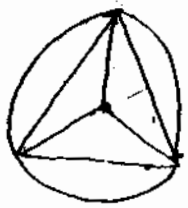
$= 616\ cm^2$.

Incircle

⇒ एक समबाहु त्रिभुज (Equilateral Triangle) के प्रत्येक भुजा की लंबाई $42\sqrt{3}$ cm है। find the Area of Incircle and Circumcircle.

Radius of Incircle $= \frac{\sqrt{3}}{2} a \times \frac{1}{3}$ [Radius of Eq. $\pi \times \frac{1}{3}$]

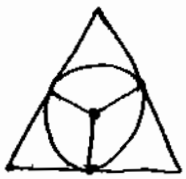
$= \frac{\sqrt{3}}{2} \times 42\sqrt{3} \times \frac{1}{3} = 21$ cm.



∴ Area $= \pi r^2 = \frac{22}{7} \times 21 \times 21 = 1386\ cm^2$

Now Radius of circum circle = 2 × Radius of Incircle.

$= 2 \times 21 = 42$ cm.



Area $= \frac{22}{7} \times 42 \times 42 = 5544\ cm^2$

⇒ एक समबाहु त्रिभुज के Incircle का Area $36\pi\ cm^2$ है Then find the Area of Equilateral Triangle.

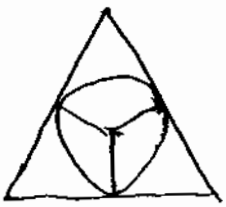
$\pi r^2 = 36\pi$

$r^2 = 36 \quad r = 6$ cm.

Now Radius of Incircle $= \frac{\sqrt{3}}{2} a \times \frac{1}{3}$

$6 = \frac{\sqrt{3}}{2} a \times \frac{1}{3}$

$a = \frac{6 \times 2 \times 3}{\sqrt{3}} = 12\sqrt{3}$ cm.



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Area of Equilateral Triangle $= \frac{\sqrt{3}}{4}a^2 = \frac{\sqrt{3}}{4} \times 12\sqrt{3} \times 12\sqrt{3}$

$= 108\sqrt{3}\ cm^2$.

⇒ एक समबाहु त्रिभुज के Circumcircle का Area $576\pi\ cm^2$ है find the Area of Equilateral Triangle.

$\pi R^2 = 576\pi$

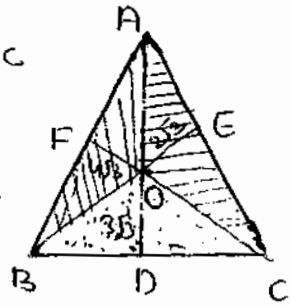
$R^2 = 576 \quad R = 24cm$

Now $R = \frac{\sqrt{3}}{2}a \times \frac{2}{3} \Rightarrow 24 = \frac{\sqrt{3}}{2}a \times \frac{2}{3}$

$\Rightarrow a = 24\sqrt{3}\ cm$.

Area $= \frac{\sqrt{3}}{4} \times 24\sqrt{3} \times 24\sqrt{3} = 432\sqrt{3}\ cm$.

⇒ एक समबाहु त्रिभुज के अन्दर किसी बिंदु से उसकी तीनों भुजाओं पर डाले गए लंब की लंबाई क्रमशः $2\sqrt{3}$, $3\sqrt{3}$ और $4\sqrt{3}\ cm$ है। त्रिभुज का परिमाप क्या होगा।



Area of ΔABC = Area of ΔAOB + Area of ΔBOC + Area of ΔAOC

$\frac{\sqrt{3}}{4}a^2 = \frac{1}{2} \times a \times 2\sqrt{3} + \frac{1}{2} \times a \times 3\sqrt{3} + \frac{1}{2} \times a \times 4\sqrt{3}$

$\frac{\sqrt{3}}{4}a^2 = \frac{1}{2}a\sqrt{3}[2+3+4]$

$\frac{\sqrt{3}}{4}a^2 = \frac{1}{2}\sqrt{3} \times a \times 9$

$\frac{a}{2} = 9 \quad a = 18\ cm$.

$\therefore$ Perimeter $= 3a = 3 \times 18 = 54cm$.

Direct $\longleftrightarrow$ $a = \frac{2}{\sqrt{3}}$ [Sum of ⊥ Height]

$= \frac{2}{\sqrt{3}}[2\sqrt{3} + 3\sqrt{3} + 4\sqrt{3}]$

$= \frac{2 \times 9\sqrt{3}}{\sqrt{3}} = 18\ cm$

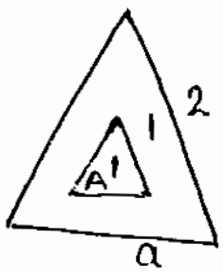
Perimeter $= 3 \times 18 = 54cm$.

⇒ एक समबाहु त्रिभुज के प्रत्येक भुजा की लंबाई $a\ cm$ है। इसके अन्दर एक और समबाहु त्रिभुज इस प्रकार बनाया जाता है कि दिया गया त्रिभुज का क्षेत्रफल दो बराबर भागों में बंट जाता है। इसकी भुजा का अनुपात क्या होगा।

Let Area of Small Triangle $(A_1) = 1$

then Area of Rest Part $= 1$

(A_2) Area of Given Triangle $= 1 + 1 = 2$



$A_1 : A_2 = 1 : 2$

$S_1 : S_2 = \sqrt{1} : \sqrt{2}$

$= 1 : \sqrt{2}$

Ans.

$\frac{\sqrt{3}}{4}A^2 = \frac{1}{2} \times \frac{\sqrt{3}}{4}a^2$

$\frac{A^2}{a^2} = \frac{1}{2}$

$\frac{A}{a} = \frac{1}{\sqrt{2}}$ $\quad A : a$

$1 : \sqrt{2}$

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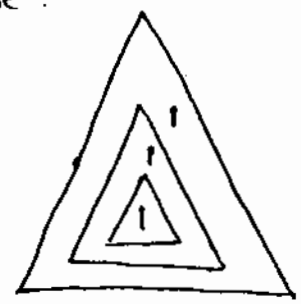
एक समबाहु त्रिभुज के प्रत्येक भुजा की लंबाई $10\sqrt{3}$ cm है इसके अन्दर दो और समबाहु त्रिभुज इस प्रकार बनाए जाते हैं कि बड़े त्रिभुज का क्षेत्रफल 3 बराबर भागों में बंट जाता है। find

(1) Ratio of Area in ascending Order.
(2) Ratio of Side in ascending order.
(3) Length of Each side of Small Equilateral Triangle.



Now Area of Smallest Triangle = 1 (A_1)
Area of Second Triangle = 1+1 = 2 (A_2)
Area of Big Triangle = 1+1+1 = 3 (A_3)

$A_1 : A_2 : A_3 = 1 : 2 : 3$

(ii) Ratio of Side.

$S_1 : S_2 : S_3 = \sqrt{1} : \sqrt{2} : \sqrt{3}$

$= 1 : \sqrt{2} : \sqrt{3}$

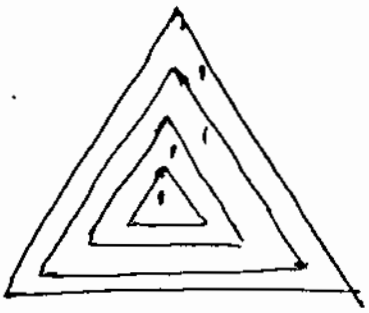
(iii) Length of Smallest side of Equilatest Triangle. $1x, \sqrt{2}x, \sqrt{3}x$

Now $\sqrt{3}x = 10\sqrt{3}$

$x = 10$ cm.

एक समबाहु त्रिभुज के प्रत्येक भुजा की लंबाई 20 है, इसके अन्दर 3 और समबाहु त्रिभुज इस प्रकार बनाए जाते हैं कि बड़े त्रिभुज का क्षेत्रफल 4 बराबर भागों में बंट जाता है। find

(i) Ratio of Area in Ascending Order.
(ii) Ratio of Side in Ascending Order.
(iii) Length of Each Side of Small Equilateral Triangle.



Let Area of Smallest Triangle (A_1) = 1
∴ Area of second Triangle (A_2) = 2
Area of Third Triangle (A_3) = 3
Area of Big Triangle (A_4) = 4.

$\therefore A_1 : A_2 : A_3 : A_4 = 1 : 2 : 3 : 4$

(ii) $S_1 : S_2 : S_3 : S_4 = \sqrt{1} : \sqrt{2} : \sqrt{3} : \sqrt{4}$

$= 1 : \sqrt{2} : \sqrt{3} : 2$

(iii) Length of Smallest side of Equilateral Traingle $x \; \sqrt{2}x \; \sqrt{3}x \; 2x$

$2x = 20$

$x = 10$ cm.

एक समबाहु त्रिभुज के प्रत्येक भुजा की लंबाई 15 cm है इसके अन्दर एक छोटा समबाहु त्रिभुज इस प्रकार बनाया जाता है कि छोटे त्रिभुज का क्षेत्रफल बड़े बचे हुए Part का $\frac{4}{5}$ है, find the Length of Each side of Smallest Triangle.

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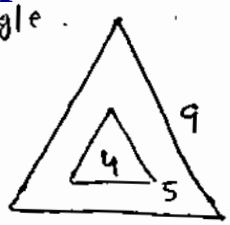
Area of Smallest Triangle = $\frac{4}{5}$ × Rest Part of Big Triangle



$\therefore$ $\frac{\text{Area of Smallest Triangle } (A_1)}{\text{Area of Rest Part } (A_{Remain})} = \frac{4}{5}$

$\therefore A_1 : A_{remain} = 4:5$

Now Area of Big Triangle (A_2) = Area of Small Triangle + Rest Part

$= 4+5 = 9$

$\therefore A_1 : A_2 = 4:9$

Ratio of side

$S_1 : S_2 = \sqrt{4} : \sqrt{9}$

$= 2:3$

Now $3 \Rightarrow 15$ $\quad \therefore 2 \Rightarrow \frac{\cancel{15}^{5}}{\cancel{3}} \times 2 = 10$

(Side) Length of Smallest Triangle = 10cm.

⇒ एक समबाहु त्रिभुज की प्रत्येक भुजा को 2cm बढ़ा दिया जाए तो त्रिभुज का क्षेत्रफल $2\sqrt{3}$ cm² बढ़ जाता है। find the length of original side of Triangle.

Let Original side of Triangle = a cm.

Area = $\frac{\sqrt{3}}{4} a^2$ cm².

Now Side of Triangle (Increase by 2) = $a+2$ cm.

Area = $\frac{\sqrt{3}}{4}(a+2)^2$

Acc. to question

$$\frac{\sqrt{3}}{4}(a+2)^2 = \frac{\sqrt{3}}{4}a^2 + 2\sqrt{3}$$

$$\cancel{\frac{\sqrt{3}}{4}a^2} + \frac{\sqrt{3}}{4}\times 4 + \frac{\sqrt{3}}{4}\times 4a = \cancel{\frac{\sqrt{3}}{4}a^2} + 2\sqrt{3}$$

$$\sqrt{3} + \sqrt{3}a = 2\sqrt{3}$$

$$\sqrt{3}a = \sqrt{3} \qquad a = 1 \text{ cm}.$$

Lenth of Original Side = 1cm.

Next Type :→ एक त्रिभुज की भुजाएं क्रमशः 9cm, 9cm और 12cm है। find the Area of Triangle

Using Heron's Formula

$$S = \frac{a+b+c}{2} = \frac{9+9+12}{2} = 15 \text{ cm}$$

$$\text{Area} = \sqrt{S(S-a)(S-b)(S-c)}$$

$$= \sqrt{15(15-9)(15-9)(15-12)}$$

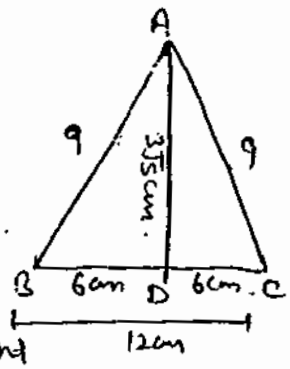
$$= \sqrt{15\times 6\times 6\times 3}$$

$$= \sqrt{3\times 5\times 3\times 2\times 3\times 2\times 3}$$

$$= 18\sqrt{5} \text{ cm}^2$$

OR

In ΔABC



$AD^2 = AC^2 - DC^2$

$AD = \sqrt{9^2 - 6^2}$

$= \sqrt{81-36}$

$= \sqrt{45} = 3\sqrt{5}$ cm.

Height = $3\sqrt{5}$ cm.

Area = $\frac{1}{2}$ × Base × Height

$= \frac{1}{\cancel{2}} \times \overset{6}{\cancel{12}} \times 3\sqrt{5}$

$= 18\sqrt{5}$ cm²

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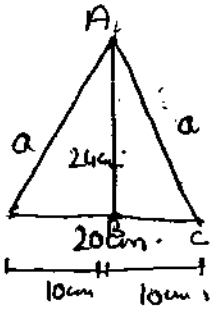
⇒ एक समद्विबाहु त्रिभुज का क्षेत्रफल $240 cm^2$ है इसके असमान भुजा की लंबाई 20cm है तो समान भुजाओं की लंबाई क्या होगी।



Using Area $= \frac{1}{2} \times$ Base $\times$ Height

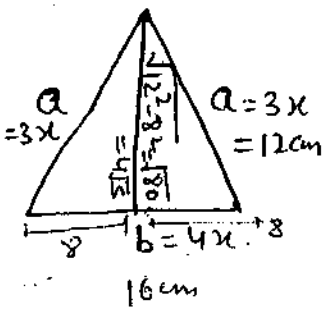
$$240 = \frac{1}{2} \times 20 \times h$$

$$h = 24 cm.$$

Now In ΔABC

$$a = \sqrt{24^2 + 10^2} = \sqrt{576+100} = \sqrt{676} = 26 cm.$$

⇒ एक समद्विबाहु त्रिभुज का परिमाप 40cm है इसकी दो बराबर भुजाओं में से एक भुजा और तीसरी भुजा का अनुपात 3:4 है find the Area of Isoceles Triangle.



$$a : b = 3 : 4$$

$$\therefore a = 3x, \; b = 4x$$

$$\therefore 3x + 3x + 4x = 40$$

$$10x = 40$$

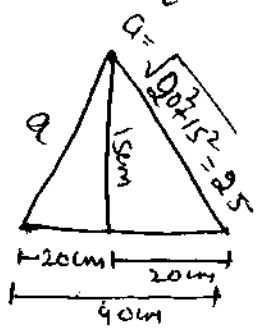
$$x = 4 cm.$$

$$\therefore a = 12 cm \quad b = 16 cm.$$

$$\therefore \text{Height} = \sqrt{12^2 - 8^2} = \sqrt{144-64} = \sqrt{80} = 4\sqrt{5} cm.$$

$$\text{Area} = \frac{1}{2} \times \text{Base} \times \text{Height} = \frac{1}{2} \times 16 \times 4\sqrt{5} = 32\sqrt{5} cm^2$$

⇒ एक समद्विबाहु त्रिभुज का क्षेत्रफल $300 cm^2$ है। Length of ⊥ on unequal side is 15cm. find the Perimeter of the Triangle.



Using Area $= \frac{1}{2} \times b \times H$

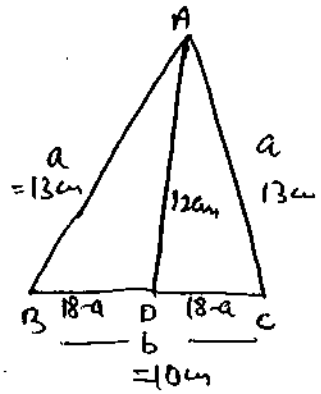
$$300 = \frac{1}{2} \times 15 \times b$$

$$b = \frac{300 \times 2}{15} = 40 cm.$$

$$\therefore a = \sqrt{20^2 + 15^2} = \sqrt{625} = 25 cm.$$

$$\text{Perimeter} = 25 + 25 + 40 = 90 cm.$$

⇒ एक समद्विबाहु त्रिभुज का परिमाप 36cm है इसके असमान भुजा पर डाले गए लंब की लंबाई 12cm है। find the Area of Triangle.



$$2a + b = 36$$

$$b = 36 - 2a$$

$$\therefore BD = BC = \frac{36-2a}{2} = 18 - a$$

In ΔADC = $AC^2 = AD^2 + DC^2$

$$a = \sqrt{12^2 + (18-a)^2}$$

$$a^2 = 144 + (18-a)^2$$

$$a^2 = 144 + 324 + a^2 - 36a, \quad 36a = 468 \quad a = \frac{468}{36} = 13$$

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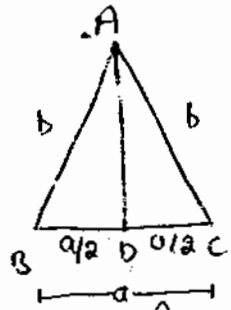
$a = 13$ cm

$b = 36 - 2a$

$= 36 - 26 = 10$ cm.

Area $= \frac{1}{2} \times \overset{5}{\cancel{10}} \times 12 = 60\ cm^2$

⇒ एक समद्विबाहु त्रिभुज के बराबर भुजाओं में से प्रत्येक की लम्बाई b cm है और असमान भुजा की लंबाई a cm है find the Area of Triangle.

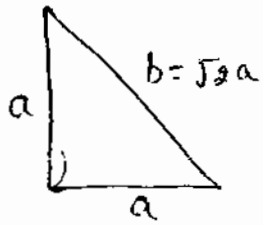


Height $AD = \sqrt{b^2 - (\frac{a}{2})^2} = \sqrt{b^2 - \frac{a^2}{4}} = \frac{\sqrt{4b^2 - a^2}}{2}$

Area $= \frac{1}{2} \times$ base $\times$ height

$= \frac{1}{2} \times a \times \frac{\sqrt{4b^2 - a^2}}{2} = \frac{1}{4} a \sqrt{4b^2 - a^2}\ cm^2$

⇒ एक समद्विबाहु समकोण त्रिभुज का परिमाप 2P है तो इसका क्षेत्रफल P की form में क्या होगा?



Now $b = \sqrt{a^2 + a^2}$
$= \sqrt{2}\, a$

Now Perimeter $= 2P$

$\therefore a + a + \sqrt{2}\, a = 2P$

$a(2 + \sqrt{2}) = 2P$

$a = \frac{2}{2 + \sqrt{2}} P = \frac{2(2 - \sqrt{2})}{2} P$

$a = (2 - \sqrt{2}) P$

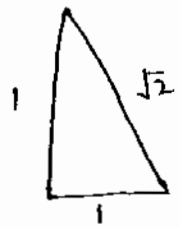
$\therefore$ Area $= \frac{1}{2} \times (2 - \sqrt{2}) P \times (2 - \sqrt{2}) P = \frac{1}{2} P^2 (4 + 2 - 4\sqrt{2})$

$= \frac{1}{2} P^2 [6 - 4\sqrt{2}] = P^2 [3 - 2\sqrt{2}]$

$\boxed{\text{Area} = P^2 [3 - 2\sqrt{2}]}$

Using this formula we can select answer with the help of Options.

Suppose $a = 1$ cm $\therefore b = \sqrt{1 + 1} = \sqrt{2}$



Area $= \frac{1}{2} \times 1 \times 1 = \frac{1}{2}\ cm^2$ — I

Perimeter $= 1 + 1 + \sqrt{2} = 2P$

$P = \frac{2 + \sqrt{2}}{2}$

Using formula $\therefore$ Area $= \left(\frac{2 + \sqrt{2}}{2}\right)^2 [3 - 2\sqrt{2}] = \frac{6 + 4\sqrt{2}}{4} (3 - 2\sqrt{2}) = \frac{(3 + 2\sqrt{2})(3 - 2\sqrt{2})}{2}$

$= \frac{9 - 8}{2} = \frac{1}{2}\ cm^2$ — II

Hence Area (both) are same.

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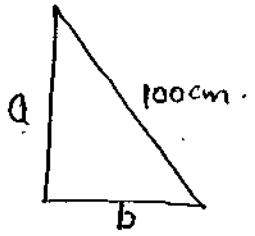
Next Type :→

एक समकोण त्रिभुज का क्षेत्रफल $2400 cm^2$ है। कर्ण की लंबाई 100 cm है। find the Difference in Length of other two side.



Detail :→ In Rightangled Triangle

$a^2+b^2=(100)^2$

$a^2+b^2=10000$ — I

Now. Area $=\frac{1}{2}\times$ Base $\times$ Height $=\frac{1}{2}\times a\times b$

$\frac{1}{2}\times a\times b=2400$ $\quad ab=4800$ — II

Now we have to find the Difference $a-b$

Using $a-b=\sqrt{a^2+b^2-2ab}$

from I or II

$a-b=\sqrt{10000-2(4800)}=\sqrt{400}=20$

∴ Difference Between two side is 20 cm.

Shortcut : $a^2+b^2=100^2$

we select two No. a and b whose square is equal 100^2

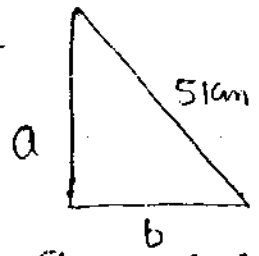
∴ $80^2+60^2=100^2$

∴ $a=80$ $b=60$

$a-b=20cm$.

∴ Area $=\frac{1}{2}\times 80\times 60=2400cm^2$

एक समकोण त्रिभुज का क्षेत्रफल $540 cm^2$ है। कर्ण की लंबाई 51cm. find the Perimeter of Triangle.



Detail :→ $a^2+b^2=51^2$

$a^2+b^2=2601$ — I

or $\frac{1}{2}\times a\times b=540$

$ab=1080$ — II

Perimeter $=a+b+51cm$.

∴ $a+b=\sqrt{a^2+b^2+2ab}$

$=\sqrt{2601+2(1080)}=\sqrt{2601+2160}=\sqrt{4761}=69cm$.

∴ Perimeter $=69+51=120cm$.

Shortcut

$(51)^2=(45)^2+(24)^2$

∴ $a=45$ $b=24$

$a+b+51$

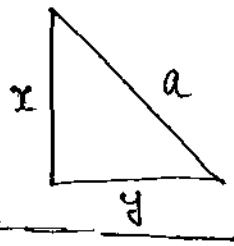
$=45+24+51$

$=120cm$.

$\frac{51}{3}=17$ or $17^2=15^2+8^2$

∴ $15\times3=45$, $8\times3=24$

एक समकोण त्रिभुज के कर्ण की लंबाई a cm है इसके समकोण बनाने वाली भुजा की लंबाई का योग b cm है find the Area of the Traingle.



Now $x+y=b$

or $x^2+y^2=a^2$

Area $=\frac{1}{2}\times x\times y$

We know that $(x+y)^2=x^2+y^2+2xy$.

∴ $2xy=\sqrt{(x+y)^2-(x^2+y^2)}$

$=\sqrt{b^2-a^2}$

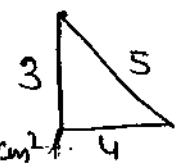
$xy=\frac{b^2-a^2}{2}$

∴ Area $=\frac{1}{2}\times\frac{b^2-a^2}{2}=\frac{b^2-a^2}{4}cm^2$

from Options.

assume side $=3,4,5$

∴ $a+b=3+4=7$



∴ Area $=\frac{1}{2}\times3\times4=6cm^2$

Now from formula $\quad b=3+4$

Area $=\frac{1}{4}\times(7^2-5^2)$

$=\frac{1}{4}\times 24=6cm^2$

Both Area are same.

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⇒ एक समकोण त्रिभुज के समकोण बनाने वाली भुजा की लंबाई 126 cm है कर्ण और दूसरी भुजा की लंबाई का Difference 42 cm है find the length of Hypotaenous.

Acc. to Given question.

$x - y = 42$ cm. —(1)

Also from Right angled Triangle.

$x^2 - y^2 = 126^2$

$(x-y)(x+y) = 126 \times 126$

$(x+y) = \frac{126 \times 126}{42} = 378$ —II

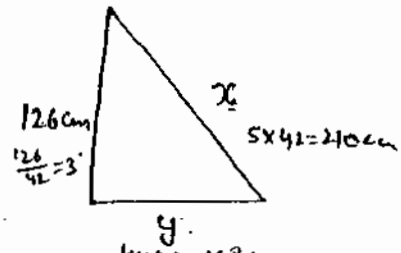
$x + y = 378$

$x - y = 42$

$2x = 420$

$x = 210$ cm.

Length of Hypo = 210 cm.



Shortcut

Let Length of One Side $= \frac{126}{42} = 3$

∴ Other = 4 and 5

Side are because $3^2 + 4^2 = 5^2$

$4 \times 42 = 168$ cm

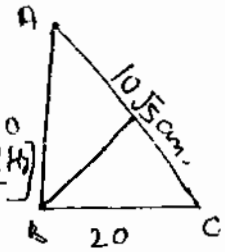
Hypo $= 5 \times 42 = 210$ cm.

⇒ एक समकोण त्रिभुज के समकोण बनाने वाली भुजा की लंबाई 10 और 20 cm है find the length of Medians and ⊥ Length of Longest side.

$AC = \sqrt{10^2 + 20^2} = \sqrt{500} = 10\sqrt{5}$ cm.

Median $= \frac{10\sqrt{5}}{2} = 5\sqrt{5}$ cm [Using = $\frac{\text{length of Longest (H)}}{2}$]

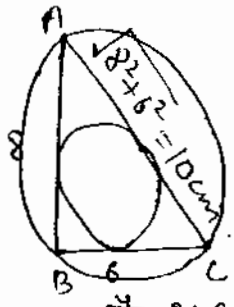
⊥ Height $= \frac{Pb}{h} = \frac{10 \times 20}{10\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} = 4\sqrt{5}$ cm.



⇒ एक समकोण त्रिभुज के समकोण बनाने वाली भुजाओं की लंबाई 8 और 6 cm है find The Radius of Incircle and Circumcircle.

r_{Circum} circle $= \frac{\text{Hypo}}{2} = \frac{10}{2} = 5$ cm.

$r_{\text{incircle}} = \frac{P+b-h}{2} = \frac{8+6-10}{2} = \frac{4}{2} = 2$ cm.

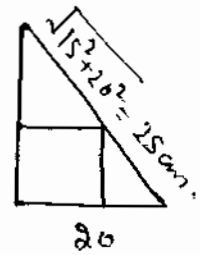


⇒ एक समकोण त्रिभुज के समकोण बनाने वाली भुजा की लंबाई 15 cm और 20 cm है। इसके अन्दर बने बड़े से बड़े वर्ग का क्षेत्रफल क्या होगा।

Area $= a^2$

$a = \frac{Pb}{P+b} = \frac{15 \times 20}{15+20} = \frac{15 \times 20}{35} = \frac{60}{7}$ cm.

Area $= \left(\frac{60}{7}\right)^2 = \frac{3600}{49}$ cm^2.



⇒ एक त्रिभुज की भुजाएं 10, 14 और 16 cm है find the Height of the longest side.

$S = \frac{10+14+16}{2} = \frac{40}{2} = 20$ cm.

Area $= \sqrt{20(20-10)(20-14)(20-16)} = \sqrt{20 \times 10 \times 6 \times 4} = 40\sqrt{3}$ cm^2

$40\sqrt{3} = \frac{1}{2} \times 16 \times h$ $\quad h = 5\sqrt{3}$ cm.

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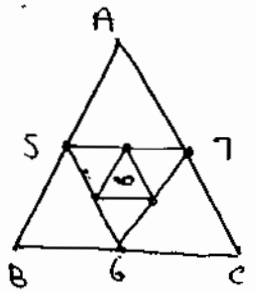
एक त्रिभुज की भुजाएं 5, 6 और 7 cm है। इस त्रिभुज की भुजाओं के मध्य बिंदु को मिलाकर एक त्रिभुज बनाया जाता है और इस प्रकार से बने त्रिभुज की भुजाओं को मिलाकर एक अन्य त्रिभुज और बनाया जाता है और इस प्रक्रिया को अनंत तक जारी रखा जाता है। Find the Sum of Area of All Triangle.



Sum of Area of Triangle

Area of ΔABC

$= \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{9 \times 4 \times 3 \times 2}$

$= 6\sqrt{6}$ cm.

$= \frac{4}{3} \times$ Area of ΔABC

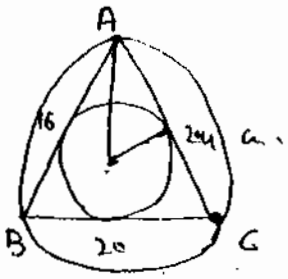
$= \frac{4}{3} \times 6\sqrt{6} = 8\sqrt{6}$ cm^2.

⇒ एक त्रिभुज की भुजाओं के मध्य बिंदु को मिलाकर एक और त्रिभुज बनाया जाता है और इस प्रकार से बने त्रिभुज के मध्य बिंदु को मिलाकर एक और त्रिभुज बनाया जाता है। इस प्रक्रिया को अनंत तक दोहराया जाता है। इस प्रकार से बने परिमाप का योग क्या होगा यदि बड़े त्रिभुज की भुजाएं क्रमशः 11, 12, 13 cm हो।

Sum of Perimeter of Triangle $= 2 \times P$

$= 2(11+12+13) = 72$ cm.

⇒ एक त्रिभुज की भुजाएं क्रमशः 16, 20 और 24 cm है। Find the Area of Incircle and Circumcircle.



Radius of Incircle $= \frac{\text{Area of } \Delta ABC}{s}$

$s = \frac{a+b+c}{2} = \frac{16+20+24}{2} = 30$ cm.

Area of $\Delta ABC = \sqrt{30(30-16)(30-20)(30-24)}$

$= \sqrt{30 \times 14 \times 10 \times 6}$

$= 60\sqrt{7}$ cm^2

$\therefore$ Rincircle $= \frac{60\sqrt{7}}{30} = 2\sqrt{7}$ cm.

Area of Incircle $= \pi r^2 = \frac{22}{7} \times 2\sqrt{7} \times 2\sqrt{7} = 88$ cm^2.

Radius of Circumcircle $= \frac{a \times b \times c}{4 \times \text{Area of } \Delta ABC} = \frac{16 \times 20 \times 24}{4 \times 60\sqrt{7}} = \frac{32}{\sqrt{7}}$ cm.

Area $= \pi r^2 = \frac{22}{7} \times \frac{32}{\sqrt{7}} \times \frac{32}{\sqrt{7}} = \frac{22528}{49}$ cm^2.

⇒ एक त्रिभुज के अंतः वृत्त की त्रिज्या 5 cm है और परिमाप 84 cm है। Find the Area of ΔABC

Using Radius $= \frac{\text{Area of } \Delta ABC}{s}$

$s = \frac{84}{2} = 42$ cm, $\quad 5 = \frac{\text{Area of } \Delta ABC}{42}$

Area of $\Delta ABC = 42 \times 5 = 210$ cm^2.

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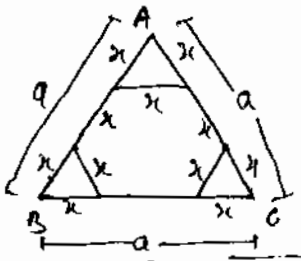
एक समबाहु त्रिभुज के प्रत्येक भुजा की लंबाई a है, इसके शीर्ष को लेकर एक समषट्भुज बनाया जाता है तो इसका Area क्या होगा।



Length of Equilateral Traingle = a cm.

Let Length of Regular Hexagonal = x cm.

$\therefore 3x = a$ cm.

$x = \frac{a}{3}$ cm.

एक समबाहु त्रिभुज के प्रत्येक भुजा की लंबाई 6 cm है, इसके शीर्ष को लेकर एक समषट्भुज बनाया जाता है, तो इसका Area क्या होगा।

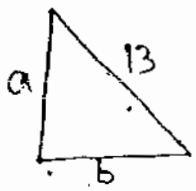
Using $x = \frac{a}{3}$

x = Side of Regular Hexagonal, a is side of Triangle

$\therefore x = \frac{a}{3} = \frac{6}{3} = 2$ cm,

Area $= \frac{\sqrt{3}}{4}a^2 \times 6 = \frac{\sqrt{3}}{4} \times \cancel{2} \times 2 \times 6 = 6\sqrt{3}$ cm^2.

Q) एक त्रिभुज का परिमाप 30 cm है। और उस त्रिभुज का क्षेत्रफल 30 cm^2 है। Length of Longest side is 13 cm है, find the length of Smallest side.



इस Type के e.g (परिमाप = दिया) हम देखते हैं कि बड़ी side (13) का square (169) किस किस के square के sum के बराबर हो सकता है। जैसे $13^2 = 12^2 + 5^2$

$\therefore$ Length of one side = 12 cm, other side = 5 cm.

Verification :→ $a + b + 13 = 30$ (होना चाहिए)

$12 + 5 + 13 = 30$

$30 = 30$. (Hence Verified)

Detail :→

$a + b + 13 = 30$

$a + b = 17$

$b = 17 - a$

or $S = \frac{\text{परिमाप}}{2} = \frac{30}{2} = 15$ cm.

$\therefore$ Area $= \sqrt{15(15-a)(15-17+a)(15-B)}$

$30 = \sqrt{30(15-a)(a-2)}$

$30 = \sqrt{30(17a - a^2 - 30)}$

Squaring Both side

$\cancel{900}\ 30 = \cancel{30}(17a - a^2 - 30)$

$30 = 17a - a^2 - 30$

$a^2 - 17a + 60 = 0$

$(a-12)(a-5) = 0$ $\quad a = 12, 5$

$\therefore b = 5, 12$

Hength Length of Side of Triangle are, 12, 5

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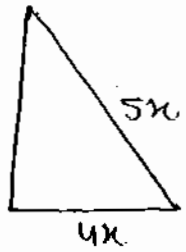
एक त्रिभुज की भुजाओं का अनुपात 3:4:5 है अगर त्रिभुज का क्षेत्रफल 72 cm² हो तो छोटी भुजा की लंबाई ज्ञात कीजिए।

Ratio of Side = 3:4:5
= 3x, 4x, 5x



इन भुजाओं से पता चलता है कि यह Right Angle Triangle है।
$[5^2 = 3^2 + 4^2]$

$\therefore$ Area $= \frac{1}{2} \times 3x \times 4x$

$72 = \frac{1}{2} \times 12 x^2$

$x^2 = 12;\ x = 2\sqrt{3}$

$\therefore$ length of Smallest Side $= 3 \times 2\sqrt{3} = 6\sqrt{3}$ cm.

OR

$S = \frac{3x+4x+5x}{2} = 6x$

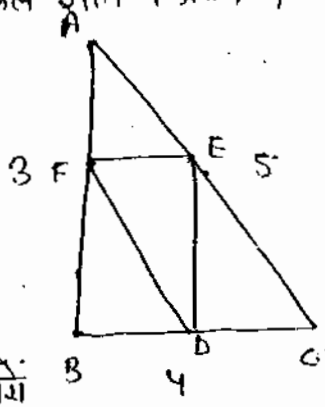
Area $= \sqrt{6x(6x-3x)(6x-4x)(6x-5x)}$

$72 = \sqrt{6x \cdot 3x \cdot 2x \cdot x}$

$72 = 6x^2,\ x^2 = 12,\ x = 2\sqrt{3}$

length of longest Side $= 6\sqrt{3}$ cm

→ एक त्रिभुज की भुजाएं 3, 4 और 5 cm है। इस त्रिभुज की भुजाओं के मध्य बिंदु को मिलाकर एक त्रिभुज बनाया जाता है तो उस त्रिभुज का क्षेत्रफल ज्ञात कीजिए।



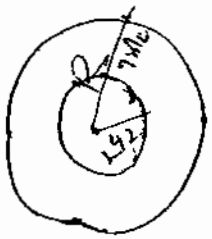
Area of Small Triangle DEF $= \frac{1}{4}$ of ΔABC

$= \frac{1}{4}\left[\frac{1}{2} \times 3 \times 4\right]$

$\frac{3}{2} = 1.5\ cm^2$

New Types:-1

एक वर्ताकार पार्क की त्रिज्या 42 m है इसके बाहर चारों ओर 7 मीटर चौड़ा रास्ता है उस रास्ते का क्षेत्रफल ज्ञात कीजिए।



$r = 42$ cm

$R = r + 7$ cm
$= 42 + 7 = 49$ cm.

Area of Path $= \pi R^2 - \pi r^2$

$= \pi(R^2 - r^2)$

$= \frac{22}{7}(49^2 - 42^2) = \frac{22}{7}(49+42)(49-42)$

$\frac{22}{7} \times 91 \times 7 = 2002\ m^2$

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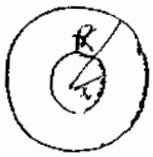
⇒ एक वर्तुलाकार पथ के भीतर और वृत्ताकार सीमा के बाहर की ओर आतल पथ क्रमशः 113 मीटर और 91 मीटर हैं। Find the Circumference of Path.



$2\pi R = 113$ मीटर

$2\pi r = 91$ m.

~~Circumference of Path~~ $= 2\pi(R) - 2\pi r$

$= 2\pi(R-r)$

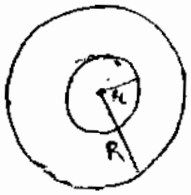
$= \frac{2\times22}{7}(113-91) = \frac{2\times22}{7}\times22$

$R-r = \quad 2\pi R - 2\pi r = 113-91$

$2\pi(R-r) = 22$

$R-r = \frac{22\times7}{2\times22} = 3.5$ m.

⇒ दो संकेंद्रीय वृत्तों का क्षेत्रफल क्रमशः 616 cm² और 154 cm² हैं find the Width of the Ring. Also find the Perimeter of Ring.



$\pi R^2 = 616$ cm², $\pi r^2 = 154$ cm²

$R^2 = \frac{616\times7}{22}$ (28), $r^2 = \frac{154\times7}{22}$ (7)

$R = \sqrt{28\times7}$, $r = \sqrt{49}$

$= \sqrt{4\times7\times7}$, $r = 7$ cm.

$= 14$ cm.

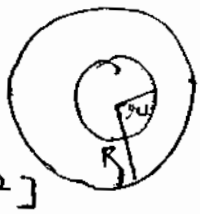
∴ Width of Ring $= R - r = 14-7 = 7$ cm.

Perimeter of Ring $= 2\pi(r+R) = 2\times\frac{22}{7}(7+14)$

$= 2\times\frac{22}{7}\times21^3 = 132$ cm.

⇒ दो संकेंद्रीय (Concentric) वृत्तों में से बड़े वृत्त की त्रिज्या 25 cm है छोटे वृत्त का क्षेत्रफल रिंग के क्षेत्रफल का $\frac{1}{4}$ भाग है find the Ratio of Their Radii. Also find the Radius of the Smallest Circle.

Detail ~~Radius~~ Area of Smallest Circle $= \frac{1}{4}\times$ Are of Ring



$\pi r^2 = \frac{1}{4}(\pi R^2 - \pi r^2)$

[∵ Area of Ring $= \pi R^2 - \pi r^2$]

$4\pi r^2 = \pi R^2 - \pi r^2$

$5\pi r^2 = \pi R^2$

$\frac{r^2}{R^2} = \frac{1}{5}$

$\frac{r}{R} = \frac{1}{\sqrt5}$ $\quad r:R = 1:\sqrt5$ Ans.

Now $\sqrt5 = 25$ cm

$1 = \frac{25}{\sqrt5} = 5\sqrt5$ cm.

∴ Radius of Smallest Circle $= 5\sqrt5$ cm.

Shortcut

Area of Small Circle : Area of Ring

$= \frac{1}{4}:1 = 1:4$

∴ Area of Big Circle $= 5 = A$

∴ $A_1 : A_2 = 1:5$

$r_1 : r_2 = \sqrt1 : \sqrt5$

$= 1:\sqrt5$ — (1)

$\sqrt5 = 25$ ∴ $1 = \frac{25}{\sqrt5} = 5\sqrt5$ (Radius)

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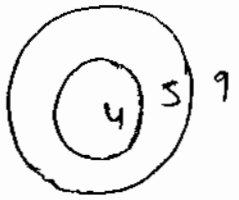
⇒ दो संकेंद्रिय वृत्तों में बड़े वृत्त का व्यास 30cm है। छोटे वृत्त का क्षेत्रफल रिंग के क्षेत्रफल का $\frac{4}{5}$ भाग है। find the Ratio of Radii of Circle. Also find the Radius of smallest Circle.



Area of Small Circle (A_1) : Area of Ring $= \frac{4}{5} : 1$

$= 4 : 5$

∴ Area of Big Circle (A_2) $= 4 + 5 = 9$.

∴ Area of Small Circle : Area of Big Circle

$A_1 : A_2 = 4 : 9$

Radio of Radii $r_1 : r_2 = \sqrt{4} : \sqrt{9}$

$= 2 : 3$

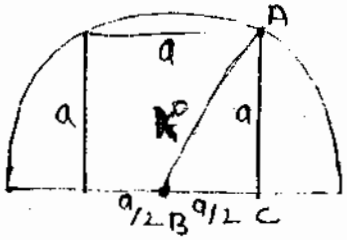
Now $3 \Rightarrow 15$

$2 \Rightarrow \frac{15}{3} \times 2 = 10cm$ [radius $= \frac{30}{2} = 15cm$]

∴ Radius of small Circle $= 10cm$.

Detail :→ Detail is same As in Previous Example.

⇒ Next Type :→ एक अर्धवृत्त (Semi-Circle) की त्रिज्या 40cm है find the Area of ~~Semicircle~~ Largest square make in a semi-Circle.



Let length of Square $= a$ cm.

Now ABC is a Right Angled Triangle. in which

$a^2 + \left(\frac{a}{2}\right)^2 = 40^2$

$a^2 + \frac{a^2}{4} = 1600 \Rightarrow 5a^2 = 6400$

$\boxed{a^2 = 1280}$ area of Square.

OR

We know that $r : a = \sqrt{5} : 2$ [$r \Rightarrow$ Radius of Semicircle, $a \Rightarrow$ Each Side of square]

Now $\sqrt{5} = 40$

$\therefore 2 = \frac{\cancel{40}^{8}}{\cancel{\sqrt{5}}} \times 2 = 16\sqrt{5}$

$\therefore a = 16\sqrt{5}$

Area of Square $= a^2 = (16\sqrt{5})^2 = 1280cm^2$.

⇒ एक वृत्त की त्रिज्या 30cm है. इसके अन्दर दो संकेंद्रिय वृत्त इस प्रकार बनाए जाते हैं कि दिए गए वृत्त का क्षेत्रफल 3 बराबर भागों में विभाजित हो जाता है find.

(i) Ratio of Area in Ascending Order.

(ii) Ratio of Radii in Ascending Order.

(iii) Radius of Smallest Circle.

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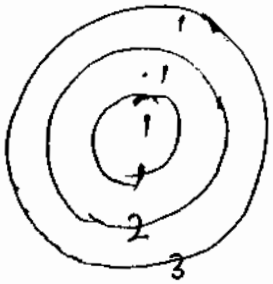
$A_1 : A_2 : A_3 = 1 : 2 : 3$

$\therefore r_1 : r_2 : r_3 = \sqrt{1} : \sqrt{2} : \sqrt{3}$

$= 1 : \sqrt{2} : \sqrt{3}$



Now $\sqrt{3} = 24$

$1 = \frac{24}{\sqrt{3}} = 8\sqrt{3}$

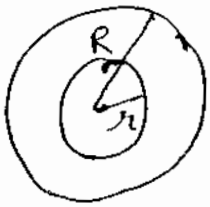
$\therefore$ Radius of Smallest Circle $= 8\sqrt{3}$ cm.

⇒ एक वर्ताकार पार्क के अन्दर में चारों ओर रास्ता है जोकि वृत्ताकार है उसके बाहरी और आंतरिक परिधि का अंतर 132 m है find the Length of Path

Given $2\pi R - 2\pi r = 132$

$2\pi(R-r) = 132$

$R - r = \frac{132 \times 7}{22 \times 2} = 21$



$\therefore$ Path की चौड़ाई $= R - r = 21$ cm.

Next Type: एक वृत्ताकार तार की त्रिज्या 56 cm है इसे मोड़ के एक वर्ग बनाया जाता है वर्ग के प्रत्येक भुजा की लंबाई क्या होगी।

तार को मोड़कर वर्ग बनाया गया है इसलिए दोनो का परिमाप बराबर होगा।

$\therefore 2\pi r = 4a$

$\Rightarrow 2 \times \frac{22}{7} \times 56 = 4a$

$a = \frac{22 \times 8 \times 2}{4} = 88$ cm.

$\therefore$ Each side of Square $= 88$ cm.

⇒ एक तार वृत्ताकार रूप में है जिसकी त्रिज्या 42 cm है, इसे मोड़कर एक आयत बनाया जाता है जिसकी Length or Breadth का Ratio 6:5 है। find the Length or Breadth of Circle.

तार को मोड़कर एक आयत बनाया जाता है।

$\therefore$ Perimeter of Wire = Perimeter of Rectangle

$2\pi r = 2(l+b)$

Now $l : b = 6 : 5$ $\quad l = 6x,\ b = 5x$

$\therefore 2 \times \frac{22}{7} \times 42 = 2(6x + 5x)$

$132 = 11x$

$x = 12$

$\therefore$ Length $(l) = 6x = 6 \times 12 = 72$ cm

Breadth $(b) = 5x = 5 \times 12 = 60$ cm

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Next Type :- तीन वृत्तों की त्रिज्या क्रमशः 12cm, 9cm और 8cm है। एक वृत्त का क्षेत्रफल इन तीनों वृत्तों के योग के क्षेत्रफल के बराबर है। बड़े वृत्त की त्रिज्या बताए।

Give $\pi(r_1^2+r_2^2+r_3^2) = \pi R^2$ [r_1, r_2, r_3 → Radius of Small Circle]

$\therefore \pi[12^2+9^2+8^2] = \pi R^2$

$289 = R^2$

$R = \sqrt{289} = 17cm$.

$\therefore$ Radius of Big Circle $(R) = 17cm$.

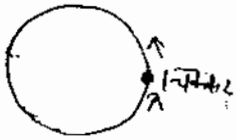
⇒ एक वृत्त के त्रिज्यारवंड (~~Arc~~ Sector) की त्रिज्या 9cm है इसके संगत चाप (Arc) की लंबाई 24cm है, find the Area of sector.

Area of sector $= \frac{1}{2} \times$ Length of Arc $\times$ Radius

$= \frac{1}{2} \times 24 \times 9 = 108 cm^2$

⇒ एक व्यक्ति एक वृत्ताकार रास्ते के चारों ओर 40 मिनट में 8 चक्कर लगाता है यदि इसके व्यास को 10 गुना कर दिया जाए तो अब वह 1 चक्कर लगाने में कितना समय लेगा।

जब व्यक्ति एक वृत्ताकार रास्ते के चारों ओर चक्कर लगाता है तो एक चक्कर में उस वृत्त की परिधि ($2\pi r$) के Equal Distance Travel करेगा।



$\therefore$ Total Distance Travelled $= 8 \times 2\pi r = 8 \times \pi d$

Shortcut :-

8 चक्कर → 40 min

$\therefore$ 10 —— $\frac{40}{8} \times 10$

$= 50 min.$

$8 \times \pi d$ Distance Travelled in → 40 minutes.

$10 \times \pi d$ —— → $\frac{40}{8 \times \pi d} \times 10 \times \pi d$

$= 50$ minutes.

Next Type :→

Total Distance Travelled By wheel $= 2\pi r \times n$
where n is No. of Rotation

⇒ एक पहिए की त्रिज्या 14cm है, 2000 चक्कर लगाने में वह कितनी दूरी तय करेगा।

Total Distance Travelled By wheel $= 2\pi r \times n$

$= 2 \times \frac{22}{7} \times 14 \times 2000$

$= 176000 cm$

$= 1760 m = 1.76 Km$.

⇒ एक पहिये की त्रिज्या 35 cm है। find the No. of Rotation to cover the Distance of 2.2 Km.

$2.2 Km = 2 \times \frac{22}{7} \times 35 \times n$

$\frac{2200}{10} \times 100 = 2 \times \frac{22}{7} \times 35 \times n$

$n = \frac{1000 \times 10}{5 \times 2}$

1000 Ans.

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⇒ एक पहिया 1250 चक्कर में 11 km की दूरी तय करता है तो Find the Radius of wheel.

Using $2\pi r \times n$ = ~~No. of~~ Total Distance.

$\Rightarrow 2 \times \frac{22}{7} \times r \times 1250 = 11 km$

$2 \times \frac{22}{7} \times r \times 1250 = 11 \times 1000 \times 100$

$r = \frac{11 \times 1000 \times 100 \times 7}{2 \times 22 \times 1250} = 140 cm$

⇒ एक पहिये के त्रिज्या 21cm है। यह पहिया 1 मिनट में 29 चक्कर लगाता है। 1.914 km की दूरी तय करने में वह कितना समय लेगा।

1 चक्कर में तय कि गई दूरी $= 2\pi r = 2 \times \frac{22}{7} \times 21 \times \frac{1}{100} m$

29 चक्कर ——— $= \frac{2 \times 22 \times 3 \times 29}{100}$

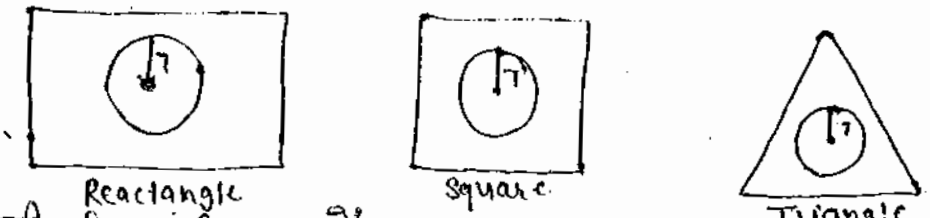
$\frac{2 \times 22 \times 3 \times 29}{100}$ Distance Travelled in 1 min.

∴ 1.914 km ⇒ 1914 m ———

$\frac{100 \times 1914}{2 \times 22 \times 3 \times 29} = 50 min.$

Next Type :→ [In The Middle of The Field]

कोई भी field दिया गया है किसी भी shape [Triangle, Rectangle, Square etc] उसमे घास उगाई गई है और एक गाय को उस field में बांध दिया गया है किसी रस्सी से तो जितनी उस रस्सी की लंबाई है वह उसी लंबाई तक एक वृत्ताकार field में घास चर सकेगी। जितनी रस्सी की लंबाई होगी वही उस वृत्त की त्रिज्या होगा। Total Grazzing Area [जिसमे घास चरी है] वह क्षेत्रफल निकल जाएगा। और जिसमें घास नहीं चरी वह Area = Area of field − Grazzing Area



माना रस्सी की लंबाई 7 cm है Then

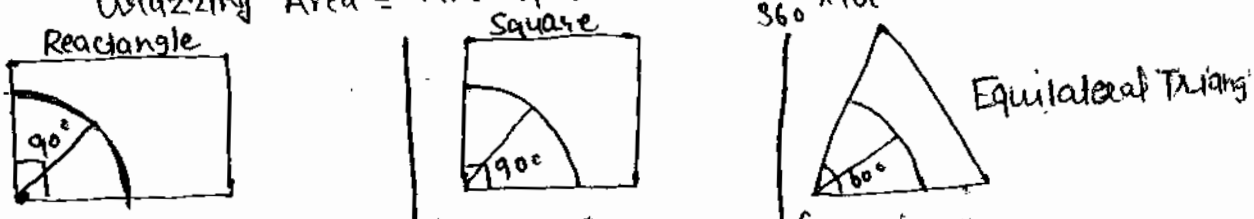
Grazzing Area $= \pi r^2 = \frac{22}{7} \times 7 \times 7 = 154 cm^2$

$$\% \text{ Area covered By Cow} = \frac{\text{Grazzing Area}}{\text{Total Area of field}} \times 100$$

Next Type :→ [At the Corner of the field]

अगर गाय को किसी भी field [Rectangle, Square, Equilateral Triangle, Polygon, Hexagonal, Other triangle] के corner पर बांध देते है Then

Grazzing Area = Area of Sector $= \frac{\theta}{360} \times \pi r^2$



Grazzing Area $= \frac{\theta}{360} \times \pi r^2 = \frac{90}{360} \times \pi r^2$

Grazzing Area $= \frac{90}{360} \times \pi r^2$

Grazzing Area $= \frac{60}{360} \times \pi r^2$

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गाय को अब जितने corner पर बांधेगे उसको Grazzing Area के साथ Multiply कर देंगे।

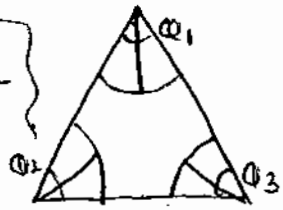
⇒ अगर Equilateral Triangle को छोड़कर अन्य कोई भी Triangle हो Then

Then

$$\text{Grazzing Area} = \frac{\theta_1}{360}\times\pi r^2 + \frac{\theta_2}{360}\times\pi r^2 + \frac{\theta_3}{360}\times\pi r^2$$

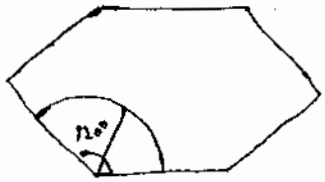
$$= \frac{\pi r^2}{360}(\theta_1+\theta_2+\theta_3)$$

$$= \frac{\pi r^2}{\cancel{360}_{2}}\times\cancel{180} = \frac{\pi r^2}{2} \quad [\theta_1+\theta_2+\theta_3 = 180^\circ]$$



⇒ अगर कोई भी Regular Hexagonal दिया गया है

$$\text{Grazzing Area} = \frac{\cancel{120}^{\circ}}{\cancel{360}_{3}}\times\pi r^2 = \frac{1}{3}\pi r^2$$



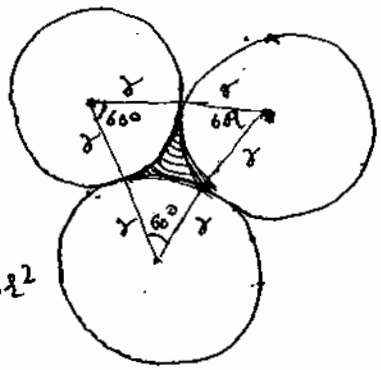
e.g. एक त्रिभुजाकार खेत के तीनों कोनों पर 14m लंबी रस्सी से तीन गायों को बांध दिया जाता है वह कितने Area में घास चरेगी।

$$\text{Grazzing Area} = \frac{1}{3}\times\pi r^2$$

$$= \frac{1}{\cancel{3}}\times\frac{22}{\cancel{7}}\times\cancel{14}\times 14 = 308\,m^2$$

Next Type:→ तीन वृत्तों को इस प्रकार से रखा गया है कि प्रत्येक Circle दूसरे दोनों Circles को Touch करता है। प्रत्येक की Radius Equal है Then find the Area of Untouched Portion.

In three circles when we coincide the centre of all it becomes an Equilateral Triangle whose each side $= r+r = 2r$



$$\text{Area of Equilateral Triangle} = \frac{\sqrt{3}}{4}a^2$$

$$= \frac{\sqrt{3}}{4}(2r)^2 = \frac{\sqrt{3}}{\cancel{4}}\times\cancel{4}r^2 = \sqrt{3}r^2$$

In this Triangle 3 sector form

$$\text{Area of 3 sector} = \cancel{3}\times\frac{60^\circ}{\cancel{360}_{2}}\times\pi r^2 = \frac{\pi r^2}{2} \qquad (I)$$

∴ Area of Untouched Portion = Area of Equilateral Δ − Area of Three Sector

$$= \sqrt{3}r^2 - \frac{\pi r^2}{2}$$

$$= \left(\sqrt{3}-\frac{\pi}{2}\right)r^2 \text{ Ans.}$$

Perimeter = 3 × length of one Arc.

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(19) तीन वृत्तों में से प्रत्येक की त्रिज्या दो (2) cm है इन वृत्तों को इस प्रकार से रखा गया है कि प्रत्येक circle एक दूसरे को Touch करता है। परिबद्ध (Untouched Portion) का Area क्या होगा। Also find the Perimeter of Untouched Portion.

Using Area of Untouched Portion $= \left(\sqrt{3}-\frac{\pi}{2}\right)r^2$

$= \sqrt{3}-\pi-\left(\sqrt{3}-\frac{\pi}{2}\right)\times 2^2 = \frac{2\sqrt{3}-\pi}{2}\times 4^2$

$= 8(2\sqrt{3}-\pi)\ cm^2$

For Perimeter we see that their are 3 arcs in Diagram (I) in Untouched Part

$\therefore$ Length of One arc $= \frac{\theta}{180}\pi r = \frac{60}{180}\pi r$

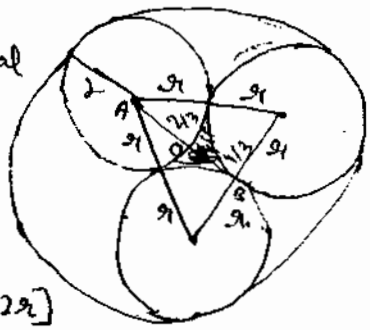
Perimeter $= 3\times$ Length of Arc $= \frac{3\times 60}{180}\times\pi\times 2 = 2\times\frac{22}{7} = \frac{44}{7}\ cm$.

Next Type:- तीन वृत्तों को इस प्रकार से दिया गया है कि प्रत्येक वृत्त दूसरे दोनों वृत्तों को Touch करता है इन सभी की Radius भी Equal है। इन सभी को Touch करता हुआ एक बड़ा वृत्त है find the Radius of Big Circle.

Let Radius of Each small Circle $= r$

By Coincide the Centre of Each Circle a Equilateral Triangle is formed.

When we drawn a Median on Equilateral Δ the.



Height $(AO) = \frac{\sqrt{3}}{2}a\times\frac{2}{3}$

$= \frac{\sqrt{3}}{2}\times 2r\times\frac{2}{3} = \frac{2\sqrt{3}}{3}r \quad [\because a=2r]$

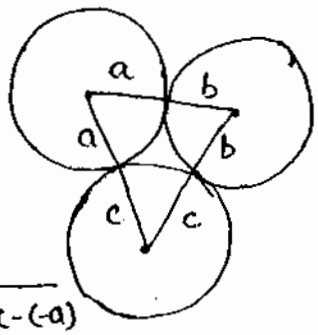
$\therefore$ Radius of Big Circle $=$ Height $(AO)+r$

$= \frac{2\sqrt{3}}{3}r + r = \left(\frac{2\sqrt{3}+3}{3}\right)r$

Next Type:- तीन वृत्तों को इस प्रकार से रखा गया है कि प्रत्येक वृत्त दूसरे दोनों वृत्तों को Touch करता है। प्रत्येक वृत्त की त्रिज्या अलग-अलग है find the Area of the Triangle formed to Joint the centre of all these Circle.

Let Radius of Circles are Respectively a, b, c

Then Three Side of Triangle are $a+b,\ b+c,\ c+a$



$\therefore$ Semi Perimeter $(S) = \frac{a+b+b+c+c+a}{2} = \frac{2(a+b+c)}{2}$

$S = a+b+c$

Area $= \sqrt{(a+b+c)(a+b+c-a-b)(a+b+c-b-c)(a+b+c-c-a)}$

$= \sqrt{(a+b+c)(c)(a)(b)}$

$\therefore$ Area $= \sqrt{abc(a+b+c)}$

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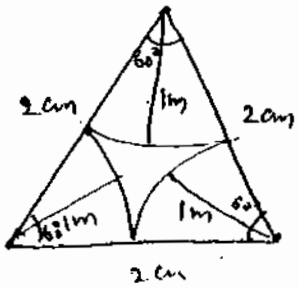
एक समबाहु खेत की प्रत्येक भुजा 2cm है इसके तीनों corner पर 1m लंबी रस्सी से गाय बांधा दी गई है गाय कितना Area चर पायगी और कितना Area शेष रह जायगी।



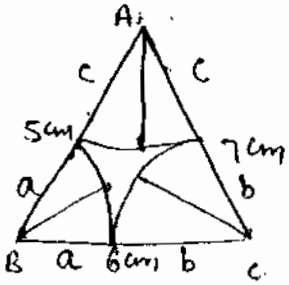
Area of the field $= \frac{\sqrt{3}}{4}(a^2) = \frac{\sqrt{3}}{4}\times 2^2 = \frac{\sqrt{3}}{4}\times 4 = \sqrt{3}\ cm^2$

Grazzing field $= 3\times\frac{60}{360}\pi r^2 = \frac{3\times 60}{360}\times\pi\times 1\times 1$

$= \frac{\pi}{2}\ cm^2$

Remaining field $= \left(\sqrt{3}-\frac{\pi}{2}\right) cm^2$.

⇒ एक त्रिभुजाकार खेत की भुजाएं क्रमशः 5, 6 और 7cm है इसके तीनों Corner पर 3 अलग-अलग लंबाई की रस्सियों से गायों को बांधा गया है वे एक दूसरे के ठीक सामने होती है find the Length of Each Rope.



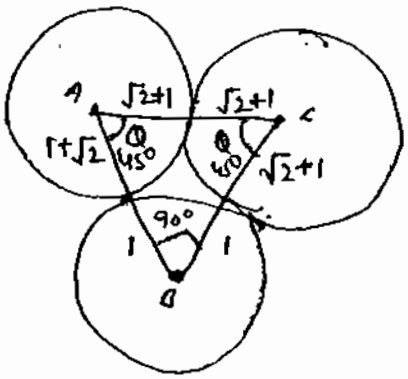
$a+b = 6\ cm$
$b+c = 7\ cm$
$c+a = 5\ cm$
+
$2(a+b+c) = 18\ cm$
$a+b+c = 9$
$6 + c = 9$
$c = 3\ cm$
$b = 4\ cm$
$a = 2\ cm$

Shortcut
$\frac{5+6+7}{2} = 9$
$9-5=4 \quad 9-6=3 \quad 9-7=2$
2cm, 3cm, 4cm.

∴ Length of Ropes are respectively 2cm, 3cm, 4cm

⇒ 3 तीन वृत्त दिए गए हैं जिनमें से दो वृत्तों की त्रिज्या $(1+\sqrt{2}\ cm)$ बराबर है और एक वृत्त की त्रिज्या 1cm है find the Perimeter or Area of Untouched Portion.

इस प्रकार के Example में हमेशा Right Angle Isoceles Triangle बनेगा।



Verification :→ $(2+\sqrt{2})^2 + (2+\sqrt{2})^2 = 2(2+\sqrt{2})^2$
$= 2(6+4\sqrt{2})$
$= 4(3+2\sqrt{2})$

or $[2(1+\sqrt{2})]^2 = 4(3+2\sqrt{2})$

$\therefore AB^2 + BC^2 = AC^2$

∴ Perimeter of Untouched Portion :→ ~~3× Length of Arce~~

$= \frac{45^{\circ}}{4\ 180}\times\pi r_1 + \frac{45^{\circ}}{180\ 4}\times\pi r_2 + \frac{90^{\circ}}{180\ 2}\times\pi r_3$

$= \frac{\pi}{4}\times\sqrt{2}+1 + \frac{\pi}{4}\times\sqrt{2}+1 + \frac{\pi}{2}\times 1$

$= \frac{\pi}{2}(\sqrt{2}+1) + \frac{\pi}{2}$

$\frac{\pi}{2}(\sqrt{2}+2)$ ans.

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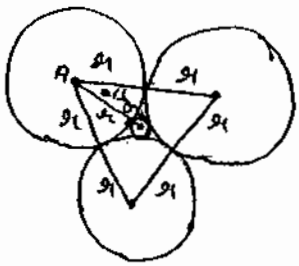
⇒ तीन वृत्त जिनमें से प्रत्येक की त्रिज्या बराबर दो गई है एक दूसरे को Touch करते हैं और इनके Untouched Portion में एक छोटा Circle बनाया गया है जो तीनों को Touch कर रहा है। find the Radius of that Small Circle.

Let Radius of Each Circle = r



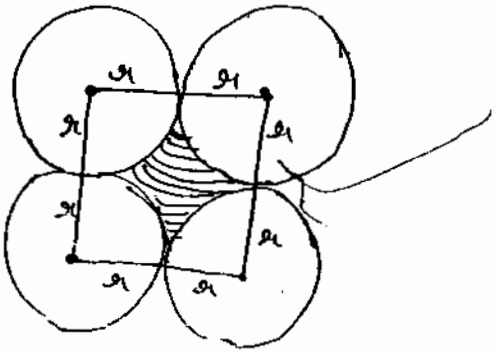
Height = AO = $\frac{\sqrt{3}}{2}(a) \times \frac{2}{3}$

$= \frac{\sqrt{3}}{2} \times 2r \times \frac{2}{3} = \frac{2\sqrt{3}}{3} r$

∴ Radius of Small Circle = $\frac{2\sqrt{3}}{3} r - r = \left(\frac{2\sqrt{3}}{3} - 1\right) r$

$$\boxed{r_{small} = \left(\frac{2\sqrt{3}-3}{3}\right) r}$$

Next Type:→ चार वृत्तों में से प्रत्येक की त्रिज्या बराबर है और एक वृत्त अन्य दो वृत्तों को Touch कर रहा है find the Area of Untouched Portion.



When we coincide the Centre of Each Circle it becomes a Square.

whose each side (a) = $r + r = 2r$

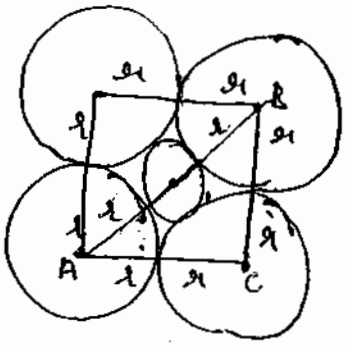
Area of Square = $a^2 = (2r)^2 = 4r^2$

Area of 4 sector = $\frac{4 \times 90}{360} \pi r^2 = \pi r^2$

∴ Area of Untouched Portion = Area of Square − Area of 4 sector

$= 4r^2 - \pi r^2 = (4 - \pi) r^2$

Next Type:→ चार वृत्तों में से प्रत्येक की त्रिज्या बराबर है और एक वृत्त अन्य दो वृत्तों का Touch कर रहा है इन सभी के Untouched portion में एक और Circle बनाया गया है find the Area of that Small Circle.



Side of Each Square = $2r = a$

AB Becomes a Diagnol

∴ Length of Diagnol = $\sqrt{2}\, a$

$= \sqrt{2} \times 2r = 2\sqrt{2}\, r$

Diameter of Small Circle = $2\sqrt{2}\, r - 2r$

$= 2r(\sqrt{2} - 1)$

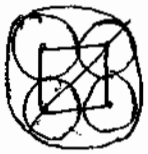
$AB = \sqrt{AC^2 + CB^2}$

$\sqrt{2r^2 + 2r^2}$

$= 2\sqrt{2}\, r$

$$\boxed{\therefore \text{Radius of Small Circle} = r(\sqrt{2} - 1)}$$

और कहे की इन सभी को Touch करते हुए एक बड़ा Circle बनाया गया तो उस Big Circle का Area क्या होगा।



Radius of Big Circle = Length of Diagnol + $2r$

$= 2\sqrt{2}\, r + 2r$

$= 2\sqrt{2}\, r + 2r$

$= 2r(\sqrt{2} + 1)$

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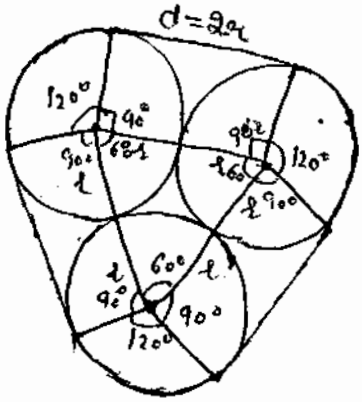
⇒ 3 वृत्त दिए गए हैं जिसमें से प्रत्येक की त्रिज्या बराबर है। इन तीनों वृत्तों को Tightly एक Pully से बांध दिया गया है। find the length of Pully.



Detail :→ जब तीनों Circle के centre को मिलाते हैं तो एक Equilateral Triangle बनता है, इन सभी को मिलाकर 3 Right angled Triangle बनते हैं जिसमें से प्रत्येक की length d = 2r है।

∴ Length of Belt = 3×d + 3× Arc length

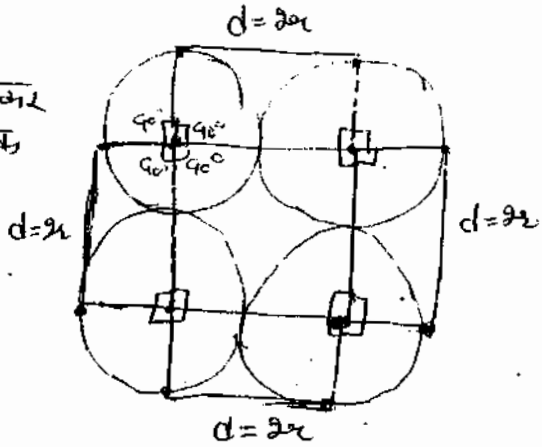
$= 3\times d + 3\times \frac{120}{360}\pi r$

$= 3d + 2\pi r$

$= 3d + d\pi = d(\pi+3)$

∴ $\boxed{\text{Length of Pully} = (\pi+3)d}$

⇒ 4 वृत्त दिए गए हैं जिसमें से प्रत्येक की त्रिज्या बराबर है इन तीन वृत्तों को Tightly एक Pully से बांध दिया गया है find the length of the Belt.



Detail जब 4 Circle के centre को मिलाते हैं तो एक Square बनता है, इन सभी को मिलाकर 4 Right Angle Triangle बनते हैं जिसमें से प्रत्येक की length d = 2r है।

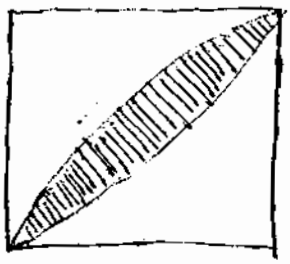
∴ Length of Belt = 4×d + 4× length of Arc.

$= 4d + 4\times \frac{90}{360}\times \pi\times r$

$4d + 2r\pi = 4d + d\pi$

$= d(4+\pi)$

⇒ एक Square के प्रत्येक भुजा की लंबाई 10 cm है और इसके अन्दर दो Sector बनाए गये हैं छायांकित भाग का क्षेत्रफल ज्ञात कीजिए। (Using π = 3.14)



Explaination

Area = Square − Sector

→ Area = Square − Sector

Area of I + II = Square − Sector + Square − Sector

= 2 square − 2 Sector

Area of III = Area of square − (2 square − 2 Sector)

= 2 × Area of Sector − Area of Square

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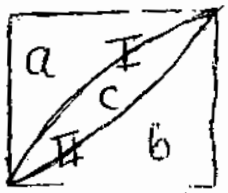
∴ Area of Shadow Portion = 2X Area of Sector − Area of square

Short :→ Area of Sector I = a+c
Area of sector II = b+c
Area of Square = a+b+c



Area of c = [a+c+b+c − (a+b+c)]
= c

∴ Area of c = 2XArea of sector − Area of square.

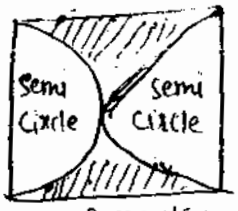
Next Type :→ e.g → एक Square की प्रत्येक भुजा 10cm है find the Area of Shadow Portion (π=3.14)

Area of c = $\cancel{2}\times\frac{\cancel{90}}{\cancel{360}_{\cancel{4}\,2}}\times\frac{\cancel{314}^{157}}{\cancel{100}}\times\cancel{10\times10}$ − 10X10

= 157 − 100 = 57 cm²

Next Type :→

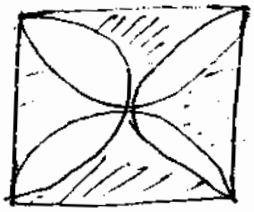
Explanation :→



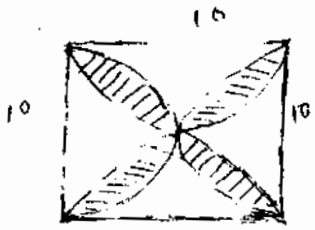
I

(Shadow) Area = Area of sq − Area of Circle

Area of Shadow = 2XArea of square − 2Area of Circle



II



III a

Total Area of Shadow Portion In II = 2XArea of square − 2 Circle (Area

Area of Shadow portion (III) = Area of square − (2XArea of square − 2Ar of Cir

= 2X Area of Circle − Area of square

e.g → एक Square की प्रत्येक भुजा 10cm है find the Area of Shadow portion Diagram (III) (3.14 = π)

Using Area = 2XArea of Circle − Area of Square

$= 2\times\pi r^2 - a^2$

$= 2\times\frac{314}{100}\times(5)^2 - (10)^2$ [∵ because = $r = \frac{a}{2} = \frac{10}{2} = 5$]

$\cancel{2}\times\frac{\cancel{314}^{157}}{\cancel{100}_{\cancel{4}\,2}}\times\cancel{25} - 100$

157 − 100 = 57cm²

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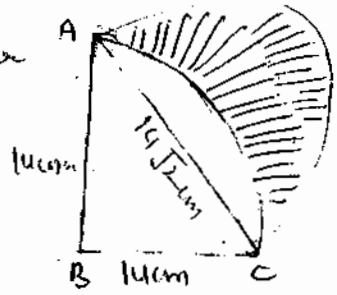
Next Type:→

• एक Sector दिया गया है जिसकी Radius 14 cm है और एक Triangle दिया गया है जिसकी खड़ी भुजा को Diameter मानकर एक Semi-circle Drawn किया गया है find the Area of Shadow portion.



In ΔABC

$AC = \sqrt{14^2+14^2} = 14\sqrt{2}$ cm.

Area of Sector $= \frac{90}{360} \times \frac{22}{7} \times 14 \times 14$

$= 154 \text{ cm}^2$.

Area of semicircle $= \frac{\pi r^2}{2} = \frac{22}{7} \times \frac{1}{2} \times 7\sqrt{2} \times 7\sqrt{2}$ $\left[r = \frac{D}{2} = \frac{14\sqrt{2}}{2} = 7\sqrt{2}\right]$

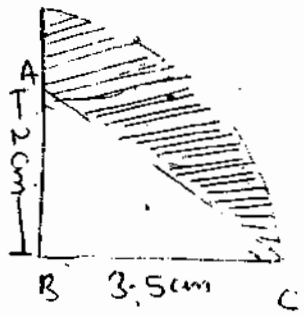
$= 154 \text{ cm}^2$

Area of $\Delta ABC = \frac{1}{2} \times 14 \times 14 = 98 \text{ cm}^2$.

∴ Area of Shadow Portion = Area of semi-circle − (Area of sector − Area of ΔABC)

$= 154 - (154 - 98) = 154 - 154 + 98 = 98 \text{ cm}^2$

Next Type:→ एक sector की Radius 3.5 cm है और इसके अन्दर एक Δ दिया गया है जिसकी Height 3 cm है और Base वही है जो Sector की Radius है, find the Area of Shadow portion.



Area of Sector $= \frac{90^\circ}{360} \times \frac{22}{7} \times 3.5 \times 3.5$

$= \frac{90}{360} \times \frac{22}{7} \times \frac{35}{10} \times \frac{35}{10} = \frac{77}{8} \text{ cm}^2$

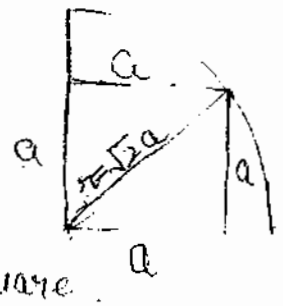
Area of $\Delta = \frac{1}{2} \times \frac{35}{10} \times 3 = \frac{7}{2} \text{ cm}^2$

Area of Shadow portion = Area of Sector − Area of ΔABC

$= \frac{77}{8} - \frac{7}{2} = \frac{77-28}{8} = \frac{49}{8}$

$= 6.12 \text{ cm}^2$

⇒ एक Sector दिया गया है और इसके अन्दर एक वृत्त बनाया गया है जिसकी प्रत्येक भुजा a है find the Ratio of Radius of sector and Side of square.



Radius of sector $= \sqrt{a^2+a^2} = \sqrt{2}a$

∴ R_{sector} : Side square $= \sqrt{2}a : a$

$= \sqrt{2} : 1$

If the Radius of sector is 10 cm find the Area of square.

$a^2 + a^2 = 10^2$

$a^2 + a^2 = 100$

$2a^2 = 100$ $\quad a^2 = 50 \text{ cm}^2$

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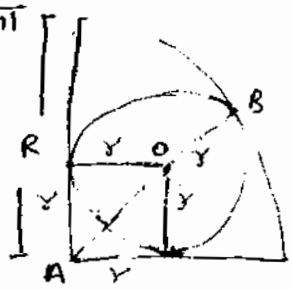
एक Sector के अन्दर एक Circle बना गया है जिसका Radius 8 है find the Radius of Big Sector R.

Explain:→ Sector की जो Radius है वो Circle के लिए Tangent का कार्य कर रही है और इससे एक वृत्त बन रहा है।



$$AO = \sqrt{r^2+r^2} = \sqrt{2r^2} = \sqrt{2}\,r$$

$$R = AO + OB = \sqrt{2}\,r + r = (\sqrt{2}+1)\,r$$

{∴ Radius of Sector $= (\sqrt{2}+1)\,r$ }

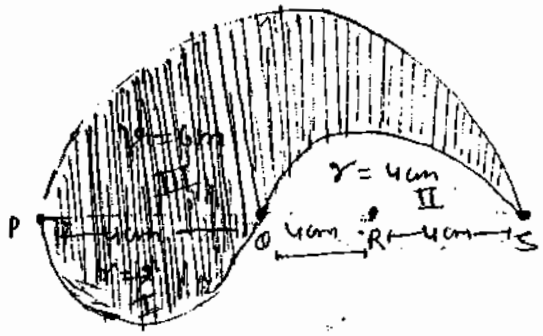
Next Type:→ Given

$PQ = QR = RS = 4cm$

find the Area and Perimeter of the Shadow portion

Ans:→ तीन Circle दिए गये हैं जिसमें

Radius of I $r = 2cm$.
Radius of II $r = 4cm$
Radius of III $r = 6cm$.



∴ Area of Shadow portion $= I + III - II$

$$= \frac{\pi}{2}(6)^2 + \frac{\pi}{2}(2)^2 - \frac{\pi}{2}(4)^2$$

$$= \frac{\pi}{2}(6^2+2^2-4^2) = \frac{\pi}{2}(36+4-16)$$

$$= \frac{\pi}{2} \times 24 = 12\pi\ cm^2.$$

Perimeter of the Shadow portion:→ Perimeter of I + II + III

$$= \pi r_1 + \pi r_2 + \pi r_3$$

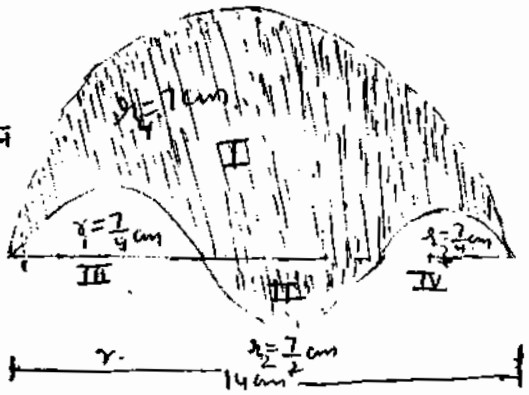
$$= \pi \times 2 + \pi \times 4 + \pi \times 6 = 12\pi\ cm.$$

Next Type:→

Given $r_1 = r_3 = \frac{7}{4}cm$

$r_2 = \frac{7}{2}cm.$

find the Area of Shadow Portion



Area of Shadow Portion = Area of I + II − (III + IV)
जो Circle बन गया

$$= \frac{1}{2}\pi \times 7^2 + \frac{1}{2}\pi \times \left(\frac{7}{2}\right)^2 - \pi \times \left(\frac{7}{4}\right)^2$$

$$= \frac{\pi \times 49}{2}\left[1 + \frac{1}{4} - \frac{1}{8}\right]$$

$$= \frac{22}{7} \times \frac{49}{2} \times \frac{9}{8} = \frac{693}{8} = 86.62\ cm^2.$$

Perimeter:→ $\pi r_2 + \pi r_4 + \pi r_1 + \pi r_3$

$$= \pi r_4 + \pi r_2 + 2\pi r$$

$$= \pi\left(7 + \frac{7}{2} + \frac{7}{2}\right) = \frac{22}{7} \times \frac{28}{2} = 44cm.$$

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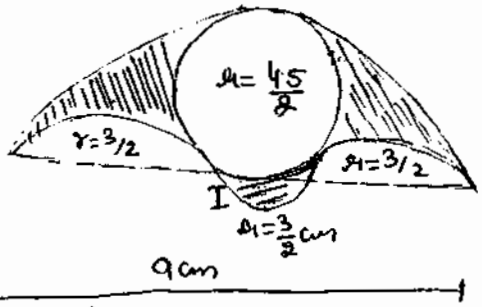
Given Radius of Circle $= \frac{4.5}{2}$ cm.

Radius of All Small semi Circle $(r) = \frac{3}{2}$ cm.

Find the Area of Shadow portion

Radius of Big semi Circle $= \frac{9}{2} = 4.5$ cm.



∴ Area of Shadow portion :→

Area of Big semi Circle + Area of I semicircle

− Area of other two semi Circle − Area of Circle.

$$= \frac{1}{2}\times\frac{22}{7}\times\frac{45}{10}\times\frac{45}{10} + \frac{1}{2}\times\frac{22}{7}\times\frac{3}{2}\times\frac{3}{2} - \frac{1}{2}\times\frac{22}{7}\times\frac{3}{2}\times\frac{3}{2} - \frac{1}{2}\times\frac{22}{7}\times\frac{45}{20}\times\frac{45}{20}$$

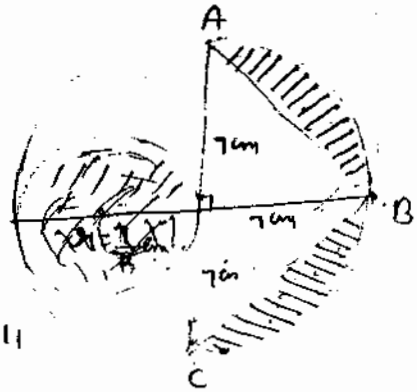
$$\frac{1}{2}\times\frac{22}{7}\times\frac{3}{2}\times\frac{3}{2}\left[9+1-2-\frac{9}{2}\right] = \frac{1}{2}\times\frac{22}{7}\times\frac{9}{4}\times\frac{7}{2} = \frac{99}{8} = 12.375 \text{ cm}^2.$$

Next Type:→

Radius of Big Circle = 7cm.

Radius of Small Circle $= \frac{7}{2}$ cm

Find the Area of Shadow Portion



Area of Shadow portion:→

(Area of Semi-Circle − Area of Triangle) + Area of Circle small

$$= \left(\frac{\pi}{2}\times7\times7 - \frac{1}{2}\times14\times7\right) + \pi\times\frac{7}{2}\times\frac{7}{2}$$

$$\left(\frac{22}{7}\times\frac{1}{2}\times7\times7 - \frac{1}{2}\times14\times7\right) + \frac{22}{7}\times\frac{7}{2}\times\frac{7}{2}$$

$$= (77-49) + \frac{77}{2} = 28 + \frac{77}{2} = \frac{56+77}{2} = \frac{133}{2} = 66.5 \text{ cm}^2$$

OR

Area of Shadow Portion :→ 2 × Area of Sector − Area of ΔABC + Area of Circle

$$= 2\times\frac{90}{360}\times\frac{22}{7}\times7\times7 - \frac{1}{2}\times14\times7 + \frac{22}{7}\times\frac{7}{2}\times\frac{7}{2}$$

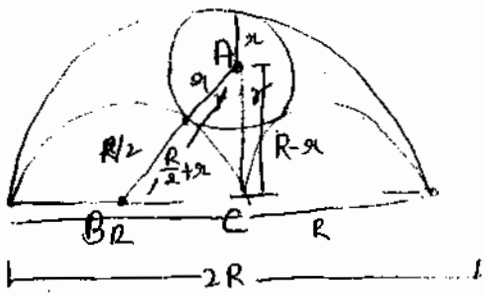
$$77 - 49 + \frac{77}{2} = \frac{133}{2} = 66.5 \text{ cm}^2.$$

Next Type:→

Given Radius of Big Semi-circle = R

Radius of Circle = r

Find the Radius Ration R : r.



In Δ Triangle ABC

$$AB^2 = AC^2 + BC^2$$

$$\left(\frac{R}{2}+r\right)^2 = (R-r)^2 + \left(\frac{R}{2}\right)^2$$

$$\frac{R^2}{4} + r^2 + Rr = R^2 + r^2 - 2Rr + \frac{R^2}{4}$$

$$R^2 = 3Rr \qquad \left\{r = \frac{R}{3},\ R = 3r\right\} \qquad R : r = 3 : 1 \text{ Ans.}$$

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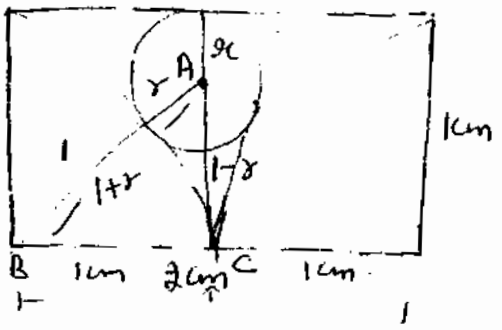
Given:→ Length And Breadth of Rectangle = 2cm, 1cm

find r of Small Circle



In Right Angle Triangle ABC

$$AB^2 = AC^2 + BC^2$$

$$(1+r)^2 = (1-r)^2 + 1^2$$

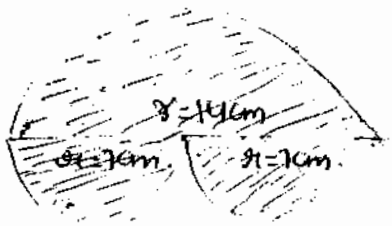
$$\cancel{1}+\cancel{r^2}+2r = \cancel{1}+\cancel{r^2}-2r+1$$

$$4r = 1$$

$$r = \frac{1}{4} \text{ (Radius of Small Circle.}$$

Next Type:→ Given Radius of Big Semi-Circle $R = 14cm$.
Radius of Small Circle $r = 7cm$.

find Area of Shadow Portion.



Area of Shadow Portion $= \frac{\pi}{2}(14)^2 + \pi(7)^2$

$$= \frac{22}{7} \times \frac{1}{2} \times 14 \times 14 + \frac{22}{7} \times 7 \times 7$$

$$308 + 154 = 462 cm^2$$

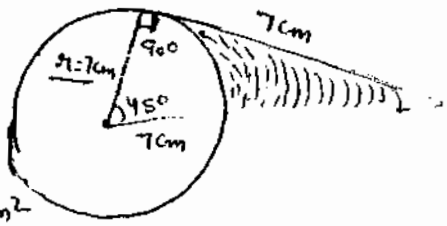
Next Type:→ Angle of Sector $= 30°$
Radius of Small Sector $= 7cm$
Radius of Big Sector $= 21cm$.

find the Area of Shadow portion.

Area of Shadow portion $= \frac{30}{360} \times \frac{22}{7} \times 21 \times 21 - \frac{30}{360} \times \frac{22}{7} \times 7 \times 7$

$$= \frac{11 \times 21}{2} - \frac{11 \times 7}{6} = \frac{11}{2}\left(21 - \frac{7}{3}\right) = \frac{11}{2} \times \frac{56}{3} = \frac{308}{3} = 102.66 cm^2$$

Next Type:→ Radius of Circle = 7cm. find the Area of Shadow portion.



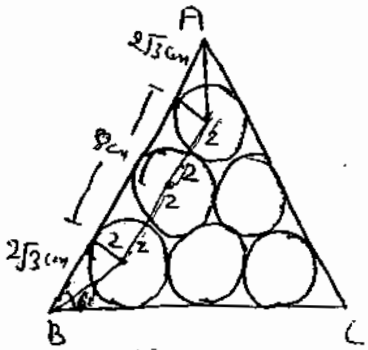
Area of Shadow Portion

$$= \text{Area of Triangle} - \text{Area of Sector}$$

$$= \frac{1}{2} \times 7 \times 7 - \frac{45}{360} \times \frac{22}{7} \times 7 \times 7$$

$$= \frac{49}{2} - \frac{77}{4} = \frac{98-77}{4} = \frac{21}{4} = 5.25 cm^2$$

Next Type:→ Radius of Each Small Circle = 2cm.
find the Radius of Big Circle. (Side of Δ)



ΔABC is a ~~Right Angled~~ Equilateral Triangle.

Now $\tan\theta = \frac{P}{B} = \frac{2}{B}$

$\tan 30° = \frac{2}{B}$ $\quad B = 2\sqrt{3}$ cm.

∴ R Side of Equilateral Triangle

$$8 + 2\sqrt{3} + 2\sqrt{3} = 8 + 4\sqrt{3} cm.$$

{Using Side of Equilateral Δ = $4r[2+\sqrt{3}] = 4r[2+\sqrt{3}]$}

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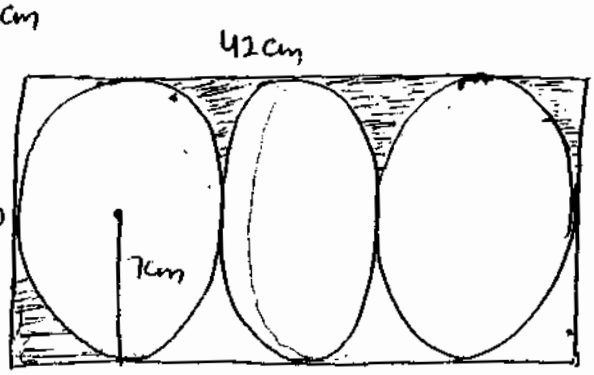
Perimeter $= 3a = 3 \times 4(2+\sqrt{3})$
$= 12(2+\sqrt{3})$ cm.

Next Type:→ Length of ~~Equilateral Triangle~~ Rightangle is 42cm and Breadth of ~~Equilateral Triangle~~ Rightangle is 14cm find the Area of Shadow portion



Area of Shadow portion

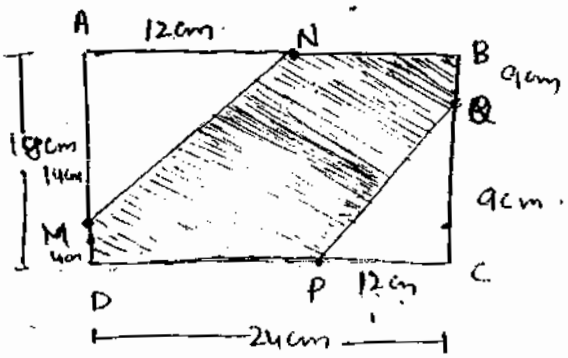
$= \frac{1}{2}$(Area of ~~Rightangle~~ Rect − 3× Area of ~~Square~~ Circle)

$= \frac{1}{2}\left[42\times14 - 3\times\frac{22}{7}\times7\times7\right]$

$= \frac{1}{2}[588 - 462] = \frac{126}{2} = 63\,cm^2$

Next Type:→

Given AB = 24 cm, BC = 18 cm, MD = 4cm
P और N are the mid point of AB and DC
∴ find the Area of Shadow portion.



Area of $\Delta AMN = \frac{1}{2}\times14\times12 = 84\,cm^2$

Area of $\Delta PCQ = \frac{1}{2}\times12\times9 = 54\,cm^2$

Area of ▭ ABCD $= 24\times18 = 432\,cm^2$

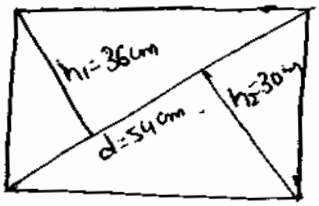
∴ Area of Shadow portion = (Area of ▭) − (Area of ΔAMN + Area of ΔPCQ)
$= 432 - (84+54)$
$= 432 - 138 = 294\,cm^2$

⇒ एक चतुर्भुज के एक विकर्ण की लंबाई 54cm है। Length of ⊥ from the Vertex on the Diagnol are 36cm है, और 30cm है find the Area of Quadrilateral

Area of Quadrilateral $= \frac{1}{2}(h_1+h_2)\times d$

$= \frac{1}{2}(36+30)\times54$

$33\frac{66}{2}\times54 = 1782\,cm^2$.



⇒ एक ~~Rightangle~~ Rectangle (आयत) का क्षेत्रफल $120\,cm^2$. यदि इसकी लंबाई को 5:6 में बढ़ा दिया जाए और इसकी चौड़ाई को 3:2 के अनुपात में कम किया जाए तो find the New Area of Reactangle.

Solution

$l \to 5:6$
$b \to 3:2$
Area $(l\times b) = 15:12$

$15 \Rightarrow 120$
$\therefore 12 \Rightarrow \frac{120}{15}\times12 = 96$

∴ Area of New $= 96\,cm^2$.

OR

Initial: $l, b = 5x, 3y$ | final: $l, b = 6x, 2y$

$5x\times3y = 120$

$xy = \frac{120}{15} = 8$

$\therefore 6x\times2y = 12xy$
$= 12\times8 = 96\,cm^2$

∴ New Area $= 96\,cm^2$

OR

$l = 5:6$ $\quad b = 3:2$ $\quad 6x \quad 2x$

$5x\times3x = 120$

$x^2 = \frac{120}{15} = 8$

$x = 2\sqrt{2}$ cm.

∴ New Length $= 6(2\sqrt{2})$, Breath $= 2(2\sqrt{2})$

$L = 12\sqrt{2}$, $B = 4\sqrt{2}$

Area $= 12\sqrt{2}\times4\sqrt{2}$
$= 96\,cm^2$.

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→ एक आयत की लंबाई और चौड़ाई का अनुपात 3:2 है। आयत की दोनों भुजाओं को दोनों side से एक मीटर घटा दिया जाए तब लंबाई और चौड़ाई का अनुपात 10:7 हो जाता है find the Initial Area

$l : b = 3 : 2$

$l = 3x, \quad b = 2x$

Acc to Condition adding 2 on L and b

New $l = 3x+2 \quad b = 2x+2$

$$\frac{3x+2}{2x+2} = \frac{10}{7}$$

$21x + 14 = 20x + 20$

$x = 6$

$\therefore l = 3 \times 6 = 18\,cm, \quad b = 2 \times 6 = 12\,cm$

Area $= 18 \times 12 = 216\,cm^2$

OR

Initial $l : b = 3 : 2$ [Multiply by 3]

diff

final $l : b = 10 : 7$

3 difference

$l : b = 9 : 6$

$l : b = 10 : 7$

$1 \Rightarrow 2 \qquad 6 \Rightarrow 2 \times 6 = 12$

$\therefore 9 \Rightarrow 18$

$\therefore$ Area $= 18 \times 12 = 216\,cm^2$.

एक आयत की लंबाई और चौड़ाई का अनुपात 3:2 है आयत की दोनों भुजाओं को दोनों side से 5 मीटर घटा दिया जाए तब लंबाई और चौड़ाई का अनुपात 5:3 हो जाता है। find the New Area.

(Each side 5+5 = 10 कम होगी)

$$\frac{3x-10}{2x-10} = \frac{5}{3}$$

$9x - 30 = 10x - 50$

$x = 20$

$\therefore$ Length $= 3x - 10 = 3(20) - 10 = 50\,cm$.

Bredth $= 2x - 10 = 2(20) - 10 = 30\,cm$.

Area $= 50 \times 30 = 1500\,cm^2$.

→ एक आयत का क्षेत्रफल $240\,cm^2$ है Sum of its Diagnol and Length is the five time of its Bredth find the Length of Reactange

Given $l \times b = 240\,cm^2$

$\therefore d + l = 5b$

$d = 5b - l$

$\sqrt{l^2 + b^2} = 5b - l$

$l^2 + b^2 = (5b - l)^2$

$\cancel{l^2} + b^2 = 25b^2 + \cancel{l^2} - 10lb$

$24b^2 = 10lb$

$24b^2 = 10 \times 240$

$b^2 = 10 \times 10$

$b = 10\,cm$.

$\therefore l \times b = 240$

$l = \frac{240}{10} = 24\,cm$.

Length of Reactangle $= 24\,cm$.

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⇒ Area of Reactangle $2250\,m^2$. If the Length of Reactangle is Increased by 5m Then Area Increased By $225\,m^2$ find the Bredth of Reactangle.

$lb = 2250$

$(l+5)b = 2250+225$

$(l+5)b = 2475$

$lb+5b = 2475$

$5b = 2475-2250$

$5b = 225$

$b = 45\,cm$

Bredth of Reactangle = 45cm

$lb = 2250$

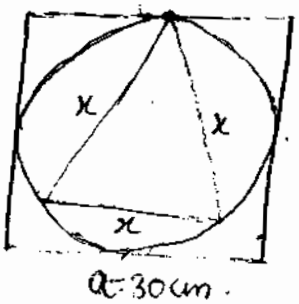
Length is Increased by 5m but base Remain Same. Area Increased by $225\,m^2$

$5 \longrightarrow 225\,m^2$

$1 \longrightarrow \frac{225}{5} = 45\,m$

∴ Base of Reactangle = 45m.

Next Type :→ Side of Big Square is 30cm. find the Area of an equilateral Triangle.



Now Diameter of Circle = Side of Square

$d = 30cm$

$\therefore r = \frac{30}{2} = 15cm.$

Now Circle is a circumcircle for equilateral Triangle.

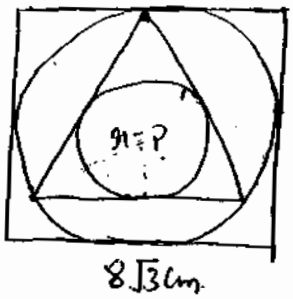
radius of Circum Circle $= \frac{\sqrt{3}}{2}a \times \frac{2}{3}$

$15 = \frac{\sqrt{3}}{2}a \times \frac{2}{3}$

$a = \frac{15\times 3}{\sqrt{3}} = 15\sqrt{3}\,cm.$

∴ Area of $\Delta = \frac{\sqrt{3}}{4}a^2 = \frac{\sqrt{3}}{4}\times 15\sqrt{3}\times 15\sqrt{3} = \frac{675\sqrt{3}}{4}\,cm^2.$

Next Type :→ Side of Square is $8\sqrt{3}$cm. find the Radius of Small Circle.



$a = 8\sqrt{3}\,cm.$

∴ ~~Radius~~ Diameter of Big Circle = Side of Square = $8\sqrt{3}$ cm.

$R = 4\sqrt{3}\,cm.$

$\because \frac{\sqrt{3}}{2}a \times \frac{2}{3} = 4\sqrt{3}$

$a = 12\,cm.$

Radius of Incircle $= \frac{\sqrt{3}}{2}a \times \frac{1}{3} = \frac{\sqrt{3}}{2}\times 12 \times \frac{1}{3} = 2\sqrt{3}\,cm.$

OR

Radius of Circumcircle (R) = $4\sqrt{3}$cm.

Radius of Incircle (r) = $\frac{\text{Radius of Circumcircle}}{2}$

$= \frac{4\sqrt{3}}{2} = 2\sqrt{3}\,cm.$

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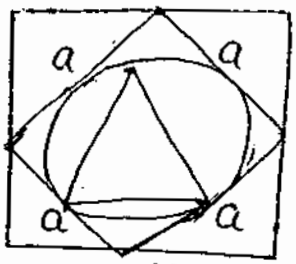
Next :→ Side of big square circumscribed an equilateral Δ.

Ans :→ Side of Big Square = Diagnol of Small Square

$8\sqrt{2} = \sqrt{2}a$

$a = 8\,cm$.

∴ Side of Small Square = 8 cm.



$a = 8\sqrt{2}$ cm

Diameter of Circle = Side of Small ~~Circle~~ Square.

$d = 8$

$\therefore r = \frac{8}{2} = 4\,cm$.

This circle act as a circumcircle for equilateral Δ.

$\therefore \frac{\sqrt{3}}{2}a \times \frac{2}{3} = 4$ [Radius of circumcircle $= \frac{\sqrt{3}}{2}a \times \frac{2}{3}$]

$a = \frac{12}{\sqrt{3}} = 4\sqrt{3}$ cm.

$\therefore$ Area $= \frac{\sqrt{3}}{4} \times 4\sqrt{3} \times 4\sqrt{3} = 12\sqrt{3}\,cm^2$.

Next Type :→ एक आयत की लंबाई उसकी चौड़ाई से 5 मीटर अधिक है। अगर लंबाई को 2 मीटर बढ़ा दिया जाए और चौड़ाई को 1 मीटर कम कर दिया जाए तो Area में कोई change नहीं होता, Find the Original Length and Breadth.

Let Breadth = b ∴ length (l) = b+5

Area (lb) = (b+5)b.

Now Apply Second condition Breadth = b-1 length (l) = b+5+2 = b+7

∴ Area ⇒ $(b+5)b = (b-1)(b+7)$

⇒ $b^2 + 5b = b^2 + 7b - b - 7$

$b = 7\,cm$.

∴ $l = b+5 = 7+5 = 12\,cm$.

Original length = 12cm, Breadth = 7cm.

(2) एक आयत की लंबाई उसकी चौड़ाई से 8 मीटर अधिक है अगर उसकी लंबाई को 7 मीटर बढ़ा दिया जाए और चौड़ाई को 4 मीटर कम कर दिया जाए तो Area में कोई Chang नहीं होता find the original length and Breadth. Also find Area.

Let Breadth = b ∴ length (l) = b+8

Now Apply Second Condition Breadth = b-4, Length = b+8+7 = b+15

Now $b(b+8) = (b-4)(b+15)$

$\cancel{b^2} + 8b = \cancel{b^2} + 11b - 60$

$3b = 60$

$b = 20$

∴ Breadth of Reactangle = 20 cm.

Length of Reactangle = 20+8 = 28cm.

∴ Area = $28 \times 20 = 560\,cm^2$.

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⇒ एक आयत की लंबाई अपनी चौड़ाई का $\frac{3}{2}$ गुना है। अगर इसकी लंबाई को 5cm बढ़ा दिया जाए और चौड़ाई को 5 cm कम कर दिया जाए तो इसका क्षेत्रफल 115 m^2 कम हो जाता है find the Original length and Breadth.

$$l : b = \frac{3}{2} : 1 = 3 : 2$$

$$3x, 2x$$

Now Acc. to Given Condition

$$(3x+5)(2x-5) = 3x \cdot 2x - 115$$

$$\cancel{6x^2} - 5x - 25 = \cancel{6x^2} - 115$$

$$5x = 90$$

$$x = 18$$

$\therefore$ Length $= 3 \times 18 = 54$ cm Breadth $= 2 \times 18 = 36$ cm.

⇒ एक आयत का क्षेत्रफल 720 cm^2 है अगर लंबाई को 4cm बढ़ा दिया जाए और चौड़ाई को 1cm कम कर दिया जाए Then Area 40 cm^2 बढ़ जाता है। find the Difference between 4 time of its breadth and length of Reactangle.

$$lb = 720 cm^2$$

$$(l+4)(b-1) = lb + 40 cm$$

$$\cancel{lb} + 4b - l - 4 = \cancel{lb} + 40$$

$$\boxed{4b - l = 44 cm}$$

⇒ एक आयत के विकर्ण की लंबाई 25cm है और आयत का परिमाप 70cm है find the Area of Reactangle.

$$\sqrt{a^2+b^2} = 25 \quad \therefore a^2+b^2 = 25^2$$

$$a^2+b^2 = 625 cm$$

$$2(a+b) = 70$$

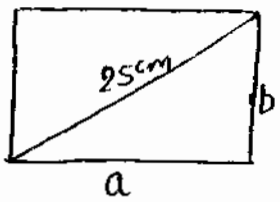
$$a+b = 35 cm$$

Using $(a+b)^2 = a^2+b^2+2ab$

$$\therefore 2ab = (35)^2 - (25)^2$$

$$= 1225 - 625$$

$$ab = 600/2 = 300 cm^2.$$



Shortcut

$$a^2+b^2 = 25^2$$

$$20^2+15^2 = 25^2$$

$$400+225 = 625$$

$\therefore a = 20 \quad b = 15$

$P = 2(20+15) = 70$ [Satisfy]

$\therefore$ Area $= 20 \times 15 = 300 cm^2$

⇒ दो व्यक्ति A और B एक ही स्थान से चलते हैं व्यक्ति A एक आयताकार मैदान को उसके विकर्णों के अनुदिश 52m/min की चाल से इसे 15 sec. में पार कर जाता है भुजा के अनुदिश 68 m/min से उतने ही समय में पार कर जाता है find the Area of the field.

विकर्ण के अनुदिश दूरी = विकर्ण की लंबाई $= \frac{\cancel{52}^{13}}{\cancel{60}_{4}} \times 15$ 19m.

भुजा के अनुदिश दूरी = length + Breadth $= 17 \frac{\cancel{68} \times 15}{\cancel{60}\ 4} = 17m$

$$l^2+b^2 = 13^2 = 169$$

$$l+b = 17$$

$$\therefore 2lb = (17)^2 - 196 = 289 - 169 = 120 \quad lb = \frac{120}{2} = 60 m^2.$$

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⇒ एक आयत की लंबाई और चौड़ाई का अनुपात 5:4 है। एक व्यक्ति 1.8 km/hr की चाल से चलते हुए इस आयताकार मैदान के चारों तरफ 3 चक्कर 18 मिनट में लगाता है find the Area of field. (31)

$l : b = 5 : 4$

$5x, 4x$

$2(5x+4x)\times 3 = 1.8\times\frac{18}{60} = \frac{18}{10}\times\frac{18}{60}\times 1000 = 540$

$54x = 540 \quad x = 10m.$

∴ Length $(l) = 5x = 5\times10 = 50m$

Breadth $(b) = 4x = 4\times10 = 40m$

Area $= l\times b = 50\times40 = 2000m^2$.

⇒ एक आयत के विकर्ण और उसकी चौड़ाई का अनुपात 17:8 है The Length of a Reactangle is 45cm find the Area of Reactangle.

Diagnol : Breadth $= 17:8$

$17x, 8x$

Now $(17x)^2 = (45)^2 + (8x)^2$

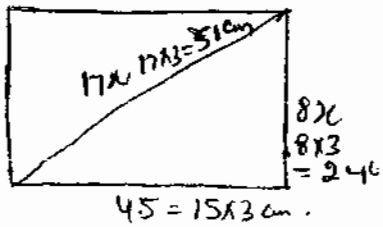
$289x^2 = 2025 + 64x^2$

$225x^2 = 2025$

$x^2 = 9 \quad x = 3cm.$

⇒ ∴ Breadth $= 8\times3 = 24cm$, Length $= 45$

Area $= 45\times24 = 1080cm^2$



Shortcut $17^2 = 15^2 + 8^2$ होता है

$(17\times3)^2 = (15\times3)^2 + (8\times3)^2$

$51^2 = 45^2 + 24^2$

Area $= 45\times24$

$= 1080\ cm^2$

⇒ आयत के एक विकर्ण की लंबाई 65cm है और आयत का परिमाप 170cm है find the Area of Reactangle.

$l^2 + b^2 = 65^2$

$l^2 + b^2 = 4225$

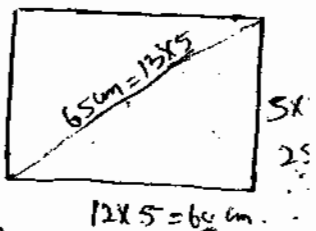
$2(l+b) = 170$

$l + b = 85$

∴ $lb = \frac{(l+b)^2 - (l^2+b^2)}{2} = \frac{(85)^2 - 4225}{2}$

$= \frac{7225-4225}{2} = \frac{3000}{2}$

$= 1500cm^2.$



Shortcut $13^2 = 12^2 + 5^2$

∴ $(13\times5)^2 = (12\times5)^2 + (5\times5)^2$

$65^2 = 60^2 + 25^2$

Area $= 60\times25 = 1500cm^2.$

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⇒ एक आयत का परिमाप और वृत्त के परिमाप की लंबाई 60cm है। आयत के क्षेत्रफल और वृत्त के क्षेत्रफल का Difference 25cm² है find the Length and Breadth of Rectangle.

$2(l+b) = 4a = 60cm$

$\therefore a = \frac{60}{4} = 15cm$. or $l+b = 30cm$.

Side of Square = 15cm.

Area of square = $(15)^2 = 225cm^2$

$a^2 - lb = 25$

$225 - lb = 25$

$lb = 200$

Now $l-b = \sqrt{(l+b)^2 - 4lb} = \sqrt{900-800} = \sqrt{100} = 10$

$l-b = 10$

$l+b = 30$

$2l = 40$ $l = 20cm$, Breadth = 10cm.

⇒ एक आयत का परिमाप और वृत्त का परिमाप बराबर है। आयत की लंबाई 48cm है आयत की चौड़ाई आयत की लंबाई का $\frac{1}{3}$ भाग है find the Area of Square

$2(l+b) = 4a$

$l+b = 2a$

$\therefore 48+16 = 2a$

$a = \frac{64}{2} = 32cm$.

$l = 48cm$

$b = \frac{1}{3}l = \frac{48}{3} = 16cm$.

Area of square = $(32)^2 = 1024cm^2$

⇒ एक आयत का क्षेत्रफल और वर्ग का क्षेत्रफल बराबर है और आयत की लंबाई वर्ग की भुजा से 25% अधिक है तो आयत की चौड़ाई वर्ग की भुजा से कितने प्रतिशत कम होगी।

Let Side of Squar (a) = x

then Length = $\frac{125}{100}x = \frac{5x}{4}$

Now

$lb = a^2$

$\frac{5x}{4} \cdot b = x \cdot x$

$b = \frac{4x}{5}$

$b = \frac{4x}{5} \times 100 = 80\%$

breadth = 100 − 80 = 20% less than side of square.

or

Side of square (a) = x

Length : Side of Square

= 125 : 100

= 5 : 4 = x , 5x , 4x

Now $l \cdot b = a^2$

$5x \cdot b = 4x \cdot 4x$

$b = \frac{16x}{5} = 3.2x$

$a - b = 4x - 3.2x = 0.8x$

% = $\frac{0.8x}{4x} \times 100 = 20\%$

Breadth is less than side of square = 20%

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⇒ एक वर्ग (Square), समबाहु त्रिभुज (Equilateral Triangle) और एक Circle (वृत्त) का क्षेत्रफल बराबर है, find the Ratio of Square of Radius of Circular Side, Triangular Side and Side of Square.

Now Area of Square = Area of Equilateral Triangle = Area of Circle

$$a_1^2 = \frac{\sqrt{3}}{4} a_2^2 = \pi r^2 = K \text{ (Constant)}$$

$$\therefore a_1^2 = K, \quad a_2^2 = \frac{4K}{\sqrt{3}}, \quad r^2 = \frac{K}{\pi} = \frac{7K}{22}$$

Now

$$r^2 : a_2^2 : a_1^2$$

$$= \frac{7K}{22} : \frac{4K}{\sqrt{3}} : K = \frac{7}{22} : \frac{4}{\sqrt{3}} : 1$$

$$= 7\sqrt{3} : 88 : 22\sqrt{3} \text{ Ans.}$$

⇒ एक Circle (वृत्त) और Square (वर्ग) का Area बराबर है find the Ratio of Diameter of Circle and Side of Square.

Now Area of Circle = Area of Square.

$$\pi r^2 = a^2$$

$$\pi \left(\frac{d}{2}\right)^2 = a^2$$

$$\pi d^2 = 4a^2 = K$$

$$d^2 = \frac{K}{\pi} = \frac{7K}{22}, \quad a^2 = \frac{K}{4}$$

$$d^2 : a^2 = \frac{7K}{22} : \frac{K}{4} = \frac{7}{22} : \frac{1}{4}$$

$$28 : 22$$

$$d : a = \sqrt{28} : \sqrt{22}$$

$$= 2\sqrt{7} : \sqrt{22}$$

$$= 2\sqrt{7} : \sqrt{2}\sqrt{11}$$

$$\sqrt{14} : \sqrt{11}$$

Or

$$\pi r^2 = a^2$$

$$\frac{22}{7} \times r^2 = a^2$$

$$\frac{r^2}{a^2} = \frac{7}{22}$$

$$\frac{r}{a} = \frac{\sqrt{7}}{\sqrt{22}}$$

$$\frac{2 \times r}{a} = \frac{2 \times \sqrt{7}}{\sqrt{22}}$$

$$\frac{d}{a} = \frac{2\sqrt{7}}{\sqrt{22}}$$

$$d : a = 2\sqrt{7} : \sqrt{2}\sqrt{11}$$

$$\sqrt{14} : \sqrt{11}$$

⇒ एक आयताकार खेत की लंबाई और चौड़ाई क्रमशः 45m और 25m है इसके बाहर चारों तरफ से 2.5m चौड़ा रास्ता है find the Area of the Road. Also find the Perimeter of Road.

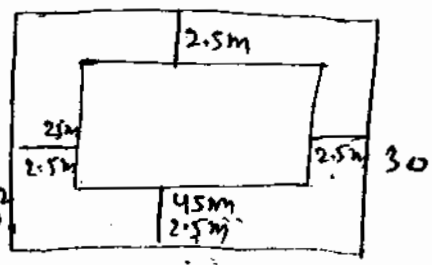
Length of field = 45m, Breadth of field = 25m.

Area of field = $45 \times 25 = 1125 m^2$.

Now length of field + Road = $45 + 2.5 + 2.5 = 50m$.

Breadth of field + Road = $25 + 2.5 + 2.5 = 30m$

Area of field + Road = $50 \times 30 = 1500 m^2$



Area of Road = $1500 - 1125 = 375 m^2$ Ans.

Perimeter of Road = $2(45+25) + 2(50+30)$

$140 + 160 = 300m$.

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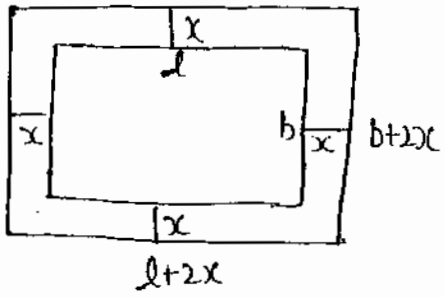
Shortcut (Using formula) [जब Road field के बाहर बनाया जाता है]



Area of Road $= (l+2x)(b+2x) - lb$

$= \cancel{lb} + 2x(l+b) + 4x^2 - \cancel{lb}$

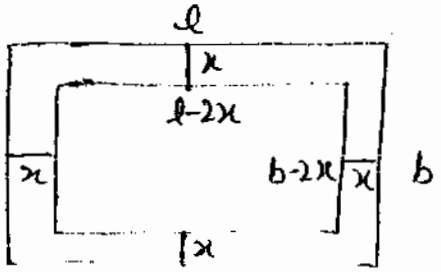
$= 2x(l+b+2x)$

e.g → $l = 45m$, $b = 25m$, x (Thickness) $= 2.5m$.

then Area of Road $= 2 \times 2.5\,(45+25+2\times 2.5)$

$= 5(45+25+5) = 5\times 75 = 375\,m^2$.

⇒ Area of Road [जब Road field के अन्दर चारों तरफ बनाया जाता है]

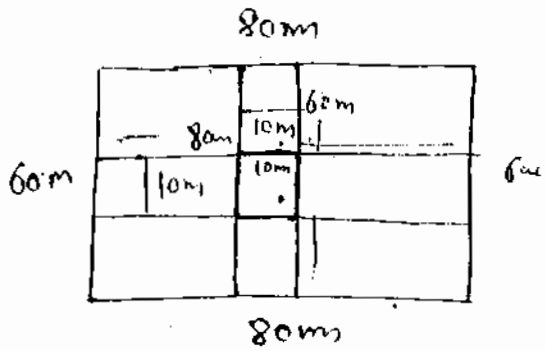


Area of Road $= lb - (l-2x)(b-2x)$

$= \cancel{lb} - \cancel{lb} + 2x(l+b) - 4x^2$

$= 2x(l+b-2x)$

⇒ एक आयताकार खेत की लंबाई और चौड़ाई क्रमशः 80m और 60m है। इसके अन्दर बीचों बीच 10m चौड़ा दो रास्ते बनाए जाते हैं जिसमें से एक लंबाई के और दूसरा चौड़ाई के parallel है find the Area of Path



Area of I path $= 80\times 10 = 800\,m^2$

Area of II path $= 60\times 10 = 600\,m^2$

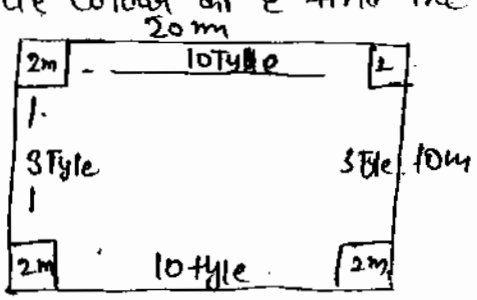
∴ Area of Both Path $= 800 + 600$

$= 1400\,m^2$.

But Blue Lined Portion come 2 times whose Area $= 10\times 10 = 100\,m^2$,

∴ Area of Path $= 1400 - 100 = 1300\,m^2$.

⇒ एक आयताकार कमरे की लंबाई और चौड़ाई क्रमशः 20cm और 10cm है। कमरे के फर्श पर 3 प्रकार की वर्गाकार Tyle लगाई जाती है जिसमें से प्रत्येक की भुजा 2m है। फर्श के चारों तरफ की पहली लाईन की पहली पंक्ति की Tyle काले रंग की है, शेष Tyle का $\frac{1}{3}$ भाग सफेद Colour का है और शेष Blue Colour की है find the No. of Blue Tyle.



Total No. of Tyle $= \dfrac{\text{Area of Room}}{\text{Area of 1 Tyle}}$

$= \dfrac{20\times 10}{2\times 2} = 50$

No. of Black Tyle $= 10+10+3+3$

$= 26$ Tyle.

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Remaining Tyle = 50 - 26 = 24

White Type = $\frac{24}{3} = 8$

∴ Blue Tyle = 24 - 8 = 16

⇒ एक आयताकार कमरे की लंबाई और चौड़ाई क्रमशः 30m और 20m है। कमरे के फर्श पर ~~30m और~~ 12cm लंबा और 10cm चौड़ा एक Tyle लगाया जाता है। The cost of 1000 Tyle is 40₹. find the Total cost.

$$\text{No. of Tyles} = \frac{\text{Area of Room}}{\text{Area of 1 Tyle}} = \frac{30 \times 100 \times 20 \times 100}{12 \times 10} = 50000 \text{ Tyle}$$

1000 ⟶ 40₹

50000 ⟶ $\frac{40}{1000} \times 50000 = 2000$₹ Ans.

⇒ If the Area of Square = Area of Reactangle, then Perimeter of Reactangle > Perimeter of Square.

e.g. Let Area of square (a^2) = Area of Reactangle (lb) = 100

∴ $a^2 = 100$ $a = 10m$.

Perimeter = 4×10 = 40m.

Now $lb = 100$

$20 \times 5 = 100$, $50 \times 2 = 100$, $25 \times 4 = 100$

Perimeter = 2(20+5) P = 2(50+2) P = 2(25+4)

50 104 58

∴ Perimeter of Reactangle > Perimeter of Square.

⇒ If the Perimeter of Square = Perimeter of Reactangle then Area of Square > Area of Reactangle.

e.g. Perimeter of Square = Perimeter of Reactangle

$4a = 2(l+b) = 100$

$4a = 100$ $a = 25$ then Area = 25×25 = 625 m²

$2(l+b) = 100$ $l+b = 50$

$30 + 20 = 50$, $35 + 15 = 50$

Area = 30×20 = 600 m² Area = 35×15 = 525 m²

∴ Area of Square > Area of Reactangle.

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⇒ एक आयताकार खेत की लंबाई और चौड़ाई क्रमशः 38m और 32m है इसके अन्दर चारों ओर एक समान चौड़ाई का रास्ता बनाया जाता है जिसका क्षेत्रफल $600 m^2$ है, तो रास्ते की चौड़ाई क्या होगी।

Using Area of Road = $2x(l+b-2x)$

[x = रास्ते की चौड़ाई]

$600 = 2x(38+32-2x)$

$300 \; \cancel{600} = \cancel{2}x(70-2x)$

$300 = 70x - 2x^2$

$2x^2 - 70x + 300 = 0$

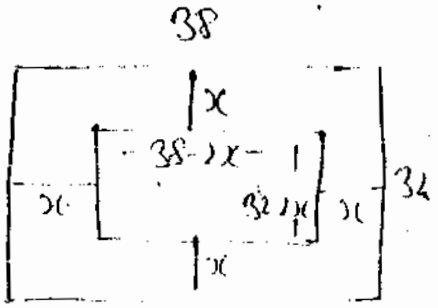
$x^2 - 35x + 150 = 0$

$(x-30)(x-5) = 0$

$x \neq 30$ or $x = 5$

∴ Breadth of Path = 5 m.

Detail



Area of Road = $[(38\times32) - (38-2x)(32-2x)]$

$600 = 38\times32 - \cancel{38\times32} + 76x + 64x - 4x^2$

$600 = 140x - 4x^2$

$x^2 - 35x + 150 = 0$

$x = 5, \; x \neq 30$

∴ Breadth of Path = 5m.

⇒ एक आयत की लंबाई को 5cm कम कर दिया जाए और उसकी चौड़ाई को 3cm बढ़ा दिया जाए Then Area Decrease by $9 cm^2$. और अगर लंबाई को 3cm बढ़ा दिया जाए और चौड़ाई को भी 2cm बढ़ा दिया जाए Then Area Increase by $67 cm^2$ find the Initial Length and Breadth.

Now ~~l = 5cm, b =~~

$lb - (l-5)(b+3) = 9$

$lb - lb - 3l + 5b + 15 = 9$

$3l - 5b = 6$ — (I)

$(l+3)(b+2) - lb = 67$

$lb + 2l + 3b + 6 - lb = 67$

$2l + 3b = 61$ — II

Multiplying Equation I by 3 and II by 5 and adding them

$9l - 15b = 18$

$10l + 15b = 305$

$19l = 323$

$l = \frac{323}{19} = 17 cm$, ∴ $b = 2\times17 + 3b = 61$

$3b = 27$ $b = 9cm$.

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Shortcut :→ from options.

(i) 15, 10 , (ii) 18, 8 (iii) 17, 9 (iv) 21, 7

$l = 17m$ $b = 9m$

Area $= 17 \times 9 = 153 m^2$.

Now $l - 5 = 17 - 5 = 12m$, $b + 3 = 9 + 3 = 12m$ [Apply Condition]

Area $= 12 \times 12 = 144 m^2$.

Area Difference $= 153 - 144 = 9 m^2$

Satisfy Condition.

⇒ The Largest Angle of a Polygen is 12^0 more than the Average of all the Angle of Polygen. If the Largest Angle is 120^0 find No. of Side.

Largest Angle $= 120^0$

Average Sum $= \frac{(n-2) \times 180}{n}$ where n = No. of sides.

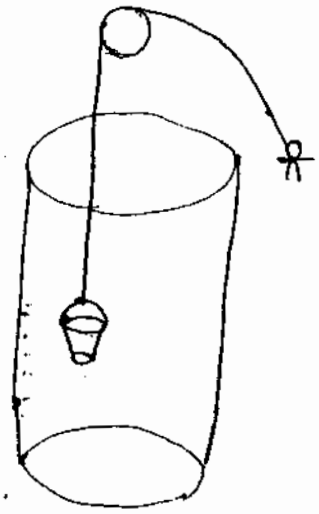
Now $\frac{(n-2) \times 180}{n} + 12 = 120^0$

$\frac{(n-2) \times \cancel{180}^{5}}{n} = \cancel{108}^{3}0$ ⇒ $5n - 10 = 3n$

$2n = 10$

$n = 5$

Next :→ length of Rope OR कुंए की गहराई



Length of Rope $= 2\pi r \times n$
where n = No. of Rotation

e.g. If the Diameter of the pully is 10.5cm and कुंए की गहराई 24.75 m है find the No. of Rotation.

Length of Rope $= 2\pi r \times n$

$D = 10.5 cm = \frac{105}{10 \times 100} m$,

$r = \frac{105}{10 \times 100 \times 2} m$

$2 \times \frac{\cancel{22}^{11}}{\cancel{7}} \times \frac{\cancel{105}^{\cancel{15}\,3}}{\cancel{10}_{2} \times \cancel{100} \times 2} \times n = \frac{2475}{\cancel{100}}$

$33 \times n = 2475$

$n = \frac{\cancel{2475}^{225}}{\cancel{33}_{3}} = 75$

No. of Rotation $= 75$

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AREA AND VOLUME [3D]

(i) Cuboid (घनाभ) :→

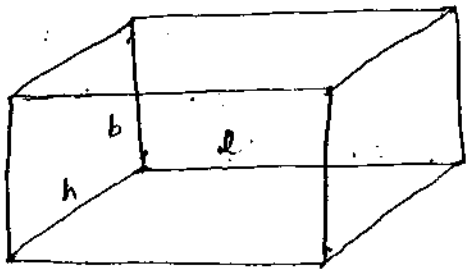
→ Lateral Surface Area (किनारे-किनारे का क्षेत्रफल)

$= 2 \cdot l \cdot h + 2 \cdot b \cdot h$

$\boxed{L.S.A = 2h(l+b)}$

$= 2 \times \text{Height} \times (l+b)$

$\boxed{L.S.A = \text{Perimeter} \times \text{Height}}$



→ Total surface Area (संपूर्ण पृष्ठ का क्षेत्रफल)

$= 2lb + 2bh + 2hl$

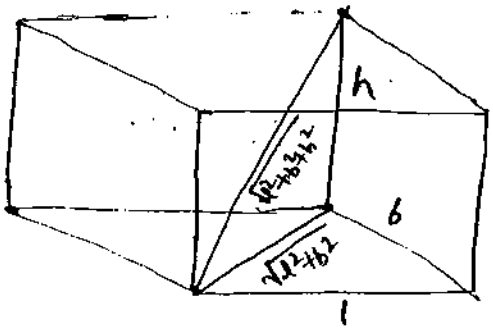
$\boxed{T.S.A = 2(lb + bh + hl)}$

→ Volume = Length × Breadth × Height $= lbh$

= Area of Base × Height

Length of Diagnol (विकर्ण)

$= \sqrt{l^2 + b^2 + h^2}$

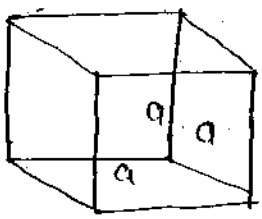


(ii) Cube (घन) :→

Lateral Surface Area (वक्र पृष्ठ का क्षेत्रफल)

$= a^2 + a^2 + a^2 + a^2 = 4a^2$

= Perimeter of Base × Height



Total Surface Area (संपूर्ण पृष्ठ का क्षेत्रफल)

$= 6a^2$

$\boxed{\text{Volume} = a^3}$

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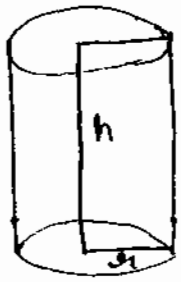
Length of Diagnol = $\sqrt{a^2+a^2+a^2} = \sqrt{3}\,a$

(iii) CYLINDER :→ (बेलन)

Lateral Surface Area (वक्र पृष्ठ का क्षेत्रफल) = $2\pi rh$

Total Surface Area (संपूर्ण पृष्ठ का क्षेत्रफल) = $2\pi rh + 2\pi r^2$
$= 2\pi r(r+h)$

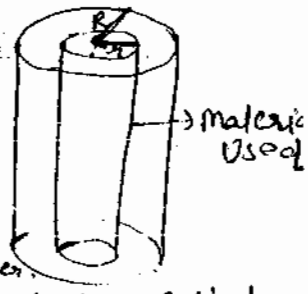
Volume = $\pi r^2 h$



(iv) Volume of Material Used in Hollow Cylinder :→

Volume = $\pi R^2 h - \pi r^2 h$
$= \pi h(R^2 - r^2)$

R = Radius of Outer Cylinder , r = Radius of Inside Cylinder



Hollow Cylinder

(IV) CONE (शंकु) :→

Slant Height (तिर्यक ऊँचाई) = $\sqrt{r^2+h^2}$

Lateral Surface Area (वक्र पृष्ठ का क्षेत्रफल) = $\pi r l$

Total Surface Area (संपूर्ण पृष्ठ का क्षेत्रफल) = $\pi r l + \pi r^2 = \pi r(r+l)$

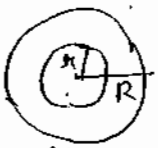
Volume = $\frac{1}{3}\pi r^2 h$

(V) SPHERE (गोला) :→

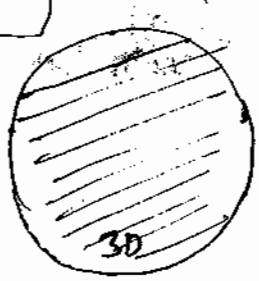
Lateral Surface Area = Total Surface Area = $4\pi r^2$

Volume = $\frac{4}{3}\pi r^3$

Volume of Material USED = $\frac{4}{3}\pi(R^3 - r^3)$



Hollow



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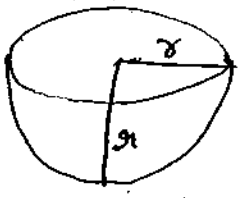
(vi) HEMI-SPHERE [अर्ध-गोला] :→



$\boxed{\text{Lateral Surface Area} = 2\pi r^2}$

$\boxed{\text{Total surface Area} = 2\pi r^2 + \pi r^2 = 3\pi r^2}$

$\boxed{\text{Volume} = \frac{2}{3}\pi r^3}$

Numerical :→

→ एक दीवार की लंबाई, चौड़ाई और ऊंचाई क्रमशः 6, 5 और 3 cm है find Total Surface Area, Volume and Length of Diagnol.

Total surface Area $= 2(lb + bh + hl)$

$= 2(6\times5 + 5\times3 + 6\times3) = 2(63) = 126\ cm^2$

Volume $= l \times b \times h$

$= 6\times5\times3 = 90\ cm^3$

Length of Diagnol $= \sqrt{l^2+b^2+h^2} = \sqrt{6^2+5^2+3^2} = \sqrt{36+25+9}$

$= \sqrt{70}\ cm$

→ एक घन की भुजा की लंबाई 8m है find Volume and T.S.Area.

Volume $= a^3 = 8^3 = 512\ cm^3$

Total Surface Area $= 6\times a^2 = 6\times8^2 = 384\ m^2$.

→ एक ठोस बेलन की त्रिज्या 14cm है और ऊंचाई 35 cm है find Volume, L.S.A and Total Surface Area.

Volume $= \pi r^2 h = \frac{22}{7}\times14\times14\times\cancel{35}^{5} = 21560\ cm^3$

L.S.A $= 2\pi rh = 2\times\frac{22}{7}\times14\times\cancel{35}^{5} = 3080\ cm^2$

T.S.A $= 2\pi r(r+h) = 2\times\frac{22}{7}\times\cancel{14}^{2}\times(35+14) = 88\times49 = 4312\ cm^2$.

→ एक ठोस शंकु की त्रिज्या 24cm है और ऊंचाई 7 cm है find Volume and T.S.A.

Volume $= \frac{1}{3}\pi r^2 h = \frac{1}{\cancel{3}}\times\frac{22}{7}\times\cancel{24}^{8}\times24\times7 = 4224\ cm^3$

$l = \sqrt{24^2+7^2} = \sqrt{625} = 25$

T.S.A $= \pi r(r+l) = \frac{22}{7}\times24\times\cancel{49}^{7} = 3696\ cm^2$.

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→ एक ठोस शंकु की त्रिज्या 12cm है और ऊंचाई 20cm है इसे पिघलाकर एक ठोस बेलन बनाया जाता है जिसकी त्रिज्या 6cm है find the Height of Cylinder. (86

Volume of Cone = Volume of Cylinder.

$\therefore \frac{1}{3}\pi \times r^2 \times h = \pi r^2 h$

$\frac{1}{3} \times \pi \times 12 \times 12 \times 20 = 6 \times 6 \times h$ $\quad h = \frac{80}{3}$ cm.

$\therefore$ Height of Cylinder $= \frac{80}{3}$ cm.

→ एक ठोस बेलन की त्रिज्या 14cm और Height 20cm है इसे पिघलाकर समान त्रिज्या वाला एक शंकु बनाया जाता है शंकु की ऊंचाई बताए।

Volume of Cylinder = Volume of Cone

$\pi \times 14 \times 14 \times 20 = \frac{1}{3} \times \pi \times 14 \times 14 \times h$

$h = 60$ cm.

Height of Cone = 60cm.

→ दो शंकुओं में से प्रत्येक की त्रिज्या 2.1cm है और ऊंचाई क्रमशः 4.1cm और 4.3cm है इन दोनों को पिघलाकर एक गोला बनाया जाता है तो गोले की त्रिज्या बताओ

Sum of Volume of Both Cone = Volume of Sphere

$\Rightarrow \frac{1}{3}\pi \times 2.1 \times 2.1 (4.1 + 4.3) = \frac{4}{3}\pi r^3$

$2.1 \times 2.1 \times 8.4 = 4r^3$

$r^3 = 2.1 \times 2.1 \times 2.1 = (2.1)^3$

$r = 2.1$

$\therefore$ Radius of sphere = 2.1cm.

→ एक ठोस शंकु की त्रिज्या 14cm और ऊंचाई 36cm है इसे पिघलाकर एक घनाभ बनाया जाता है जिसकी चौड़ाई और ऊंचाई क्रमशः 6cm और 7cm है। Find the Length of Cuboid.

Volume of Cone = Volume of Cuboid

$\frac{1}{3} \times \pi \times r^2 \times h = l \cdot B \cdot H$

$\frac{1}{3} \times \frac{22}{7} \times 14 \times 14 \times 36 = 6 \times 7 \times l$

$176 = l$

$\therefore$ Length of Cuboid = 176cm.

→ 3 Sphere की Radii क्रमशः 6cm, 8cm और 10cm है इसे पिघलाकर एक बड़ा Sphere बनाया जाता है, find the Radius of Sphere.

$\frac{4}{3}\pi (6^3 + 8^3 + 10^3) = \frac{4}{3}\pi R^3$

$R^3 = 216 + 512 + 1000 = 1728$

$R = 12$cm.

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→ एक ठोस घनाभ की लंबाई, चौड़ाई और ऊंचाई क्रमशः 36 cm, 22 cm और 10 cm है इसे पिघलाकर एक खोखला बेलन बनाया जाता है जिसकी Height 28 cm है and Thickness is 5 cm find the External Diameter of Cylinder.

Volume of Cuboid = Volume of Hollow Cylinder

$$36 \times 22 \times 10 = \frac{22}{7} \times 28 \times (R^2 - r^2)$$

$$R^2 - r^2 = 90 \qquad \text{Now } R - r = 5 \text{ (Given)}$$

$$(R-r)(R+r) = 90$$

$$5 \cdot (R+r) = 90$$

$$R + r = 18$$

$$R - r = 5$$

$$2R = 23$$

$$d = 23$$

∴ External Diameter of Cylinder = 23 cm.

→ एक ठोस गोले की त्रिज्या 10 cm है इसे पिघलाकर 0.2 cm वाली त्रिज्या की कितनी गोलिया बनाई जा सकती है।

$$n = \frac{\frac{4}{3}\pi \times 10 \times 10 \times 10}{\frac{4}{3}\pi \times 0.2 \times 0.2 \times 0.2} = \frac{10 \times 10 \times 10 \times 10 \times 10 \times 10}{2 \times 2 \times 2}$$

$$= 125000$$

125000 गोलिया बनाई जा सकती है।

→ एक ठोस शंकु की त्रिज्या 2 m है और ऊंचाई 3 m है इसे पिघलाकर 2 cm त्रिज्या वाली छोटी-छोटी कितनी गोलिया बनाई जा सकती है।

$$n = \frac{\frac{1}{3}\pi r^2 h}{\frac{4}{3}\pi r^3} = \frac{\frac{1}{3}\pi \times 200 \times 200 \times 300}{\frac{4}{3}\pi \times 2 \times 2 \times 2}$$

$$= 375000$$

→ एक ठोस गोले का व्यास 9 cm है इसे पिघलाकर 2 mm व्यास का तार बनाया जाता है, तार की लंबाई क्या होगी।

$d = 2\text{mm} = \frac{2}{10}\text{cm}$

$r = \frac{2}{10 \times 2} = \frac{1}{10}\text{cm}.$

Volume of wire = Volum of Sphere.

$$\pi \times \frac{1}{10} \times \frac{1}{10} \times h = \frac{4}{3} \times \pi \times \frac{9}{2} \times \frac{9}{2} \times \frac{9}{2}$$

$$h = \frac{3 \times 9 \times 9 \times 10 \times 10 \times 5}{2} = 12150\text{ cm}$$

$$= \frac{12150}{100} = 121.5\text{ m}.$$

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⇒ एक घनाभ की लंबाई, चौड़ाई और ऊंचाई क्रमशः 10m, 10cm, 15cm है। If the weight of $1cm^3$ is 4.5 gm then find the whole weight of cubiod. (In kg) (37)

Volume of cubiod = lbh
$= 1000 \times 10 \times 15$
$= 150000\ cm^3$.

Mass = Density × Volume

whole weight of Cubiod = Volume of Cubiod × Weight/cm^3
$= 150000 \times \frac{45}{10} \times \frac{1}{1000} = 675kg$.

⇒ एक ठोस गोले की त्रिज्या 10cm है और इस ठोस गोले का घनत्व (Density) $8.4g/cm^3$ है। सपूर्ण गोले का भार क्या है।

Volume of sphere $= \frac{4}{3} \times \frac{22}{7} \times 10 \times 10 \times 10 = \frac{88000}{21} cm^3$

Density $= 8.4 g/cm^3$

∴ Total Weight = Density × Volume $= \frac{88000}{21} \times \frac{84}{10} = 35200g$

$= \frac{35200}{1000} = 35.2kg$

⇒ एक ठोस गोले का सपूर्ण भार 194.04 kg है और इसकी $1cm^3$ वस्तु का वजन 5g है तो गोले की त्रिज्या क्या है।

Total Weight = Volume × Density [Weight/cm^3]

$1000 \times \frac{19404}{100} gm = \frac{4}{3} \times \frac{22}{7} \times r^3 \times 5$

$r^3 = \frac{19404 \times 10 \times 7 \times 3}{5 \times 22 \times 4}$

$r^3 = 441 \times 21$

$r^3 = 21^3 \quad r = 21$

∴ Radius of sphere = 21 cm

⇒ एक खोखले गोला की बाहरी और आंतरिक त्रिज्याएं क्रमशः 12cm और 8cm है यदि $1cm^3$ वस्तु का वजन 6.3g हो खोखले गोले का कुल वजन ज्ञात्ओ।

Volume of Hollow sphere $= \frac{4}{3} \times \frac{22}{7} (R^3 - r^3)$

$= \frac{4}{3} \times \frac{22}{7} (12^3 - 8^3) = \frac{4}{3} \times \frac{22}{7} (1728 - 512)$

Total Weight $= \frac{4}{3} \times \frac{22}{7} \times 1216 \times \frac{63}{10} \times \frac{1}{1000} = \frac{321024}{10000}$

$= 32.1024$ kg.

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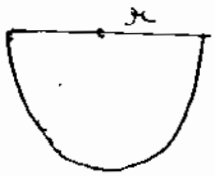
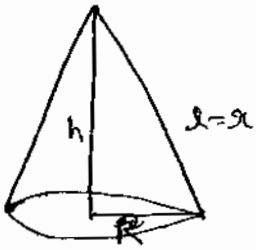
Next Type :→ IMPORTANT

जब अर्धवृत्त को मोड़कर एक शंकु बनाया जाता है तो अर्धवृत्त की त्रिज्या (r) और शंकु की तिर्यक ऊंचाई (l) बराबर होती है।



$$\boxed{r = l}$$

∴ L.S.A of Cone = Area of semicircle



$$\pi R l = \frac{\pi r^2}{2}$$

$$\pi R r = \frac{\pi r^2}{2} \quad [\because l = r]$$

$$\boxed{R = \frac{r}{2}}$$

$$\text{Height } (h) = \sqrt{l^2 - R^2} = \sqrt{r^2 - \left(\frac{r}{2}\right)^2} = \sqrt{r^2 - \frac{r^2}{4}}$$

$$= \sqrt{\frac{3r^2}{4}} = \frac{\sqrt{3}\,r}{2} = \sqrt{3}R$$

$$\boxed{\therefore \text{Height of Cone} = \frac{\sqrt{3}\,r}{2} = \sqrt{3}R}$$

→ एक अर्धवृत्त कागज के टुकड़े की त्रिज्या 42cm है इसे मोड़कर एक शंकु बनाया जाता है तो शंकु का आयतन क्या होगा।

$$\text{Radius of Cone } (R) = \frac{\text{Radius of Hemisphere } (r)}{2} = \frac{42}{2} = 21\text{cm}.$$

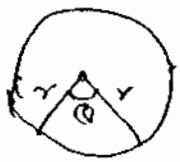
$$\text{Height of Cone } (h) = \sqrt{3}R = \sqrt{3} \times 21 = 21\sqrt{3}\text{ cm}.$$

$$\therefore \text{Volume of Cone} = \frac{1}{3} \times \frac{22}{7} \times 21 \times 21 \times 21\sqrt{3}$$

$$= 9702\sqrt{3}\text{ cm}^3$$

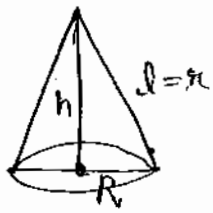
Next Type :→

जब sector नुमा आकार के कागज को मोड़कर एक शंकु बनाते हैं तो sector की त्रिज्या और शंकु की तिर्यक ऊंचाई बराबर होती है।



$$\boxed{r = l}$$

∴ L.S.A of Cone = Area of Sector



$$\pi R l = \frac{\theta}{360} \times \pi \times r^2$$

$$\pi R r = \frac{\theta}{360} \pi r^2$$

$$\boxed{R = \frac{\theta}{360} r}$$

$$\text{height} = \sqrt{r^2 - R^2}$$

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एक वर्तुलाकार कागज [illegible] है और यह Sect केन्द्र पर 120° का कोण बनाता है तो शंकु का आयतन बताओ यदि इस Sector को मोड़कर एक शंकु बनाया जाता है। (38)

Radius of Sector = तिर्यक ऊँचाई = 9 cm.

$$\text{Radius of Cone } (R) = \frac{\theta}{360} \times r = \frac{120}{360} \times 9 = 3\text{cm}$$

$$\text{Height} = \sqrt{9^2 - 3^2} = \sqrt{81-9} = \sqrt{72} = 6\sqrt{2}\text{ cm}.$$

$$\text{Volume of Cone} = \frac{1}{3} \times \pi \times 3 \times 3 \times 6\sqrt{2} = 18\sqrt{2}\pi\ \text{cm}^3.$$

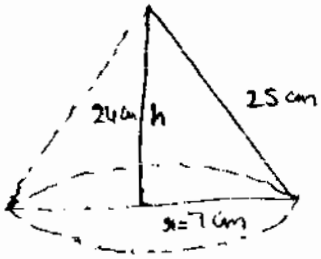
Next Type:→ एक समकोण त्रिभुज के समकोण बनाने वाली भुजा की लंबाई 24cm और 7cm है इसे 24cm भुजा के चारों तरफ घुमाया जाता है और एक शंकु बनाया जाता है Find the Volume of Cone.

Note:→ जिस भुजा के अनुदिश घुमाया जाता है वह Cone की Height का कार्य करती है। दूसरी भुजा Radius का कार्य करती है।

Height of Cone = 24cm, Radius = 7cm.

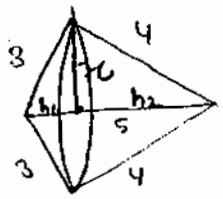
$$\text{Volume} = \frac{1}{3} \times \frac{22}{7} \times 7 \times 7 \times 24$$

$$= 1232\ \text{cm}^3$$



Next Type:→ अगर Hypotaneous के अनुदिश चारों तरफ घुमाया जाता है तो Hypotaneous Height की तरह कार्य करता है। और दो Cone बनते हैं।

किसी भी Right Angle Triangle में Height Drawn करते हैं



B = 3cm
P = 4cm
h = 5cm

Then = $x = \frac{Pb}{h}$

$$\therefore x = \frac{3 \times 4}{5} = \frac{12}{5}\text{ cm}.$$

$$\text{Volume} = \frac{1}{3}\pi r^2 h_1 + \frac{1}{3}\pi r^2 h_2 = \frac{1}{3}\pi r^2 (h_1 + h_2)$$

$$= \frac{1}{3}\pi r^2 h$$

$$= \frac{1}{3} \times \pi \times \frac{12}{5} \times \frac{12}{5} \times 5 = \frac{48}{5}\pi\ \text{cm}^3.$$

OR

$$\text{Volume} = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi \times \left(\frac{Pb}{h}\right)^2 \times h = \frac{1}{3} \times \pi \times \frac{P^2 b^2}{h^2} \times h$$

$$\boxed{= \frac{1}{3}\pi \frac{P^2 b^2}{h}}$$

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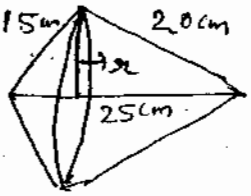
एक समकोण त्रिभुज के समकोण बनाने वाली भुजा की लंबाई 15cm और 20cm है। इसके कर्ण के चारों तरफ घुमाया जाता है और एक Cone बनाया जाता है find the Volume of Cone. ($\pi = 3.14$)



$h = \text{Hypo} = \sqrt{15^2+20^2} = \sqrt{625} = 25\text{cm}$.

$r = \frac{Pb}{h} = \frac{15 \times 20}{25} = 12\text{cm}$.

$\text{Volume} = \frac{1}{3} \times \frac{314}{100} \times 12 \times 12 \times 25 = 3768\text{ cm}^3$.

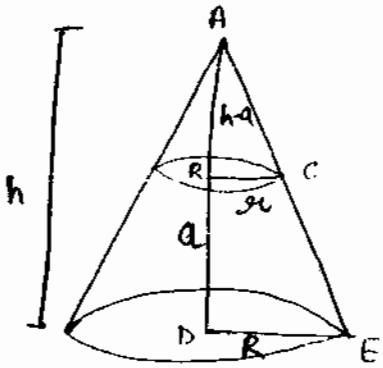
OR

$\text{Volume} = \frac{1}{3}\pi \frac{P^2 b^2}{h} = \frac{1}{3} \times \frac{314}{100} \times \frac{15 \times 15 \times 20 \times 20}{25}$

$= 3768\text{ cm}^2$

Next Type:→

एक शंकु के आधार के समांतर एक छोटा शंकु काटा जाता है किसी भी a Height पर find the Radius of Small Cone

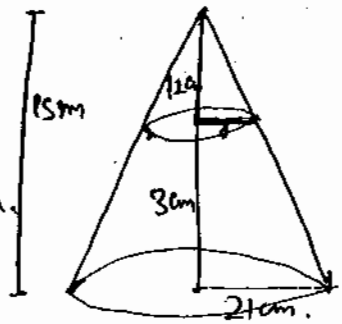


$\Delta ABC \sim \Delta ADE$

$\therefore \frac{h-a}{h} = \frac{r}{R}$

$\boxed{r = \frac{h}{h-a} R}$

एक शंकु की त्रिज्या 21cm और Height 15cm है इसे आधार के समांतर 3cm की ऊंचाई पर एक शंकु काटा जाता है find the Volume of Small Cone.



$\therefore \frac{r}{21} = \frac{12}{15}$, $r = \frac{12}{15} \times 21 = \frac{84}{5}$

Radius of Small Cone $= \frac{84}{5}$ cm, Height $= 15-3 = 12$ cm.

$\therefore \text{Volume} = \frac{1}{3} \times \frac{22}{7} \times \frac{84}{5} \times \frac{84}{5} \times 12$

$= \frac{88704}{25} = 3548.16\text{cm}^3$.

एक शंकु की त्रिज्या 21cm और Height 18cm है इसके आधार के समांतर 6cm की ऊंचाई पर एक Cone काटा जाता है find the Volume of Cone Small.

$\frac{r}{21} = \frac{18-6}{18} \Rightarrow \frac{r}{21} = \frac{12}{18}$

$r = \frac{21 \times 12}{18}$ $r = 14\text{cm}$, Height $= 12\text{cm}$.

$\text{Volume} = \cancel{\text{Volume}} = \frac{1}{3} \times \frac{22}{7} \times 14 \times 14 \times 12 = 2464\text{cm}^3$.

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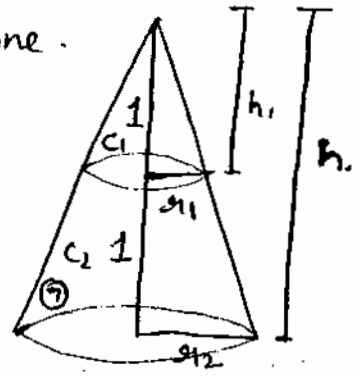
→ एक शंकु को आधार के समांतर ~~2 छोटे~~ 1 छोटा शंकु इस प्रकार काटा जाता है कि शंकु की ऊंचाई दो बराबर भागों में बंट जाती है Find the Ratio of Height, Radius, Volume, Volume of Both Part of Both Cone (39)

h_1 = Height of Small Cone, h_2 = Height of Big Cone.



Let $h_1 = 1$ then Height of Remain Part is also 1

$\therefore h_2 = 1+1 = 2$

$$\boxed{h_1 : h_2 = 1:2}$$

Now $\frac{r_1}{r_2} = \frac{h_1}{h_2} \Rightarrow \frac{r_1}{r_2} = \frac{1}{2}$

$$\boxed{r_1 : r_2 = 1:2}$$

$$V_1 : V_2 = r_1^3 : r_2^3 = 1^3 : 2^3 = 1:8$$

$$\boxed{V_1 : V_2 = 1:8}$$

Detail for $(V_1 : V_2)$

Now $\frac{r_1}{r_2} = \frac{1}{2} \Rightarrow 2r_1 = r_2$.

$$V_1 : V_2 = \frac{1}{3}\pi r_1^2 h_1 : \frac{1}{3}\pi r_2^2 h_2$$

$$= r_1^2 \cdot 1 : (2r_1)^2 \cdot 2.$$

$$r_1^2 : 4 \cdot r_1^2 \cdot 8 \Rightarrow 1:8$$

✓ Volume of Both Parts. $(C_1 : C_2)$

$$C_1 : C_2 = 1 : (8-1)$$

$$= 1:7$$

एक शंकु की ऊंचाई 60cm है इसके आधार के समांतर एक छोटा शंकु इस प्रकार काटा जाता है कि छोटे शंकु की Height बड़े शंकु की Height का $\frac{1}{3}$ भाग है। दोनों शंकु के आयतनों का अनुपात बताओ।

$$h_1 : h_2 = 1:3$$

$$r_1 : r_2 = 1:3 \quad \left[\frac{r_1}{r_2} = \frac{h_1}{h_2} = \frac{1}{3} \Rightarrow r_1 : r_2 = 1:3\right]$$

$$V_1 : V_2 = 1^3 : 3^2$$

$$= 1:27 \text{ Ans.}$$

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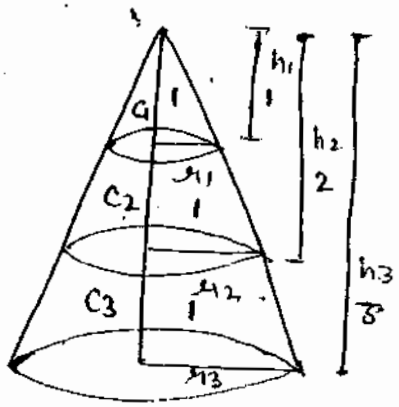
एक शंकु की ऊँचाई 30cm है। इसे आधार के समांतर 2 शंकु इस प्रकार काटे जाते हैं कि शंकु की ऊँचाई 3 बराबर भागों में बंट जाती है।

(i) find the Ratio of Volume of the Cone in Ascending Order.

(ii) Find the Ratio of Volume of all three Parts.



$$h_1 : h_2 : h_3 = 1 : 2 : 3$$

$$\therefore r_1 : r_2 : r_3 = 1 : 2 : 3$$

$$\boxed{\therefore V_1 : V_2 : V_3 = 1 : 8 : 27}$$

Volume Ratio Parts.

$$C_1 : C_2 : C_3 = 1 : (8-1) : (27-8)$$

$$= 1 : 7 : 19$$

एक शंकु की ऊँचाई 30cm है इसके आधार के साथ समांतर एक छोटा शंकु काटा जाता है If the Volume of Smaller cone is $\frac{1}{27}$ Part of Big Cone. Find the Height of Smaller Cone.

$$V_1 : V_2 = 1 : 27$$

$$\therefore h_1 : h_2 = \sqrt[3]{1} : \sqrt[3]{27} = 1 : 3$$

Now $3 \rightarrow 30$ $\quad \therefore 1 \rightarrow \frac{30}{3} = 10cm$.

$\therefore$ Height of Smaller Cone = 10cm.

एक शंकु की ऊँचाई 45cm है। इसके आधार के ॥ एक छोटा शंकु काटा जाता है जिसकी Volume बड़े शंकु की Volume का 8/27 भाग है find the Height of Smaller Cone.

$$V_{smaller} : V_{Bigger} = 8 : 27$$

$$\therefore h_{small} : h_{Big} = \sqrt[3]{8} : \sqrt[3]{27}$$

$$= 2 : 3$$

Now $3 \rightarrow 30$

$$\therefore 2 \rightarrow \frac{\cancel{30}^{10}}{\cancel{3}} \times 2 = 20.$$

Height of smaller Cone = 20cm.

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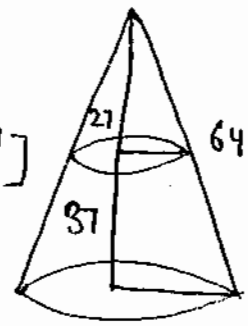
एक शंकु की ऊँचाई 60cm है इसके आधार के समांतर एक छोटा शंकु काटा जाता है और इसे काटकर हटा दिया जाता है If the Volume of the Rest Part is 37/64 Times the Volume of Larger cone. Find the Height of Smaller Cone. (40)



$V_{Rest\ Part} : V_{Big\ cone} = 37 : 64$

$V_{small} : V_{Big\ cone} = 27 : 64$ $[V_{small} = 64 - 37 = 27]$

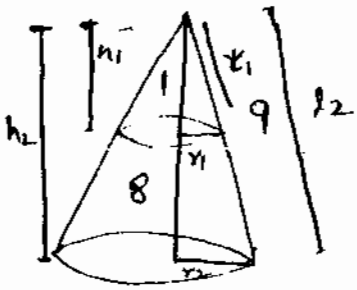
$\therefore h_{small} : h_{Big} = 3 : 4$

Now $4 \rightarrow 60$

$\therefore 3 \rightarrow \frac{60}{4} \times 3 = 45\,cm.$

$\therefore$ Height of Smaller Cone = 45 cm.

→ एक शंकु की ऊँचाई 30cm है इसके आधार के समांतर एक छोटा शंकु काट लिया जाता है और इसके ऊपरी भाग को हटा दिया जाता है। If the L.S.A of the Rest Part is 8/9 Times the L.S.A of Larger Cone. Find the Height of Smaller Cone



$L_{Rest} : L_{Big\ cone} = 8 : 9$

$\therefore L_{(small\ cone)} : L_{Big\ Cone} = 1 : 9$ $[9 - 8 = 1]$

$\therefore h_{small} : h_{Big} = \sqrt{1} : \sqrt{9}$

$= 1 : 3$

Now $3 \rightarrow 30$

$1 \rightarrow \frac{30}{3} = 10\,cm.$

$\therefore$ Height of Smaller Cone = 10cm.

Detail :→ $\frac{r_1}{r_2} = \frac{h_1}{h_2} = \frac{l_1}{l_2}$

Now L.S.A of Smaller cone = $\frac{1}{9}$ × L.S.A of Larger cone.

$\pi r_1 \cdot l_1 = \frac{1}{9} \times \pi \times r_2 \times l_2$

$\frac{r_1 \cdot l_1}{r_2 l_2} = \frac{1}{9}$ $\Rightarrow \frac{h_1}{h_2} \cdot \frac{h_1}{h_2} = \frac{1}{9}$

$\frac{h_1}{h_2} = \frac{1}{3}$ $h_1 : h_2 = 1 : 3$

$3 \rightarrow 30$ $\therefore$ $1 \rightarrow$ 10cm (Height of Smaller cone)

एक शंकु की ऊँचाई 80cm है इसके आधार के समांतर एक छोटा शंकु काटकर हटा दिया जाता है। If L.S.A of Rest Part is 16/25 Times the L.S.A of Larger Cone. find the Height of Smaller Cone.

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$L_{Rest} : L_{Big\ cone} = 16 : 25$

$\therefore h\,(Small : Big\ Cone) = 9 : 25 \quad [25-16=9]$

$\because h\,(Small : Big\ Cone) = \sqrt{9} : \sqrt{25}$

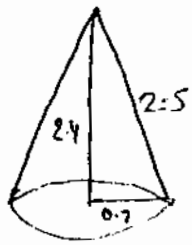
$= 3 : 5$

Now $5 \rightarrow 80$

$3 \rightarrow \frac{\cancel{80}^{16}}{\cancel{5}} \times 3 = 48$

Height of Smaller cone $= 48\,cm$.

⇒ एक शंकु के आधार का क्षेत्रफल $1.54\,m^2$ है और तिर्यक ऊँचाई $2.5\,m$ है Find the Height of Cone.



Now $\pi r^2 = 1.54$

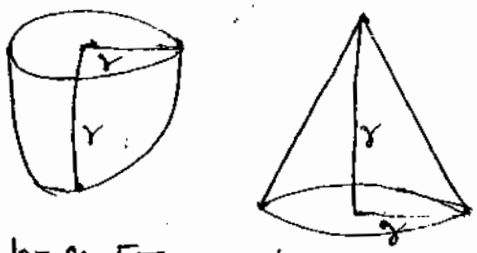
$r^2 = \frac{\cancel{154}^{7}}{\cancel{22}} \times \frac{7}{100}$

$r = \frac{7}{10} = 0.7 \qquad l = 2.5$

$\therefore h = \sqrt{l^2 - r^2} = \sqrt{(2.5)^2 - (0.7)^2} = \sqrt{6.25 - 0.49} = \sqrt{5.76}$

$= 2.4\,cm$

⇒ एक अर्धगोले और शंकु के आधार की त्रिज्या बराबर है और इनकी ऊँचाई भी बराबर है find the Ratio of their L.S.A.



$h = r$ [In Hemi-sphere]

$\therefore h = r$ [In cone]

L.S.A Hemisphere : L.S.A. cone

$= 2\pi r^2 : \pi r l$

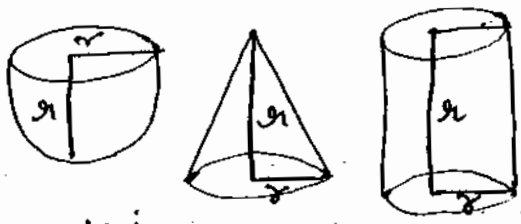
$= 2r : l$

$= 2r : \sqrt{r^2 + h^2}$

$= 2r : \sqrt{r^2 + r^2} \Rightarrow 2r : \sqrt{2}\,r$

$\Rightarrow \sqrt{2} : 1$ Ans.

एक शंकु, अर्धगोले और बेलन की त्रिज्याएँ बराबर है और इनकी ऊँचाई भी आपस में बराबर है find the Ratio of their Volume.



$V_{cone} : V_{Hemi\text{-}sphere} : V_{Cylinder}$

$= \frac{1}{3}\pi r^2 h : \frac{2}{3}\pi r^3 : \pi r^2 h$

$= \frac{1}{3}\cancel{\pi r^2} r : \frac{2}{3}\cancel{\pi r^3} : \cancel{\pi r^2} r$

$= \frac{1}{3} : \frac{2}{3} : 1$

$= 1 : 2 : 3$

Height of Hemi sphere = Radius of Hemi-Sphere.

$\therefore$ Height of Hemisphere = Height of Cone = Height of Cylinder = Radius.

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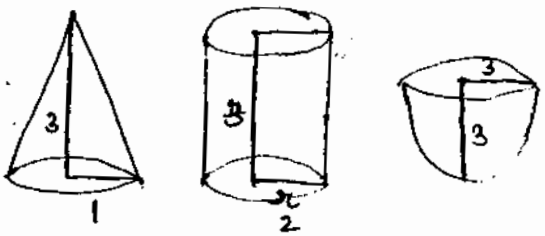
→ एक शंकु, बेलन और अर्धगोले की त्रिज्या का अनुपात 1 : 2 : 3 है और ऊंचाई भी आपस में बराबर हैं, find the Ratio of their Volume. (ग)

अर्धगोले की त्रिज्या = अर्धगोले की ऊंचाई = 3 cm.

∴ बेलन का ऊंचाई = शंकु की ऊंचाई = अर्धगोले की ऊंचाई = 3cm.



$\therefore r_1 : r_2 : r_3 = 1 : 2 : 3$

$\frac{1}{3}\pi r_1^2 h : \pi \times r_2^2 \times h : \frac{2}{3} \times \pi \times r_3^3$

$= \frac{1}{3} \times 1^2 \times 3 : 2^2 \times 3 : \frac{2}{3} \times 3^3$

$V_1 : V_2 : V_3 = 1 : 12 : 18$ Ans.

एक गोले और एक शंकु का आयतन बराबर है और शंकु की ऊंचाई अपनी त्रिज्या की आधी है Find the Ratio of LSA of cone and sphere

Now $h : r = 1 : 2$ [In Cone] $h = 1, r = 2$ $\left[h = \frac{r}{2},\ h : r = 1 : 2\right]$

Volume of sphere = Volume of cone

$\frac{4}{3}\pi r_1^3 = \frac{1}{3} \times \pi \times r_2^2 \times h$

$4 r_1^3 = 2^2 \cdot 1$ $r^3 = 1$ $r = 1$ (Radius of sphere)

Now LSA (cone : sphere) $= \pi r l : 4\pi (r^2)$

$= 2 \cdot \sqrt{2^2+1} : 4(1)^2$

$\Rightarrow 2\sqrt{5} : 4$

$\Rightarrow \sqrt{5} : 2$ Ans.

एक घन और गोले के संपूर्ण पृष्ठ का क्षेत्रफल बराबर है find the Ratio of Volume of Cube and sphere.

T.S.A of Cube = T.S.A of sphere

$6a^2 = 4 \times \frac{22}{7} \times r^2$

$\frac{a^2}{r^2} = \frac{4 \times 22}{7 \times 6} \quad \frac{a^2}{r^2} = \frac{44}{21} \Rightarrow \frac{a}{r} = \frac{2\sqrt{11}}{\sqrt{21}}$

Now Volume of Cube : Volume of Sphere $a : r = 2\sqrt{11} : \sqrt{21}$

$= a^3 : \frac{4}{3}\pi r^3$

$\Rightarrow (2\sqrt{11})^3 : \frac{4}{3} \times \frac{22}{7} \times (\sqrt{21})^3$

$\Rightarrow 8 \times 11 \times 2\sqrt{11} : \frac{4}{3} \times \frac{22}{7} \times 21 \times \sqrt{21}$

$\sqrt{11} : \sqrt{21}$

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एक शंकु का आयतन अपने Lateral Surface Area के बराबर है, If h and r are height and Radius of Cone find $\frac{1}{h^2}+\frac{1}{r^2}$

Now $\frac{1}{3}\pi r^2 h = \pi r l$

$rh = 3l$

$rh = 3\sqrt{r^2+h^2}$

Squaring Both side

$r^2h^2 = 9(r^2+h^2)$

$\frac{r^2+h^2}{r^2h^2} = \frac{1}{9}$, $\frac{1}{h^2}+\frac{1}{r^2} = \frac{1}{9}$

एक बेलन का आयतन $V\,cm^3$ है और Total surface are is S, h and r are respectively height and Radius find $V\left(\frac{1}{h}+\frac{1}{r}\right) : S$

$V = \pi r^2 h$ $\quad S = 2\pi r(r+h)$

$\therefore \pi r^2 h\left(\frac{1}{h}+\frac{1}{r}\right) : 2\pi r(r+h)$

$\Rightarrow \cancel{\pi} \times \cancel{r} \times \cancel{r} \times \cancel{h} \frac{\cancel{(r+h)}}{\cancel{rh}} : 2\cancel{\pi}\cancel{r}\cancel{(r+h)}$

$1:2$ Ans.

⇒ एक बेलन का Lateral Surface Area $264\,m^2$ है और आयतन $924\,m^3$ है find the Ratio of Diameter and Height

$2\pi rh = 264\,m^2$ — I

$\pi r^2 h \quad \cancel{2\pi r(r+h)} = 924\,m^3$ — II

Divide II by I

$\frac{\cancel{\pi r^2 h}\ \cancel{2\pi r(r+h)}}{\cancel{2\pi r h}} = \frac{924}{264}$

$\frac{r+h}{h} = \frac{924}{264}$

$\frac{r}{2} = \frac{924}{264}$

$r = \frac{\cancel{924}^{7} \times \cancel{2}}{\cancel{264}_{\cancel{132}}}$

$r = 7\,m$

$\therefore 2\pi rh = 264$

$2 \times \frac{22}{\cancel{7}} \times \cancel{7} \times h = 264$

$h = \frac{\cancel{264}^{\cancel{12}\,6}}{\cancel{22} \times \cancel{2}}$ $\quad h = 6$

$\therefore 2r : h = D : h = \cancel{2} \times 7 : \cancel{6}\,3$

$= 7:3$

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एक बेलन की त्रिज्या और ऊँचाई का योग 53 m है और इसके संपूर्ण पृष्ठ का क्षेत्रफल 4664 m² है find the Volume of Cylinder.

(पृ२)

$$r + h = 53 \quad \text{—} (1)$$

$$2\pi r(r+h) = 4664$$

$$\therefore r = \frac{4664 \times 7}{2 \times 22 \times 53}$$

$$r = 14 \text{ m.}$$

$$\therefore h = 53 - 14 = 39 \text{ m.}$$

$$\text{Volume of Cylinder} = \frac{22}{7} \times 14 \times 14 \times 39 = 24024 \text{ m}^3.$$

एक बेलन के LSA और T.S.A का अनुपात 1 : 3 है और L.S.A 1386 m² है find the Height of Cylinder.

$$\frac{2\pi rh}{2\pi r(r+h)} = \frac{1}{3}$$

$$\frac{h}{r+h} = \frac{1}{3} \Rightarrow 3h = r + h$$

$$\boxed{2h = r}$$

$$\therefore 2\pi rh = 1386$$

$$\Rightarrow 2 \times \frac{22}{7} \times 2h \times h = 1386$$

$$h^2 = \frac{7 \times 9 \times 7}{2 \times 2} \quad \therefore h = \frac{7 \times 3}{2} = \frac{21}{2} = 10.5 \text{ m.}$$

एक बेलन का Total surface Area 165 cm² है LSA is $\frac{8}{15}$ times of TSA. find The Radius of Cylinder.

$$2\pi r(r+h) = 165 \text{ cm}^2 \quad \text{— I}$$

$$LSA = \frac{8}{15} \times TSA$$

$$2\pi rh = \frac{8}{15} \times 165$$

$$2\pi rh = 88 \quad \text{— II}$$

✓ Divide I by II

$$\frac{2\pi r(r+h)}{2\pi rh} = \frac{165}{88}$$

$$8r + 8h = 15h$$

$$8r = 7h$$

$$h = \frac{8}{7}r.$$

$$\therefore 2\pi rh = 88 \Rightarrow 2 \times \frac{22}{7} \times r \times \frac{8}{7}r = 88$$

$$r^2 = \frac{49}{4}, \quad r = \frac{7}{2} \text{ cm.}$$

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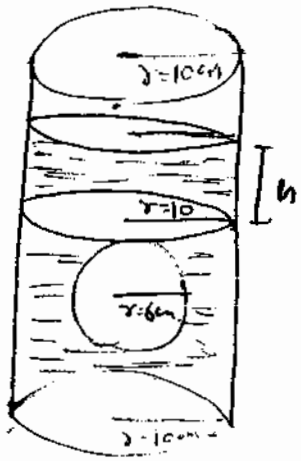
एक बेलनाकार बर्तन की त्रिज्या 10cm है जो आधा पानी से भरा है इसके अन्दर एक ठोस Sphere डुबा दिया जाता है जिसकी Radius 6cm है find the Height of Rising Water.

Volume of the Rising Water = Volume of Sphere Emmersed



$$\pi \times 10 \times 10 \times h = \frac{4}{3} \times \pi \times 6 \times 6 \times 6$$

$$h = \frac{288}{100} = 2.88 \text{ cm}.$$

∴ Height of Rising Water = 2.88 cm.

⇒ एक बेलनाकार बर्तन जिसका व्यास 42cm है आधा पानी से भरा है इसके अन्दर एक ठोस घनाभ जिसके Dimensions 24cm, 21cm और 11cm है डुबा दिया जाता है find the Height of Rising Water. $r = \frac{42}{2} = 21$cm.

Volume of Rising Water = Volume of Cuboid

$$\frac{22}{7} \times 21 \times 21 \times h = 24 \times 21 \times 11$$

$$h = 4 \text{ cm}.$$

∴ Height of Rising Water = 4cm.

एक बेलनाकार बर्तन की त्रिज्या 40cm है आधा पानी से भरा है, 20cm व्यास के दो गोले इस बर्तन में डुबा दिये जाते हैं find the Height of Rising Level.

Volume of Rising Water = Volume of Both Sphere.

$$\pi \times 40 \times 40 \times h = 2 \times \frac{4}{3} \pi \times 10 \times 10 \times 10$$

$$h = \frac{5}{3} \text{ cm}.$$

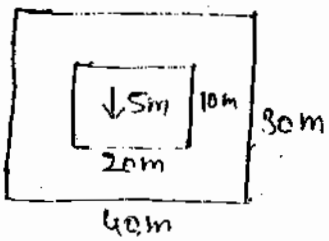
Height of Rising Water = $\frac{5}{3}$ cm.

Next Type :→ एक आयताकार मैदान की लंबाई और चौड़ाई क्रमशः 40 और 30 m है इसके अन्दर एक तालाब बनाया जाता है जिसके Dimensions 20m, 10m और 0.5m है. इस तालाब से निकली हुई मिट्टी को समान रूप से पूरे मैदान में बिछा दिया जाता है तो मैदान का Level कितना Increase हो जाएगा।

Area of the field = $40 \times 30 = 1200 m^2$

Area of Water Tank = $20 \times 10 = 200 m^2$

Area of Remaining Portion = $1200 - 200$
$= 1000 m^2$



Volume of Remain Portion = $1000 \times h$

Now Volume of Remain portion = Volume of water tank

$$1000 \times h = 200 \times 10 \times 5$$

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$h = 1m$.

∴ Height (Level) 1m बढ़ जाएगा।

एक आयताकार मैदान की लंबाई और चौड़ाई क्रमशः 30m और 20m है इसमें 10m लंबा, 10m चौड़ा और 20m गहरा एक water tank बनाया जाता है इससे निकली मिट्टी को Rest भाग पर एक समान रूप से फैला दिया जाता है find the Increasing Level (h) of field.

Area of field $= 30 \times 20 = 600 m^2$

Area of water tank $= 10 \times 10 = 100 m^2$

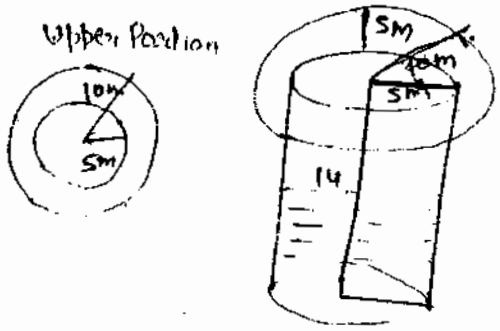
Area of Remain portion $= 600 - 100 = 500 m^2$.

Now $\cancel{500} \times h = 10 \times 10 \times 20$ 4

$h = 4m$.

एक कुएँ की त्रिज्या 5m है और 14 मीटर गहरा है इससे निकली हुई मिट्टी को बिछाकर एक चबूतरा बनाया जाता है। जो 5 मीटर चौड़ा है find चबूतरे की ऊँचाई बताओ

$r = 5m \quad R = 5+5 = 10m$.



Area of Ring $= \pi(R+r)(R-r)$

Volume of Ring $= \pi(R+r)(R-r)h$

Volume of Well $= \pi r^2 H$

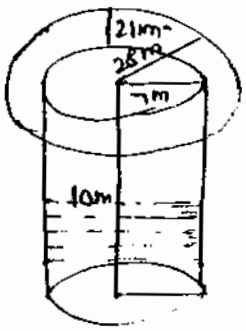
$\therefore \pi(R+r)(R-r) \times h = \pi r^2 h$

$(10+5)(10-5) \times h = 5 \times 5 \times 14$

$h = \frac{5 \times 5 \times 14}{15 \times 5} = \frac{14}{3} m$.

एक कुएँ का व्यास 14m है और वह 10m गहरा है इससे निकली मिट्टी को चारों तरफ बिछाकर 21m चौड़ा चबूतरा बनाया जाता है चबूतरे की ऊँचाई बताओ।

$r = 7m, \quad R = 21+7 = 28m$.



Now $\pi(R+r)(R-r) \times h = \pi r^2 H$

$(28+7)(28-7) \times h = 7 \times 7 \times H$

$35 \times 21 \times h = 7 \times 7 \times 10$

$h = \frac{2}{3} m$.

∴ चबूतरे की ऊँचाई $= \frac{2}{3} m$.

→ एक रोलर की त्रिज्या 1.68 m है और उसकी लंबाई 4.5m है यह रोलर 150 चक्करों में मैदान के 72% Area को समतल करता है find the Total Area of field.

$72\% \longrightarrow 2\pi r h \times 150$

$100\% \longrightarrow \frac{2\pi r h \times 150 \times 100}{72}$

$= 2 \times \frac{22}{7} \times \frac{168}{100} \times \frac{45}{10} \times \frac{150 \times 100}{72} = 9900 m^2$

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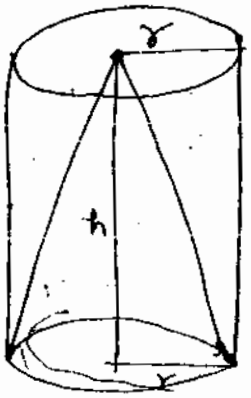
एक रोलर की लंबाई 28 m है यह 300 चक्कर में 75% Area समतल करता है। If the Area of the field is 12320 m^2. find the Diameter of Roller.

$$75\% \longrightarrow 2\pi rh \times 300$$
$$100\% \longrightarrow \frac{2\pi rh \times 300 \times 100}{75} = 12320$$
$$= 2 \times \frac{22}{7} \times r \times \frac{28}{10} \times \frac{300 \times 100}{75} = 12320$$
$$r = \frac{12320 \quad 56 \quad 4 \quad 7}{2 \times 22 \times 4 \times 4} = \frac{7}{4}$$
$$\text{Diameter} = \frac{2 \times 7}{4} = 3.5\,m.$$

Next Type:→ एक ठोस बेलन की त्रिज्या r है और ऊँचाई h है इसके अन्दर उतनी ही त्रिज्या और ऊँचाई एक शंकवाकार ठोस निकाला जाता है find the TSA.



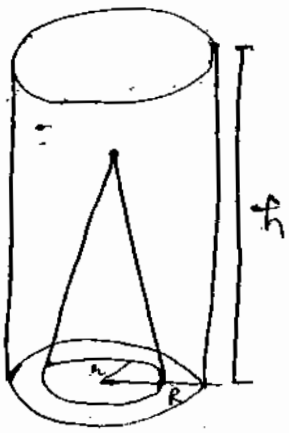
$$\text{Area of Cylinder} = 2\pi rh + \pi r^2 + \pi r^2$$
$$\therefore \text{Area of Remain Position} = \text{L.S.A of Cylinder} + \text{Area of one Base} + \text{L.S.A of Cone}$$
$$= 2\pi rh + \pi r^2 + \pi r l$$
$$= \boxed{\pi r(2h + r + l)}$$

→ एक ठोस बेलन की त्रिज्या और ऊँचाई क्रमशः 3cm और 4cm है इसके अन्दर 3cm त्रिज्या और 4cm ऊँचाई का एक शंकु बनाया जाता है find Total Surface Area

$$\text{TSA of Remain Portion} = \pi r(2h + r + l)$$
$$= \frac{22}{7} \times 3\,(2 \times 4 + 3 + 5)$$
$$= \frac{22}{7} \times 3 \times 16 = 48\pi\,cm^2.$$
$$\left[l = \sqrt{3^2 + 4^2} = \sqrt{25} = 5\right]$$

Next Type:→

Radius of Cylinder = R
Radius of Cone = r



$$\text{TSA of Portion} = \text{LSA of Cylinder} + \text{Area of one Base} + \text{L.S.A of Cone} + \text{Area of Ring}$$
$$= 2\pi Rh + \pi R^2 + \pi r l + \pi(R+r)(R-r)$$
$$\boxed{\pi(2Rh + R^2 + rl + (R+r)(R-r))}$$

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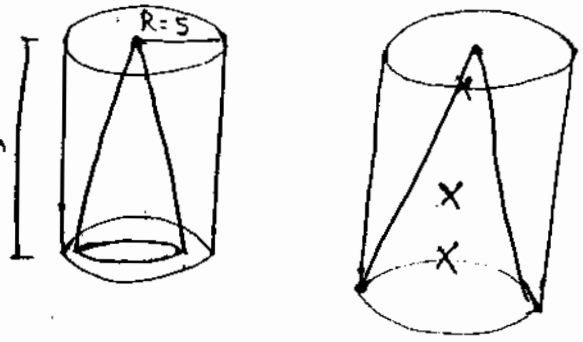
एक ठोस बेलन की त्रिज्या और ऊंचाई क्रमशः 5cm और 4cm है एक 3cm त्रिज्या और 4cm ऊंचाई के एक शंक्वाकार भाग को काट कर निकाल दिया जाता है find the Total Surface Area.

(4cm)



TSA of Remain Portion

$= \pi(2Rh + R^2 + rl + (R+r)(l-r))$

$= \pi(2\times5\times4 + 5\times5 + 3\times5 + 8\times2)$

$= \pi[40+25+15+16]$

$= 96\pi\ cm^2$.

Next Type:- एक घनाभ के तीन आसन्न फलकों का क्षेत्रफल $a, b, c\ cm^2$ है find the Relation between Volume and a, b, c.

Now $L\times B = a$ — I

$B\times H = b$ — II

$H\times L = c$ — III

Multiplying I, II and III

$L^2\times B^2\times H^2 = abc$

$(LBH)^2 = abc$

$V^2 = abc$

OR

$V = \sqrt{abc}$

एक घनाभ के तीन आसन्न फलकों का क्षेत्रफल $144cm^2$, $196cm^2$, $225cm^2$ है find the Volume and Dimension of Cuboid

$a = 144cm^2$ (lb), $b = 196cm^2$ (bh), $c = 225\ cm^2$ (hl)

$V = \sqrt{abc}$

$= \sqrt{144\times196\times225}$

$V = 12\times14\times15 = 2520\ cm^3$.

$h = \frac{lbh}{lb} = \frac{2520}{144} = 17.5cm$

$b = \frac{lbh}{hl} = \frac{2520}{225} = \frac{56}{5} = 11.2cm$.

$l = \frac{lbh}{bh} = \frac{2520}{196} = \frac{90}{7} = 12.85\ cm$

एक घनाभ के तीन आसन्न फलकों का क्षेत्रफल $100cm^2$, $64cm^2$, $81cm^2$ find the Volume and Dimensions of Cuboid.

$V = \sqrt{100\times64\times81}$

$= 10\times8\times9 = 720$

$h = \frac{lbh}{lb} = \frac{720}{100} = 7.2cm$

$l = \frac{lbh}{bh} = \frac{720}{64} = 11.25cm$

$b = \frac{lbh}{lh} = \frac{720}{81} = 8.88cm$.

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एक घनाभ की लंबाई और चौड़ाई और ऊँचाई का योग 25cm है। And Length of its Diagnol is 15 cm. find the Total Surface Area of cubiod.

$$l+b+h = 25cm$$
$$l^2+b^2+h^2 = (15)^2$$
$$l^2+b^2+h^2 = 225$$

Now Using $(l+b+h)^2 = l^2+b^2+h^2+2(lb+bh+hl)$

$$(25)^2 = 225 + T.SA.$$
$$TSA = 625-225 = 400cm^2.$$

एक घनाभ की लंबाई, चौड़ाई और ऊँचाई का अनुपात 4:3:2 है और Total Surface Area is 1872 cm². find the Volume of Cuboid.

$$l:b:h = 4:3:2$$
$$4x,\ 3x,\ 2x$$

Now $2(lb+bh+hl) = 2(4x\cdot 3x + 3x\cdot 2x + 4x\cdot 2x) = 1872$

$$2(12x^2+6x^2+8x^2) = 1872$$
$$2\times 26x^2 = 1872$$
$$x^2 = \frac{1872}{2\times 26} = 36$$
$$x = 6$$

$\therefore l = 4\times 6 = 24cm,\quad b = 3\times 6 = 18cm,\quad h = 2\times 6 = 12cm.$

$$Volume = 24\times 18\times 12 = 5184cm^3.$$

एक घनाभ का Total Surface Area $x\ cm^2$ है और लंबाई चौड़ाई और ऊँचाई का योग $y\ cm$ है। find the length of its Diagnol.

$$2(lb+bh+hl) = x$$
$$l+b+h = y$$
$$Diagnol = \sqrt{l^2+b^2+h^2}$$

$$\therefore (l+b+h)^2 = l^2+b^2+h^2+2(lb+bh+hl)$$
$$y^2 = l^2+b^2+h^2+x$$
$$l^2+b^2+h^2 = y^2-x$$
$$\sqrt{l^2+b^2+h^2} = \sqrt{y^2-x}$$

एक आयताकार षट्फलक की भुजाओं का योग 24cm है और विकर्ण की लंबाई 15cm है। find the Total Surface Area.

$$l+b+h = 24cm$$
$$l^2+b^2+h^2 = 15^2 = 225$$

Now $2(lb+bh+hl) = (l+b+h)^2 - (l^2+b^2+h^2)$

$$= (24)^2 - 225$$
$$= 576-225 = 351cm^2$$

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एक आयताकार [illegible] और इसकी ऊंचाई 6 है और Total Surface Area $558\,m^2$ है find its Length

$$l : b = 5 : 3$$
$$= 5x, 3x$$
$$h = 6\,cm$$

Now $2(lb + hb + hl) = 558$

$$2(5x \cdot 3x + 6 \times 3x + 6 \times 5x) = \cancel{558}\ 279$$

$$15x^2 + 48x - 279 = 0$$
$$5x^2 + 16x - 93 = 0$$
$$5x^2 + 31x - 15x - 93 = 0$$
$$5x^2 - 15x + 31x - 93 = 0$$
$$5x(x-3) + 31(x-3) = 0$$
$$x = 3,\ x \neq -\frac{31}{5}$$

$\therefore$ Length $(l) = 5x = 5 \times 3 = 15\,cm$.

एक बेलन की ऊंचाई h cm है और इसकी Volume और Total Surface Area बराबर है। Find the Radius in the form of Height.

$$\cancel{\pi} r^{\cancel{2}} h = 2\cancel{\pi} r(r + h)$$
$$rh = 2r + 2h$$
$$rh - 2r = 2h$$
$$r(h - 2) = 2h$$
$$\boxed{r = \frac{2h}{h-2}}$$

एक घनाभ के Dimensions 10cm, 7cm और 5cm है, इसको काटकर एक Cone बनाया जाता है जिसकी Radius 7/2 cm और height 3cm है इसको बनाने में कितने % Material Waste होगा।

Volume of Material Used (in %)

$$= \frac{\text{Volume of Cone}}{\text{Volume of Cuboid}} \times 100\%$$
$$= \frac{1}{3} \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times \frac{3 \times 100}{10 \times 7 \times 5}$$
$$= 11\%$$

Waster Material $= 100 - 11 = 89\%$

Volume of Wasted Material

$$= \text{Volume of Cuboid} - \text{Volume of Cone}$$
$$= 10 \times 7 \times 5 - \frac{1}{3} \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times 3$$
$$350 - \frac{77}{2} = \frac{700 - 77}{2} = \frac{623}{2}$$

Volume of ~~wate~~ Waster Material (%)

$$= \frac{623 \times 100}{2 \times 350} = 89\%$$

एक घनाभ की लंबाई, चौड़ाई और ऊंचाई क्रमशः 10cm, 5cm और 2cm है इसे काटकर एक Cm त्रिज्या और 7cm Height का एक Cone बनाया जाता है, इसको बनाने में कितने % Material Use होगा और कितना Waste होगा।

$$\text{Volume of Material Used in \%} = \frac{1}{3} \times \frac{22}{7} \times 1 \times 1 \times 7 \times 100 \,/\, 10 \times 5 \times 2 = \frac{22}{3}\% = 7.33\%$$

Waster Material $= 100 - 7.33 = 92.67\%$

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Next Type :→

$$1 \text{ Lit} = 1000 \text{ cm}^3$$
$$\therefore 1 \text{cm}^3 = \frac{1}{1000} \text{ Lit}$$

$$1 \text{ Lit} = 1000 \text{ cm}^3 = 1000 \left(\frac{1}{100} \text{m}\right)^3$$
$$= 1000 \times \frac{1}{100} \times \frac{1}{100} \times \frac{1}{100} \text{ m}^3 = \frac{1}{1000} \text{ m}^3$$

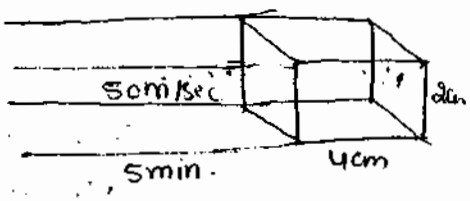
$$\therefore 1 \text{ Lit} = \frac{1}{1000} \text{ m}^3$$
$$1 \text{m}^3 = 1000 \text{ Lit}$$

Next Type :→ कोई भी Pipe है Circular / Cuboid के Dimensions / Radius दी गई हो और उसमें बहने वाली पानी की speed दी गई और Time दिया गया है तो हम Volume निकाल सकते हैं।

$$\text{Volume} = \text{Area} \times \text{Speed} \times \text{Time}$$
$$= \text{Area} \times \text{Distance}$$
$$= \text{Area} \times \text{Length}$$

Eg → एक घनाभाकार Pipe की breadth और Height क्रमशः 4cm और 2cm है। और इसमें पानी 50m/sec की चाल से बहता है और 5min के लिए बहता है Find the Volume.

$$\text{Area} = \frac{4 \times 2}{100 \times 100} \text{ cm}^2$$



$$(\text{Length}) \text{ Distance} = \text{Speed} \times \text{Time}$$
$$= 50 \times 5 \times 60$$

$$\text{Volume} = \text{Area} \times \text{Length}$$
$$= \frac{4}{100} \times \frac{2}{100} \times 50 \times 5 \times 60 = 12 \text{m}^3 = 12000 \text{ Lit}.$$

अगर कहे कि यह पानी एक टंकी (आयताकार षट्फलक) में डाला जाता है जिसकी लंबाई 4m और चौड़ाई 2m है find the Height

$$\text{Volume of Tank} = \text{Volume of Pipe}$$
$$4 \times 2 \times h = 3$$
$$h = \frac{3}{2} \text{ m}.$$

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एक वृत्ताकार नाली जिसका व्यास 14cm है इससे पानी 5km/hr की चाल से बहते हुए एक आयताकार टंकी में गिराया जाता है जिसकी लंबाई 50m और चौड़ाई 44m है, 7cm तक टंकी भरने में कितना समय लगेगा।

$r = \frac{14}{2} = 7cm = \frac{7}{100}m$

Speed $= 5 \times \frac{5}{18} = \frac{25}{18}$ m/sec

Volume of Tank = Volume of Pipe

$50 \times 44 \times \frac{7}{100}$ = Area × Speed × Time

$\cancel{50}^{2} \times 44 \times \frac{7}{\cancel{100}} = \frac{22}{7} \times \frac{7}{100} \times \frac{7}{100} \times \frac{25}{18} \times t$

$t = 2 \times 2 \times 18 \times 100 = 7200$ sec.

$= \frac{\cancel{7200}^{\cancel{30}\,2}}{\cancel{60} \times \cancel{60}} = 2$ Hour.

एक वृत्ताकार नल जिसका व्यास 20cm है इससे पानी 3km/hr की Speed से बहते हुए एक बेलनाकार टंकी में गिराया जाता है जिसका व्यास 10m और गहराई 2m है कितनी देर में टंकी पूरी भर जाएगी।

Volume of tank = Volume of Pipe.

Area × Speed × Time = Volume of tank

$\pi \times \frac{10}{100} \times \frac{10}{100} \times 3 \times \frac{5}{18} \times t = \pi \times 5 \times 5 \times 2$

$t = \frac{100 \times \overset{6}{\cancel{18}} \times 5 \times 2}{\cancel{3}} = 6000$ sec.

$= \frac{6000}{60} = 100$ min

$= 1$ Hour 40 min.

एक वृत्ताकार नल जिसका व्यास 5mm है इससे पानी 10m/min की चाल से बहते हुए एक शंक्वाकार टंकी में गिराया जाता है जिसका व्यास 40cm और Height 24cm है कितनी देर में टंकी पूरी भर जाएगी।

Volume of Pipe = Volume of Tank

Area × Speed × Time $= \frac{1}{3} \times \pi \times 20 \times 20 \times 24$

$\pi \times \frac{5}{2 \times 10} \times \frac{5}{2 \times 10} \times 10 \times 100 \times t = \frac{1}{\cancel{3}} \times \pi \times \overset{4}{\cancel{20}} \times \overset{4}{\cancel{20}} \times \overset{8}{\cancel{24}}$

$t = \frac{4 \times 4 \times 8 \times 2 \times 2}{10} = \frac{512}{10} = 51 \text{ min } \frac{2}{10} \times 60$

$= 51$ min 12 sec.

एक वृत्ताकार नल जिसका व्यास 2cm है इससे पानी 7m/sec से बहते हुए एक वृत्ताकार टंकी में गिराया जाता है जिसकी त्रिज्या 40cm है आधे घंटे में टंकी में कितना पानी भर जाएगा।

Area × Speed × Time $= \pi \times r^2 \times h$

$\pi \times 1 \times 1 \times 7 \times 100 \times \overset{30}{\cancel{30}} \times \overset{60}{\cancel{60}}$ (15, 15) $= \pi \times 40 \times \overset{40}{\cancel{40}} \times h$ (2)

$h = \frac{7 \times 15 \times 15}{2} = \frac{1575}{2} = 787.5$ cm

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एक वृत्ताकार नल जिसका व्यास 7cm है इससे पानी 72 cm/sec की speed से बहता है। 1 घंटे में कितने Lit पानी बाहर निकल जाएगा।

$$\text{Volume of water come out} = \text{Area} \times \text{Speed} \times \text{Time}$$

$$= \frac{22}{7} \times \frac{7}{2} \times \frac{1}{2} \times \frac{72 \times 60 \times 60}{100}$$

Time = 1 Hour = 60×60 sec

$$= \frac{99792}{10} = 9979.2 \text{ Lit.}$$

→ एक वृत्ताकार नल जिसका व्यास 7cm है इसमें पानी 192.5 Lit/min की speed से बह रहा है पानी की speed km/hr में क्या होगी।

$$\text{Volume} = \text{Area} \times \text{Speed} \times \text{Time}$$

$$\text{Now Volume} = 192.5 \times \frac{1000 \text{ cm}^3}{1 \text{ min}} \times 60 \text{ min}$$

[∵ हमें speed km/hr में बतानी है ∴ 60 min से Multiply किया]

$$\therefore \frac{1925}{10} \times 1000 \times 60 = \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times \text{Speed} \times 1 \text{ Hour}$$

$$\text{Speed} = 300000 \text{ cm/hr}$$

$$= \frac{300000}{100 \times 1000} \text{ km/hr} = 3 \text{ km/hr.}$$

→ एक अर्धगोलाकार टंकी का व्यास 3m है जोकि पानी से भरा है यह एक पाईप से जुड़ा है यह $3\frac{4}{7}$ l/sec की दर से बाहर निकलता है आधी टंकी खाली होने में कितना समय लगेगा।

$$\text{Volume of water tank} = 3\frac{4}{7} \frac{\text{lit}}{\text{time}} \times \text{time}$$

[∵ $\frac{\text{Volume}}{\text{time}} \times \text{time} = \text{Volume}$]

$$\frac{2}{3} \times \frac{22}{7} \times \frac{300}{2} \times \frac{300}{2} \times \frac{300}{2} = \frac{25}{7} \times 1000 \times \text{time}$$

[1 Lit = 1000 cm³]

$$\text{Time (for full tank)} = 22 \times 30 \times 3 \text{ sec.}$$

$$\therefore \text{for Half Water Tank} = \text{Time} = \frac{22 \times 30 \times 3}{2} = 990 \text{ sec.}$$

$$= \frac{990}{60} = 16.5 \text{ min.}$$

एक अर्धगोलाकार टंकी की त्रिज्या 1.75 m है पानी से भरी है यह एक पाईप से जुड़ी है यह पाईप 7 Lit/sec की speed से पानी बाहर निकालती है टंकी को खाली होने में कितने समय लगेगा।

$$\frac{2}{3} \pi r^3 = \frac{7 \text{ Lit}}{\text{time}} \times \text{time}$$

$$\Rightarrow \frac{2}{3} \times \frac{22}{7} \times 175 \times 175 \times 175 = 7 \times 1000 \times \text{time}$$

$$t = \frac{2 \times 22 \times 5 \times 175}{3 \times 4} = \frac{9625}{6} \text{ sec.}$$

$$= \frac{9625}{6 \times 60} = \frac{1925}{72} = 26.73 \text{ min.}$$

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VOLUME

एक आयताकार टंकी की लंबाई 50m और गहराई 29 m है इसमें से 1000m³ पानी निकाल लिया जाता है. जिससे पानी का level 2 m नीचे चला जाता है Find the Volume of Water tank and Find the Breadth of tank.

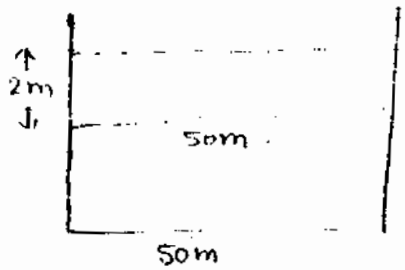
Volume of Water Decrease = 1000m³

$\therefore 50 \times b \times 2 = 1000$

$b = 10m$

$\therefore$ Volume of water Tank $= 50 \times b \times h$

$= 50 \times 10 \times 29 = \underline{14500 m^3}$



OR

$50 \times b \times (29-2) + 1000 = 50 \times b \times 29$

$50 \times b \times 27 + 1000 = 50 \times b \times 29$

$50 \times b \times 29 - 50 \times b \times 27 = 1000$

$50 \times b \times 2 = 1000 \quad \therefore b = 10m$

$\therefore$ Volume of water tank $= 50 \times 10 \times 29 = 14500m^3$.

एक आयताकार टंकी की लंबाई 15m और गहराई 27m है इसमें से 450m³ पानी निकाल लिया जाता है जिससे पानी का level 3m नीचे चला जाता है Find the Volume of Water tank and find the Breadth of tank.

Volume of Water Decreas = 450m³ Heigrd = 3m, Length = 15m

$15 \times b \times 3 = 450$

$b = \frac{450}{45} = 10m$

$\therefore$ Volume of water tank $= 15 \times 10 \times 27 = \underline{4050m^3}$.

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Next Type :→

In Closed Box

$l_{ext} = l_{int} + 2x$

$b_{ext} = b_{int} + 2x$

$h_{ext} = h_{ext} + 2x$

In Open Box

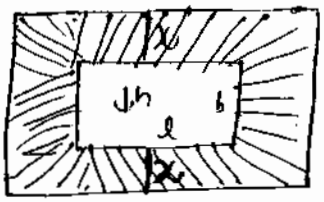
$l_{ext} = l_{int} + 2x$

$b_{ext} = b_{int} + 2x$

$h_{ext} = h_{int} + x$

[∵ Open Box में ऊपर की Height Same रहती है इसलिए एक बार Thickness को Subtract करेंगे]

Box



$$\boxed{\text{Volume of Material Used} = \text{External Volume} - \text{Internal Volume}}$$

→ एक Closed Box के बाहरी माप क्रमशः 115, 75 और 35cm है और इसकी मोटाई 2.5 cm है. Find the Volume of the Material Used.

External Volume of Box $= 115 \times 75 \times 35 = 301875\ cm^3$

Internal Volume of Box $= (115 - 2\times2.5)(75 - 2\times2.5) \times (35 - 2\times2.5)$

$= 110 \times 70 \times 30 = 231000\ cm^3$

Volume of Material Used $= 301875 - 231000 = 70875\ cm^3$

→ एक खुली टंकी की धारिता 8000 Lit पानी की है. इसके बाहरी माप 3.3 m, 2.6 m और 1.1 m है. इसकी दीवारों की मोटाई 5cm है, Find the thickness of Bottom.

Internal Volume = 8000 Lit

External Dimensions = 330 cm, 260 cm, 110 cm.

∴ Internal Dimensions : $(330 - 2\times5)$, $(260 - 2\times5)$, $(110 - x)$ [Open Box]

$= 320, 250, 110 - x$

Now

$320 \times 250 \times (110 - x) = 8000$ Lit

$\Rightarrow 320 \times 250 \times (110 - x) = 8000 \times 1000$

$110 - x = \dfrac{\cancel{8000}^{100} \times \cancel{1000}\ 4}{\cancel{320}_{8} \times \cancel{250}}$

$x = 110 - 100 = 10cm$ ✓

∴ Thickness of Bottom = 10cm.

Next Type :→ एक निश्चित क्षेत्र में 2cm तक बारिश हुई और यह 6 Hect. Area को Cover करती है. Find the Volume of Water ~~be~~

$$\boxed{1\ \text{Hect} = 10000\ m^2}$$

Height $= \frac{2}{100} m$

Volume = Area × Height

$= 6 \times 10000 \times \frac{2}{100} = 1200 m^3$

$= 1200 \times 1000 = 1200000$ Lit.

एक निश्चित क्षेत्र में 2cm तक बारिश हुई और यह 1 Square km Area को Cover करती है 50% पानी को एक बर्तन में जमा किया जाता है उस बर्तन की Length 100 m और Breadth 10 m है Find the Level of water.

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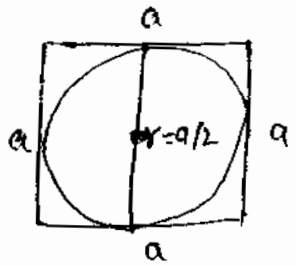
Total Volume of water flow = $1000 \times 1000 \times \frac{2}{100} = 20000\ m^3$

Volume of water tank = $20000 \times \frac{50}{100} = 10000\ m^3$

$1000 \times h \times 10 = 10000$

$h = 10\ m$.

एक घन के अन्दर 1 गोला इस प्रकार से रखा जाता है वह घन की सभी भुजाओं को स्पर्श करता है तो गोले ने घन की कितनी % Volume को cover कर रखा है।



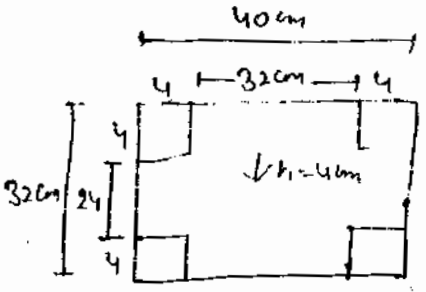
Area of Cube = a^3

Volume ~~Area~~ of Sphere = $\frac{4}{3} \times \pi \times \frac{a}{2} \times \frac{a}{2} \times \frac{a}{2}$

% Area Covered by Sphere = $\frac{4}{3} \times \frac{\pi \times a \times a \times a}{2 \times 2 \times 2} \times \frac{50}{100} \times \frac{1}{a^3}$

$= \frac{50\pi}{8}\%$

एक आयताकार कागज की टुकड़े की लंबाई और चौड़ाई क्रमशः 40cm और 32cm है इसके चारों corner पर 4cm के square काट लिए जाते हैं और इसे मोड़कर एक दानाभ बनाया जाता है Find the Volume and Area of Cuboid



Length = $40 - 2 \times 4 = 32\ cm$

Breadth = $32 - 2 \times 4 = 32 - 8 = 24\ cm$.

Height = 4cm.

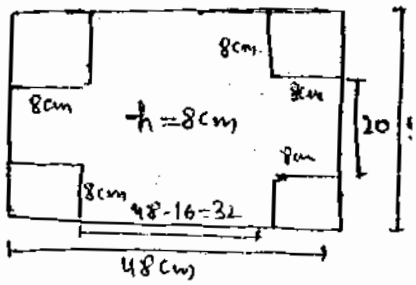
Volume of Cuboid = $32 \times 24 \times 4 = 3072\ cm^3$

Area = $lb + 2bh + 2hl$ [∵ उपर का surface खुला होता है, तो एक बार lb होगा।]

$= 32 \times 24 + 2 \times 24 \times 4 + 2 \times 32 \times 4$

$= 768 + 192 + 256 = 1216\ cm^2$

एक आयताकार कागज के टुकड़े की लंबाई और चौड़ाई क्रमशः 48cm और 36cm है इसके प्रत्येक corner पर एक वर्ग काटा जाता है जिसकी प्रत्येक भुजा की लंबाई 8cm है इसे मोड़कर एक दानाभ बनाया जाता है Find the Volume and Total Surface Area of Cuboid.



Length = $48 - 2 \times 8 = 32\ cm$

Breadth = $36 - 2 \times 8 = 20\ cm$

Height = 8cm

Volume ~~Area~~ = $32 \times 20 \times 8 = 5120\ cm^3$

Area = $lb + 2bh + 2hl$

$= 32 \times 20 + 2 \times 20 \times 8 + 2 \times 8 \times 32$

$= 640 + 320 + 2 \times 256$

$= 640 + 320 + 512 = 1472\ cm^2$

एक 8 मीटर लंबे, 6 मीटर चौड़े और 22.5m ऊंची दीवार बनाने के लिए कितनी ईंटों की आवश्यकता होगी अगर ईंट की लंबाई, चौड़ाई और ऊंचाई क्रमशः 25cm, 11.25cm और 6cm है।

No of Bricks = $\frac{\text{Volume of Wall}}{\text{Volume of 1 Bricks}}$

$= \frac{8 \times 6 \times 22.5 \times 100 \times 100 \times 100 \times 100}{25 \times 11.25 \times 6} = 640000$

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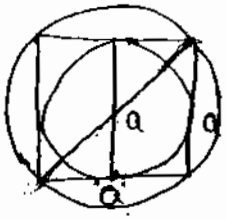
एक गोले का व्यास $500\sqrt{3}$ मीटर है इसके अन्दर बड़े से बड़े आकार का एक घन बनाया जाता है और इस घन के अन्दर बड़े से बड़े आकार का गोला बनाया जाता है Find the Ratio of Volume of Cube and Volume of Small Sphere.



Now Diagnol of Cube = Diameter of Big Sphere

$$\sqrt{3}a = 500\sqrt{3}$$

$$a = 500\,m.$$

$\therefore$ Side of Cube = 500m.

Now Diameter of Small Sphere = Side of Cube

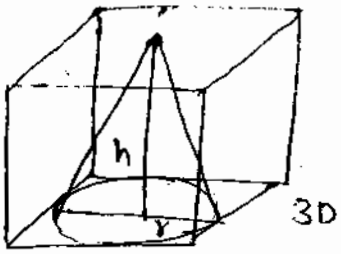
$$r = \frac{500}{2} = 250m.$$

$$V_{cube} : V_{sphere} = a^3 : \frac{4}{3}\pi r^3$$

$$= \cancel{500}\times\cancel{500}\times\cancel{500} : \frac{\cancel{4}^{2}}{3}\times\frac{22}{7}\times\cancel{250}\times\cancel{250}\times\cancel{250}$$

$$= 21 : 11 \quad Ans.$$

एक घन के प्रत्येक भुजा की लंबाई 42cm है इसके अन्दर बड़े से बड़े आकार का शंकु बनाया जाता है Find the Volume of Cone



Height of Cone = Side of Cube = 42cm

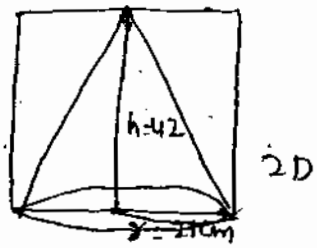
$$\text{Radius} = \frac{a}{2} = \frac{42}{2} = 21cm.$$

$\therefore$ Volume of ~~Cube~~ Cone $= \frac{1}{3}\pi r^2 h$

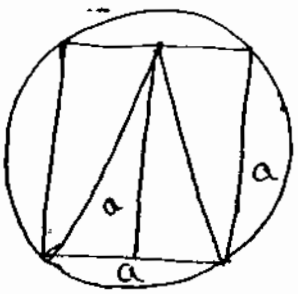
$$= \frac{1}{3}\times\frac{22}{7}\times 21\times 21\times 42$$

$$= 22\times 21\times 42$$

$$= 19404\,cm^3.$$



एक गोले का व्यास $28\sqrt{3}$ cm है इसके अन्दर बड़े से बड़े आकार का घन बनाया जाता है और इस घन के अन्दर बड़े से बड़े आकार का शंकु बनाया जाता है Find the Ratio of Volume of Cube and Cone.



Diagnol of Cube = Diameter of Big Sphere

$$\sqrt{3}a = 28\sqrt{3}$$

$$a = 28\,cm.$$

$\therefore$ Side of Cube = 28cm.

Height of Cone = Side of Cube = 28cm

$$\text{Radius} = \frac{a}{2} = \frac{28}{2} = 14cm$$

$$V_{cube} : V_{cone} = \cancel{28}\times\cancel{28}\times\cancel{28} : \frac{1}{3}\times\frac{22}{7}\times\cancel{14}\times\cancel{14}\times\cancel{28}$$

$$= 42 : 11$$

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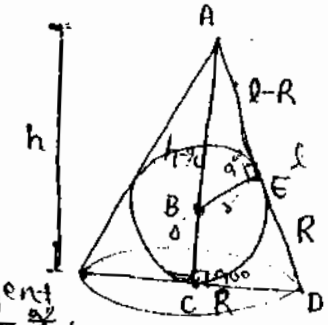
अगर एक Cone [illegible] तो find the Volume of Sphere.

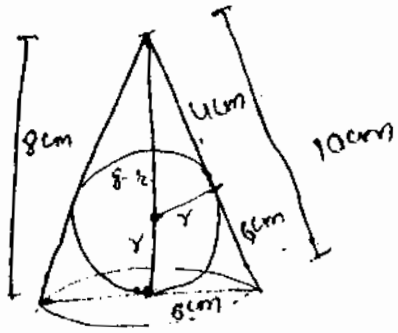


Now $\Delta ABE \sim \Delta ACD$

$$\therefore \frac{h-r}{l} = \frac{r}{R} = \frac{l-R}{h}$$

$ED = CD = R$ [किसी भी Exterior Point से Circle पर 2 Tangent Drawn करते हैं, तो दोनों Equal Length के होते हैं।

→ एक शंकु की त्रिज्या 6cm है और Height 8cm है इसके अन्दर बड़े से बड़े आकार का गोला बनाया जाता है find the Volume of that Sphere.



Now $l = \sqrt{8^2+6^2} = \sqrt{100} = 10cm$.

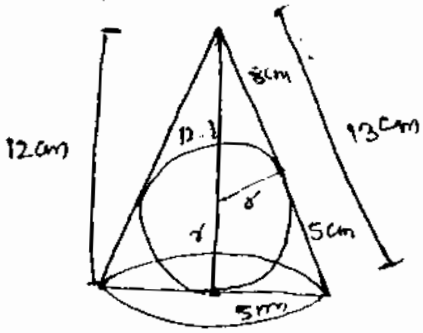
Now Both Δ are same.

$$\therefore \frac{r}{R} = \frac{l-R}{h} \Rightarrow \frac{r}{6} = \frac{10-6}{8}$$

$$\frac{r}{6} = \frac{4}{8} \qquad r = 3$$

Volume of Sphere $= \frac{4}{3} \times \frac{22}{7} \times 3 \times 3 \times 3 = 36 \times \frac{22}{7} = 36\pi\ cm^3$

एक शंक्वाकार बर्तन की त्रिज्या 5cm है और ऊंचाई 12cm है इसके अन्दर बड़े से बड़े आकार का Sphere बनाया जाता है find the Ratio of Volume of Cone and Sphere.



$$l = \sqrt{12^2+5^2} = \sqrt{169} = 13cm$$

$$\therefore \frac{r}{5} = \frac{13-5}{12} \qquad \frac{r}{5} = \frac{8}{12} = \frac{2}{3}$$

$$r = \frac{10}{3} cm.$$

Now $V_{cone} : V_{sphere} = \frac{1}{3}\pi R^2 h : \frac{4}{3}\pi r^3$

$$= \frac{1}{3}\pi \times 5 \times 5 \times 12 : \frac{4}{3}\pi \times \frac{10}{3} \times \frac{10}{3} \times \frac{10}{3}$$

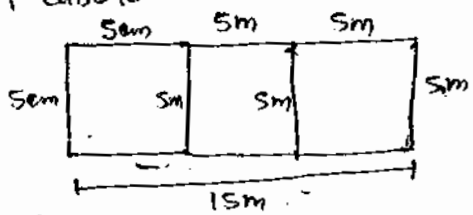
$$\Rightarrow 81 : 40$$

एक अर्द्धगोलाकार कटोरे का व्यास 36cm है यह पानी से भरा है यह Liquid एक बेलनाकार बर्तन में भरा जाता है जिसकी त्रिज्या 3cm है और Height 6cm है पूरे पानी को इन बर्तनों में डालने के लिए ऐसे कितने बर्तनों की आवश्यकता होगी।

$$\text{No. of Vessel} = \frac{\text{Volume of Hemisphere}}{\text{Volume of Cylinder}} = \frac{\frac{2}{3} \times \pi \times 18 \times 18 \times 18}{\pi \times 3 \times 3 \times 6}$$

$$= 72.$$

एक घन के प्रत्येक भुजा की लंबाई 5m है ऐसे तीन घन को आपस में जोड़कर एक घनाभ बनाया जाता है find the Volume of Cuboid and T.S.A of Cuboid.



∴ For a Cuboid

$l = 3 \times 5 = 15m, \quad b = 5m, \quad h = 5m$

Volume $= 15 \times 5 \times 5 = 375\ m^3$

T.S.A $= 2(15 \cdot 5 + 5 \cdot 5 + 5 \cdot 15) = 2(175) = 350\ m^2$

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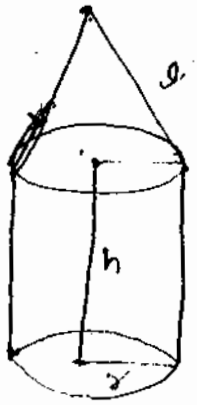
or

Volume of Cuboid $= 3 \times$ Volume of 1 Cube
$= 3 \times s^3 = 3 \times 125 = 375\ m^3$

T.S.A $= 3 \times$ T.S.A of Cube
$= 3 \times 6a^2 = 3 \times 6 \times 5^2 = 850\ m^2$

Next Type :→

किसी भी बेलनाकार टेंट के ऊपर शंक्वाकार भाग और बनाया जाता है Then



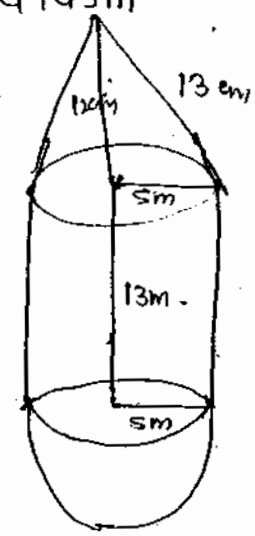
$$\boxed{\text{Total Volume} = \frac{1}{3}\pi r^2 h + \pi r^2 h}$$

$$\boxed{\text{कपड़े की आवश्यकता} = \text{L.S.A of Cylinder} + \text{L.S.A of Cone} = 2\pi r h + \pi r l}$$

एक Circus का टेंट 3m ऊंचाई तक बेलनाकार है इसके ऊपर शंक्वाकार है आधार का व्यास यदि 10m है और शंकु की तिर्यक ऊंचाई 53m हो तो इस टेंट को बनाने में कितने कपड़े की आवश्यकता होगी।

कपड़े की आवश्यकता $= 2\pi r h + \pi r l$
$= \pi r(2h + l)$
$= \frac{22}{7} \times \frac{105}{2}(2 \times 3 + 53) = \frac{\cancel{22}^{11}}{7} \times \frac{\cancel{105}^{15}}{\cancel{2}} \times 59$
$= 9745\ m^2$

एक खिलौने की shape बेलनाकार है उसके एक किनारे पर शंकु है और दूसरे किनारे पर अर्द्धगोला है यदि बेलन की ऊंचाई और त्रिज्या क्रमशः 13cm और 5cm है। अर्द्धगोले और शंकु की त्रिज्या बेलन की त्रिज्या के बराबर है और शंक्वाकार भाग की ऊंचाई 12cm है, find the T.S.A.



T.S.A = L.S.A of Cylinder + L.S.A of Cone + L.S.A of Hemisphere
$= 2\pi r h + \pi r l + 2\pi r^2$
$= \pi r(2h + l + 2r)$

Now Slant Height (l) of Cone $= \sqrt{12^2 + 5^2} = 13$ cm.

$\therefore$ T.S.A of Toy $= \frac{22}{7} \times 5(2 \times 13 + 13 + 2 \times 5)$
$= \frac{22}{7} \times 5 \times 49 = 770\ cm^2$.

एक खिलौने की shape बेलनाकार है इसके दोनों किनारों पर अर्द्धगोला है यदि खिलौने की कुल ऊंचाई 19cm है और आधार का व्यास 7cm find the Volume and T.S.A of Toy.

Now TSA = L.S.A of Cylinder + LSA of Hemisphere + L.S.A of Hemisphere
$= 2\pi r h + 2\pi r^2 + 2\pi r^2$
$= 2\pi r h + 4\pi r^2$
$= 2\pi r(h + 2r)$

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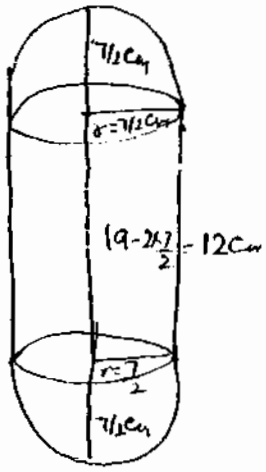


Volume = Volume of Cylinder + 2 × Volume of Hemisphere

$= \pi r^2 h + 2 \times \frac{2}{3}\pi r^3$

$= \pi r^2 h + \frac{4}{3}\pi r^3$

Height of Cylinder $= 19 - 2 \times \frac{7}{2} = 12$ cm.

$\therefore$ T.S.A of Toy $= 2\pi r(h + 2r)$

$= 2 \times \frac{22}{7} \times \frac{7}{2}\left(12 + \frac{7}{2} \times 4\right) = \cancel{\frac{22 \times 31}{7}} = 22 \times 19$

$= 418\ cm^2$.

Volume $= \pi r^2\left(h + \frac{4}{3}r\right)$

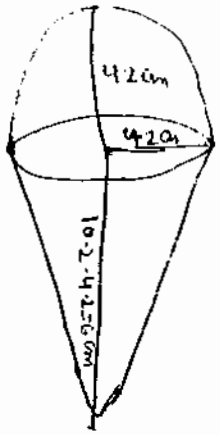
$= \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2}\left(12 + \frac{4}{3} \times \frac{7}{2}\right)$

$= \frac{77}{2}\left(12 + \frac{14}{3}\right) = \frac{77}{2} \times \frac{50}{3} = \frac{1925}{3} = 641\frac{2}{3}\ cm^3$

एक खिलौना शंकु के आकार का है और उसके सिर के ऊपर अर्धगोला है यदि खिलौने की पूरी ऊँचाई 10.2 cm है, Radius of Hemisphere is 4.2 cm. find total Volume of Toy.

Radius = 4.2 cm

$\therefore$ Height of Cone $= 10.2 - 4.2 = 6$ cm.



Volume of Toy $= \frac{1}{3}\pi r^2 h + \frac{2}{3}\pi r^3$

$= \frac{1}{3}\pi r^2(h + 2r)$

$= \frac{1}{3} \times \frac{22}{7} \times \frac{42}{10} \times \frac{42}{10} \times \left(6 + \frac{84}{10}\right)$

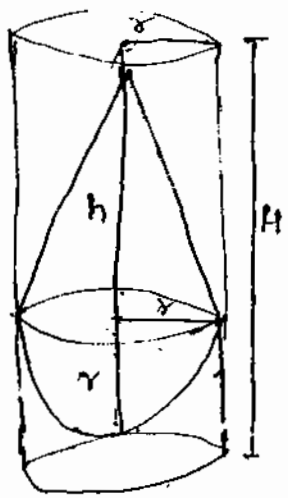
$= \frac{22 \times 2 \times 42 \times 144}{10 \times 10 \times 10} = \frac{266112}{1000} = 266.112\ cm^3$.

एक खिलौना अर्धगोलाकार है इसके सिर के ऊपर शंकु है और शंकु की ऊँचाई 2 cm है आधार का व्यास 4 cm है। खिलौने को एक बेलनाकार डिब्बे इस प्रकार से रखा जाता है कि यह खिलौना पूरी तरह से फिट हो जाता है। Find the Volume of the Remaining Portion.

Volume of Remaining Portion

= Volume of Cylinder − (Volume of Cone + Volume of Hemisphere)

$= \pi r^2 H - \left(\frac{2}{3}\pi r^3 + \frac{1}{3}\pi r^2 h\right)$



Now h (Height of Cone) = 2 cm.

r = 2 cm.

$\therefore$ Height of Cylinder (H) = h + r = 2 + 2 = 4 cm

$\therefore$ Volume $= \pi \times 2 \times 2 \times 4 - \left(\frac{2}{3}\pi \times 2^3 + \frac{1}{3}\pi \times 2 \times 2 \times 2\right)$

$16\pi - \left(\frac{16}{3}\pi + \frac{8\pi}{3}\right)$

$\frac{48\pi - 24\pi}{3} = \frac{24\pi}{3} = 8\pi\ cm^3$.

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एक ठोस गोले की त्रिज्या 14cm है। इसे 4 बराबर भागों में काट दिया जाता है। इन चारों भाग का T.S.A क्या होगा।

$$T.S.A \text{ of All Parts} = 8\pi r^2$$
$$= 8 \times \frac{22}{7} \times 14 \times 14 = 4928 cm^2$$

Next Type:- एक दालान के आकार की टंकी की लंबाई, चौड़ाई और ऊँचाई क्रमशः 30m, 20m और 10m है। इसमें 5m तक पानी भरा जाता है कितना Area उसका भिगा हुआ है।

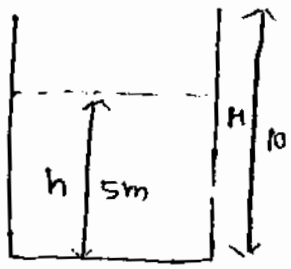
$l = 30, b = 20m$

भिगा हुआ Area $= lb + 2bh + 2hl$

$= 2(l+b)h + lb$ [h = Height जहां तक पानी है]

$= 2(30+20) \times 5 + 30 \times 20$

$= 500 + 600 = 1100m^2$.



एक घनाभ की लंबाई, चौड़ाई और ऊँचाई का अनुपात क्रमशः 4:3:2 है और T.S.A 1872 cm^2 है find the Volume of Cuboid

$l : b : h = 4 : 3 : 2$

$\Rightarrow 4x, 3x, 2x$

Now $T.S.A = 2(lb + bh + hl)$

$\Rightarrow 2(4x \cdot 3x + 3x \cdot 2x + 2x \cdot 4x) = 1872$

$2(12x^2 + 6x^2 + 8x^2) = 1872$

$2 \cdot 26x^2 = 1872$

$x^2 = \frac{1872}{2 \times 26}$ 72 36 $\quad x = 6cm$.

$\therefore l = 6 \times 4 = 24cm$, $b = 3 \times 6 = 18cm$, $h = 2 \times 6 = 12cm$.

$\therefore$ Volume $= 24 \times 18 \times 12 = 5184cm^2$

Next Type:- If a Reactangular Paper is folded Along the length to form a Cylinder Then

$$\boxed{2\pi r = l \quad \text{and} \quad h = b}$$

and if Paper is folded Along the Bredth

$$\boxed{2\pi r = b \quad \text{and} \quad h = l}$$

- एक आयताकार कागज के टुकड़े की लंबाई और चौड़ाई क्रमशः 22cm और 14cm है इसे क्रमशः लंबाई और चौड़ाई की दिशा में मोड़कर एक बेलन बनाया जाता है find the Volume in Both case.

1 folded Along the Length

then $2\pi r = 22 \qquad h = b$

$2 \times \frac{22}{7} \times r = 22 \qquad h = 14cm$

$r = \frac{7}{2} cm$

$\therefore$ Volume $= \pi r^2 h = \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times 14 = 539 cm^3$

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(ii) folded Along the Breadth

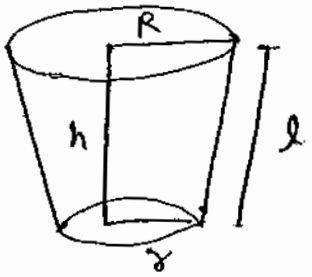
then $2\pi r = b$ $\quad h = l$

$2\pi r = 14$ $\quad h = 22\,cm$

$r = \frac{7}{\pi}$ $\quad h = 22\,cm$

$$\text{Volume} = \cancel{\pi} \times \frac{7}{\cancel{\pi}} \times \frac{7}{\pi} \times 22 = \frac{7 \times 7 \times 7 \times \cancel{22}}{\cancel{22}} = 343\,cm^3$$

Next Type :→ FRUSTUM OF A CONE [छिन्नक]



$$\boxed{\text{Slant Height} = \sqrt{h^2 + (R-r)^2}}$$

$$\boxed{L.S.A = \pi(R+r)\,l}$$

$$\text{T.S.A [Hollow form]} = \pi(R+r)\,l + \pi r^2$$

$$\text{TSA [Solid form]} = \pi(R+r)\,l + \pi r^2 + \pi R^2$$

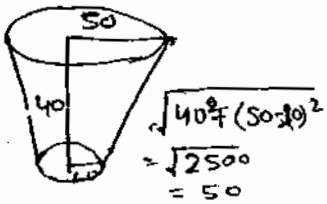
$$\boxed{\text{Volume} = \frac{1}{3}\pi(R^2 + r^2 + Rr)\,h}$$

एक छिन्नक के ऊपरी भाग और निचले भाग की त्रिज्या 50cm और 20cm है और Height 40cm है, find the slant Height.

$$\text{Slant Height } (l) = \sqrt{h^2 + (R-r)^2}$$

$$= \sqrt{40^2 + (50-20)^2}$$

$$= \sqrt{1600 + 30^2} = \sqrt{2500} = 50\,cm.$$



एक बाल्टी के ऊपरी भाग और निचले भाग की त्रिज्या 21cm और 14cm है बाल्टी की ऊँचाई 60cm है; Find the Volume.

$$\text{Volume} = \frac{1}{3}\pi(R^2 + r^2 + Rr)\,h$$

$$= \frac{1}{\cancel{3}} \times \frac{22}{7}(21^2 + 14^2 + 21 \cdot 14) \times 60$$

$$= \frac{1}{3} \times \frac{22}{7}(441 + 196 + 294) \times 60$$

$$= \frac{1}{3} \times \frac{22}{7} \times \overset{133}{\cancel{931}} \times \overset{20}{\cancel{60}} = 58520\,cm^3$$

एक बाल्टी के ऊपरी भाग और निचले भाग की परिधि क्रमशः 18cm और 6cm है और Slant Height 4cm है find the LSA of frustum.

$2\pi R = 18$ $\quad 2\pi r = 6$ $\quad l = 4$

$R = \frac{9}{\pi}$ $\quad r = \frac{3}{\pi}$ $\quad l = 4$

$$L.S.A = \pi(R+r)\,l = \pi\left(\frac{9}{\pi} + \frac{3}{\pi}\right) \times 4 = \pi \times \frac{12}{\pi} \times 4 = 48\,cm^2$$

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एक बाल्टी के ऊपरी भाग और निचले भाग की त्रिज्याएं क्रमशः 14cm और 6cm है, और Height 6 cm Find L.S.A.

$$l = \sqrt{6^2 + (14-6)^2} = \sqrt{6^2+8^2} = \sqrt{100} = 10\,cm$$

$$LSA = \pi(R+r)l = \pi(14+6)\times 10 = \pi \times 20 \times 10 = 200\pi\ cm^2.$$

एक बाल्टी के ऊपरी भाग और नीचले भाग की परिधि क्रमशः 48cm और 36 cm है और उसकी त्रिज्या Height 11cm है Find Volume

$$2\pi R = 48 \quad (24) \qquad 2\pi r = 36$$

$$R = \frac{24}{\pi} \qquad r = \frac{18}{\pi}$$

$$Volume = \frac{1}{3}\pi(R^2+r^2+Rr)h = \frac{1}{3}\pi\left[\left(\frac{24}{\pi}\right)^2+\left(\frac{18}{\pi}\right)^2+\left(\frac{24}{\pi}\right)\left(\frac{18}{\pi}\right)\right]\times 11$$

$$= \frac{1}{3}\pi\left[\frac{576}{\pi^2}+\frac{324}{\pi^2}+\frac{432}{\pi^2}\right]\times 11$$

$$= \frac{1}{3}\pi \times \frac{1332}{\pi^2}\times 11 = \frac{1}{3}\times\frac{1332}{22}\times 7\times 11$$

$$= 1554\,cm^2$$

एक बाल्टी का आयतन 28.49 lit है इसकी ऊपरी और निचले भाग की त्रिज्या 28cm और 21cm है find the Height

$$Volume = 28.49\,lit = \frac{2849}{100}\times 1000 = 28490\,cm^3$$

$$28490 = \frac{1}{3}\times\frac{22}{7}(28^2+21^2+28\cdot 21)\times h = \frac{1}{3}\times\frac{22}{7}(784+441+588)\times h$$

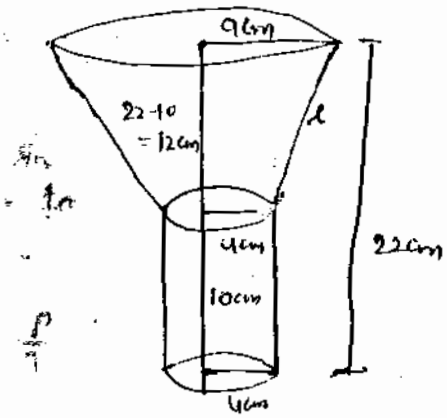
$$28490 = \frac{1}{3}\times\frac{22}{7}\times 1813\times h$$

$$h = \frac{2849\times 10\times 3\times 7}{22\times 1813} = 15\,cm$$

एक कीप 10cm ऊँचाई तक बेलनाकार है और उसका ऊपरी भाग शंक्वाकार छिन्नक है। यदि कीप की कुल ऊँचाई 22cm है और बेलनाकार भाग का व्यास 8cm है। funnel के ऊपरी भाग का व्यास 18cm है कीप बनाने के लिए कितना Area of टीन चाहिए।

$$l = \sqrt{h^2+(R-r)^2}$$

$$= \sqrt{12^2+(9-4)^2} = \sqrt{144+25} = 13\,cm.$$



$$Area\ of\ funnel = 2\pi rh + \pi(R+r)l$$

$$\Rightarrow \pi(2\times 4\times 10 + (9+4)\,13)$$

$$= \pi(80+169)$$

$$= 249\pi\ cm^2$$

Area = $2\pi rh + \pi(R+r)l$

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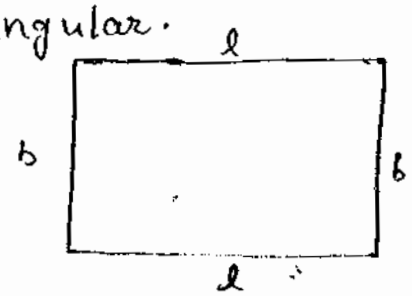
Important PRISM

L.S.A of Prism = Perimeter of Base × Height of Prism

T.S.A of Prism = L.S.A + 2 × Area of Base

Volume of Prism = Area of Base × Height of Prism

e.g → If the Base of Prism is ~~Circular~~ Rectangular.



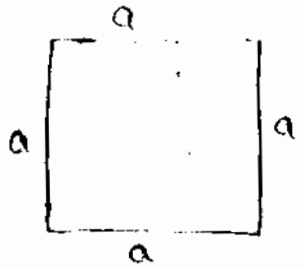
$L.S.A = \underbrace{2(l+b)}_{\text{Perimeter of Base}} \times h$

$T.S.A = 2(l+b)\times h + \underbrace{2lb}_{\text{Area of Base}}$

$= 2(lb+bh+hl)$

Volume $= lbh$

If the Base of Prism is Square.



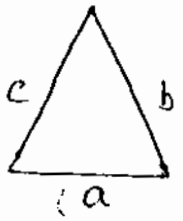
$L.S.A = 4ah$

$T.S.A = 4ah + 2\times a^2$

$= 2a(h+a)$

Volume $= a^2 \times h$.

If the Base of Prism is Triangular

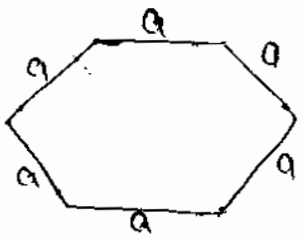


$L.S.A = (a+b+c)h$

$T.S.A = (a+b+c)h + 2$ Area of Base

Volume = Area of Base × h

If the Base of Prism is Regular Hexagonal



$L.S.A = 6ah$

$T.S.A = 6ah + 2$ Area of Base

$= 6ah + 2\times 6\times \frac{\sqrt{3}}{4}a^2$

Volume $= 6\times\frac{\sqrt{3}}{4}a^2\times h$

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एक प्रिज्म का आधार समबाहु त्रिभुजाकार का है जिसमें प्रत्येक भुजा की लंबाई 6cm है और प्रिज्म की ऊंचाई 15cm है Then Find T.S.A, L.S.A and Volume

L.S.A = Perimeter of Base × Height [Perimeter = 3a]

$= 3 \times 6 \times 15 = 270 cm^2$

T.S.A = LSA + 2 × Area of Base [Area of Base of Equilateral Δ $= \frac{\sqrt{3}}{4}a^2$]

$= 270 + 2 \times \frac{\sqrt{3}}{4} \times 6 \times 6$

$= 270 + 18\sqrt{3}\ cm^2$

Volume = Area of Base × Height

$= \frac{\sqrt{3}}{4} \times 6 \times 6 \times 15 = 135\sqrt{3}\ cm^3$

एक प्रिज्म का आधार वर्गाकार है जिसमें प्रत्येक भुजा की लंबाई 10cm और ऊंचाई 8cm है Then Find L.S.A, T.S.A and Volume of Prism.

L.S.A = Perimeter of Base × Height [Perimeter = 4a]

$= 4 \times 10 \times 8 = 320 cm^2$

T.S.A = L.S.A + 2 × Area of Base [Area of Base of Square $= a^2$]

$= 320 + 2 \times 10^2 = 320 + 200$

$= 520 cm^2$

Volume = Area of Base × Height

$= 10 \times 10 \times 8 = 800 cm^3$

एक प्रिज्म का आधार आयताकार है इसकी लंबाई और चौड़ाई क्रमशः 8 और 6cm है और ऊंचाई 12cm है Then Find L.S.A, T.S.A and Volume

L.S.A = ~~Area of~~ Perimeter of Base × Height

$= 2(8+6) \times 12$ [Perimeter = 2(l+b)]

$= 28 \times 12 = 336 cm^2$

T.S.A = L.S.A + 2 × Area of Base

$= 336 + 2 \times 8 \times 6 = 432 cm^2$

Volume = Area of Base × Height

$\Rightarrow 8 \times 6 \times 12 = 576 cm^3$

एक प्रिज्म का आधार सम षट्भुज का है यदि प्रत्येक भुजा की लंबाई 2m है और Prism की ऊंचाई 5m है find L.S.A, T.S.A and Volume

L.S.A = Perimeter of Base × Height

$= 6 \times 2 \times 5 = 60 cm^2$ [Perimeter of Hexagonal = 6a]

T.S.A = L.S.A + 2 × Area of Base

$= 60 + 2 \times 6 \times \frac{\sqrt{3}}{4} \times 2 \times 2$ [Area of Base $= 6 \times \frac{\sqrt{3}}{4}a^2$]

$= 60 + 12\sqrt{3}\ cm^2$

Volume $= 6 \times \frac{\sqrt{3}}{4} \times 2 \times 2 \times 5 = 30\sqrt{3}\ m^3$

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एक प्रिज्म का आधार समबाहु त्रिभुजाकार का है इसके प्रत्येक भुजा की लंबाई 6 cm है प्रिज्म का आयतन $108\sqrt{3}\ cm^3$ है find the L.S.A of Prism.

$$\text{Volume} = \text{Area of Base} \times \text{Height}$$

$$108\sqrt{3} = \frac{\sqrt{3}}{4} \times 6 \times 6 \times h \qquad \left[\text{Area of Equilateral } \Delta = \frac{\sqrt{3}}{4}a^2\right]$$

$$h = \frac{\cancel{108}^{3} \times 4}{\cancel{6 \times 6}} = 12\ cm$$

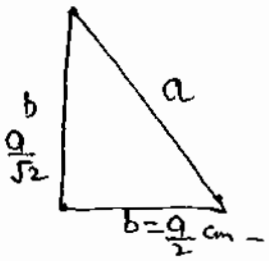
$$L.S.A = \text{Perimeter of Base} \times \text{Height}$$
$$= 3 \times 6 \times 12 = 216\ cm^2$$

एक प्रिज्म का आधार समबाहु त्रिभुजाकार है जिसका क्षेत्रफल $173\ cm^2$. If the Volume of the Prism is $10380\ cm^3$ find L.S.A of Prism [use $\sqrt{3} = 1.73$

$$\text{Area of Equilateral Triangle} = \frac{\sqrt{3}}{4}a^2 = 173$$
$$\Rightarrow \frac{\cancel{1.73}}{4}a^2 = \cancel{173}\ 100$$
$$a^2 = 400 \Rightarrow a = 20\ cm.$$

$$\text{Now Volume} = \text{Area of Base} \times \text{Height}$$
$$\text{Volume} = \frac{\sqrt{3}}{4}a^2 \times h$$
$$60\ \cancel{10380} = \cancel{173} \times h$$
$$h = 60\ cm$$
$$L.S.A = 3 \times 20 \times 60 = 3600\ cm^2$$

एक प्रिज्म का आधार समद्विबाहु समकोण त्रिभुज के आकार का है जिसके कर्ण की लंबाई a cm है और प्रिज्म की ऊँचाई h cm है Find the Volume.



$$\text{Now } b^2 + b^2 = a^2$$
$$2b^2 = a^2$$
$$b = \sqrt{\frac{a^2}{2}} = \frac{a}{\sqrt{2}}$$

$$\text{Volume} = \text{Area of Base} \times \text{Height}$$
$$= \frac{1}{2} \times \frac{a}{\sqrt{2}} \times \frac{a}{\sqrt{2}} \times h = \frac{1}{4}a^2h\ cm^3$$

एक त्रिभुज का आधार वर्काकार है यदि प्रिज्म की ऊँचाई 15 cm है If the TSA of Prism is $608\ cm^2$. Find the Volume.

$$T.S.A = L.S.A + 2\,\text{Area of Base}$$
$$608 = 4 \times a \times 15 + 2 \times a^2$$

$$2a^2 + 60a - 608 = 0$$
$$a^2 + 30a - 304 = 0$$
$$a(a+38) - 8(a+38) = 0$$
$$a = 8,\ a \neq -38$$
$$\text{Volume} = a^2h = 8 \times 8 \times 15 = 960\ cm^3.$$

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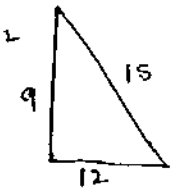
एक प्रिज्म का आधार त्रिभुजाकार है इसकी भुजा की लंबाई 9, 12, 15 cm है। और ऊंचाई 5cm है Find the T.S.A.



Area of Base $= \frac{1}{\cancel{2}} \times 9 \times \cancel{12}^{6} = 54\,cm^2$

Height = 5cm

T.S.A = L.S.A + 2 · Area of Base

$= (9+12+15) \times 5 + 2 \cdot 54$

$= 180 + 108 = 288\,cm^2$

$S = \frac{9+12+15}{2} = 18\,cm$

Area of Base $= \sqrt{18(18-9)(18-12)(18-15)}$

$= \sqrt{2 \cdot 9 \cdot 9 \cdot 2 \cdot 3 \cdot 3}$

$= 54\,cm^2$

T.S.A $= (9+12+15) \times 5 + 2 \cdot 54$

$= 180 + 108 = 288\,cm^2$

एक प्रिज्म का आधार समकोण त्रिभुज के आकार का है जिसके समकोण बनाने वाली भुजा की लंबाई 10 और 12cm है यदि इसकी ऊंचाई 20cm है If the Density of Material is 6g/cm³. Find the Total Weight.

Area of Base $= \frac{1}{\cancel{2}} \times 10 \times \cancel{12}^{6} = 60\,cm^2$

Volume = Area of Base × Height

$= 60 \times 20 = 1200\,cm^3$

$1\,cm^3 \longrightarrow 6g$

$1200\,cm^3 \rightarrow 1200 \times 6 = 7200\,g$

$= 7.2\,kg$.

एक प्रिज्म का आधार त्रिभुजाकार है जिसकी भुजा 8cm, 10cm और 12cm है यदि प्रिज्म की ऊंचाई 24cm हो, Find L.S.A and Volume.

$S = \frac{8+10+12}{2} = 15\,cm$

Area of Base $= \sqrt{15(15-8)(15-10)(15-12)}$

$= \sqrt{15 \cdot 7 \cdot 5 \cdot 3} = 15\sqrt{7}\,cm^2$

L.S.A = Perimeter of Base × Height $= (8+10+12) \times 24$

$= 30 \times 24 = 720\,cm^2$

Volume = Area of Base × Height

$= 15\sqrt{7} \times 24 = 360\sqrt{7}\,cm^3$

एक प्रिज्म का आधार समबाहु त्रिभुज के आधार का है जिसकी प्रत्येक भुजा की लंबाई 12cm है। If the Volume of Prism is 1080cm³ Then find T.S.A.

Volume = Area of Base × Height

$30\ \cancel{1080}\ \cancel{90} = \frac{\sqrt{3}}{4} \times 12 \times \cancel{12}^{3} \times h$

$h = \frac{30}{\sqrt{3}} = 10\sqrt{3}\,cm$

T.S.A = L.S.A + 2 × Area of Base

$= 3 \times 12 \times 10\sqrt{3} + 2 \times \frac{\sqrt{3}}{4} \times \cancel{12}^{3} \times 12$

$= 360\sqrt{3} + 72\sqrt{3} = 432\sqrt{3}\,cm^3$

एक प्रिज्म का आधार समकोण त्रिभुज के आकार का है जिसकी भुजा की लंबाई 5, 12 और 13cm है, यदि T.S.A of Prism is 360 cm² find the Volume of Prism.

$$\text{Area of Base} = \frac{1}{2} \times 5 \times 12 = 30 cm^2$$

$$T.S.A = L.S.A + 2 \times \text{Area of Base}$$

$$\Rightarrow 360 = (5+12+13) \times h + 2 \times 30$$

$$360 = 30h + 60$$

$$30h = 300 \quad h = 10 cm.$$

$$\text{Volume} = 30 \times 10 = 300 cm^3$$

एक प्रिज्म का आधार त्रिभुजाकार है जिसका परिमाप 28cm है Radius of Incircle of Triangle is 4cm. If the Volume of Prism 366 cm³ Then Find L.S.A

$$\text{Radius of Incircle} = \frac{\text{Area of } \Delta}{S}$$

$$4 = \frac{\text{Area of } \Delta}{14} \qquad \left[S = \frac{28}{2} = 14 cm\right]$$

$$\text{Area of } \Delta = 56 cm^2$$

$$\text{Volume} = \text{Area of Base} \times \text{Height}$$

$$\Rightarrow 366 = 56 \times h$$

$$h = \frac{\cancel{366}^{183}}{\cancel{56}_{28}} = \frac{183}{28}$$

$$L.S.A = \text{Perimeter of Base} \times \text{height}$$

$$= \cancel{28} \times \frac{183}{\cancel{28}} = 183 cm^2$$

एक प्रिज्म का आधार समचतुर्भुज के आकार का है जिसके दोनों विकर्णों की लंबाई 14cm और 18cm है। प्रिज्म की ऊंचाई 20cm है Find the Volume.

$$\text{Area of Base} = \frac{1}{2} \times 14 \times 18 = 126 cm^2$$

$$\text{Volume} = \text{Area of Base} \times \text{Height}$$

$$= 126 \times 20$$

$$= 2520 cm^3$$

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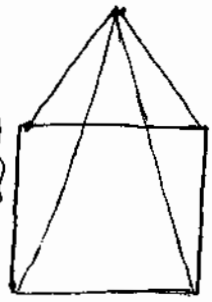
PYRAMID

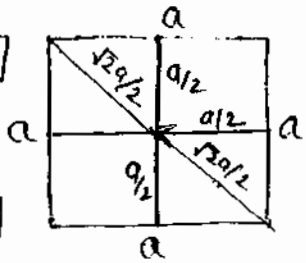


L.S.A. Of A Pyramid $= \frac{1}{2} \times$ Perimeter of Base $\times$ Height of Surface (h)

T.S.A of A Pyramide = L.S.A + Area of Base

Volume of a Pyramid $= \frac{1}{3} \times$ Area of Base $\times$ Height of Pyramid

e.g. When the Base of Pyramid is Square :→



$L.S.A = \frac{1}{2} \times 4a \times h$ [h = Height of Triangle Surface]

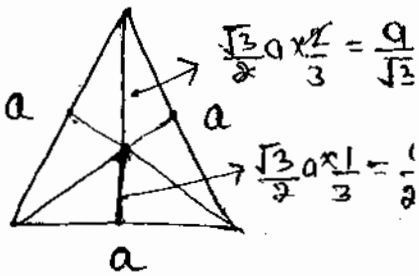
$T.S.A = L.S.A + \text{Area of Base } (a^2)$

$\text{Volume} = \frac{1}{3} \times a^2 \times H$ [H = Height of Pyramid]

Height of Triangular Surface (h) $= \sqrt{H^2 + \left(\frac{a}{2}\right)^2}$

Slant Height of Pyramid (l) $= \sqrt{H^2 + \left(\frac{\sqrt{2}a}{2}\right)^2} = \sqrt{H^2 + \left(\frac{a}{\sqrt{2}}\right)^2}$

When The Base of Pyramid is Equilateral Triangular :→



$L.S.A = \frac{1}{2} \times 3a \times h$

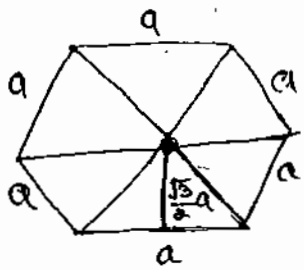
$T.S.A = LSA + \text{Area of Base } \left(\frac{\sqrt{3}}{4}a^2\right)$

$\text{Volume} = \frac{1}{3} \times \frac{\sqrt{3}}{4} a^2 \times H$

Height of Triangular Surface (h) $= \sqrt{H^2 + \left(\frac{a}{2\sqrt{3}}\right)^2}$

Slant Height (l) $= \sqrt{H^2 + \left(\frac{a}{\sqrt{3}}\right)^2}$

When the Base of Pyramid is Regular Hexagonal :→



$L.S.A = \frac{1}{2} \times 6a \times h$

$T.S.A = L.S.A + \text{Area of Base } \left(6 \times \frac{\sqrt{3}}{4}a^2\right)$

$\text{Volume} = \frac{1}{3} \times 6 \times \frac{\sqrt{3}}{4} a^2 \times H$

Height of Triangular face (h) $= \sqrt{H^2 + \left(\frac{\sqrt{3}}{2}a\right)^2}$

Slant Height $= \sqrt{H^2 + a^2}$

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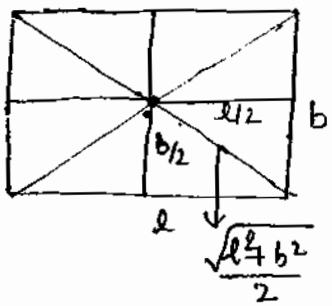
When The Base of Pyramid is Reactangular :- h_1, h_2 are height of Δ Surface.



$$L.S.A = \frac{1}{2} \times l \times h_1 \times 2 + \frac{1}{2} \times b \times h_2 \times 2$$

$$= lh_1 + bh_2$$

$$T.S.A = L.S.A + \text{Area of Base}$$

$$= lh_1 + bh_2 + lb$$

$$\text{Volume} = \frac{1}{3} \times \text{Area of Base} \times \text{Height of Pyramid}$$

$$h_1 = \sqrt{H^2 + \left(\frac{b}{2}\right)^2} \quad \text{and} \quad h_2 = \sqrt{H^2 + \left(\frac{l}{2}\right)^2}$$

$$\text{Slant Height} = \sqrt{H^2 + \left(\frac{\sqrt{l^2+b^2}}{2}\right)^2}$$

एक पिरामिड का आधार वर्गाकार है जिसकी प्रत्येक भुजा की लंबाई 10cm है। पिरामिड की ऊंचाई 12cm है, Find L.S.A, T.S.A and Volume.

$$\text{Height of triangular Surface} = \sqrt{H^2 + \left(\frac{a}{2}\right)^2} = \sqrt{12^2 + \left(\frac{10}{2}\right)^2} = \sqrt{144+25}$$

$$= 13\,cm.$$

$$L.S.A = \frac{1}{2} \times \text{Perimeter of Base} \times \text{Height of Triangular Surface}$$

$$= \frac{1}{2} \times \cancel{4}^{2} \times 10 \times 13 = 260\,cm^2$$

$$T.S.A = L.S.A + \text{Area of Base} = 260 + 10 \times 10 = 360\,cm^2$$

$$\text{Volume} = \frac{1}{3} \times \text{Area of Base} \times \text{Height of Prism} = \frac{1}{3} \times 10 \times 10 \times \cancel{12}^{4} = 400\,cm^3$$

एक पिरामिड का आधार समषट्भुज आकार का है जिसकी प्रत्येक भुजा की लंबाई 20cm है। पिरामिड की ऊंचाई 10cm है Find L.S.A, T.S.A and Volume.

$$\text{Height of } \Delta \text{ Surface } (h) = \sqrt{H^2 + \left(\frac{\sqrt{3}}{2}a\right)^2} = \sqrt{10^2 + \left(\frac{\sqrt{3}}{2} \times \cancel{20}^{10}\right)^2}$$

$$= \sqrt{100+300} = 20\,cm.$$

$$L.S.A = \frac{1}{2} \times \text{Perimeter of Base} \times h = \frac{1}{2} \times \cancel{6}^{3} \times 20 \times 20 = 1200\,cm^2$$

$$T.S.A = L.S.A + \text{Area of Base} = 1200 + 6 \times \frac{\sqrt{3}}{4} \times \cancel{20}^{5} \times 20$$

$$= 1200 + 600\sqrt{3}\,cm^2$$

$$\text{Volume} = \frac{1}{3} \times \text{Area of Base} \times H$$

$$= \frac{1}{3} \times \cancel{6}^{2} \times \frac{\sqrt{3}}{4} \times \cancel{20}^{5} \times 20 \times 10 = 2000\sqrt{3}\,cm^3$$

एक पिरामिड का आधार वर्गाकार है जिसका क्षेत्रफल $441 cm^2$ है यदि पिरामिड का ~~क्षेत्रफल~~ आयतन $1470 cm^3$ है Find L.S.A.

Area of Base $= 441 cm^2$

$\therefore a^2 = 441$

$a = 21 cm$.

Now Volume of Pyramid $= 1470$

$\therefore 1470 = \frac{1}{3} \times$ Area of Base $\times$ Height of Pyramid

$10\,\cancel{70}\ \cancel{1470} = \frac{1}{\cancel{3}} \times \cancel{21} \times \overset{7}{\cancel{21}} \times H$

$H = 10 cm$.

Height of Δ Surface $(h) = \sqrt{H^2 + \left(\frac{a}{2}\right)^2} = \sqrt{10^2 + \left(\frac{21}{2}\right)^2}$

$= \sqrt{100 + \frac{441}{4}} = \sqrt{\frac{841}{4}} = \frac{29}{2} cm$.

$\therefore$ L.S.A $= \frac{1}{\cancel{2}} \times \cancel{4} \times 21 \times \frac{29}{\cancel{2}} = 609 cm^2$

एक पिरामिड का आधार समबाहु त्रिभुज के आकार का है जिसकी प्रत्येक भुजा की लंबाई $10\sqrt{3}$ है T.S.A of Pyramid is $270\sqrt{3} cm^2$. Find the Volume.

T.S.A = L.S.A + Area of Base

$270\sqrt{3} = \frac{1}{\cancel{2}} \times 3 \times \overset{5}{\cancel{10}}\sqrt{3} \times h + \frac{\sqrt{3}}{\cancel{4}_{2}} \times 10\sqrt{3} \times \overset{5}{\cancel{10}}\sqrt{3}$

$270\sqrt{3} = 15\sqrt{3}\,h + 75\sqrt{3}$

$270 = 15h + 75$

$15h = 195 \quad h = \frac{195}{15} = 13 cm$.

Now Height of Pyramid $H = \sqrt{h^2 - \left(\frac{10\sqrt{3}}{2\sqrt{3}}\right)^2} = \sqrt{169 - 25} = 12 cm$.

Volume $= \frac{1}{\cancel{3}} \times \frac{\sqrt{3}}{4} \times 10\sqrt{3} \times 10\sqrt{3} \times \cancel{12}\ 3 = 300\sqrt{3} cm^3$

एक पिरामिड का आधार समबाहु त्रिभुजाकार है if the Volume of Pyramid is $48\sqrt{3} cm^3$ and Height of Pyramid is 4cm. Find L.S.A.

Volume $= \frac{1}{3} \times$ Area of Base $\times$ Height of Pyramid

$48\sqrt{3} = \frac{1}{3} \times \frac{\sqrt{3}}{4} a^2 \times 4$

$a^2 = 48 \times 3 = 144$

$\therefore a = 12 cm$.

Now Height of Δ Surface $= \sqrt{H^2 + \left(\frac{a}{2\sqrt{3}}\right)^2} = \sqrt{4^2 + \left(\frac{12}{2\sqrt{3}}\right)^2}$

$= \sqrt{16 + \frac{36}{3}} = \sqrt{16 + 12}$

$= 2\sqrt{7} cm$.

L.S.A $= \frac{1}{\cancel{2}} \times$ ~~Area of Base~~ $3 \times 12 \times \cancel{2}\sqrt{7}$

$= 36\sqrt{7} cm^2$

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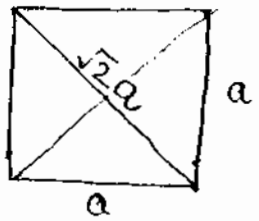
एक पिरामिड का आधार वर्गाकार है Length of whose Diagnol is $10\sqrt{2}$ cm. If Height of Pyramid is 12cm. Find L.S.A.



Now Diagnol $= \sqrt{2}a = 10\sqrt{2}$ cm

$\therefore a = 10$ cm.

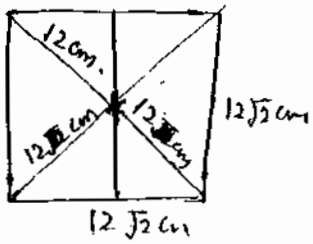
hight of Δ surface $= \sqrt{H^2 + (\frac{a}{2})^2}$

$= \sqrt{12^2 + (\frac{10}{2})^2} = \sqrt{169} = 13$ cm.

L.S.A $= \frac{1}{\cancel{2}} \times \overset{2}{\cancel{4}} \times 10 \times 13 = 260 \text{ cm}^2$

एक पिरामिड का आधार वर्गाकार है जिसकी प्रत्येक भुजा $12\sqrt{2}$ cm है। पिरामिड का पृष्ठ समबाहु त्रिभुज के आकार का है। find the L.S.A and Volume.

पिरामिड के प्रत्येक पृष्ठ की भुजा की लंबाई = Side of square = $12\sqrt{2}$ cm



Length of Diagnol $= \sqrt{2}a = \sqrt{2} \times 12\sqrt{2} = 24$ cm.

$\therefore$ Height of Pyramid $= \sqrt{(12\sqrt{2})^2 - (12)^2}$

$= \sqrt{288 - 144} = 12$ cm.

$\therefore$ L.S.A $= 4 \times$ Area of Equilateral Δ

$= \cancel{4} \times \frac{\sqrt{3}}{\cancel{4}} \times 12\sqrt{2} \times 12\sqrt{2} = 288\sqrt{3} \text{ cm}^2$

Volume $= \frac{1}{3} \times$ Area of Base $\times H = \frac{1}{\cancel{3}} \times \frac{\sqrt{3}}{\cancel{4}} \times 12\sqrt{2} \times 12\sqrt{2} \times \overset{4}{\cancel{12}}$

$= 1152 \text{ cm}^3$

OR L.S.A $= \frac{1}{2} \times$ Perimeter of Base $\times h$

$= \frac{1}{2} \times 4 \times 12\sqrt{2} \times 6\sqrt{6}$

$= \frac{1}{\cancel{2}} \times \overset{2}{\cancel{4}} \times 12\sqrt{2} \times 6\sqrt{2}\sqrt{3} = 288\sqrt{3} \text{ cm}^2$.

$h = \sqrt{12^2 + (\frac{12\sqrt{2}}{2})^2}$

$= \sqrt{144 + 72} = \sqrt{216}$

$= 6\sqrt{6}$ cm.

एक पिरामिड का आधार समषट्भुज आकार का है जिसके प्रत्येक भुजा की लंबाई 2a cm है। If the slant Height of Pyramid is $\frac{5a}{2}$ cm. Then find Volume and L.S.A.

Height of Pyramid $= \sqrt{l^2 - a^2} = \sqrt{(\frac{5a}{2})^2 - (2a)^2}$

$= \sqrt{\frac{25a^2}{4} - 4a^2} = \frac{3a}{2}$ cm.

Volume $= \frac{1}{\cancel{3}} \times \overset{3}{\cancel{6}} \times \frac{\sqrt{3}}{4} \times 2a \times 2a \times \frac{3a}{\cancel{2}} = 3\sqrt{3}\,a^3 \text{ cm}^3$

hlight of Δ surface $= \sqrt{H^2 + (\frac{\sqrt{3}}{2}a)^2} = \sqrt{(\frac{3a}{2})^2 + (\frac{\sqrt{3}}{2} \times 2a)^2}$

$= \frac{9a^2}{4} + \frac{12}{4}a^2 = \sqrt{\frac{21}{4}a^2} = \frac{a}{2}\sqrt{21}$

L.S.A $= \frac{1}{\cancel{2}} \times \overset{3}{\cancel{6}} \times 2a \times \frac{a}{\cancel{2}}\sqrt{21} = 3\sqrt{21}\,a^2 \text{ cm}^2$

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एक पिरामिड का आधार समकोण त्रिभुज के आकार का है जिसकी भुजाओं की लंबाई 5, 12 और 13 cm है If the Volume of Pyramid is 330 cm³. Find the Height of Pyramid.

$$\text{Volume} = \frac{1}{3} \times \text{Area of Base} \times H$$

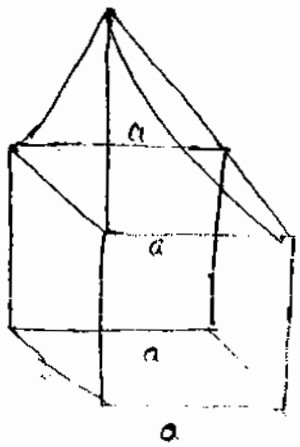
$$330 = \frac{1}{3} \times \frac{1}{2} \times 5 \times 12 \times H$$

$$H = \frac{330}{10} = 33 \text{ cm}.$$

एक घन के प्रत्येक भुजा की लंबाई a cm है इसके ऊपर एक पिरामिड है. Height of Pyramid is one half of the side of Cube. Find the Ratio of Volume of Cube and Pyramid

$$\text{Height of Pyramid} = \frac{1}{2} \times \text{side of Cube}$$

$$H = \frac{a}{2}$$



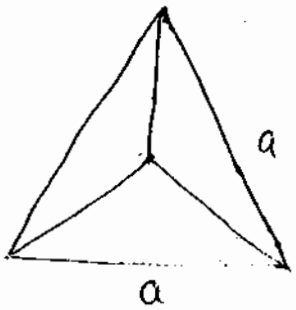
$$\text{Volume cube} : \text{Volume Pyramide}$$

$$= a^3 : \frac{1}{3} \times a^2 \times \frac{a}{2}$$

$$\Rightarrow 1 : \frac{1}{6} = \boxed{6:1}$$

TETRAHEDRAL [चतुष्फलक]

In Which Bottom And Surface Are in the form of Equilateral Triangle.



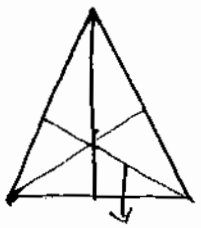
$$L.S.A = 3 \times \frac{\sqrt{3}}{4} \times a^2$$

$$T.S.A = L.S.A + \text{Area of Base}$$

$$= 3 \times \frac{\sqrt{3}}{4} a^2 + \frac{\sqrt{3}}{4} a^2 = 4 \times \frac{\sqrt{3}}{4} a^2$$

$$= \sqrt{3}\, a^2$$

Volume - Height of Pyramid (Tetrahedral) =



$$\frac{\sqrt{3}}{2} a \times \frac{2}{3} = \frac{a}{\sqrt{3}}$$

$$H = \sqrt{a^2 - \left(\frac{a}{\sqrt{3}}\right)^2} = \sqrt{a^2 - \frac{a^2}{3}}$$

$$= \frac{\sqrt{2}}{\sqrt{3}} a$$

$$\text{Volume} = \frac{1}{3} \times \text{Area of Base} \times H$$

$$= \frac{1}{3} \times \frac{\sqrt{3}}{4} a^2 \times \frac{\sqrt{2}}{\sqrt{3}} a$$

$$= \frac{\sqrt{2}}{12} a^3$$

एक Tetrahedral के प्रत्येक भुजा की लंबाई 12cm है find L.S.A, T.S.A, Volume.

$$L.S.A = 3 \times \frac{\sqrt{3}}{4} a^2 = 3 \times \frac{\sqrt{3}}{\cancel{4}} \times \overset{3}{\cancel{12}} \times 12 = 108\sqrt{3}\ cm^2$$

$$T.S.A = 4 \times \frac{\sqrt{3}}{4} a^2 = \cancel{4} \times \frac{\sqrt{3}}{\cancel{4}} \times 12 \times 12 = 144\sqrt{3}\ cm^2$$

$$Volume = \frac{\sqrt{2}}{12} a^3 = \frac{\sqrt{2}}{\cancel{12}} \times 12 \times 12 \times \cancel{12} = 144\sqrt{2}\ cm^3$$

THE END

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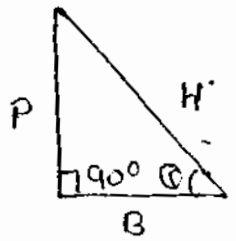
TRIGNOMETRY [Part - I]

P	B	P	H	H	B
H	H	B	P	B	P
↓	↓	↓	↓	↓	↓
$\sin\theta$	$\cos\theta$	$\tan\theta$	$\operatorname{cosec}\theta$	$\sec\theta$	$\cot\theta$



$H^2 = P^2 + B^2$

Rule → पाकिस्तान मुरवा प्यासा, हरा-हरा भारत

FORMULA

(1) $\sin\theta = \frac{1}{\operatorname{cosec}\theta}$, (2) $\cos\theta = \frac{1}{\sec\theta}$, (3) $\tan\theta = \frac{1}{\cot\theta}$

$\sin\theta \operatorname{cosec}\theta = 1$ $\cos\theta \sec\theta = 1$ $\tan\theta \cdot \cot\theta = 1$

(4) $\boxed{\sin^2\theta + \cos^2\theta = 1}$ [Power or Angle Same होने चाहिए]

Proof :→

$$\sin^2\theta + \cos^2\theta = \left(\frac{P}{H}\right)^2 + \left(\frac{B}{H}\right)^2 = \frac{P^2+B^2}{H^2} = \frac{H^2}{H^2} = 1$$

$\therefore \sin^2\theta = 1 - \cos^2\theta$ or $\cos^2\theta = 1 - \sin^2\theta$

$\sin\theta = \sqrt{1-\cos^2\theta}$ or $\cos\theta = \sqrt{1-\sin^2\theta}$

(5) $\boxed{\sec^2\theta - \tan^2\theta = 1}$

Proof :→

$$\left(\frac{H}{B}\right)^2 - \left(\frac{P}{B}\right)^2 = \frac{H^2}{B^2} - \frac{P^2}{B^2} = \frac{H^2-P^2}{B^2} = \frac{B^2}{B^2} = 1$$

$\therefore \sec^2\theta = 1 + \tan^2\theta$ or $\tan^2\theta = \sec^2\theta - 1$

$\sec\theta = \sqrt{1+\tan^2\theta}$, $\tan\theta = \sqrt{\sec^2\theta - 1}$

(6) $\boxed{\operatorname{cosec}^2\theta - \cot^2\theta = 1}$

Proof :→

$$\left(\frac{H}{P}\right)^2 - \left(\frac{B}{P}\right)^2 = \frac{H^2}{P^2} - \frac{B^2}{P^2} = \frac{H^2-B^2}{P^2} = \frac{P^2}{P^2} = 1$$

$\therefore \operatorname{cosec}^2\theta = 1 + \cot^2\theta$ or $\cot^2\theta = \operatorname{cosec}^2\theta - 1$

$\operatorname{cosec}\theta = \sqrt{1+\cot^2\theta}$, $\cot\theta = \sqrt{\operatorname{cosec}^2\theta - 1}$

(7) $\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{P}{h} \times \frac{h}{B} = \frac{P}{B}$

(8) $\cot\theta = \frac{\cos\theta}{\sin\theta} = \frac{B}{h} \times \frac{h}{P} = \frac{B}{P}$

(9) $\sin(A+B) = \sin A\cos B + \cos A \sin B$

$\sin(A-B) = \sin A \cos B - \cos A \sin B$

(10) $\cos(A+B) = \cos A \cos B - \sin A \sin B$

$\cos(A-B) = \cos A \cos B + \sin A \sin B$

(11) $\tan(A+B) = \dfrac{\tan A + \tan B}{1 - \tan A \tan B}$

(12) $\tan(A-B) = \dfrac{\tan A - \tan B}{1 + \tan A \tan B}$

(13) $\cot(A+B) = \dfrac{\cot A \cot B - 1}{\cot B + \cot A}$

(14) $\cot(A-B) = \dfrac{\cot A \cot B + 1}{\cot B - \cot A}$

(15) $\sin 2\theta = 2\sin\theta\cos\theta = \dfrac{2\tan\theta}{1+\tan^2\theta}$

Proof :→ $\sin 2\theta = \dfrac{2\sin\theta\cos\theta}{1} = \dfrac{2\sin\theta\cos\theta}{\sin^2\theta + \cos^2\theta}$

Dividing Num. and Deno. By $\cos^2\theta$.

$$\Rightarrow \frac{\dfrac{2\sin\theta \cdot \cancel{\cos\theta}}{\cos^2\theta}}{\dfrac{\sin^2\theta}{\cos^2\theta} + \dfrac{\cos^2\theta}{\cos^2\theta}} = \frac{2\tan\theta}{1+\tan^2\theta}$$

(16) $\cos 2\theta = \cos^2\theta - \sin^2\theta = 1 - 2\sin^2\theta = 2\cos^2\theta - 1 = \dfrac{1-\tan^2\theta}{1+\tan^2\theta}$

Proof :→ $\cos 2\theta = \cos^2\theta - \sin^2\theta = 1 - \sin^2\theta - \sin^2\theta = 1 - 2\sin^2\theta$

or $\cos^2\theta - \sin^2\theta = \cos^2\theta - (1 - \cos^2\theta) = 2\cos^2\theta - 1$

or $\cos^2\theta - \sin^2\theta = \dfrac{\cos^2\theta - \sin^2\theta}{\cos^2\theta + \sin^2\theta} = \dfrac{\dfrac{\cos^2\theta}{\cos^2\theta} - \dfrac{\sin^2\theta}{\cos^2\theta}}{\dfrac{\cos^2\theta}{\cos^2\theta} + \dfrac{\sin^2\theta}{\cos^2\theta}} = \dfrac{1-\tan^2\theta}{1+\tan^2\theta}$

(17) $\tan 2\theta = \dfrac{2\tan\theta}{1-\tan^2\theta}$

(18) $\cot 2\theta = \dfrac{\cot^2\theta - 1}{2\cot\theta}$

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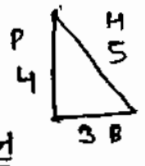
(EXAMPLE)

1 if $\sin\theta = \frac{4}{5}$ Find $\frac{\sin\theta + \cos\theta}{\tan\theta + \cot\theta}$

Now $\sin\theta = \frac{P}{H} = \frac{4}{5}$

$\therefore B = \sqrt{5^2 - 4^2} = \sqrt{9} = 3$



$\cos\theta = \frac{B}{H} = \frac{3}{5}$, $\tan\theta = \frac{P}{B} = \frac{4}{3}$

$\cot\theta = \frac{B}{P} = \frac{3}{4}$

$\therefore \frac{\sin\theta + \cos\theta}{\tan\theta + \cot\theta} = \frac{\frac{4}{5} + \frac{3}{5}}{\frac{4}{3} + \frac{3}{4}} = \frac{\frac{7}{5}}{\frac{25}{12}} = \frac{7}{5} \times \frac{12}{25}$

$= \frac{84}{125}$ Ans.

(2) $\sec\theta = \frac{5}{4}$ Find $\frac{\sin\theta - 2\cos\theta}{\tan\theta - \cot\theta}$

Now $\sec\theta = \frac{H}{B} = \frac{5}{4}$

$\therefore P = \sqrt{5^2 - 4^2} = 3$

$\therefore \sin\theta = \frac{3}{5}$, $\cos\theta = \frac{4}{5}$, $\tan\theta = \frac{3}{4}$, $\cot\theta = \frac{4}{3}$

$\therefore \frac{\sin\theta - 2\cos\theta}{\tan\theta - \cot\theta} = \frac{\frac{3}{5} - \frac{2\times 4}{5}}{\frac{3}{4} - \frac{4}{3}} = \frac{\frac{3-8}{5}}{\frac{9-16}{12}}$

$= \frac{-5}{5} \times \frac{12}{-7} = \frac{12}{7}$ Ans.

(3) $\tan\theta = \frac{a}{b}$ Find $\frac{a\sin\theta - b\cos\theta}{a\sin\theta + b\cos\theta}$

$\tan\theta = \frac{P}{B} = \frac{a}{b}$

$\therefore P = a$, $B = b$

इस Type के Example में जब Num. और Den. में $\sin\theta$, $\cos\theta$ दिये हो तो हम $\sin\theta = P$, $\cos\theta = B$ लेते हैं। Because जब हम LCM लेते हैं तो Solve करके Num. और Deno. में (H) रहेगा जो एक दूसरे से Cancel हो जाएगा।

e.g $\frac{a\sin\theta - b\cos\theta}{a\sin\theta + b\cos\theta} = \frac{a \times \frac{P}{H} - b \times \frac{B}{H}}{a \times \frac{P}{H} + b \times \frac{B}{H}}$

$= \frac{aP - bB}{H} \times \frac{H}{aP + bB}$

$$\boxed{= \frac{aP - bB}{aP + bB}}$$

[Put $\sin\theta = P = a$, $\cos\theta = B = b$]

$\therefore \frac{a\sin\theta - b\cos\theta}{a\sin\theta + b\cos\theta}$

$= \frac{a \cdot a - b \cdot b}{a \cdot a + b \cdot b} = \frac{a^2 - b^2}{a^2 + b^2}$

(4) $3\tan\theta = 4$ Find $\frac{4\cos\theta - \sin\theta}{2\cos\theta + \sin\theta}$

$3\tan\theta = 4$

$\tan\theta = \frac{4}{3} = \frac{P}{B}$

$\therefore$ Put $\sin\theta = 4$, $\cos\theta = 3$

$\therefore \frac{4\cos\theta - \sin\theta}{2\cos\theta + \sin\theta} = \frac{4\times 3 - 4}{2\times 3 + 4} = \frac{8}{10} = \frac{4}{5}$

(5) $\sin\theta = \frac{3}{4}$ Find $\sqrt{\frac{\operatorname{cosec}^2\theta - \cot^2\theta}{\sec^2\theta - 1}}$

$\sin\theta = \frac{P}{H} = \frac{3}{4}$

$\therefore B = \sqrt{4^2 - 3^2} = \sqrt{7}$

$\therefore \cos\theta = \frac{\sqrt{7}}{4}$

Now $\sqrt{\frac{\operatorname{cosec}^2\theta - \cot^2\theta}{\sec^2\theta - 1}} = \sqrt{\frac{1}{\tan^2\theta}} = \sqrt{\cot^2\theta}$

$= \cot\theta$

$\cot\theta = \frac{\cos\theta}{\sin\theta} = \frac{\sqrt{7}}{4} \times \frac{4}{3} = \frac{\sqrt{7}}{3}$ Ans

(6) If $\tan\theta = \sqrt{2} - 1$ Find $\sin\theta \cdot \cos\theta$

$\tan\theta = \frac{\sqrt{2} - 1}{1} = \frac{P}{B}$

$\therefore H = \sqrt{(\sqrt{2} - 1)^2 + 1^2}$

$= \sqrt{2 + 1 - 2\sqrt{2} + 1} = \sqrt{4 - 2\sqrt{2}}$

$\therefore \sin\theta \cdot \cos\theta = \frac{\sqrt{2} - 1}{\sqrt{4 - 2\sqrt{2}}} \times \frac{1}{\sqrt{4 - 2\sqrt{2}}}$

$= \frac{\sqrt{2} - 1}{4 - 2\sqrt{2}} = \frac{\sqrt{2} - 1}{2\sqrt{2}(\sqrt{2} - 1)} = \frac{1}{2\sqrt{2}}$

(7) $\operatorname{cosec}\theta = 2$ Find $\frac{1}{\tan\theta} + \frac{\sin\theta}{1 + \cos\theta}$

$\operatorname{cosec}\theta = \frac{H}{P} = \frac{2}{1}$

$\therefore B = \sqrt{2^2 - 1^2} = \sqrt{3}$

$\therefore \frac{1}{\tan\theta} + \frac{\sin\theta}{1 + \cos\theta} = \frac{\sqrt{3}}{1} + \frac{\frac{1}{2}}{1 + \frac{\sqrt{3}}{2}}$

$= \sqrt{3} + \frac{1}{2 + \sqrt{3}} = \sqrt{3} + \frac{2 - \sqrt{3}}{4 - 3} = 2$

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8) If $\sin\theta = \frac{a^2-b^2}{a^2+b^2}$ Find $\tan\theta$

$\sin\theta = \frac{P}{H} = \frac{a^2-b^2}{a^2+b^2}$

$B = \sqrt{(a^2+b^2)^2-(a^2-b^2)^2}$

$= \sqrt{4a^2b^2} = 2ab$

$\therefore \tan\theta = \frac{P}{B} = \frac{a^2-b^2}{2ab}$

(9) $\cot\theta = \frac{7}{8}$ Find $\frac{(1+\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(1-\cos\theta)}$

$\cot\theta = \frac{B}{P} = \frac{7}{8}$, ~~$P = \sqrt{64-49} = \sqrt{15} = 8$~~

$\therefore \frac{(1+\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(1-\cos\theta)} = \frac{1-\sin^2\theta}{1-\cos^2\theta} = \frac{\cos^2\theta}{\sin^2\theta}$

$= \cot^2\theta$

$= \left(\frac{7}{8}\right)^2 = \frac{49}{64}$ Ans.

(10) $\tan\theta = \frac{a}{b}$ Find $\frac{\sin\theta+\cos\theta}{\sin\theta-\cos\theta}$

Put $\sin\theta = a$, $\cos\theta = b$

$\therefore \frac{\sin\theta+\cos\theta}{\sin\theta-\cos\theta} = \frac{a+b}{a-b}$

(11) $\tan\theta = \frac{3}{4}$ Find $\frac{2\sin\theta+3\cos\theta}{3\sin\theta-2\cos\theta}$

Put $\sin\theta = 3$, $\cos\theta = 4$

$\Rightarrow \frac{2\sin\theta+3\cos\theta}{3\sin\theta-2\cos\theta} = \frac{2\times3+3\times4}{3\times3-2\times4} = 18$

(12) $\cot\theta = \frac{3}{4}$ Find $\frac{1+\sin\theta+2\cos\theta}{1-2\sin\theta+\cos\theta}$

Now $\frac{1+\sin\theta+2\cos\theta}{1-2\sin\theta+\cos\theta} = \frac{1+\frac{P}{h}+\frac{2B}{H}}{1-2\times\frac{P}{h}+\frac{B}{H}}$

$\boxed{= \frac{H+P+2B}{H-2P+B}}$

$\therefore \cot\theta = \frac{3}{4} = \frac{B}{P}$

$\therefore H = \sqrt{4^2+3^2} = 5$

$\therefore \frac{1+\sin\theta+2\cos\theta}{1-2\sin\theta+\cos\theta} = \frac{5+4+2\times3}{5-2\times4+3}$

$= \frac{15}{0} = \infty$

(13) $\frac{\sin\theta+\cos\theta}{\sin\theta-\cos\theta} = \frac{5}{3}$, Find $\tan\theta$

$3\sin\theta+3\cos\theta = 5\sin\theta-5\cos\theta$

$2\sin\theta = 8\cos\theta$, $\frac{\sin\theta}{\cos\theta} = \frac{8}{2}$

$\tan\theta = 4$

(14) $\cos\theta = \frac{3}{5}$ Find $\frac{\sin\theta-\cot\theta}{2\tan\theta}$

$\cos\theta = \frac{B}{H} = \frac{3}{5}$

$P = \sqrt{5^2-3^2} = \sqrt{25-9} = 4$

$\therefore \frac{\sin\theta-\cot\theta}{2\tan\theta} = \frac{\frac{4}{5}-\frac{3}{4}}{2\times\frac{4}{3}} = \frac{1}{20}\times\frac{3}{8}$

$= \frac{3}{160}$

(15) $\cot\theta = \frac{1}{\sqrt{3}}$ Find $\frac{1-\cos^2\theta}{2-\sin^2\theta}$

$\cot\theta = \frac{B}{P} = \frac{1}{\sqrt{3}}$ $\therefore H = \sqrt{3+1} = 2$

$\therefore \frac{1-\cos^2\theta}{2-\sin^2\theta} = \frac{\sin^2\theta}{2-\sin^2\theta} = \frac{\left(\frac{\sqrt{3}}{2}\right)^2}{2-\left(\frac{\sqrt{3}}{2}\right)^2}$

$= \frac{3}{4}\times\frac{4}{8-3} = \frac{3}{5}$

(16) $\tan\theta = \frac{24}{7}$ Find $\sin\theta+\cos\theta$

$\tan\theta = \frac{P}{B} = \frac{24}{7}$

$\therefore H = \sqrt{24^2+7^2} = \sqrt{625} = 25$

$\therefore \sin\theta+\cos\theta = \frac{P}{H}+\frac{B}{H} = \frac{24}{25}+\frac{7}{25}$

$= \frac{31}{25}$ Ans.

(17) $\sin\theta = \frac{a}{b}$ Find $\sec\theta+\tan\theta$

$\sec\theta+\tan\theta = \frac{1}{\cos\theta}+\frac{\sin\theta}{\cos\theta} = \frac{1+\sin\theta}{\cancel{\cos\theta}\ \cos^2\theta}$

$= \frac{1+\sin\theta}{\sqrt{1-\sin^2\theta}} = \frac{1+\sin\theta}{\sqrt{(1+\sin\theta)(1-\sin\theta)}}$

$= \sqrt{\frac{1+\sin\theta}{1-\sin\theta}}$

$= \sqrt{\frac{1+\frac{a}{b}}{1-\frac{a}{b}}} = \sqrt{\frac{b+a}{b-a}}$

(18) $3\cos\theta - 4\sin\theta = 2\cos\theta + \sin\theta$ find $\tan\theta$

$\Rightarrow 3\cos\theta - 4\sin\theta = 2\cos\theta + \sin\theta$

$\cos\theta = 5\sin\theta$

$\tan\theta = \frac{1}{5}$ Ans.

(19) $\cot\theta = \frac{3}{4}$ Find $\sqrt{\frac{\sec\theta - \operatorname{cosec}\theta}{\sec\theta + \operatorname{cosec}\theta}}$

$\cot\theta = \frac{B}{P} = \frac{3}{4}$, $H = \sqrt{4^2+3^2} = 5$

$\therefore \sqrt{\frac{\sec\theta - \operatorname{cosec}\theta}{\sec\theta + \operatorname{cosec}\theta}} = \sqrt{\frac{\frac{5}{3} - \frac{5}{4}}{\frac{5}{3} + \frac{5}{4}}} = \sqrt{\frac{5}{12} \times \frac{12}{35}} = \frac{1}{\sqrt{7}}$

OR

$\sqrt{\frac{\sec\theta - \operatorname{cosec}\theta}{\sec\theta + \operatorname{cosec}\theta}} = \sqrt{\frac{\frac{1}{\cos\theta} - \frac{1}{\sin\theta}}{\frac{1}{\cos\theta} + \frac{1}{\sin\theta}}}$

$= \sqrt{\frac{\sin\theta - \cos\theta}{\sin\theta + \cos\theta}}$

Put $\sin\theta = 4$ $\cos\theta = 3$

$= \sqrt{\frac{4-3}{4+3}} = \frac{1}{\sqrt{7}}$ Ans.

(20) $\frac{2\sin\theta - 3\cos\theta}{3\sin\theta + 2\cos\theta} = \frac{2}{5}$ Find $\tan\theta$

$10\sin\theta - 15\cos\theta = 6\sin\theta + 4\cos\theta$

$4\sin\theta = 19\cos\theta$

$\frac{\sin\theta}{\cos\theta} = \frac{19}{4}$, $\tan\theta = \frac{19}{4}$

(21) $\frac{2\sin\theta - 3\tan\theta}{3\sin\theta + 2\tan\theta} = \frac{1}{3}$ Find $\cos\theta$

$6\sin\theta - 9\tan\theta = 3\sin\theta + 2\tan\theta$

$3\sin\theta = 11\tan\theta$

$3\sin\theta = 11\frac{\sin\theta}{\cos\theta}$

$\cos\theta = \frac{11}{3}$

(22) $\tan\theta = \frac{20}{21}$, Find $\frac{1 - \sin\theta + \cos\theta}{1 + \sin\theta + \cos\theta}$

$\tan\theta = \frac{P}{B} = \frac{20}{21}$

$H = \sqrt{21^2+20^2} = \sqrt{841} = 29$

$\therefore \frac{1 - \sin\theta + \cos\theta}{1 + \sin\theta + \cos\theta} = \frac{29 - 20 + 21}{29 + 20 + 21} = \frac{30}{70} = \frac{3}{7}$

(23) $\sec\theta = \frac{5}{4}$ Find $\frac{2 - \sin\theta + \cos\theta}{1 - 2\sin\theta + 3\cos\theta}$

$\sec\theta = \frac{H}{B} = \frac{5}{4}$

$P = \sqrt{5^2 - 4^2} = \sqrt{9} = 3$

Put $\sin\theta = 3$, $\cos\theta = 4$

$\therefore \frac{2 - \sin\theta + \cos\theta}{1 - 2\sin\theta + 3\cos\theta} = \frac{2(5) - 3 + 4}{5 - 2(3) + 3 \times 4} = \frac{11}{11} = 1$

(24) $\frac{\tan\theta + \cot\theta}{\tan\theta - \cot\theta} = 2$ Find $\sin\theta$

$\tan\theta + \cot\theta = 2\tan\theta - 2\cot\theta$

$\tan\theta = 3\cot\theta$

$\tan\theta = 3 \cdot \frac{1}{\tan\theta} \Rightarrow \tan^2\theta = 3$

$\tan\theta = \frac{\sqrt{3}}{1} = \frac{P}{B}$

$H = \sqrt{3+1} = 2$

$\therefore \sin\theta = \frac{P}{H} = \frac{\sqrt{3}}{2}$ Ans.

(25) $\sec\theta = x + \frac{1}{4x}$ Find $\sec\theta + \tan\theta$

$\sec\theta = \frac{4x^2+1}{4x} = \frac{H}{B}$

$\therefore P = \sqrt{(4x^2+1)^2 - (4x)^2}$

$= \sqrt{16x^2 + 1 + 8x^2 - 16x^2}$

$= \sqrt{16x^4 + 1 - 8x^2} = \sqrt{(4x^2-1)^2}$

$= 4x^2 - 1$

$\therefore \tan\theta = \frac{4x^2-1}{4x}$

$\therefore \sec\theta + \tan\theta = \frac{4x^2+1}{4x} + \frac{4x^2-1}{4x}$

$= \frac{8x^2}{4x} = 2x$ Ans.

(26) $\tan\theta = \frac{4}{3}$ Find $\frac{3\sin\theta + 2\cos\theta}{3\sin\theta - 2\cos\theta}$

Put $\sin\theta = 4$, $\cos\theta = 3$

$\Rightarrow \frac{3\sin\theta + 2\cos\theta}{3\sin\theta - 2\cos\theta} = \frac{3 \times 4 + 2 \times 3}{3 \times 4 - 2 \times 3}$

$= \frac{18}{6} = 3$

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27) $\frac{\sin\theta+\cos\theta}{\sin\theta-\cos\theta}=3$ Find $\sin^4\theta-\cos^4\theta$

$3\sin\theta-3\cos\theta=\sin\theta+\cos\theta$

$2\sin\theta=4\cos\theta$

$\tan\theta=\frac{2}{1}=\frac{P}{B}$

$H=\sqrt{2^2+1^2}=\sqrt{5}$

$\therefore \sin^4\theta-\cos^4\theta=\left(\frac{2}{\sqrt5}\right)^4-\left(\frac{1}{\sqrt5}\right)^4$

$=\frac{16}{25}-\frac{1}{25}=\frac{15}{25}=\frac{3}{5}$

28) $\frac{2\sin\theta-\cos\theta}{\cos\theta+\sin\theta}=1$ Find $\cot\theta$

$2\sin\theta-\cos\theta=\cos\theta+\sin\theta$

$\sin\theta=2\cos\theta$, $\frac{\cos\theta}{\sin\theta}=\frac{1}{2}$

$\cot\theta=\frac{1}{2}$

29) $\tan\theta=2$ find $\frac{8\sin\theta+5\cos\theta}{\sin^3\theta+2\cos^3\theta+3\cos\theta}$

Si $\tan\theta=\frac{2}{1}=\frac{P}{B}$, $H=\sqrt{2^2+1^2}=\sqrt5$

$\sin\theta=\frac{2}{\sqrt5}$, $\cos\theta=\frac{1}{\sqrt5}$

$\therefore \frac{8\sin\theta+5\cos\theta}{\sin^3\theta+2\cos^3\theta+3\cos\theta}=\frac{\frac{8\times2}{\sqrt5}+\frac{5}{\sqrt5}}{\left(\frac{2}{\sqrt5}\right)^3+2\left(\frac{1}{\sqrt5}\right)^3+\frac{3}{\sqrt5}}$

$=\frac{\frac{21}{\sqrt5}}{\frac{8}{5\sqrt5}+\frac{2}{5\sqrt5}+\frac{3}{\sqrt5}}=\frac{21}{\sqrt5}\times\frac{5\sqrt5}{25}$

$=\frac{21}{5}$ Ans

30) $\tan\theta=\frac{1}{\sqrt7}$ find $\frac{\operatorname{cosec}^2\theta-\sec^2\theta}{\operatorname{cosec}^2\theta+\sec^2\theta}$

$\rightarrow \frac{\operatorname{cosec}^2\theta-\sec^2\theta}{\operatorname{cosec}^2\theta+\sec^2\theta}=\frac{\frac{1}{\sin^2\theta}-\frac{1}{\cos^2\theta}}{\frac{1}{\sin^2\theta}+\frac{1}{\cos^2\theta}}$

$=\frac{\cos^2\theta-\sin^2\theta}{\cos^2\theta+\sin^2\theta}$

$=\frac{(\sqrt7)^2-1^2}{(\sqrt7)^2+1}=\frac{6}{8}=\frac{3}{4}$

31) $\tan\theta=\frac{12}{13}$ Find $\frac{2\sin\theta}{\cos^2\theta-\sin^2\theta}$

$\tan\theta=\frac{12}{13}=\frac{P}{B}$ $H=\sqrt{144+169}$

$=\sqrt{313}$

$\therefore \frac{2\sin\theta}{\cos^2\theta-\sin^2\theta}=\frac{\frac{2\times12}{\sqrt{313}}}{\left(\frac{13}{\sqrt{313}}\right)^2-\left(\frac{12}{\sqrt{313}}\right)^2}$

$=\frac{24}{\sqrt{313}}\times\frac{313}{169-144}$

$=\frac{24}{25}\sqrt{313}$ Ans.

(32) $\tan\theta=\frac{12}{13}$ Find $\frac{2\sin\theta\cos\theta}{\cos^2\theta-\sin^2\theta}$

$\sin\theta=\frac{12}{\sqrt{313}}$ $\cos\theta=\frac{13}{\sqrt{313}}$

$\therefore \frac{2\sin\theta\cos\theta}{\cos^2\theta-\sin^2\theta}=\frac{\frac{2\times12\times13}{313}}{\frac{169-144}{313}}$

$=\frac{2\times12\times13}{313}\times\frac{313}{25}=\frac{312}{25}$ Ans.

(33) $\sec\theta=\frac{17}{8}$ Find $\frac{8-4\sin^2\theta}{4\cos^2\theta-3}$

$\sec\theta=\frac{17}{8}=\frac{H}{B}$

$\therefore P=\sqrt{17^2-8^2}=\sqrt{225}=15$

$\therefore \cos\theta=\frac{8}{17}$, $\sin\theta=\frac{15}{17}$

$\therefore \frac{3-4\sin^2\theta}{4\cos^2\theta-3}=\frac{3-4\left(\frac{15}{17}\right)^2}{4\left(\frac{8}{17}\right)^2-3}$

$=\frac{3(17)^2-4(15)^2}{4(8)^2-3(17)^2}$

$=\frac{867-900}{256-867}=\frac{-33}{-611}=\frac{33}{611}$ Ans.

(34) $\sec\theta=\frac{5}{4}$ Find $\frac{3\sin\theta-4\sin^3\theta}{4\cos^3\theta-3\cos\theta}$

$\sec\theta=\frac{5}{4}=\frac{H}{B}$

$\therefore P=\sqrt{25-16}=3$

$\therefore \sin\theta=\frac{3}{5}$, $\cos\theta=\frac{4}{5}$

(35) $\therefore \dfrac{3\sin\theta - 4\sin^3\theta}{4\cos^3\theta - 3\cos\theta} = \dfrac{3\times\frac{3}{5} - 4\left(\frac{3}{5}\right)^3}{4\left(\frac{4}{5}\right)^3 - 3\left(\frac{4}{5}\right)} = \dfrac{9\times25 - 4\times27}{4\times4^3 - 3\times4\times25}$

$= \dfrac{9(25-12)}{4(64-75)} = \dfrac{9\times13}{4\times-11}$

$= -\dfrac{117}{44}$ Ans

35 $5\tan\theta - 4 = 0$ Find $\dfrac{5\sin\theta - 3\cos\theta}{5\sin\theta + 2\cos\theta}$

$5\tan\theta = 4$

$\tan\theta = \frac{4}{5}$ Put $\sin\theta = 4$, $\cos\theta = 5$

$\therefore \dfrac{5\sin\theta - 3\cos\theta}{5\sin\theta + 2\cos\theta} = \dfrac{5\times4 - 3\times5}{5\times4 + 2\times5} = \dfrac{5}{30} = \dfrac{1}{6}$

(36) $\tan\theta = 2$ Find $\sec\theta\sin\theta + \tan^2\theta - \operatorname{cosec}\theta$

$\tan\theta = \frac{2}{1}$

$H = \sqrt{4+1} = \sqrt{5}$ $\therefore \operatorname{cosec}\theta = \frac{\sqrt{5}}{2}$

Now $\sec\theta\sin\theta + \tan^2\theta - \operatorname{cosec}\theta = \tan\theta + \tan^2\theta - \operatorname{cosec}\theta$

$= 2 + (2)^2 - \frac{\sqrt{5}}{2} = 6 - \frac{\sqrt{5}}{2} = \frac{12-\sqrt{5}}{2}$

(37) $\sec\theta = \frac{5}{4}$ Find $\dfrac{1+\tan\theta}{1-\tan\theta}$

$\therefore \sec\theta = \frac{5}{4} = \frac{H}{B}$

$\therefore P = \sqrt{5^2 - 4^2} = \sqrt{9} = 3$, $\tan\theta = \frac{3}{4}$

$\therefore \dfrac{1+\tan\theta}{1-\tan\theta} = \dfrac{1+\frac{3}{4}}{1-\frac{3}{4}} = \dfrac{4+3}{4-3} = \dfrac{7}{1}$

Similarly $\boxed{\dfrac{1-\tan\theta}{1+\tan\theta} = \dfrac{1}{7}}$

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TRIGNOMETRY [PART-II]

	0°	30°	45°	60°	90°
$\sin\theta$	0	$1/2$	$1/\sqrt{2}$	$\sqrt{3}/2$	1
$\cos\theta$	1	$\sqrt{3}/2$	$1/\sqrt{2}$	$1/2$	0
$\tan\theta$	0	$1/\sqrt{3}$	1	$\sqrt{3}$	∞
$\operatorname{cosec}\theta$	∞	2	$\sqrt{2}$	$2/\sqrt{3}$	1
$\sec\theta$	1	$2/\sqrt{3}$	$\sqrt{2}$	2	∞
$\cot\theta$	∞	$\sqrt{3}$	1	$1/\sqrt{3}$	0

Find the VALUE OF :→

(i) $\sin 45^\circ . \sin 30^\circ + \cos 45^\circ \cos 30^\circ$

$= \frac{1}{\sqrt{2}} \cdot \frac{1}{2} + \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{3}}{2} = \frac{1+\sqrt{3}}{2\sqrt{2}}$

(ii) $\sin 60^\circ \cdot \cos 30^\circ + \cos 60^\circ \sin 30^\circ$

$= \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{3}}{2} + \frac{1}{2} \cdot \frac{1}{2} = \frac{3}{4} + \frac{1}{4} = \frac{4}{4} = 1$

(iii) $\sin^2 30^\circ + \sin^2 45^\circ + \sin^2 60^\circ + \sin^2 90^\circ$

$= \left(\frac{1}{2}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2 + (1)^2$

$= \frac{1}{4} + \frac{1}{2} + \frac{3}{4} + 1 = \frac{1+2+3+4}{4} = \frac{10}{4} = \frac{5}{2}$

(iv) $2\sin^2 30^\circ - 3\cos^2 45^\circ + \tan^2 60^\circ$

$= 2\left(\frac{1}{2}\right)^2 - 3\left(\frac{1}{\sqrt{2}}\right)^2 + (\sqrt{3})^2$

$= \frac{1}{2} - \frac{3}{2} + 3 = \frac{1-3+6}{2} = \frac{4}{2} = 2$

(v) $\sin^2 30^\circ \cos^2 45^\circ + 4\tan^2 30^\circ + \frac{1}{2}\sin^3 90^\circ + 2\cos^2 90^\circ + \frac{1}{24}\cos^2 0^\circ$

$= \left(\frac{1}{2}\right)^2\left(\frac{1}{\sqrt{2}}\right)^2 + 4\left(\frac{1}{\sqrt{3}}\right)^2 + \frac{1}{2} \times (1)^3 + 2(0)^2 + \frac{1}{24}(1)^2$

$= \frac{1}{8} + \frac{4}{3} + \frac{1}{2} + \frac{1}{24} = \frac{3+32+12+1}{24}$

$= \frac{48}{24} = 2$

(vi) $4(\sin^4 60^\circ + \cos^4 30^\circ) - 3(\tan^2 60^\circ - \tan^2 45^\circ) + 5\cos^2 45^\circ$

$= 4\left[\left(\frac{\sqrt{3}}{2}\right)^4 + \left(\frac{\sqrt{3}}{2}\right)^4\right] - 3\left[(\sqrt{3})^2 - (1)^2\right] + 5\left(\frac{1}{\sqrt{2}}\right)^2$

$= 4\left[\frac{9}{16} + \frac{9}{16}\right] - 3[3-1] + \frac{5}{2}$

$= \cancel{4} \times \frac{\cancel{18}^{9}}{\cancel{16}_{\cancel{8}\,2}} - 6 + \frac{5}{2} = \frac{9}{2} - 6 + \frac{5}{2}$

$= \frac{2}{2} = 1$

(vii) $4(\sin^4 30^\circ + \cos^2 60^\circ) - 3(\cos^2 45^\circ - \sin^2 90^\circ) - \sin^2 60^\circ$

$= 4\left[\left(\frac{1}{2}\right)^4 + \left(\frac{1}{2}\right)^2\right] - 3\left[\left(\frac{1}{\sqrt{2}}\right)^2 - 1\right] - \left[\frac{\sqrt{3}}{2}\right]^2$

$= 4\left[\frac{1}{16} + \frac{1}{4}\right] - 3\left[\frac{1}{2} - 1\right] - \frac{3}{4}$

$= \cancel{4} \times \frac{5}{\cancel{16}_{4}} + \frac{3}{2} - \frac{3}{4} = \frac{5+6-3}{4}$

$= \frac{8}{4} = 2$ Ans.

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find the Value of

(i) $2(\cos^2 45^\circ + \tan^2 60^\circ) - 6(\sin^2 45^\circ - \tan^2 30^\circ)$

(ii) $2(\cos^4 60^\circ + \sin^4 30^\circ) - (\tan^2 60^\circ + \cot^2 30^\circ) + 3\sec^2 30^\circ$

(iii) $\dfrac{\tan^2 60^\circ + 4\cos^2 45^\circ + 3\sec^2 30^\circ + 5\cos^2 90^\circ}{\operatorname{cosec} 30^\circ + \sec 60^\circ - \cot^2 30^\circ}$

(iv) $\dfrac{\tan 45^\circ}{\operatorname{cosec} 30^\circ} + \dfrac{\sec 60^\circ}{\cot 45^\circ} - \dfrac{5\sin 90^\circ}{2\cos 0^\circ}$

Ans

(i) $\Rightarrow 2\left[\left(\frac{1}{\sqrt{2}}\right)^2 + (\sqrt{3})^2\right] - 6\left[\left(\frac{1}{\sqrt{2}}\right)^2 - \left(\frac{1}{\sqrt{3}}\right)^2\right]$

$2\left[\frac{1}{2} + 3\right] - 6\left[\frac{1}{2} - \frac{1}{3}\right]$

$2\left[\frac{7}{2}\right] - 6\left[\frac{3-2}{6}\right]$

$= 7 - 1 = 6$

(ii) $\Rightarrow 2\left(\left(\frac{1}{2}\right)^4 + \left(\frac{1}{2}\right)^4\right] - (3+1) + 3 \times \frac{4}{3}$

$= 2\left(\frac{1}{16} + \frac{1}{16}\right) - 4 + 4$

$= \cancel{2} \times \frac{2}{\cancel{16}_{4}} = \frac{1}{4}$

(iii) $\dfrac{(\sqrt{3})^2 + 4\left(\frac{1}{\sqrt{2}}\right)^2 + 3\left(\frac{2}{\sqrt{3}}\right)^2 + 5(0)}{2 + 2 - (\sqrt{3})^2}$

$= \dfrac{3 + 2 + 12}{1} = 9$ Ans.

iv $\Rightarrow \dfrac{1}{2} + \dfrac{2}{1} - \dfrac{5 \times 1}{2}$

$= \dfrac{1}{2} + \dfrac{2}{1} - \dfrac{5}{2} = \dfrac{5}{2} - \dfrac{5}{2} = 0$

Next

$2\sin\frac{\theta}{2} = 1$ find Value of θ

$\Rightarrow \sin\frac{\theta}{2} = \frac{1}{2}$

$\Rightarrow \sin\frac{\theta}{2} = \sin 30^\circ$ $\left[\because \sin 30^\circ = \frac{1}{2}\right]$

$\frac{\theta}{2} = 30^\circ \Rightarrow \boxed{\theta = 60^\circ}$

(2) $\sqrt{3}\sin\theta = \cos\theta$ Find Value of θ

$\sqrt{3}\sin\theta = \cos\theta$

$\frac{\sin\theta}{\cos\theta} = \frac{1}{\sqrt{3}} \Rightarrow \tan\theta = \frac{1}{\sqrt{3}}$

$\tan\theta = \tan 30^\circ$ $\therefore \boxed{\theta = 30^\circ}$

(3) $\tan\theta = \sin 45^\circ \cos 45^\circ + \sin 30^\circ$ find θ

$\tan\theta = \sin 45^\circ \cdot \cos 45^\circ + \sin 30^\circ$

$= \frac{1}{\sqrt{2}} \cdot \frac{1}{\sqrt{2}} + \frac{1}{2} \Rightarrow \frac{1}{2} + \frac{1}{2} = 1$

$\tan\theta = 1 \Rightarrow \tan\theta = \tan 45^\circ$

$\boxed{\theta = 45^\circ}$

(4) $\sqrt{3}\tan 2\theta = \cos 60^\circ + \sin 45^\circ \cos 45^\circ$ find θ

$\sqrt{3}\tan 2\theta = \frac{1}{2} + \frac{1}{\sqrt{2}} \cdot \frac{1}{\sqrt{2}}$

$\sqrt{3}\tan 2\theta = \frac{1}{2} + \frac{1}{2} = 1$

$\sqrt{3}\tan 2\theta = 1$

$\tan 2\theta = \frac{1}{\sqrt{3}} \Rightarrow \tan 2\theta = \tan 30^\circ$

$\therefore 2\theta = 30^\circ$

$\boxed{\theta = 15^\circ}$

(5) $\cos 2\theta = \cos 60^\circ \cos 30^\circ + \sin 60^\circ \sin 30^\circ$ find θ

$\cos 2\theta = \cos(60^\circ - 30^\circ)$

$[\because \cos A \sin B + \sin A \cos B = \cos(A-B)]$

$\Rightarrow \cos 2\theta = \cos 30^\circ$

$2\theta = 30^\circ$

$\boxed{\theta = 15^\circ}$

Next Type :-

$\sin(A+B) = 1$ $\sin(A-B) = \frac{1}{2}$ find A and B

$\sin(A+B) = 1$, $\sin(A-B) = \frac{1}{2}$

$\sin(A+B) = \sin 90^\circ$, $\sin(A-B) = \sin 30^\circ$

$\Rightarrow A+B = 90^\circ$ $A = 60^\circ$

$\Rightarrow A-B = 30^\circ$ $\therefore B = 90 - 60 = 30^\circ$

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$\Rightarrow \tan(A-B)=\frac{1}{\sqrt{3}}$, $\tan(A+B)=\sqrt{3}$ find A and B

$\tan(A-B)=\frac{1}{\sqrt{3}}$

$\tan(A-B)=\tan 30^\circ$

$A-B=30^\circ$ — I

$\tan(A+B)=\sqrt{3}$

$\tan(A+B)=\tan 60^\circ$

$A+B=60$ — II

$A+\cancel{B}=60^\circ$
$A-\cancel{B}=30^\circ$

$2A=90^\circ \Rightarrow A=45^\circ$

$\therefore B=60-45=15^\circ$

$\Rightarrow \sin(A-B)=\frac{1}{2}$, $\cos(A+B)=\frac{1}{2}$ find A and B

$\sin(A-B)=\frac{1}{2}$

$\sin(A-B)=\sin 30^\circ$

$A-B=30^\circ$ — I

$\cos(A+B)=\frac{1}{2}$

$\cos(A+B)=\cos 60^\circ$

$A+B=60^\circ$ — II

$A+\cancel{B}=60^\circ$
$A-\cancel{B}=30^\circ$

$2A=90^\circ$ $A=45^\circ$

$\therefore B=60^\circ-45^\circ=15^\circ$

$\Rightarrow \sin(A+2B)=\frac{\sqrt{3}}{2}$ $\cos(A+4B)=0$ find A and B

$\sin(A+2B)=\frac{\sqrt{3}}{2}$

$\sin(A+2B)=\sin 60^\circ$

$A+2B=60^\circ$ — I

$\cos(A+4B)=0$

$\cos(A+4B)=\cos 90^\circ$

$A+4B=90^\circ$ — II

$A+4B=90^\circ$
$A+2B=60^\circ$
$- \quad - \quad -$

$2B=30^\circ$ $B=15^\circ$

$\therefore A=90^\circ-4(15^\circ)=90^\circ-60^\circ$
$=30^\circ$

$\Rightarrow \tan(A-B)=\frac{1}{\sqrt{3}}$, $\tan(A+B)=\sqrt{3}$ find A and B

$\tan(A-B)=\tan 30^\circ$

$A-B=30^\circ$

$\tan(A+B)=\tan 60^\circ$

$A+B=60^\circ$

$A+\cancel{B}=60^\circ$
$A-\cancel{B}=30$

$2A=90^\circ$

$A=45^\circ$, $B=15^\circ$

Next Type :→

$\Rightarrow \frac{\cos\theta-\sin\theta}{\cos\theta+\sin\theta}=\frac{1-\sqrt{3}}{1+\sqrt{3}}$ find θ.

Applying Componando and Dividingo Rule

$\frac{\cos\theta-\cancel{\sin\theta}+\cos\theta+\cancel{\sin\theta}}{\cancel{\cos\theta}+\sin\theta-\cancel{\cos\theta}-\sin\theta}=\frac{1-\cancel{\sqrt{3}}+1+\cancel{\sqrt{3}}}{1-\sqrt{3}-1-\sqrt{3}}$

$\frac{\cancel{2}\cos\theta}{-\cancel{2}\sin\theta}=\frac{\cancel{2}}{-\cancel{2}\sqrt{3}}$

$\cot\theta=\frac{1}{\sqrt{3}}$ $\cot\theta=\cot 60^\circ$

$\theta=60^\circ$

OR

$\frac{\cos\theta-\sin\theta}{\cos\theta+\sin\theta}=\frac{1-\sqrt{3}}{1+\sqrt{3}}$

$\Rightarrow \frac{1-\tan\theta}{1+\tan\theta}=\frac{1-\sqrt{3}}{1+\sqrt{3}}$

Comparing Both side

$\tan\theta=\sqrt{3}$

$\tan\theta=\tan 60^\circ$

$\theta=60^\circ$ Ans.

$\Rightarrow \sin\theta=\cos\theta$ Find $2\tan^2\theta+\sin^2\theta-1$

$\Rightarrow \sin\theta=\cos\theta$

$\frac{\sin\theta}{\cos\theta}=1 \Rightarrow \tan\theta=1$

$\Rightarrow \tan\theta=\tan 45^\circ$

$\theta=45^\circ$

$\therefore 2\tan^2\theta+\sin^2\theta-1$

$=2\tan^2 45^\circ+\sin^2 45^\circ-1$

$=2(1)+\frac{1}{\sqrt{2}}\cdot\frac{1}{\sqrt{2}}-1 \Rightarrow 2+\frac{1}{2}-1$

$=\frac{4+1-2}{2}=\frac{3}{2}$ Ans.

$\Rightarrow \tan\theta + \cot\theta = 2$ find $\sin^2\theta + \operatorname{cosec}^2\theta$

Now $\tan\theta + \cot\theta = 2$

Put $\tan\theta = 1$, $\cot\theta = 1$

$\therefore \tan\theta = \tan 45^\circ$, $\cot\theta = \cot 45^\circ$

$\theta = 45^\circ$

$\therefore \sin^2\theta + \operatorname{cosec}^2\theta = \sin^2 45^\circ + \operatorname{cosec}^2 45^\circ$

$= \left(\frac{1}{\sqrt{2}}\right)^2 + (\sqrt{2})^2 = \frac{1}{2} + 2 = \frac{5}{2}$ Ans.

$\Rightarrow \sin^7\theta + \operatorname{cosec}^7\theta = 2$ Find $\sin^{99}\theta + \operatorname{cosec}^{99}\theta$

Put $\sin^7\theta = 1$ $\operatorname{cosec}^7\theta = 1$

$\sin^7\theta = \sin 90^\circ$, $\operatorname{cosec}\theta = \operatorname{cosec} 90^\circ$

$\theta = 90^\circ$

$\therefore \sin^{99}(\theta) + \operatorname{cosec}^{99}\theta = \sin^{99}(90^\circ) + \operatorname{cosec}^{99}(90^\circ)$

$= 1 + 1 = 2$

$\Rightarrow \cos\theta + \sec\theta = 2$ Find $\cos^{19}\theta + \sec^{19}\theta$

$\cos\theta = 1$ $\sec\theta = 1$

$\cos\theta = \cos 0^\circ$ $\sec\theta = \sec 0^\circ$

$\theta = 0^\circ$

$\therefore \sec^{19}\theta + \cos^{19}\theta = \sec^{19}(0) + \cos^{19}(0)$

$= 1 + 1 = 2$

$\rightarrow \sin(\alpha + \beta - \gamma) = \frac{1}{2}$, $\cos(\beta + \gamma - \alpha) = \frac{1}{2}$

$\tan(\gamma + \alpha - \beta) = 1$ find α, β, γ

$\sin(\alpha + \beta - \gamma) = \frac{1}{2}$

$\sin(\alpha + \beta - \gamma) = \sin 30^\circ$

$\alpha + \beta - \gamma = 30^\circ$ — I

$\cos(\beta + \gamma - \alpha) = \frac{1}{2}$

$\cos(\beta + \gamma - \alpha) = \cos 60^\circ$

$\beta + \gamma - \alpha = 60^\circ$ — II

$\tan(\gamma + \alpha - \beta) = 1$

$\tan(\gamma + \alpha - \beta) = \tan 45^\circ$

$\gamma + \alpha - \beta = 45^\circ$ — III

$\rightarrow$ Adding I and II

$\alpha + \beta - \gamma + \beta + \gamma - \alpha = 30^\circ + 60^\circ$

$2\beta = 90^\circ \Rightarrow \boxed{\beta = 45^\circ}$

Adding II and III

$\beta + \gamma - \alpha + \gamma + \alpha - \beta = 60 + 45$

$2\gamma = 105^\circ$ $\boxed{\gamma = 52\frac{1}{2}^\circ}$

Adding I and III

$\alpha + \beta - \gamma + \gamma + \alpha - \beta = 30^\circ + 45^\circ$

$2\alpha = 75^\circ$ $\alpha = \frac{75}{2} = \boxed{37\frac{1}{2}^\circ}$

In a ΔABC

$\tan(A + B - C) = 1$

$\sec(B + C - A) = 2$

find A, B and C

In a ΔABC

$A + B + C = 180^\circ$ — I

Now

$\tan(A + B - C) = 1$

$\tan(A + B - C) = \tan 45^\circ$

$A + B - C = 45^\circ$ — II

$\sec(B + C - A) = 2$

$\sec(B + C - A) = \sec 60^\circ$

$B + C - A = 60^\circ$ — III

Adding II and III

$A + B - C + B + C - A = 105^\circ$

$2B = 105^\circ$

$\boxed{B = 52\frac{1}{2}^\circ}$

Subtracting I − II

$A + B + C - A - B + C = 180^\circ - 45^\circ$

$2C = 135^\circ$

$\boxed{C = 67\frac{1}{2}^\circ}$

Now $A = 180^\circ - (B + C)$

$= 180^\circ - \left(52\frac{1}{2} + 67\frac{1}{2}^\circ\right)$

$= 180^\circ - 120^\circ = 60^\circ$

$\boxed{A = 60^\circ}$

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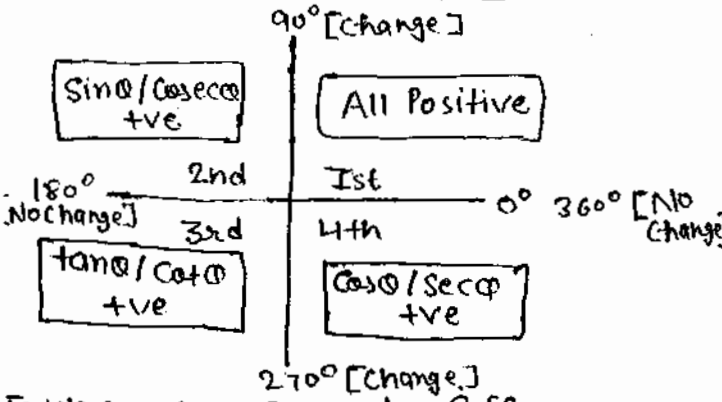
(Important)

TRIGNOMETRY [Part-III]



Trick :→ Add Sugar to Coffee
Add → All +ve; Sugar → Sinθ/Cosecθ +ve; to → tanθ/Cotθ +ve; Coffee → Cosθ/Secθ +ve

Ist Quadrent :→

All Positive

$\sin(90-\theta) = \cos\theta$
$\cos(90^\circ-\theta) = \sin\theta$
$\tan(90^\circ-\theta) = \cot\theta$
$\cot(90^\circ-\theta) = \tan\theta$
$\sec(90^\circ-\theta) = \operatorname{cosec}\theta$
$\operatorname{cosec}(90^\circ-\theta) = \sec\theta$

[All Change at 90°]

2nd Quadrent :→

in/Cosecθ +ve

$\sin(90^\circ+\theta) = \cos\theta$
$\cos(90^\circ+\theta) = -\sin\theta$
$\tan(90^\circ+\theta) = -\cot\theta$
$\cot(90^\circ+\theta) = -\tan\theta$
$\sec(90^\circ+\theta) = -\operatorname{cosec}\theta$
$\operatorname{cosec}(90^\circ+\theta) = \sec\theta$

[Value Change And Sign Change]

nd Quadrent :→

$\sin(180^\circ-\theta) = \sin\theta$
$\cos(180^\circ-\theta) = -\cos\theta$
$\tan(180^\circ-\theta) = -\tan\theta$
$\cot(180^\circ-\theta) = -\cot\theta$
$\sec(180^\circ-\theta) = -\sec\theta$
$\operatorname{cosec}(180^\circ-\theta) = \operatorname{cosec}\theta$

[No change at 180° But Sign Change]

3rd Quadrent :→

tanθ / Cotθ +ve

$\sin(180^\circ+\theta) = -\sin\theta$
$\cos(180^\circ+\theta) = -\cos\theta$
$\tan(180^\circ+\theta) = +\tan\theta$
$\cot(180^\circ+\theta) = \cot\theta$
$\sec(180^\circ+\theta) = -\sec\theta$
$\operatorname{cosec}(180^\circ+\theta) = -\operatorname{cosec}\theta$

[No Change at 180° But Sign Change]

3rd Quadrent :→

$\sin(270^\circ-\theta) = -\cos\theta$
$\cos(270^\circ-\theta) = -\sin\theta$
$\tan(270^\circ-\theta) = \cot\theta$
$\cot(270^\circ-\theta) = \tan\theta$
$\sec(270^\circ-\theta) = -\operatorname{cosec}\theta$
$\operatorname{cosec}(270^\circ-\theta) = -\sec\theta$

[Value change and Sign Change]

4th Quadrent :→

Cosθ / Secθ +ve

$\sin(270^\circ+\theta) = -\cos\theta$
$\cos(270^\circ+\theta) = \sin\theta$
$\tan(270^\circ+\theta) = -\cot\theta$
$\cot(270^\circ+\theta) = -\tan\theta$
$\sec(270^\circ+\theta) = \operatorname{cosec}\theta$
$\operatorname{cosec}(270^\circ+\theta) = -\sec\theta$

[Value and Sign Both Change]

4th Quadrent

Cosθ / Secθ +ve

$\sin(360^\circ-\theta) = -\sin\theta$
$\cos(360^\circ-\theta) = \cos\theta$
$\tan(360^\circ-\theta) = -\tan\theta$
$\cot(360^\circ-\theta) = -\cot\theta$
$\sec(360^\circ-\theta) = \sec\theta$
$\operatorname{cosec}(360^\circ-\theta) = -\operatorname{cosec}\theta$

[No Value Change But sign Change]

Ist Quadrent

All Positive

$\sin(360^\circ+\theta) = \sin\theta$
$\cos(360^\circ+\theta) = \cos\theta$
$\tan(360^\circ+\theta) = \tan\theta$
$\cot(360^\circ+\theta) = \cot\theta$
$\sec(360^\circ+\theta) = \sec\theta$
$\operatorname{cosec}(360^\circ+\theta) = \operatorname{cosec}\theta$

[All +ve No Value Change]

→

$\sin(360^\circ \times n+\theta) = \sin\theta$
$\cos(360^\circ \times n+\theta) = \cos\theta$
$\tan(360^\circ \times n+\theta) = \tan\theta$
$\cot(360^\circ \times n+\theta) = \cot\theta$
$\sec(360^\circ \times n+\theta) = \sec\theta$
$\operatorname{cosec}(360^\circ \times n+\theta) = \operatorname{cosec}\theta$

find the Value of following :→

$\sin 120^\circ = \sin(90^\circ+30^\circ)$
$= \cos 30^\circ = \frac{\sqrt{3}}{2}$

$\cos 120^\circ = \cos(90^\circ+30^\circ)$
$= -\sin 30^\circ = \frac{-1}{2}$

$\sin 420^\circ = \sin(360^\circ + 60^\circ)$

$= \sin 60^\circ = \frac{\sqrt{3}}{2}$

$\sin 480^\circ = \sin(360^\circ + 120^\circ)$

$= \sin 120^\circ$

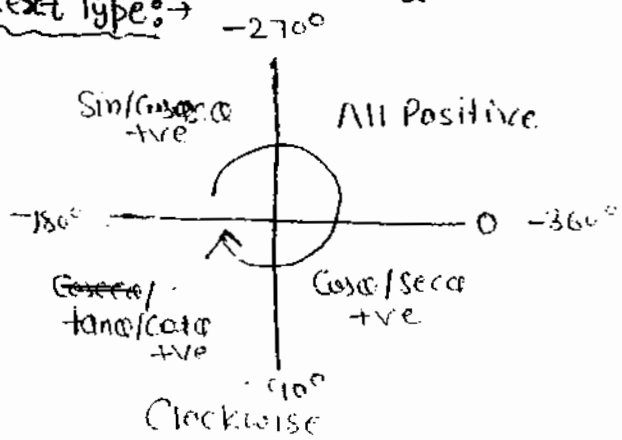
$= \sin(180^\circ - 60^\circ)$

$= \sin 60^\circ = \frac{\sqrt{3}}{2}$

$\sin 1860^\circ = \sin(360 \times 5 + 60^\circ)$

$= \sin 60^\circ = \frac{\sqrt{3}}{2}$

Next Type:→



Formula (Important)

$\sin(-\theta) = -\sin\theta$

$\cos(-\theta) = \cos\theta$

$\tan(-\theta) = -\tan\theta$

$\cot(-\theta) = -\cot\theta$

$\sec(-\theta) = \sec\theta$

$\operatorname{cosec}(-\theta) = -\operatorname{cosec}\theta$

Find the Value of following

1. $\sin(-120^\circ) = -\sin 120^\circ$

$= -\sin(180^\circ - 60^\circ)$

$= -\sin 60^\circ$

$= -\frac{\sqrt{3}}{2}$ Ans.

⇒ $\tan(-120^\circ) = -\tan 120^\circ$

$= -\tan(180^\circ - 60^\circ)$

$= -(-\tan 60^\circ)$

$= +\tan 60^\circ = \sqrt{3}$ Ans.

⇒ $\cos(-120^\circ) = \cos 120^\circ$

$= \cos(180^\circ - 60^\circ)$

$= -\cos 60^\circ = -\frac{1}{2}$ Ans.

⇒ $\sin(-300^\circ) = -\sin(300^\circ)$

$= -\sin(360^\circ - 60^\circ)$

$= -(-\sin 60^\circ) = +\frac{\sqrt{3}}{2}$ Ans.

Next Type:

If $A + B = 90^\circ$ [Complementary Angle]
पूरक कोण

Then
$$\boxed{\begin{aligned} \sin A &= \cos B \\ \sec A &= \operatorname{cosec} B \\ \tan A &= \cot B \end{aligned}}$$

Proof:→ $\sin A = \cos B$

Now $A + B = 90^\circ$, $A = 90^\circ - B$

∴ L.H.S

$\sin A = \sin(90^\circ - B)$

$= \cos B$

Hence $\sin A = \cos B$

$\sec A = \operatorname{cosec} B$

$\sec A = \sec(90^\circ - B)$

$\sec A = \operatorname{cosec} B$

Similarly $\tan A = \cot B$

Example

Find the Value of following

→ $\frac{\sin 40^\circ}{\cos 50^\circ}$

Ans = 1 [Sum of $40^\circ + 50^\circ = 90^\circ$ ∴ Ans = 1]

Detail ⇒ $\frac{\sin 40^\circ}{\cos 50^\circ} = \frac{\cos(90^\circ - 40^\circ)}{\cos 50^\circ} = \frac{\cos 50^\circ}{\cos 50^\circ} = 1$

→ $\frac{\tan 28^\circ}{\tan 62^\circ}$ Ans = 1

$\frac{\tan(28^\circ)}{\tan 62^\circ} = \frac{\cot(90^\circ - 28^\circ)}{\tan 62^\circ} = \frac{\tan 62^\circ}{\tan 62^\circ} = 1$

→ $\frac{\sin(40^\circ + \theta)}{\cos(50^\circ - \theta)} = 1$

Now $40 + \theta + 50 - \theta = 90^\circ$

Proof

⇒ $\frac{\sin(40^\circ + \theta)}{\cos(50^\circ - \theta)} = \frac{\cos(90^\circ - (40^\circ + \theta))}{\cos(50^\circ - \theta)}$

$= \frac{\cos(50^\circ - \theta)}{\cos(50^\circ - \theta)} = 1$ Ans.

→ $\cos 48^\circ - \sin 42^\circ$

⇒ $\cos 48^\circ - \sin(90^\circ - 48^\circ)$

⇒ $\cos 48^\circ - \cos 48^\circ = 0$

→ $\operatorname{cosec} 31^\circ - \sec 59^\circ$

⇒ $\operatorname{cosec} 31^\circ - \sec(90^\circ - 31^\circ)$

⇒ $\operatorname{cosec} 31^\circ - \operatorname{cosec} 31^\circ$

$= 0$ Ans.

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(9) $(\sin 72^\circ + \cos 18^\circ)(\sin 72^\circ - \cos 18^\circ)$.

$\Rightarrow (\sin 72^\circ + \cos 18^\circ)(\sin 72^\circ - \cos(90^\circ - 72^\circ))$

$\Rightarrow (\sin 72^\circ + \cos 18^\circ)(\sin 72^\circ - \sin 72^\circ)$

$= (\sin 72^\circ + \cos 18^\circ)0 = 0$

$\Rightarrow \sin 35^\circ \sin 55^\circ - \cos(35^\circ)\cos 55^\circ$

~~Sin~~ $\cos(55^\circ + 35^\circ) = \cos 90^\circ = 0$ Ans.

OR $\sin 35^\circ \cos(90 - 55^\circ) - \cos 35^\circ \cos(90^\circ - 55^\circ)$

$\sin 35^\circ \cos 35^\circ - \cos 35^\circ \sin 35^\circ$

$= 1 - 1 = 0$

$\Rightarrow \sec 50^\circ \sin 40^\circ + \cos 40^\circ \operatorname{cosec} 50^\circ$

$\Rightarrow \sec 50^\circ \sin(90^\circ - 50^\circ) + \cos 40^\circ \sec(90^\circ - 50^\circ)$

$\Rightarrow \sec 50^\circ \cos 50^\circ + \cos 40^\circ \sec 40^\circ$

$\Rightarrow 1 + 1 = 2$

$\Rightarrow \operatorname{cosec}(65 + \theta) - \sec(25 - \theta)$

Now $65 + \theta + 25 - \theta = 90^\circ$

$\therefore$ Ans $= 0$

Detail :→ $\operatorname{cosec}(65 + \theta) - \operatorname{cosec}[90^\circ - (25 - \theta)]$

$\Rightarrow \operatorname{cosec}(65 + \theta) - \operatorname{cosec}(65 + \theta)$

$= 0$ Ans.

$\Rightarrow \dfrac{\cos(90^\circ - \theta) \cdot \sin(90^\circ - \theta)}{\tan(90^\circ - \theta)}$

$= \dfrac{\sin\theta \cdot \cos\theta}{\cot\theta} = \dfrac{\sin\theta \cdot \cos\theta \cdot \sin\theta}{\cos\theta}$

$= \sin^2\theta$.

$\Rightarrow \tan 40^\circ \tan 50^\circ$

$\Rightarrow \tan 40^\circ \cot(90^\circ - 50^\circ)$ $\quad [40 + 50 = 90^\circ$, Ans $= 1]$

$\Rightarrow \tan 40^\circ \cot 40^\circ = 1$

$\Rightarrow \tan(32 + \theta)\tan(58 - \theta)$

$\Rightarrow \tan(32 + \theta)\cot[90^\circ - (58 - \theta)]$

$\Rightarrow \tan(32 + \theta)\cot(90^\circ - 58 + \theta)$

$\Rightarrow \tan(32^\circ + \theta)\cot(32^\circ + \theta)$

$\Rightarrow 1$ Ans.

Text $\tan 10^\circ \times \tan 20^\circ \times \tan 70^\circ \times \tan 80^\circ$

$\Rightarrow \tan(90^\circ - 80^\circ) \times \tan(90^\circ - 70^\circ) \times \tan 70^\circ \times \tan 80^\circ$

$\Rightarrow \cot 80^\circ \times \cot 70^\circ \times \tan 70^\circ \times \tan 80^\circ$

$= 1 \times 1 = 1$

$\tan 1^\circ \times \tan 2^\circ \times \tan 3^\circ \times \cdots \tan 89^\circ$

$\Rightarrow \tan 1^\circ = \cot 89^\circ$

$\tan 2^\circ = \cot 88^\circ$

$= \cot 89^\circ \times \cot 88^\circ \times \cdots \tan 45^\circ \times \cdots \tan 88^\circ \times \tan 89^\circ$

$= 1 \times \tan 45^\circ = 1 \times 1 = 1$

$\tan 5^\circ \times \tan 10^\circ \times \tan 15^\circ \times \cdots \tan 85^\circ$

$\Rightarrow \cot 85^\circ \times \cot 80^\circ \times \cdots$ ~~cot 80°~~ $\tan 80^\circ \times \tan 85^\circ$

$= 1 \times 1 \times 1 \cdots 1 = 1$

$\cot 15^\circ \times \cot 20^\circ \times \cot 25^\circ \times \cdots \cot 75^\circ$

$\Rightarrow \tan 75^\circ \times \tan 70^\circ \times \cdots \cot 70^\circ \times \cot 75^\circ$

$= 1 \times 1 = 1$

$\tan(10 + \alpha)\tan(20 + \beta) \times \tan(30 + \gamma)$
$\tan(80 - \alpha)\tan(70 - \beta) \times \tan(60 - \gamma)$

$= \tan(10 + \alpha) \cdot \tan(20 + \beta) \cdot \tan(30 + \gamma)$
$\cot(10 + \alpha)\cot(20 + \beta)\cot(30 + \gamma)$

$= 1 \cdot 1 \cdot 1 = 1$

$\rightarrow \sin^2 10^\circ + \sin^2 80^\circ = 1$

Proof $\sin^2 10^\circ + \sin^2 80^\circ$

$\cos^2(90^\circ - 10^\circ) + \sin^2 80^\circ$

$\cos^2 80^\circ + \sin^2 80^\circ = 1$

$\rightarrow \sin^2 8^\circ + \sin^2 42^\circ + \sin^2 48^\circ + \sin^2 82^\circ$

Now $\sin^2 8^\circ + \sin^2 82^\circ = 1$

$\sin^2 42^\circ + \sin^2 48^\circ = 1$

$\therefore 1 + 1 = 2$

[इस Type के Example में हम No. of Terms को देखते हैं कितनी Terms होगी उसको 2 से Divide कर देंगे 2 से Divide करने पर जो बचेगा वही Ans. होगा]

Here No of Terms $= 4$

Ans $= \frac{4}{2} = 2$

$\rightarrow \sin^2(48 + \theta) + \sin^2 30^\circ + \sin^2(42 - \theta)$

Now $\sin^2(48 + \theta) + \sin^2(42 - \theta) = 1$

$\therefore 1 + \sin^2 30^\circ = 1 + \frac{1}{4} = \frac{5}{4}$

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$\sin^2 1^\circ + \sin^2 2^\circ + \sin^2 3^\circ + \sin^2 4^\circ + \dots + \sin^2 89^\circ$

Here No. of Terms = 89

[Using
$T_n = a + (n-1)d$
$89 = 1 + (n-1)1$
$89 = 1 + n - 1$
$n = 89$]

$\text{Ans} = \frac{89}{2}$
$= 44\frac{1}{2}$

OR

$\sin^2 1^\circ + \sin^2 2^\circ + \dots + \sin^2 89^\circ$
$= \cos^2 89^\circ + \cos^2 88^\circ + \dots \sin^2 45^\circ + \dots \sin^2 88^\circ + \sin^2 89^\circ$

$44 + \sin^2 45^\circ$

$44 + \frac{1}{2} = \frac{89}{2} = 44\frac{1}{2}^\circ$

→ $\sin^2 5^\circ + \sin^2 10^\circ + \sin^2 15^\circ + \dots + \sin^2 85^\circ + \sin^2 90^\circ$

हम 5° से 85° तक No. of Terms देखते हैं।

∴ No. of Terms = 17

[$85 = 5 + (n-1)5$
$n - 1 = 16$
$n = 17$]

$\text{Ans} = \frac{17}{2} + \sin^2 90^\circ$
$= \frac{17+1}{2} = 8.5 + 1 = 9.5$

→ $\sin^2 10^\circ + \sin^2 20^\circ + \dots + \sin^2 80^\circ$

Here No. of Terms = 8

[$80 = 10 + (n-1)10$
$n - 1 = 7$
$n = 8$]

$\text{Ans} = \frac{8}{2} = 4$

→ $\sin^2 15^\circ + \sin^2 20^\circ + \sin^2 25^\circ + \dots + \sin^2 75^\circ$

→ No. of Terms = 13

[$75 = 15 + (n-1)5$
$(n-1)5 = 60$
$n = 13$]

Here
$\text{Ans} = \frac{13}{2} = 6.5$

→ $\cos^2 1^\circ + \cos^2 2^\circ + \dots + \cos^2 89^\circ$

No. of Terms = 89

$\text{Ans} = \frac{89}{2} = 44\frac{1}{2}$

→ $\cos^2 10^\circ + \cos^2 15^\circ + \dots + \cos^2 80^\circ$

No. of Terms = 15

$\text{Ans} = \frac{15}{2} = 7.5$

Next Type :→

In ΔABC $A + B + C = 180^\circ \Rightarrow A + B = 180^\circ - C$

$\frac{A+B}{2} = \frac{180^\circ - C}{2}$

$\Rightarrow \frac{A+B}{2} = 90^\circ - \frac{C}{2}$

Taking sinθ on Both side

$\sin\left(\frac{A+B}{2}\right) = \sin\left(90^\circ - \frac{C}{2}\right)$

$\boxed{\Rightarrow \sin\left(\frac{A+B}{2}\right) = \cos\frac{C}{2}}$

Similarly अगर L.H.S में कुछ भी दिया जैसे $\left(\frac{B+C}{2}\right)$ Then Ans में Remaining Ratio $\frac{A}{2}$ आ जायेगा है। एक side sinθ है तो दूसरी side cosθ आएगा।

$\Rightarrow$
$\operatorname{cosec}\left(\frac{A+B}{2}\right) = \sec\frac{C}{2}$
$\tan\left(\frac{A+B}{2}\right) = \cot\frac{C}{2}$

Next Type :→

If $\sin\theta = \cos 2\theta$ find the Value of θ

Detail
$\sin\theta = \cos 2\theta$
$\Rightarrow \sin\theta = \sin(90^\circ - 2\theta)$
$\Rightarrow \theta = 90^\circ - 2\theta$
$3\theta = 90^\circ$
$\boxed{\theta = 30^\circ}$

Short :→ Add Both θ and 2θ and Put equal to 90°
$\theta + 2\theta = 90^\circ$
$3\theta = 90^\circ$
$\boxed{\theta = 30^\circ}$

If $\tan 2\theta = \cot 3\theta$ find the Value of θ

Detail
$\tan 2\theta = \cot 3\theta$
$\tan 2\theta = \tan(90^\circ - 3\theta)$
$2\theta = 90^\circ - 3\theta$
$5\theta = 90^\circ$
$\boxed{\theta = 18^\circ}$

Short :→
$2\theta + 3\theta = 90^\circ$
$\theta = \frac{90^\circ}{5}$
$\boxed{\theta = 18^\circ}$

If $\sec 2\theta = \operatorname{cosec} 4\theta$ find the Value of θ

Short :→ $2\theta + 4\theta = 90^\circ$
$\theta = 90^\circ/6 = 15^\circ$

D→
$\sec 2\theta = \operatorname{cosec} 4\theta$
$\Rightarrow \sec 2\theta = \sec(90^\circ - 4\theta)$
$2\theta = 90 - 4\theta$
$\theta = 90^\circ/6 = 15^\circ$

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$\sec(\theta-20^\circ) = \operatorname{cosec}(25-2\theta)$ find θ

Detail is same as in Previous Example

$\theta-20+25-2\theta = 90^\circ$

$-\theta = 85^\circ$

$\boxed{\theta = -85^\circ}$

$\sin(\theta+30^\circ) = \cos 2\theta$ find θ

$\theta+30^\circ+2\theta = 90^\circ$

$3\theta = 60^\circ$ $\boxed{\theta = 20^\circ}$

Detail

$\sin(\theta+30^\circ) = \sin(90^\circ-2\theta)$

$\theta+30^\circ = 90^\circ-2\theta$

$3\theta = 60^\circ \Rightarrow \boxed{\theta = 20^\circ}$

Next Type :→

$\tan\theta \cdot \tan 2\theta = 1$ find the Value of θ

Now Detail

$\tan\theta \tan 2\theta = 1$

$\tan\theta \cdot \frac{1}{\cot 2\theta} = 1$

$\tan\theta = \cot 2\theta$

$\tan\theta = \tan(90-2\theta)$

$\theta = 90^\circ-2\theta$

$3\theta = 90^\circ$ $\theta = 30^\circ$

Short

$\tan\theta \tan 2\theta = 1$

$\theta+2\theta = 90^\circ$

$3\theta = 90^\circ$

$\boxed{\theta = 30^\circ}$

$\tan 2\theta \cdot \tan 3\theta = 1$ find θ

$2\theta+3\theta = 90^\circ$

$5\theta = 90^\circ$ $\boxed{\theta = 18^\circ}$

→ $\sin\theta \sec 3\theta = 1$

$\theta+3\theta = 90^\circ$ $4\theta = 90^\circ$

$\theta = \frac{90}{4} = 22\frac{1}{2}^\circ$

Detail

$\sin\theta \cdot \sec 3\theta = 1$

$\sin\theta \cdot \frac{1}{\cos 3\theta} = 1$

$\sin\theta = \cos 3\theta$

$= \sin(90^\circ-3\theta)$

$\Rightarrow \theta = 90^\circ-3\theta$

$4\theta = 90^\circ$

$\boxed{\theta = 22\frac{1}{2}^\circ}$

→ $\cot 5\theta \cdot \cot\theta = 1$ Find $\tan 3\theta$

Now Using Shortcut

$5\theta+\theta = 90^\circ$

$\theta = 90^\circ/6 = 15^\circ$

$\therefore \tan 3\theta = \tan 3\times 15^\circ = \tan 45^\circ = 1$

→ $\sin\theta = \cos(\theta-45^\circ)$ find θ

$\theta+\theta-45^\circ = 90^\circ$

$2\theta = 135^\circ$

$\theta = \frac{135}{2} = 67\frac{1}{2}^\circ$

→ $\sin(2\theta+45^\circ) = \cos(30-\theta)$ find

$\sin^2 3\theta + \cos^2 4\theta$

Now $2\theta+45^\circ+30^\circ-\theta = 90^\circ$

$\theta = 90^\circ-75^\circ$

$\theta = 15^\circ$

$\therefore \sin^2 3\theta + \cos^2 4\theta$

$= \sin^2 3\times 15 + \cos^2 4\times 15 = \sin^2 45^\circ + \cos^2 60^\circ$

$= \left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{1}{2}\right)^2 = \frac{1}{2}+\frac{1}{4} = \frac{3}{4}$

→ $\sin 3\theta = \cos(\theta-6^\circ)$ find θ

$3\theta+\theta-6^\circ = 90^\circ$

$4\theta = 96^\circ$

$\theta = \frac{96}{4} \Rightarrow \boxed{\theta = 24^\circ}$

→ $\sec 2\theta = \operatorname{cosec}(\theta-42^\circ)$

$2\theta+\theta-42^\circ = 90^\circ$

$3\theta = 132^\circ$ $\boxed{\theta = 44^\circ}$

→ $\cot 5\theta \times \cot(\theta-36^\circ) = 1$ find θ

$5\theta+\theta-36^\circ = 90^\circ$

$6\theta = 126^\circ$ $\boxed{\theta = 21^\circ}$

→ $\sin(x+y) = \cos[3(x+y)]$ Find $\tan[2(x+y)]$

Now $(x+y)+3(x+y) = 90^\circ$

$4(x+y) = 90^\circ$

$(x+y) = \frac{90^\circ}{4} = \frac{45^\circ}{2}$

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$\therefore \tan[2(x+y)] = \tan\left[2\times\frac{45^\circ}{2}\right]$

$= \tan 45^\circ = 1$ Ans.

→ $\tan(x+y)\cdot\tan(x-y) = 1$ find $\tan x^\circ$

Now $x+\cancel{y}+x-\cancel{y} = 90^\circ$

$2x = 90^\circ \quad x = 45^\circ$

$\therefore \tan x^\circ = \tan 45^\circ = 1$

→ $\cos x = \sin y$ and $\cot(x-40^\circ) = \tan(50-y)$ find x and y

Now $x+y = 90^\circ$ — I

$(x-40)+(50-y) = 90^\circ$ — II

$x-y = 80^\circ$ — II

$x+y = 90^\circ$

$x-y = 80^\circ$

$2x = 170^\circ \quad x = 85^\circ$

$\therefore y = 90-85^\circ = 5^\circ$

→ $\sin(3x-20) = \cos(3y+20)$ find $x+y$

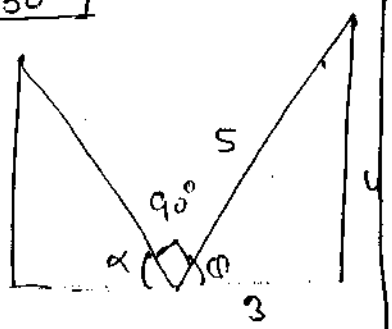
$3x-\cancel{20} + 3y+\cancel{20} = 90^\circ$

$3(x+y) = 90^\circ$

$\boxed{x+y = 30^\circ}$

Next Type :→

find the Value of $\sin\alpha$



Now

$\alpha+\theta = 180^\circ - 90^\circ$

$\alpha+\theta = 90^\circ$

$\alpha = 90^\circ - \theta$

Taking sin on Both side

$\sin\alpha = \sin(90^\circ-\theta)$

$\sin\alpha = \cos\theta = \frac{3}{5}$ $\quad \left[\cos\theta = \frac{B}{H} = \frac{3}{5}\right]$

$\therefore \boxed{\sin\alpha = \frac{3}{5}}$

If $A+B = 90^\circ$

find $\sqrt{\frac{\tan A\tan B + \tan A\cot B}{\sin A\sec B} - \frac{\sin^2 B}{\cos^2 A}}$

Now $A+B = 90^\circ \quad \therefore B = 90^\circ - A$

$= \sqrt{\frac{\tan A\tan(90^\circ-A) + \tan A\cot(90^\circ-A)}{\sin A\sec(90^\circ-A)} - \frac{\sin^2(90^\circ-A)}{\cos^2 A}}$

$= \sqrt{\frac{\tan A\cot A + \tan A\tan A}{\sin A\operatorname{cosec} A} - \frac{\cos^2 A}{\cos^2 A}}$

$= \sqrt{\frac{1+\tan^2 A}{1} - 1}$

$= \sqrt{\cancel{1}+\tan^2 A - \cancel{1}} = \sqrt{\tan^2 A}$

$= \tan A$.

If $A+B = 90^\circ$

find $\sin A\cos B + \cos A\sin B - \tan A\tan B + \sec^2 A - \cot^2 B$

$\sin(A+B) - \tan A\tan(90^\circ-A) + \sec^2 A - \cot^2(90^\circ-A)$

$[\because \sin A\cos B + \cos A\sin B = \sin(A+B)]$

$= \underset{1}{\sin 90^\circ} - \underset{1}{\tan A\cot A} + \underset{1}{(\sec^2 A - \tan^2 A)}$

$1 - 1 + 1 = 1$

If $A+B = 90^\circ$ find

$(1-\sin^2 A)(1-\cos^2 A)(1+\cot^2 B)(1+\tan^2 B)$

$= \cos^2 A\cdot\sin^2 A\ \operatorname{cosec}^2 B\cdot\sec^2 B$

$= \cos^2 A\ \sin^2 A\ \sec^2 A\ \operatorname{cosec}^2 A$

$= \underbrace{\quad\quad}_{1}\ \underbrace{\quad\quad}_{1}$

$= 1$ Ans.

If $A+B = 90^\circ$ find $\sec^2 A + \sec^2 B - \sec^2 A\times\sec^2 B$

$\Rightarrow \sec^2 A + \operatorname{cosec}^2 A - \sec^2 A\ \operatorname{cosec}^2 A$

$[\because \sec^2 B = \sec^2(90^\circ-A) = \operatorname{cosec}^2 A]$

$\Rightarrow \frac{1}{\sin^2 A} + \frac{1}{\cos^2 A} - \frac{1}{\cos^2 A\sin^2 A}$

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$$\frac{\sin^2 A+\cos^2 A}{\sin^2 A\cdot\cos^2 A} - \frac{1}{\sin^2 A\cos^2 A}$$

$$= \frac{1}{\sin^2 A\cos^2 A} - \frac{1}{\sin^2 A\cos^2 A}$$

$$= 1-1 = 0$$

→ $\sin 2\theta = 2\cos\theta \sin\theta$

$\sin 2\theta = 2\cos\theta$

$\Rightarrow 2\sin\theta\cos\theta = 2\cos\theta$

$\sin\theta = 1$

$\sin\theta = \sin 90^\circ$

$\boxed{\theta = 90^\circ}$

→ $\sin 2\theta = 2\sin\theta$ find θ

$\Rightarrow 2\sin\theta\cos\theta = 2\sin\theta$

$\cos\theta = 1$

$\cos\theta = \cos 0^\circ$

$\boxed{\theta = 0^\circ}$

→ $\cos(A-B) = \frac{1}{2}$, $\sin(A+B) = \frac{1}{2}$.

find A and B Both A and B are (+ve)

$\sin(A+B) = \frac{1}{2}$

$\sin(A+B) = \sin 30^\circ$

$\sin(A+B) = \sin(180^\circ - 150^\circ)$

$\sin(A+B) = \sin 150^\circ$

$A+B = 150^\circ$ — I

or $\cos(A-B) = \cos 60^\circ$

$A-B = 60^\circ$ — II

$A+B = 150^\circ$

$A-B = 60^\circ$

$2A = 210^\circ$ $\boxed{A = 105^\circ}$

$\therefore B = 150^\circ - 105^\circ = 45^\circ$

$\boxed{B = 45^\circ}$

① Solve the following:

$(1-\sin^2\theta)\sec^2\theta$

$= \cos^2\theta\cdot\sec^2\theta$ $[\because 1-\sin^2\theta = \cos^2\theta]$

$= \frac{1}{\sec^2\theta}\cdot\sec^2\theta = 1$

② $\cos^2\theta + \frac{1}{1+\cot^2\theta} = ?$

$\Rightarrow \cos^2\theta + \frac{1}{1+\cot^2\theta} = \cos^2\theta + \frac{1}{\operatorname{cosec}^2\theta}$

$\Rightarrow \cos^2\theta + \sin^2\theta$

$= 1$

③ $\cos^2\theta(1+\tan^2\theta) = ?$

$\cos^2\theta(1+\tan^2\theta)$

$= \cos^2\theta\cdot\sec^2\theta = \cos^2\theta\cdot\frac{1}{\cos^2\theta}$

$= 1$

④ $\cot^2\theta - \frac{1}{\sin^2\theta} = ?$

$\cot^2\theta - \frac{1}{\sin^2\theta} \Rightarrow \cot^2\theta - \operatorname{cosec}^2\theta$

$= -1(\operatorname{cosec}^2\theta - \cot^2\theta)$

$= -1$ Ans.

⑤ $(1+\tan^2\theta)(1+\sin\theta)(1-\sin\theta)$

$= (1+\tan^2\theta)(1+\sin\theta)(1-\sin\theta)$

$= \sec^2\theta\cdot(1-\sin^2\theta)$

$= \sec^2\theta\cdot\cos^2\theta = 1$

⑥ $(1+\cot^2\theta)(1-\cos\theta)(1+\cos\theta)$

$\Rightarrow \operatorname{cosec}^2\theta(1-\cos^2\theta)$

$\Rightarrow \operatorname{cosec}^2\theta\cdot\sin^2\theta = 1$

⑦ $\tan^2\theta - \frac{1}{\cos^2\theta} = ?$

$\Rightarrow \tan^2\theta - \sec^2\theta = -1(\sec^2\theta - \tan^2\theta)$

$= -1$ Ans.

TRIGNOMETRY [PART-IV]

Prove the following

→ Prove $\frac{\sin\theta}{1-\cos\theta} \Rightarrow \operatorname{cosec}\theta + \cos\theta$

L.H.S

$$\frac{\sin\theta}{1-\cos\theta} \Rightarrow \frac{\sin\theta}{1-\cos\theta} \times \frac{1+\cos\theta}{1+\cos\theta}$$

$$= \frac{\sin\theta(1+\cos\theta)}{1-\cos^2\theta}$$

$$= \frac{\sin\theta(1+\cos\theta)}{\sin^2\theta}$$

$$= \frac{1}{\sin\theta} + \frac{\cos\theta}{\sin\theta} = \operatorname{cosec}\theta + \cot\theta$$

(2) Prove $\frac{1-\sin\theta}{1+\sin\theta} = (\sec\theta - \tan\theta)^2$

L.H.S

$$\frac{1-\sin\theta}{1+\sin\theta} \Rightarrow \frac{(1-\sin\theta)(1-\sin\theta)}{(1+\sin\theta)(1-\sin\theta)}$$

$$= \frac{(1-\sin\theta)^2}{1-\sin^2\theta} = \cancel{1+\sin^2\theta - 2\sin\theta}$$

$$= \frac{(1-\sin\theta)^2}{\cos^2\theta} \Rightarrow \left(\frac{1-\sin\theta}{\cos\theta}\right)^2$$

$$= (\sec\theta - \tan\theta)^2$$

OR Put $\theta = 0°$

$$L.H.S = \frac{1-\sin\theta}{1+\sin\theta} = \frac{1-\sin 0°}{1+\sin 0°} = \frac{1-0}{1+0} = 1$$

$$R.H.S = (\sec\theta - \tan\theta)^2$$

$$= (\sec 0° - \tan 0°)^2$$

$$\Rightarrow (1-0)^2 = 1$$

(3) $\frac{1+\cos\theta}{1-\cos\theta} \Rightarrow (\operatorname{cosec}\theta + \cot\theta)^2$

$$\Rightarrow \frac{1+\cos\theta}{1-\cos\theta} \times \frac{1+\cos\theta}{1+\cos\theta}$$

$$\Rightarrow \frac{(1+\cos\theta)^2}{1-\cos^2\theta} = \left(\frac{1+\cos\theta}{\sin\theta}\right)^2$$

$$= (\operatorname{cosec}\theta + \cot\theta)^2$$

(4) $\frac{\cos\theta}{1-\sin\theta} \Rightarrow \sec\theta + \tan\theta$

$$\Rightarrow L.H.S \quad \frac{\cos\theta}{1-\sin\theta} \times \frac{(1+\sin\theta)}{1+\sin\theta}$$

$$\Rightarrow \frac{\cos\theta(1+\sin\theta)}{1-\sin^2\theta} \Rightarrow \frac{\cos\theta(1+\sin\theta)}{\cos^2\theta}$$

$$\Rightarrow \frac{1}{\cos\theta} + \frac{\sin\theta}{\cos\theta} \Rightarrow \sec\theta + \tan\theta$$

(5) $\sqrt{\frac{1+\sin\theta}{1-\sin\theta}} \Rightarrow \sec\theta + \tan\theta$

L.H.S

$$\sqrt{\frac{1+\sin\theta}{1-\sin\theta} \times \frac{1+\sin\theta}{1+\sin\theta}} = \sqrt{\frac{(1+\sin\theta)^2}{\cos^2\theta}}$$

$$= \sqrt{\left(\frac{1+\sin\theta}{\cos\theta}\right)^2} = \frac{1}{\cos\theta} + \frac{\sin\theta}{\cos\theta}$$

$$\Rightarrow \sec\theta + \tan\theta = R.H.S$$

OR Put $\theta = 0°$

$$L.H.S = \sqrt{\frac{1+\sin 0°}{1-\sin 0°}} = \sqrt{\frac{1+0}{1-0}} = 1$$

R.H.S. $\sec\theta + \tan\theta$

$$\Rightarrow \sec 0° + \tan 0° = 1+0 = 1$$

L.H.S = R.H.S

(6) $\sqrt{\frac{1-\cos\theta}{1+\cos\theta}} = \operatorname{cosec}\theta - \cot\theta$

L.H.S

$$\sqrt{\frac{(1-\cos\theta}{1+\cos\theta} \times \frac{1-\cos\theta}{1-\cos\theta}} = \sqrt{\frac{(1-\cos\theta)^2}{1-\cos^2\theta}}$$

$$= \sqrt{\left(\frac{1-\cos\theta}{\sin\theta}\right)^2} = \frac{1-\cos\theta}{\sin\theta}$$

$$= \frac{1}{\sin\theta} - \frac{\cos\theta}{\sin\theta} = \operatorname{cosec}\theta - \cot\theta$$

OR Put $\theta = 90°$

$$L.H.S = \sqrt{\frac{1-\cos 90°}{1+\cos 90°}} = \sqrt{\frac{1-0}{1+0}} = 1$$

$$R.H.S = \operatorname{cosec} 90° - \cot 90° = 1-0 = 1$$

L.H.S = R.H.S

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$\rightarrow$ K $\frac{1}{1+\sin\theta} + \frac{1}{1-\sin\theta} \Rightarrow 2\sec^2\theta$

$\Rightarrow \frac{1}{1+\sin\theta} + \frac{1}{1-\sin\theta}$

$\Rightarrow \frac{1-\sin\theta+1+\sin\theta}{1-\sin^2\theta} = \frac{2}{\cos^2\theta}$

$= 2\sec^2\theta.$

$\rightarrow \frac{\cos\theta}{1-\sin\theta} + \frac{\cos\theta}{1+\sin\theta} = 2\sec\theta$

$\Rightarrow \frac{\cos\theta}{1-\sin\theta} + \frac{\cos\theta}{1+\sin\theta} \Rightarrow \cos\theta\left(\frac{1}{1-\sin\theta} + \frac{1}{1+\sin\theta}\right)$

$= \cos\theta\left(\frac{1+\sin\theta+1-\sin\theta}{1-\sin^2\theta}\right)$

$2\cos\theta \cdot \frac{1}{\cos^2\theta} = 2\sec\theta$

$\rightarrow \frac{\sin\theta}{1-\cos\theta} + \frac{\sin\theta}{1+\cos\theta} = 2\operatorname{cosec}\theta$

$\Rightarrow \sin\theta\left(\frac{1}{1-\cos\theta} + \frac{1}{1+\cos\theta}\right)$

$\Rightarrow \sin\theta\left(\frac{1+\cos\theta+1-\cos\theta}{1-\cos^2\theta}\right)$

$= 2\sin\theta \cdot \frac{1}{\sin^2\theta} = 2\operatorname{cosec}\theta.$

$\rightarrow (\operatorname{cosec}\theta - \cot\theta)^2 \Rightarrow \frac{1-\cos\theta}{1+\cos\theta}$

L.H.S

$\Rightarrow \left(\frac{1}{\sin\theta} - \frac{\cos\theta}{\sin\theta}\right)^2$

$\Rightarrow \frac{(1-\cos\theta)^2}{\sin^2\theta} = \frac{(1-\cos\theta)^2}{1-\cos^2\theta}$

$= \frac{(1-\cos\theta)(1-\cos\theta)}{(1+\cos\theta)(1-\cos\theta)}$

$\Rightarrow \frac{1-\cos\theta}{1+\cos\theta} = R.H.S.$

$\rightarrow \frac{\sin\theta - 2\sin^3\theta}{2\cos^3\theta - \cos\theta} \Rightarrow \tan\theta$

L.H.S $= \frac{\sin\theta - 2\sin^3\theta}{2\cos^3\theta - \cos\theta}$

$= \frac{\sin\theta(1-2\sin^2\theta)}{\cos\theta(2\cos^2\theta - 1)}$

$= \frac{\sin\theta(\cos^2\theta - \sin^2\theta)}{\cos\theta(\cos^2\theta - \sin^2\theta)} = \tan\theta$

$\Rightarrow \sec^4\theta - \sec^2\theta \Rightarrow \tan^4\theta + \tan^2\theta$

L.H.S $= \sec^4\theta - \sec^2\theta$

$= \sec^2\theta(\sec^2\theta - 1)$

$= (1+\tan^2\theta)\tan^2\theta$

$= \tan^2\theta + \tan^4\theta = R.H.S.$

$\Rightarrow (\sin\theta + \operatorname{cosec}\theta)^2 + (\cos\theta + \sec\theta)^2$

$= 7 + \tan^2\theta + \cot^2\theta$

L.H.S

$(\sin\theta + \operatorname{cosec}\theta)^2 + (\cos\theta + \sec\theta)^2$

$\Rightarrow \sin^2\theta + \operatorname{cosec}^2\theta + 2\sin\theta\operatorname{cosec}\theta + \cos^2\theta$

$+ \sec^2\theta + 2\cos\theta\sec\theta$

$= \sin^2\theta + \cos^2\theta + 2\cdot 1 + \operatorname{cosec}^2\theta + \sec^2\theta + 2\cdot 1$

$= 1 + 2 + 2 + 1 + \cot^2\theta + 1 + \tan^2\theta$

$= 7 + \tan^2\theta + \cot^2\theta = R.H.S.$

$\rightarrow (\sin\theta + \sec\theta)^2 + (\operatorname{cosec}\theta + \cos\theta)^2$

$= (1 + \sec\theta\operatorname{cosec}\theta)^2$

L.H.S $(\sin\theta + \sec\theta)^2 + (\operatorname{cosec}\theta + \cos\theta)^2$

$= \left(\frac{1}{\operatorname{cosec}\theta} + \sec\theta\right)^2 + \left(\operatorname{cosec}\theta + \frac{1}{\sec\theta}\right)^2$

$= \left(\frac{1+\sec\theta\operatorname{cosec}\theta}{\operatorname{cosec}\theta}\right)^2 + \left(\frac{1+\operatorname{cosec}\theta\sec\theta}{\sec\theta}\right)^2$

$= (1+\sec\theta\operatorname{cosec}\theta)^2\left[\frac{1}{\operatorname{cosec}^2\theta} + \frac{1}{\sec^2\theta}\right]$

$= (1+\sec\theta\operatorname{cosec}\theta)^2\left[\sin^2\theta + \cos^2\theta\right]$

$= (1+\sec\theta\operatorname{cosec}\theta)^2$

$\Rightarrow (\sin\theta - \sec\theta)^2 + (\cos\theta - \operatorname{cosec}\theta)^2$

$= (1-\sec\theta\operatorname{cosec}\theta)^2$

$\Rightarrow$ L.H.S

$\left(\frac{1}{\operatorname{cosec}\theta} - \sec\theta\right)^2 + \left(\frac{1}{\sec\theta} - \operatorname{cosec}\theta\right)^2$

$\Rightarrow \left(\frac{1-\sec\theta\operatorname{cosec}\theta}{\operatorname{cosec}\theta}\right)^2 + \left(\frac{1-\operatorname{cosec}\theta\sec\theta}{\sec\theta}\right)^2$

$= (1-\sec\theta\operatorname{cosec}\theta)^2\left[\frac{1}{\operatorname{cosec}^2\theta} + \frac{1}{\sec^2\theta}\right]$

$= (1-\sec\theta\operatorname{cosec}\theta)^2[1] = (1-\sec\theta\operatorname{cosec}\theta)^2$

Next Type :→

① Prove the following :→

$$\frac{\tan\theta - \cot\theta}{\sin\theta\cos\theta} \Rightarrow \tan^2\theta - \cot^2\theta$$

L.H.S

$$\frac{\tan\theta - \cot\theta}{\sin\theta\cos\theta} = \frac{\frac{\sin\theta}{\cos\theta} - \frac{\cos\theta}{\sin\theta}}{\sin\theta\cos\theta}$$

$$\Rightarrow \frac{\sin^2\theta - \cos^2\theta}{\sin^2\theta\cos^2\theta}$$

$$\Rightarrow \frac{\sin^2\theta}{\sin^2\theta\cos^2\theta} - \frac{\cos^2\theta}{\sin^2\theta\cos^2\theta}$$

$$= \frac{1}{\cos^2\theta} - \frac{1}{\sin^2\theta} \Rightarrow \sec^2\theta - \operatorname{cosec}^2\theta$$

$$\Rightarrow 1 + \tan^2\theta - 1 - \cot^2\theta$$

$$\Rightarrow \tan^2\theta - \cot^2\theta = R.H.S$$

② $\tan^2\theta + \cot^2\theta + 2 = \sec^2\theta \cdot \operatorname{cosec}^2\theta$

L.H.S $\tan^2\theta + \cot^2\theta + 2$

$$\Rightarrow \tan^2\theta + \cot^2\theta + 2\tan\theta\cot\theta$$

$$= (\tan\theta + \cot\theta)^2$$

$$= \left(\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}\right)^2 = \left(\frac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta}\right)^2$$

$$= \left[\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}\right]^2$$

$$= \cancel{\tan\theta} \left[\frac{1}{\sin\theta\cos\theta}\right]^2$$

$$= \sec^2\theta \cdot \operatorname{cosec}^2\theta.$$

OR

Put $\theta = 45^\circ$

$$L.H.S = \tan^2 45^\circ + \cot^2 45^\circ + 2$$

$$= 1 + 1 + 2 = 4$$

$$R.H.S = \sec^2 45^\circ \operatorname{cosec}^2 45^\circ$$

$$= (\sqrt{2})^2 \cdot (\sqrt{2})^2 \Rightarrow 2 \cdot 2 = 4 \text{ Ans}$$

$$L.H.S = R.H.S$$

③ $\sqrt{\sec^2\theta + \operatorname{cosec}^2\theta} = \tan\theta + \cot\theta$

$\Rightarrow$ L.H.S $\sqrt{\sec^2\theta + \operatorname{cosec}^2\theta}$

$$= \sqrt{1 + \tan^2\theta + 1 + \cot^2\theta}$$

$$= \sqrt{\tan^2\theta + \cot^2\theta + 2}$$

$$= \sqrt{\tan^2\theta + \cot^2\theta + 2\tan\theta\cot\theta}$$

$$= \sqrt{(\tan\theta + \cot\theta)^2} = \tan\theta + \cot\theta$$

$$= R.H.S$$

OR Put $\theta = 45^\circ$

$$L.H.S = \sqrt{\sec^2 45^\circ + \operatorname{cosec}^2 45^\circ}$$

$$= \sqrt{(\sqrt{2})^2 + (\sqrt{2})^2} = \sqrt{2+2} = 2 \text{ Ans.}$$

$$R.H.S = \tan 45^\circ + \cot 45^\circ = 1 + 1 = 2$$

$$L.H.S = R.H.S.$$

④ $\cot^2\theta\left(\frac{\sec\theta - 1}{\sin\theta + 1}\right) + \sec^2\theta\left(\frac{\sin\theta - 1}{\sec\theta + 1}\right) \Rightarrow 0$

L.H.S

$$= \frac{\cot^2\theta(\sec^2\theta - 1) + \sec^2\theta(\sin^2\theta - 1)}{(\sin\theta + 1)(\sec\theta + 1)}$$

$$= \frac{\cot^2\theta \cdot \tan^2\theta + \sec^2\theta(-\cos^2\theta)}{(\sin\theta + 1)(\sec\theta + 1)}$$

$$= \frac{1 - 1}{(\sin\theta + 1)(\sec\theta + 1)} = 0 \text{ Ans.}$$

OR Put $\theta = 45^\circ$

$$L.H.S = \cot^2 45^\circ\left(\frac{\sec 45^\circ - 1}{\sin 45^\circ + 1}\right) + \sec^2 45^\circ\left(\frac{\sin 45^\circ - 1}{\sec 45^\circ + 1}\right)$$

$$= 1\left(\frac{\sqrt{2} - 1}{\frac{1}{\sqrt{2}} + 1}\right) + 2\left(\frac{\frac{1}{\sqrt{2}} - 1}{\sqrt{2} + 1}\right)$$

$$= \frac{\sqrt{2}(\sqrt{2} - 1)}{\sqrt{2} + 1} + \frac{2}{\sqrt{2}}\frac{(1 - \sqrt{2})}{\sqrt{2} + 1}$$

$$= \frac{\sqrt{2}(\cancel{\sqrt{2} - 1})}{\sqrt{2} + 1} - \frac{\sqrt{2}(\cancel{\sqrt{2} - 1})}{\sqrt{2} + 1} = 0$$

$$L.H.S = R.H.S.$$

⑤ $\frac{\sin^3\theta + \cos^3\theta}{\sin\theta + \cos\theta} + \sin\theta\cos\theta \Rightarrow 1$

$$\Rightarrow \frac{\cancel{(\sin\theta + \cos\theta)}(\sin^2\theta + \cos^2\theta - \sin\theta\cos\theta)}{\cancel{\sin\theta + \cos\theta}} + \sin\theta\cos\theta$$

$$= 1 - \cancel{\sin\theta\cos\theta} + \cancel{\sin\theta\cos\theta}$$

$$\Rightarrow 1 \text{ Ans.}$$

$$L.H.S = R.H.S$$

Put $\theta = 0^\circ \Rightarrow \frac{\sin^3 0^\circ + \cos^3 0^\circ}{\sin 0^\circ + \cos 0^\circ} + \sin 0^\circ \cos 0^\circ$

$$\Rightarrow \frac{1}{1} + 0 = 1 = R.H.S$$

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$\rightarrow \frac{\cos^2\theta}{1-\tan\theta} + \frac{\sin^3\theta}{\sin\theta-\cos\theta} \Rightarrow 1+\sin\theta\cos\theta$

$\Rightarrow$ L.H.S $\Rightarrow \frac{\cos^2\theta}{1-\frac{\sin\theta}{\cos\theta}} + \frac{\sin^3\theta}{\sin\theta-\cos\theta}$

$\Rightarrow \frac{\cos^3\theta}{\cos\theta-\sin\theta} - \frac{\sin^3\theta}{\cos\theta-\sin\theta}$

$= \frac{\cos^3\theta-\sin^3\theta}{\cos\theta-\sin\theta}$

$\Rightarrow \frac{\cancel{(\cos\theta-\sin\theta)}(\cos^2\theta+\sin^2\theta+\sin\theta\cos\theta)}{\cancel{(\cos\theta-\sin\theta)}}$

$\Rightarrow 1+\sin\theta\cos\theta =$ R.H.S

$\rightarrow \frac{\sin A-\sin B}{\cos A+\cos B} + \frac{\cos A-\cos B}{\sin A+\sin B} \Rightarrow 0$

L.H.S $\frac{\sin^2A-\sin^2B+\cos^2A-\cos^2B}{(\cos A+\cos B)(\sin A+\sin B)}$

$\Rightarrow \frac{1-1}{(\cos A+\cos B)(\sin A+\sin B)} = 0 =$ R.H.S

L.H.S = R.H.S.

$\rightarrow \frac{\cos\theta}{1-\tan\theta} + \frac{\sin^2\theta}{\sin\theta-\cos\theta} \Rightarrow \sin\theta+\cos\theta$

L.H.S $\Rightarrow \frac{\cos\theta}{1-\frac{\sin\theta}{\cos\theta}} + \frac{\sin^2\theta}{\sin\theta-\cos\theta}$

$\Rightarrow \frac{\cos^2\theta}{\cos\theta-\sin\theta} - \frac{\sin^2\theta}{\cos\theta-\sin\theta}$

$\Rightarrow \frac{\cos^2\theta-\sin^2\theta}{\cos\theta-\sin\theta}$

$\Rightarrow \frac{(\cos\theta+\sin\theta)\cancel{(\cos\theta-\sin\theta)}}{\cancel{\cos\theta-\sin\theta}}$

$\Rightarrow \sin\theta+\cos\theta =$ R.H.S

$\rightarrow \frac{\cos\theta}{1-\tan\theta} + \frac{\sin\theta}{1-\cot\theta} \Rightarrow \cos\theta+\sin\theta$

L.H.S $\Rightarrow \frac{\cos\theta}{1-\frac{\sin\theta}{\cos\theta}} + \frac{\sin\theta}{1-\frac{\cos\theta}{\sin\theta}}$

$\Rightarrow \frac{\cos^2\theta}{\cos\theta-\sin\theta} + \frac{\sin^2\theta}{\sin\theta-\cos\theta}$

$\Rightarrow \frac{\cos^2\theta-\sin^2\theta}{\cos\theta-\sin\theta} \Rightarrow \frac{\cancel{(\cos\theta-\sin\theta)}(\cos\theta+\sin\theta)}{\cancel{\cos\theta-\sin\theta}}$

$\rightarrow \cos\theta+\sin\theta$

$\rightarrow \frac{\tan\theta}{1-\cot\theta} + \frac{\cot\theta}{1-\tan\theta} = 1+\sec\theta\,\mathrm{cosec}\,\theta$

L.H.S

$= \frac{\tan\theta(1-\tan\theta)+\cot\theta(1-\cot\theta)}{(1-\cot\theta)(1-\tan\theta)}$

$\Rightarrow \frac{\tan\theta-\tan^2\theta+\cot\theta-\cot^2\theta}{(1-\cot\theta)(1-\tan\theta)}$

$\Rightarrow \frac{\tan\theta-\tan^2\theta+\frac{1}{\tan\theta}-\frac{1}{\tan^2\theta}}{\left(1-\frac{1}{\tan\theta}\right)(1-\tan\theta)}$

$= \cancel{\tan^2\theta}$

$\Rightarrow \frac{\tan\theta}{1-\frac{1}{\tan\theta}} + \frac{\frac{1}{\tan\theta}}{1-\tan\theta}$

$= \frac{\tan^2\theta}{\tan\theta-1} + \frac{1}{\tan\theta(1-\tan\theta)}$

$= \frac{\tan^3\theta-1}{(\tan\theta-1)\tan\theta}$

$\Rightarrow \frac{\cancel{(\tan\theta-1)}(\tan^2\theta+1+\tan\theta)}{\cancel{(\tan\theta-1)}\tan\theta}$

$= \tan\theta+\frac{1}{\tan\theta}+1$

$\Rightarrow \frac{\sin\theta}{\cos\theta}+\frac{\cos\theta}{\sin\theta}+1$

$\Rightarrow \frac{\sin^2\theta+\cos^2\theta+\sin\theta\cos\theta}{\sin\theta\cos\theta}$

$\Rightarrow \frac{1+\sin\theta\cos\theta}{\sin\theta\cos\theta} = \sec\theta\,\mathrm{cosec}\,\theta+1$

$=$ R.H.S.

OR

L.H.S $\Rightarrow \frac{\frac{\sin\theta}{\cos\theta}}{1-\frac{\cos\theta}{\sin\theta}} + \frac{\frac{\cos\theta}{\sin\theta}}{1-\frac{\sin\theta}{\cos\theta}}$

$= \frac{\sin^2\theta}{\cos\theta(\sin\theta-\cos\theta)} + \frac{\cos^2\theta}{\sin\theta(\cos\theta-\sin\theta)}$

$\Rightarrow \frac{\sin^3-\cos^3\theta}{\sin\theta\cos\theta(\sin\theta-\cos\theta)}$

$= \frac{\cancel{(\sin\theta-\cos\theta)}(\sin^2\theta+\cos^2\theta+\sin\theta\cos\theta)}{(\sin\theta\cos\theta)\cancel{(\sin\theta-\cos\theta)}}$

③ $\frac{\sin\theta}{1+\cos\theta} + \frac{1+\cos\theta}{\sin\theta} \Rightarrow 2\operatorname{cosec}\theta$

$$\frac{\sin^2\theta + (1+\cos\theta)^2}{(1+\cos\theta)\sin\theta}$$

$$\Rightarrow \frac{\sin^2\theta + 1 + \cos^2\theta + 2\cos\theta}{(1+\cos\theta)\sin\theta}$$

$$= \frac{2+2\cos\theta}{(1+\cos\theta)\sin\theta} \Rightarrow \frac{2\cancel{(1+\cos\theta)}}{\cancel{(1+\cos\theta)}\sin\theta}$$

$$\Rightarrow 2\operatorname{cosec}\theta$$

④ $(\operatorname{cosec}\theta - \sin\theta)(\sec\theta - \cos\theta) = \frac{1}{\tan\theta + \cot\theta}$

L.H.S

$$\Rightarrow \left(\frac{1}{\sin\theta} - \sin\theta\right)\left(\frac{1}{\cos\theta} - \cos\theta\right)$$

$$\Rightarrow \frac{1-\sin^2\theta}{\sin\theta} \cdot \frac{1-\cos^2\theta}{\cos\theta}$$

$$\Rightarrow \frac{\cos^2\theta}{\sin\theta} \cdot \frac{\sin^2\theta}{\cos\theta} = \frac{\sin\theta\cos\theta}{1}$$

$$\Rightarrow \frac{\sin\theta\cos\theta}{\sin^2\theta + \cos^2\theta}$$

$$= \frac{1}{\frac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta}} = \frac{1}{\tan\theta + \cot\theta}$$

Next Type :→ IMPORTANT

① $\frac{\tan\theta + \sec\theta - 1}{\tan\theta - \sec\theta + 1} \Rightarrow \frac{1+\sin\theta}{\cos\theta}$

L.H.S $\frac{\tan\theta + \sec\theta - 1}{\tan\theta - \sec\theta + 1}$

$$= \frac{\tan\theta + \sec\theta - (\sec^2\theta - \tan^2\theta)}{\tan\theta - \sec\theta + 1}$$

$$= \frac{\tan\theta + \sec\theta - (\sec\theta + \tan\theta)(\sec\theta - \tan\theta)}{\tan\theta - \sec\theta + 1}$$

$$= \frac{(\sec\theta + \tan\theta)(\cancel{1 - \sec\theta + \tan\theta})}{\cancel{\tan\theta - \sec\theta + 1}}$$

$$\Rightarrow \sec\theta + \tan\theta$$

$$\Rightarrow \frac{1}{\cos\theta} + \frac{\sin\theta}{\cos\theta} \Rightarrow \frac{1+\sin\theta}{\cos\theta}.$$

Put $\theta = 45^\circ$

L.H.S

$$\frac{\tan 45^\circ + \sec 45^\circ - 1}{\tan 45^\circ - \sec 45^\circ + 1}$$

$$= \frac{1+\sqrt{2}-1}{1-\sqrt{2}+1} = \frac{\sqrt{2}}{2-\sqrt{2}} = \frac{\cancel{\sqrt{2}}}{\cancel{\sqrt{2}}(\sqrt{2}-1)} \times \frac{\sqrt{2}+1}{\sqrt{2}+1}$$

$$= \frac{\sqrt{2}+1}{1}$$

R.H.S

$$\frac{1+\sin 45^\circ}{\cos 45^\circ} = \frac{1+\frac{1}{\sqrt{2}}}{\frac{1}{\sqrt{2}}} \quad \frac{\sqrt{2}+1}{\cancel{\sqrt{2}}} \times \frac{\cancel{\sqrt{2}}}{1}$$

$$= \sqrt{2}+1$$

L.H.S = R.H.S.

② $\frac{\cot\theta + \operatorname{cosec}\theta - 1}{\cot\theta - \operatorname{cosec}\theta + 1} \Rightarrow \frac{1+\cos\theta}{\sin\theta}$

$$\text{L.H.S.} = \frac{\cot\theta + \operatorname{cosec}\theta - (\operatorname{cosec}^2\theta - \cot^2\theta)}{\cot\theta - \operatorname{cosec}\theta + 1}$$

$$= \frac{\cot\theta + \operatorname{cosec}\theta - (\operatorname{cosec}\theta + \cot\theta)(\operatorname{cosec}\theta - \cot\theta)}{\cot\theta - \operatorname{cosec}\theta + 1}$$

$$\Rightarrow \frac{\cot\theta + \operatorname{cosec}\theta\,(\cancel{1 - \operatorname{cosec}\theta + \cot\theta})}{\cancel{\cot\theta - \operatorname{cosec}\theta + 1}}$$

$$\Rightarrow \cot\theta + \operatorname{cosec}\theta = \frac{\cos\theta}{\sin\theta} + \frac{1}{\sin\theta}$$

$$= \frac{1+\cos\theta}{\sin\theta} = \text{R.H.S}$$

OR

We can Also Put $\theta = 45^\circ$

③ $\frac{1 - \cos\theta + \sin\theta}{\sin\theta + \cos\theta - 1} \Rightarrow \sec\theta + \tan\theta.$

[We can Put $\theta = 45^\circ$] →

Or Dividing Num. and Den. by $\cos\theta$

$$\frac{\frac{1}{\cos\theta} - \frac{\cos\theta}{\cos\theta} + \frac{\sin\theta}{\cos\theta}}{\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\cos\theta} - \frac{1}{\cos\theta}} \Rightarrow \frac{\sec\theta - 1 + \tan\theta}{\tan\theta + 1 - \sec\theta}$$

$$\Rightarrow \frac{\tan\theta + \sec\theta - 1}{\tan\theta - \sec\theta + 1}$$

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(4) $\dfrac{\cos\theta-\sin\theta+1}{\cos\theta+\sin\theta-1} \Rightarrow \operatorname{cosec}\theta+\cot\theta$

[Numerator मे जो Sign ⊖ दिया हो उसी से Divide किया जाता है यहा $\sin\theta$ के आगे ⊖ve Sign है तो $\sin\theta$ से Divide करेंगे]

$$\frac{\frac{\cos\theta}{\sin\theta}-\frac{\sin\theta}{\sin\theta}+\frac{1}{\sin\theta}}{\frac{\cos\theta}{\sin\theta}+\frac{\sin\theta}{\sin\theta}-\frac{1}{\sin\theta}} \Rightarrow \frac{\cot\theta+\operatorname{cosec}\theta-1}{\cot\theta-\operatorname{cosec}\theta+1}$$

[Same as in 2nd Example]

Next Type:→ Prove

$(1+\cot\theta-\operatorname{cosec}\theta)(1+\tan\theta+\sec\theta)=2$

Put $\theta=45^\circ$

$(1+\cos 45^\circ-\operatorname{cosec}45^\circ)(1+\tan 45^\circ+\sec 45^\circ)$

$(1+1-\sqrt{2})(1+1+\sqrt{2})$

$(2-\sqrt{2})(2+\sqrt{2}) = 2^2-\sqrt{2}^2 = 4-2=2$

L.H.S = R.H.S

Detail :→ L.H.S

$(1+\cot\theta-\operatorname{cosec}\theta)(1+\tan\theta+\sec\theta)$

$\Rightarrow \left(1+\frac{\cos\theta}{\sin\theta}-\frac{1}{\sin\theta}\right)\left(1+\frac{\sin\theta}{\cos\theta}+\frac{1}{\cos\theta}\right)$

$\frac{(\sin\theta+\cos\theta-1)}{\sin\theta}\frac{(\cos\theta+\sin\theta+1)}{\cos\theta}$

$\Rightarrow \frac{(\sin\theta+\cos\theta-1)(\sin\theta+\cos\theta+1)}{\sin\theta\cos\theta}$

$\Rightarrow \frac{(\sin\theta+\cos\theta)^2-1^2}{\sin\theta\cos\theta}$

$\Rightarrow \frac{\sin^2\theta+\cos^2\theta+2\sin\theta\cos\theta-1}{\sin\theta\cos\theta}$

$= \frac{1+2\sin\theta\cos\theta-1}{\sin\theta\cos\theta} = \frac{2\cancel{\sin\theta\cos\theta}}{\cancel{\sin\theta\cos\theta}}$

$=2$.

(2) $(1+\tan\theta-\sec\theta)(1+\cot\theta+\operatorname{cosec}\theta)=2$

Put $\theta=45^\circ$

$(1+\tan 45^\circ-\sec 45^\circ)(1+\cot 45^\circ+\operatorname{cosec}45^\circ)$

$=(1+1-\sqrt{2})(1+1+\sqrt{2})$

$4-2=2=$ RHS

(3) $(1+\tan 42^\circ-\sec 42^\circ)(1+\tan 48^\circ+\operatorname{cosec}42^\circ) \Rightarrow 2$

L.H.S

$(1+\tan 42^\circ-\sec 42^\circ)(1+\cot 48^\circ+\operatorname{cosec}42^\circ)$

$=\left(1+\frac{\sin 42^\circ}{\cos 42^\circ}-\frac{1}{\cos 42^\circ}\right)\left(1+\frac{\cos 42^\circ}{\sin 42^\circ}+\frac{1}{\sin 42^\circ}\right)$

$=\left(\frac{\cos 42^\circ+\sin 42^\circ-1}{\cos 42^\circ}\right)\left(\frac{\sin 42^\circ+\cos 42^\circ+1}{\sin 42^\circ}\right)$

$=\frac{(\sin 42^\circ+\cos 42^\circ)^2-1}{\sin 42^\circ\cos 42^\circ}$

$=\frac{\sin^2 42^\circ+\cos^2 42^\circ+2\sin 42^\circ\cos 42^\circ-1}{\sin 42^\circ\cos 42^\circ}$

$=\frac{\cancel{1}+2\sin 42^\circ\cos 42^\circ-\cancel{1}}{\sin 42^\circ\cos 42^\circ} = 2$ Ans.

Next Type :→

(1) $\frac{\tan\theta}{1-\cot\theta}+\frac{\cot\theta}{1-\tan\theta} \Rightarrow 1+\tan\theta+\cot\theta$

L.H.S $\frac{\tan\theta}{1-\frac{1}{\tan\theta}}+\frac{\frac{1}{\tan\theta}}{1-\tan\theta}$

$\frac{\tan^2\theta}{\tan\theta-1}+\frac{1}{\tan\theta(1-\tan\theta)}$

$\Rightarrow \frac{\tan^3\theta-1}{\tan\theta(\tan\theta-1)}$

$\Rightarrow \frac{\cancel{(\tan\theta-1)}(\tan^2\theta+1+\tan\theta)}{\tan\theta\cancel{(\tan\theta-1)}}$

$=\tan\theta+\cot\theta+1=$ R.H.S

(2) $\frac{\cot\theta}{\cot\theta-\cot 3\theta}+\frac{\tan\theta}{\tan\theta-\tan 3\theta} = 1+\tan\theta+\cot\theta$

$\Rightarrow \frac{\frac{1}{\tan\theta}}{\frac{1}{\tan\theta}-\frac{1}{\tan 3\theta}}+\frac{\tan\theta}{\tan\theta-\tan 3\theta}$

$=\frac{\frac{1}{\tan\theta}}{\frac{\tan 3\theta-\tan\theta}{\tan\theta\tan 3\theta}}+\frac{\tan\theta}{\tan\theta-\tan 3\theta}$

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→ MENSURATION
- Area
- Volume

→ TRIGNOMETRY

→ GEOMETRY

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$$\frac{1}{\operatorname{cosec}\theta} + \frac{1}{\sec\theta} = \frac{13}{2}$$

$$\frac{\operatorname{cosec}\theta + \sec\theta}{\ldots}$$

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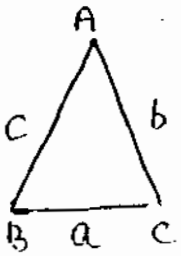
MENSURATION

Meansuration
- 2D
  - Area
  - Perimeter
- 3D
  - Area
  - Volume

Area :→ Area is the Whole space Covered. of any Diagram

Perimeter :→ Perimeter is the Total Length of any Diagram.



जो Angle जितना बड़ा होगा उसके सामने की भुजा का Size उतना ही बड़ा होगा। और जो Angle छोटा होगा उसके सामने की भुजा उतनी ही छोटी होगी।

Sum of any two side of a traingle is always greater than 3rd side $a+b>c$

Difference of any two side of a traingle is always less than 3rd side $a-b<c$

e.g 8 5 4 → This is a Triangle.

$8+5>4$, $5+4>4$, $8+4>5$, $5+4>8$ $\therefore a+b>c$

$8-5<4$, or $5-4<8$, $8-4<5$ $\therefore a-b<c$

→ Two Side of a Triangle is 6, 8. find the Rangle of third Side.

We know for a Triangle $a+b>c$, $a-b<c$

$\therefore 6+8>c$ $\quad$ $8-6<c$

$14>c$ $\quad$ $2<c$

$\therefore 2<c<14$. [Range of Third Side]

→ Sum of All three Angles of a Triangle is 180°

→ The Ratio of angle of a Triangle is 1:2:3 find the Smallest angle

Total Value of Angle $= 1+2+3=6$

Let Smallest angle $= a = \frac{1}{6}\times 180^\circ = 30^\circ$ Ans.

$b = 2\times 30^\circ = 60^\circ$ $\quad$ $c = 3\times 30^\circ = 90^\circ$ Ans

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⇒ If Each Angle of a Triangle is Decreased By 10° then the Ratio of the angle Become 1:2:3. Find the each Angle of a Triangle:

We know that $\angle a + \angle b + \angle c = 180°$

Now Every Angle is Decreased By 10°.

$\therefore \angle a - 10° + \angle b - 10° + \angle c - 10° = 180° - 30°$

$\angle a - 10° + \angle b - 10° + \angle c - 10° = 150°$

$\therefore a : b : c = 1 : 2 : 3 \Rightarrow 6$

$\therefore a = \frac{1}{6} \times 150 = 25°$ $\quad b = 2 \times 25° = 50°$ $\quad c = 3 \times 25° = 75°$

अब जो Decrease किया था उसको Add भी करेंगे

$\therefore \angle a = 25° + 10° = 35°$ $\quad \angle b = 50° + 10° = 60°$ $\quad \angle c = 75° + 10° = 85°$

⇒ If Each Angle of a Triangle is Decreased By 20° then the Ratio of Angle Becomes 2:3:5. find the Each Angle of a Triangle.

$\angle a + \angle b + \angle c = 180°$

$\angle a - 20° + \angle b - 20° + \angle c - 20° = 180° - 60 = 120°$

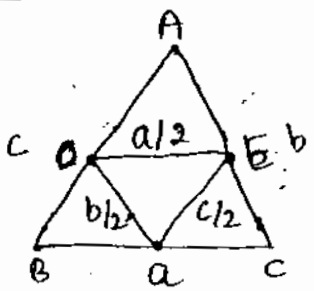
$\therefore a : b : c = 2 : 3 : 5 = 10$

$\angle a = \frac{2}{10} \times 120° = 12 \times 2 = 24°$ $\quad \angle b = \frac{3}{10} \times 120° = 36°$ $\quad \angle c = \frac{5}{10} \times 120 = 60°$

Ab अब 20° Add भी करेंगे

$\angle a = 24° + 20 = 44°$ $\quad \angle b = 36° + 20° = 56°$ $\quad \angle c = 60° + 20° = 80°$

Next



अगर किसी दो भुजाओं के mid Point को मिलाते हैं तो वह भुजा तीसरी भुजा के Parallel होती है

$DE \parallel BC$

और वह भुजा तीसरी भुजा से आधी होती है।

$DE = \frac{1}{2} \times BC$

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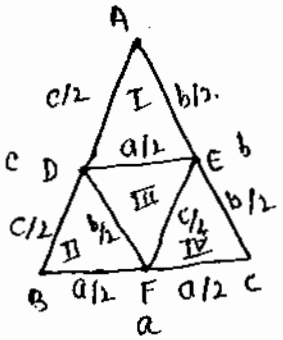


Triangle की सभी भुजाओं के mid point को मिलाते हैं तो उस Triangle के अंदर 4 Triangle बनते हैं और इन सभी का Area Equal होता है। सभी की Side भी बराबर होती हैं और Perimeter भी बराबर होता है।

$\therefore$ Area of ΔADE = Area of ΔDBF = Area of ΔDEF = Area of ΔEFC

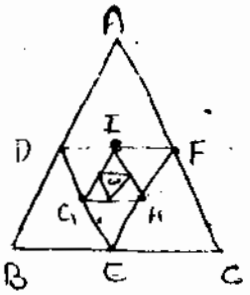
और इन सभी Traingle का Area पूरे Triangle के Area का $\frac{1}{4}$th होता

$\therefore$ Area of ΔADE = Area of ΔDEF = Area of ΔDBF = Area of ΔEFC

$= \frac{1}{4} \times$ Area of ΔABC

→ एक त्रिभुज की भुजाओं के मध्य बिंदु को मिलाकर एक त्रिभुज बनाया जाता है और उस त्रिभुज के मध्य बिन्दु को मिलाकर एक और त्रिभुज बनाया जाता है और यह प्रक्रिया अनंत तक दोहराई जाती है। find the sum of area of All Triangle formed.

$\therefore$ Let Area of Triangle ABC = $\underline{A}$



$\therefore$ Sum of Area of all Triangle

$= A + A \times \frac{1}{4} + A \times \frac{1}{4} \times \frac{1}{4} + \cdots \infty$

$= A\left[1 + \frac{1}{4} + \left(\frac{1}{4}\right)^2 + \cdots \infty\right]$ becomes an G.P

$= A\left[\frac{1}{1-\frac{1}{4}}\right]$ $\left[\because S_\infty = \frac{a}{1-r}\right]$

$A\left[\frac{4}{3}\right] = \frac{4}{3}A$

Area of $\Delta DEF = \frac{1}{4} \times \Delta A$
$\therefore$ Area of ΔGHE
$= \frac{1}{4} \times \Delta DEF$
$= \frac{1}{4} \times \frac{1}{4} \times \Delta ABC$
Continuos

$\therefore$ Sum of area of All Triangle formed = $\frac{4}{3}A$

OR

Sum of area of All Traingle formed = $\frac{4}{3} \times$ Area of ΔABC

→ समबाहु त्रिभुज (Equilateral Triangle) के प्रत्येक भुजा की लंबाई 12 cm है। इस त्रिभुज की भुजाओं के मध्य बिंदु को मिलाकर एक त्रिभुज बनाया जाता है, और इस प्रकार से बने त्रिभुज के मध्य बिंदु को मिलाकर एक त्रिभुज और बनाया जाता है और यह प्रक्रिया अनंत तक दोहराई जाती है। इस प्रकार से बने त्रिभुज के क्षेत्रफल का योग क्या होगा?

Area of Equilateral Triangle = $\frac{\sqrt{3}}{4}(\text{side})^2$

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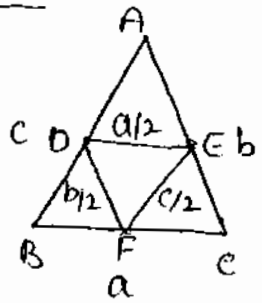
$\therefore$ Area $= \frac{\sqrt{3}}{4} \times 12 \times 12 = 36\sqrt{3}$

$\therefore$ Sum of Area of whole Triangle $= \frac{4}{3} \times 36\sqrt{3} = 48\sqrt{3}$

Next :-



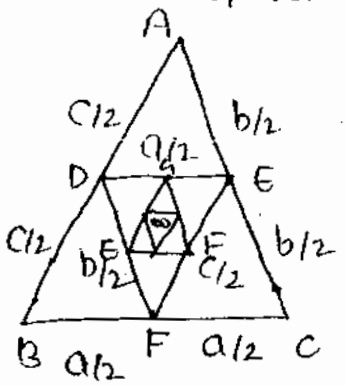
$\therefore$ Perimeter of $\Delta ABC = a+b+c$

$\therefore$ Perimeter of $\Delta DEF = \frac{a}{2} + \frac{b}{2} + \frac{c}{2} = \frac{a+b+c}{2}$

$\therefore$ Perimeter of $\Delta DEF = \frac{1}{2} \times$ Perimeter of ΔABC

Just

एक त्रिभुज की भुजाओं के मध्य बिंदु को मिलाकर एक त्रिभुज बनाया जाता है और इस प्रकार से बने त्रिभुज के मध्य बिंदु को मिलाकर एक और त्रिभुज बनाया जाता है और यह प्रक्रिया अनंत तक दोहराई जाती है। find the sum of Perimeter of triangle formed.



Let Perimeter of $\Delta ABC = P$

$\therefore$ Sum of Perimeter of whole Triangle formed

$= P + \frac{1}{2} \times P + \frac{1}{2} \times \frac{1}{2} \times P + \cdots \infty$

$= P\left[1 + \frac{1}{2} + \left(\frac{1}{2}\right)^2 + \cdots \infty\right]$

$= P\left[\frac{1}{1-\frac{1}{2}}\right] = 2P$

[$\because$ Perimeter of $DEF = \frac{1}{2}\Delta ABC$
Perimeter of $\Delta EFG = \frac{1}{2}\Delta DEF$
$= \frac{1}{2} \times \frac{1}{2} \times \Delta ABC$
continuslly]

$\therefore$ Sum of Perimeter of whole Traingle
$= 2 \times$ Perimeter of ΔABC

$\Rightarrow$ एक त्रिभुज की भुजाओं के मध्य बिंदु को मिलाकर एक त्रिभुज बनाया जाता है इस प्रकार से बने त्रिभुज की भुजाओं के मध्य बिन्दु को मिलाकर एक अन्य त्रिभुज बनाया जाता है और यह प्रक्रिया अनंत तक दोहराई जाती है इस प्रकार से बने त्रिभुजो के परिमाप का योग क्या होगा यदि बड़े त्रिभुज की भुजाएं 5, 6, 7 cm हो।

Perimeter of Triangle $= 5+6+7 = 18$ cm.

$\therefore$ Sum of Perimeter of whole Triangle $= 2 \times P = 2 \times 18 = 36$ cm.

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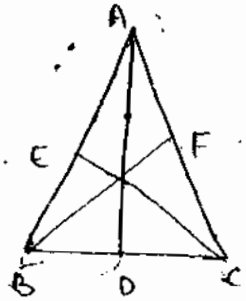
Next Type :→

Median (माध्यिका) :→ (Vertex (शीर्ष-बिंदु) से आने वाली वह रेखा जो किसी भी Triangle को समान भागों में बांट देती है, भुजाओं को दो समान भागों में और Area को भी दो समान भागों में बांट देती है।



Area of ΔADB = Area of ΔADC

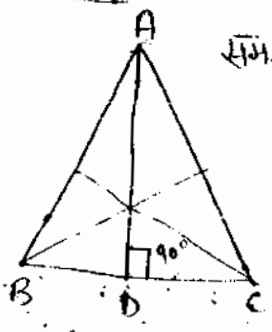
Similarly Area of ΔBFC = Area of ΔAFB

Area of ΔAEC = Area of ΔBEC

OR $BD = DC$, $AF = FC$, $AE = EB$

यह जरूरी नहीं कि माध्यिका किसी भी Angle को दो बराबर भागों में बांटती है और किसी भी Side पर माध्यिका डाले तो वह 90° का Angle बनाए।

Next Type :→



समद्विबाहु त्रिभुज

If $AB = BC$ then अगर हम AD Median डालते हैं BC पर तो AD BC को Bisect करती है और 90° का Angle बनाती है।

$\therefore BD = DC$

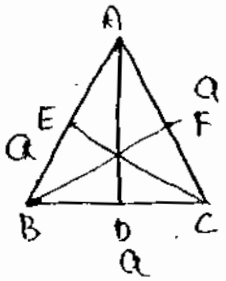
or $\angle ADB = \angle ADC = 90°$

पर यह जरूरी नहीं कि B और C से Median डाले तो वह भी 90° का Angle बनाए।

if $AB = BC$ then Median AD, Angle $\angle A$ का Bisector भी होगा।

$\therefore \angle BAD = \angle CAD$

Next Type :→



Equilateral Triangle (समबाहु त्रिभुज)

if $AB = BC = CA$

then $AD \perp BC$, $BF \perp AC$ or $CE \perp AB$

$\angle BAD = \angle CAD$, $\angle BCE = \angle ACE$, $\angle CBF = \angle ABF$

Median जिस Point पर एक दूसरे को काटती है उसे केन्द्रक (Centroid) कहते हैं। वह Point Median को 2:1 में काटती है।

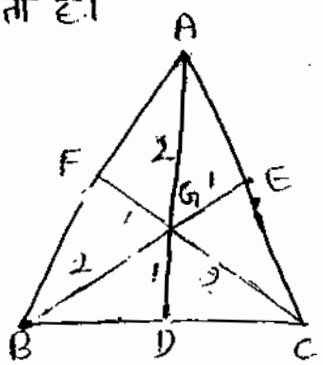
$AD \neq BE \neq CF$

$AG : GD = 2:1$

$BG : GE = 2:1$

$CG : GF = 2:1$

यहां केवल Ratio बराबर होगा, Length may be Various.



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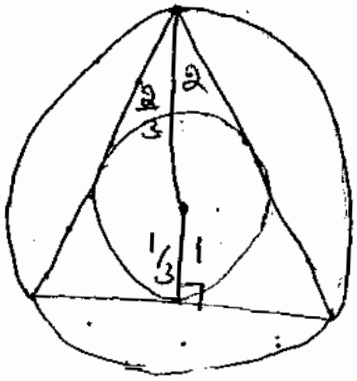
समबाहु त्रिभुज में अंत वृत्त (Incentre) का केन्द्र ~~होगा~~ वही होगा जो Circumcentre (परिकेन्द्र) का होगा।

Incentre की जो Radius होगी वो $\frac{1}{3}$ होगी और Circumcentre की जो Radius होगी वो $\frac{2}{3}$ होगी



Radius of incentre : Radius of Circumcentre

$= \frac{1}{3} : \frac{2}{3}$

$= 1 : 2$

Diameter $d_1 : d_2 = 1 : 2$

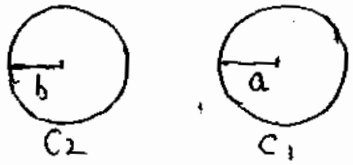
Circumcentre $C_1 : C_2 = 1 : 2$

Area $A_1 : A_2 = 1 : 2$

जब दो वृत्त दिए हो उनकी Radius का Ratio वही होगा जो उनके Diameter और परिधी का होगा।

Let $r_1 : r_2 = a : b$

$d_1 : d_2 = 2a : 2b = a : b$



(Circum) $C_1 : C_2 = 2\pi a : 2\pi b = a : b$

⇒ दो Circle में उनके Area का Ratio उनके परिधि व त्रिज्या के Ratio का Square के Equal होता है।

Let $r_1 : r_2 = a : b$ Or $C_1 : C_2 = a : b$

$\therefore A_1 : A_2 = \pi a^2 : \pi b^2$

$= a^2 : b^2$

⇒ जब दो वृत्तों के क्षेत्रफल का अनुपात दिया गया हो तो उनके त्रिज्या/परिधी का अनुपात क्षेत्रफल के अनुपात के Square Root के बराबर होता है।

e.g यदि दो वृत्त के क्षेत्रफल का अनुपात 25 : 16 है। तब परिधि/त्रिज्या का अनुपात 5 : 4 होगा।

⇒ जब दो square दिए हो तब उनकी Side का Ratio जो होगा, वही उनके Perimeter व Diagnol का Ratio होगा।

Side $S_1 : S_2 = a : b$

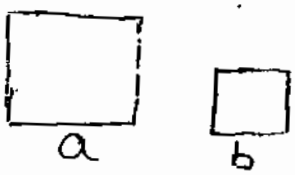
Perimeter $P_1 : P_2 = 4a : 4b$

$= a : b$

Diagnol $d_1 : d_2 = \sqrt{2}a : \sqrt{2}b$

$= a : b$

Area $A_1 : A_2 = a^2 : b^2$



अगर Area का Ratio $a^2 : b^2$ है तो

$$\boxed{\text{Ratio of Side} = \sqrt{\text{Ratio of Area}}}$$

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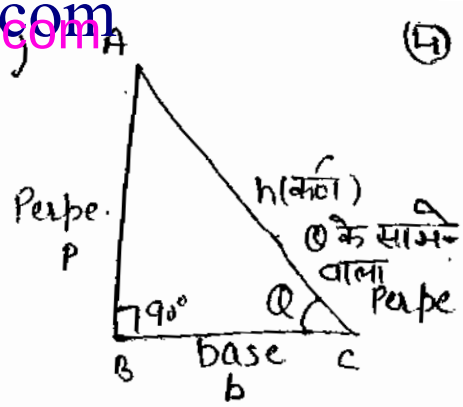
⇒ In Right Angle Triangle (समकोण त्रिभुज में)

$\angle ABC = 90^\circ$

According To Pythagorus Theorem

$h^2 = P^2 + b^2$ $[\because AC^2 = AB^2 + BC^2]$

$\Rightarrow \boxed{h = \sqrt{P^2 + b^2}}$



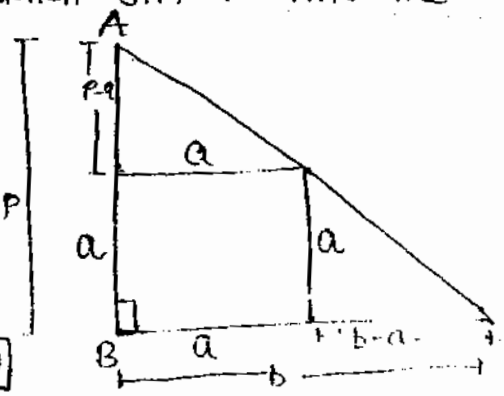
$\boxed{\text{Area} = \frac{1}{2}[\text{Base} \times \text{Height}]}$

$\boxed{\text{परिमाप} = h + P + b \text{ [सभी भुजाओं का योग]}}$

समकोण त्रिभुज में बड़े से बड़े आकार का square बनाया जाए तो find the Length of Side of Square.

Let length of Each Side of square = a

Area of Square = Area of whole ΔABC − Area of Two Small Triangle



$a^2 = \frac{1}{2} \times P \times b - \left[\frac{1}{2} \times a \times (b-a) + \frac{1}{2} \times a \times (P-a)\right]$

$2a^2 = Pb - [ab - a^2 + ab - a^2]$

$= Pb - [ab + aP - 2a^2]$

$2a^2 = Pb - ab - aP + 2a^2$

$0 = Pb - ab - aP$

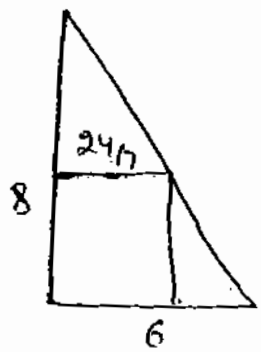
$Pb = ab + aP \Rightarrow a(b+P)$

$\boxed{a = \frac{Pb}{b+P}}$ where P और b is the Side of Triangle, a is the Side of square.

⇒ एक समकोण त्रिभुज के समकोण बनाने वाली भुजा की लंबाई 8 व 6 cm है। इस त्रिभुज के अन्दर बड़े से बड़े आकार का वर्ग बनाया जाता है तब वर्ग की भुजा व क्षेत्रफल ज्ञात कीजिए।

Using $a = \frac{Pb}{P+b}$ (a = Side of square)

$a = \frac{8 \times 6}{8+6} = \frac{48}{14} = \frac{24}{7}$ cm.



Area $= \left(\frac{24}{7}\right)^2 = \frac{576}{49}$ cm²

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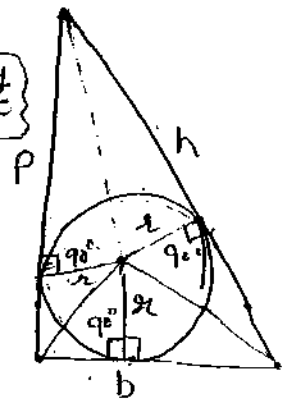
एक समकोण त्रिभुज के अन्दर बड़े से बड़े आकार का Circle बनाया जाता है।



Radius हमेशा स्पर्श रेखा पर 90° का Angle बनाती है

$$\frac{1}{2}\times P\times b = \frac{1}{2}\times \underset{\text{Base}}{P}\times r + \frac{1}{2}\times \underset{\text{Base}}{b}\times r + \frac{1}{2}\times \underset{\text{Base}}{h}\times r \ (\text{Height})$$

$$\frac{1}{2}\times P\times b = \frac{1}{2} r[P+b+h]$$

$$Pb = r[P+b+h]$$

$$\boxed{r = \frac{Pb}{P+b+h}} = \frac{Pb}{(P+b)+h}\times\frac{(P+b)-h}{(P+b)-h} = \frac{Pb(P+b-h)}{(P+b)^2-h^2}$$

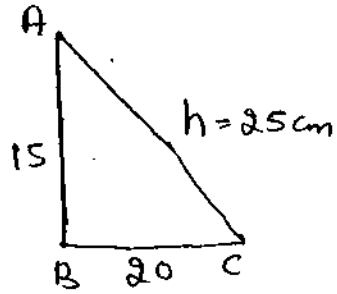
$$= \frac{Pb(P+b-h)}{P^2+b^2+2Pb-h^2}$$

$$= \frac{Pb(P+b-h)}{h^2+2Pb-h^2} \quad [\because P^2+b^2=h^2]$$

$$= \frac{Pb(P+b-h)}{2Pb} = \frac{P+b-h}{2}$$

$$\boxed{r = \frac{P+b-h}{2}}$$

⇒ एक समकोण त्रिभुज में समकोण बनाने वाली भुजा की लंबाई 15cm, 20cm है find the Radius of Incircle.



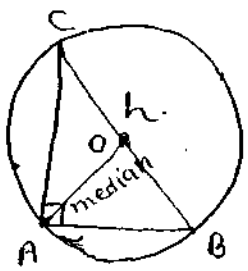
Here ⇒ $P = 15cm$, $b = 20cm$

$$\therefore h = \sqrt{P^2+b^2} = \sqrt{(20)^2+(15)^2} = \sqrt{625} = 25cm$$

$$\therefore \text{Radius of incircle} = \frac{15+20-25}{2} = \frac{10}{2} = 5cm$$

$$\left[\text{using } r = \frac{P+b-h}{2}\right]$$

समकोण त्रिभुज (में परिवृत (Circumcentre) का centre कर्ण का मध्य बिंदु होता है।



Diameter = h = BC

radius = (OB = OC) = $h/2$

केंद्र से परिधि तक सभी त्रिज्या बराबर होती है।

$\therefore OC = OB = OA = h/2$, AO → Median भी है।

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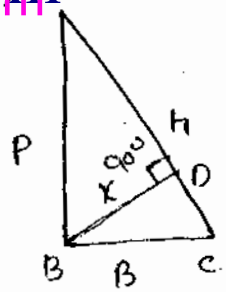
31/8/15 Given BD ⊥ [illegible]



Area of Triangle ABC $= \frac{1}{2} \times P \times b$ $\left[\because \frac{1}{2} \times \text{base} \times \text{Height}\right]$ —(1)

Let $x = BD =$ Perpendicular Height

Then $h \rightarrow$ Base

$\therefore$ Area of $\Delta ABC = \frac{1}{2} \times x \times h$ — II

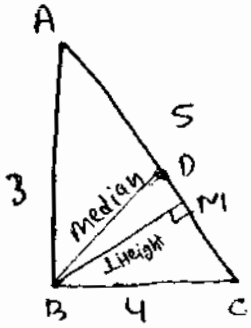
from I and II

$$\frac{1}{2} \times P \times b = \frac{1}{2} \times x \times h \Rightarrow \boxed{x = \frac{Pb}{h}}$$

Next :→ Here Perp. (P) = 3cm, Base (b) = 4cm

Hypo (h) = 5 Cm

Median BD $= \frac{1}{2} \times$ Hypotaneous $= \frac{1}{2} \times 5 = \frac{5}{2}$ cm

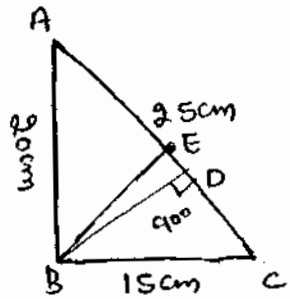


⊥ Height BM $= \frac{P \times b}{h} = \frac{3 \times 4}{5} = \frac{12}{5} = 2.4$ cm

{Median की लंबाई कर्ण की लंबाई का आधा होती है।}

Next :→ {Median व Perpendicular बराबर भी हो सकते हैं जब Right angle side equal हो।}

⇒ एक समकोण त्रिभुज के समकोण बनाने वाली भुजा की लंबाई 15cm और 20cm है। Find the Length of Median and Longest Side ⊥ Height.



$$AC = \sqrt{20^2 + 15^2} = \sqrt{400 + 225} = \sqrt{625} = 25 \text{ cm}$$

Perpendicular Height

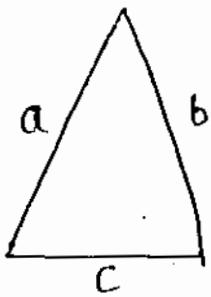
$$= \frac{P \times b}{h} = \frac{\overset{4}{\cancel{20}} \times \overset{3}{\cancel{15}}}{\underset{5}{\cancel{25}}} = 12 \text{ cm}$$

Median (BE) $= \frac{1}{2} \times h = \frac{1}{2} \times 25 = 12.5$ cm

Next Type :→ Scalane Triangle [विषमबाहु त्रिभुज]

in Scalane Triangle

Perimeter $= \boxed{a + b + c}$



By Heron's Formula

$$S = \frac{a + b + c}{2} \quad \text{[Semiperimeter]}$$

$$\boxed{\text{Area} = \sqrt{S(S-a)(S-b)(S-c)}}$$ or $\frac{1}{2} \times$ Base $\times$ Height जब ⊥ Height दी गई हो

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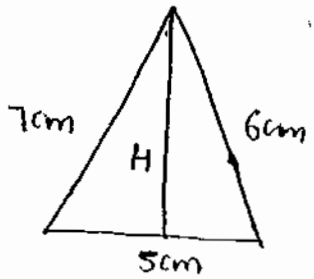
⇒ If the Side of a Scalane Triangle is 5, 6, 7 cm. find the length of Perpendicular drawn on smallest side.



$$S = \frac{a+b+c}{2} = \frac{5+6+7}{2} = 9\text{ cm}.$$

$$\text{Area} = \sqrt{S(S-a)(S-b)(S-c)} = \frac{1}{2} \times \text{Base} \times \text{Height}$$

$$\Rightarrow \sqrt{9(9-7)(9-6)(9-5)} = \frac{1}{2} \times 5 \times H$$

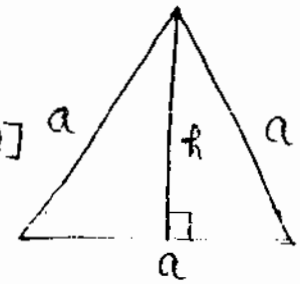
$$\sqrt{9 \cdot 2 \cdot 3 \cdot 4} = \frac{1}{2} \times 5 \times H \Rightarrow \frac{1}{2} \times 5 \times H = 3 \times 2\sqrt{6}$$

$$H = \frac{2 \times 3 \times 2\sqrt{6}}{5} = \frac{12}{5}\sqrt{6}$$

$$= 2.4\sqrt{6}\text{ cm}$$

Next:→

Equilateral Triangle (समबाहु त्रिभुज):→



$$\text{Perimeter} = a + a + a = 3a$$

[All sides are equal]

$$\text{Area} = \frac{\sqrt{3}}{4} a^2 \quad \text{--- I}$$

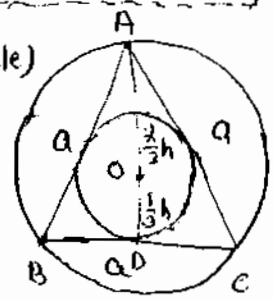
और जब ⊥ Height दी हो. ∴ $\text{Area} = \frac{1}{2} \times \text{Base} \times \text{Height}$

$$= \frac{1}{2} \times a \times h \quad \text{--- II}$$

from I and II

$$\frac{\sqrt{3}}{4} a^2 = \frac{1}{2} \times a \times h, \quad \boxed{h = \frac{\sqrt{3}}{\cancel{4}_2} a^{\cancel{2}} \times \frac{\cancel{2}}{\cancel{a}} = \frac{\sqrt{3}}{2} a}$$

Next:→ अगर किसी भी समबाहु त्रिभुज में अंतवृत्त (Incircle) और Circumcircle (बाह्यवृत्त) दिए गए हो



then Radius of Circumcircle

$$= \frac{2}{3} \times h = \frac{2}{3} \times \frac{\sqrt{3}}{2} a \quad \left[\because h = \frac{\sqrt{3}}{2} a\right]$$

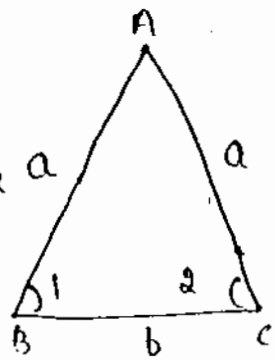
$$= \frac{1}{\sqrt{3}} a$$

Radius of Incircle

$$= \frac{1}{3} \times h = \frac{1}{3} \times \frac{\sqrt{3}}{2} a = \frac{1}{2\sqrt{3}} a$$

Next:→ Isoceles Triangle (समद्विबाहु त्रिभुज):→

समबाहु त्रिभुज में दो भुजाएँ बराबर होती हैं और जो side equal होती हैं उनके opposite वाला Angle बराबर होता है।



$$AB = AC$$

$$\therefore \angle 1 = \angle 2$$

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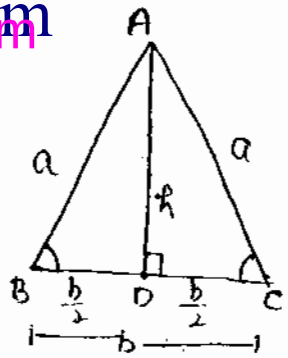
In Isoceles Δ ABC



Here $AB = AC$, $\angle B = \angle C$

Perimeter $= a + a + b = 2a + b$

Area $= \sqrt{S(S-a)(S-b)(S-c)}$

$= \sqrt{S(S-a)(S-a)(S-b)}$

$= (S-a)\sqrt{S(S-b)}$ where $S \Rightarrow$ Semiperimeter

In Diagram AD दो Equal side के बीच का Perpendicular है तो Median है और BC को 2 Equal parts में Divide करता है।

$\therefore BD = DC = b/2$

Now In ΔABD

$$h^2 = a^2 - \frac{b^2}{4} = \frac{4a^2 - b^2}{4}$$

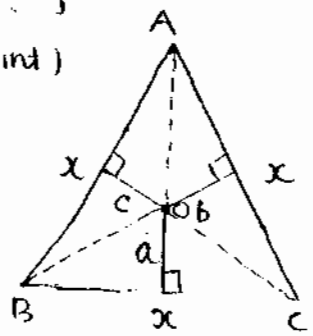
$$h = \sqrt{\frac{4a^2-b^2}{4}} = \frac{\sqrt{4a^2-b^2}}{2}$$

$$\text{Area} = \frac{1}{2} \times \text{Base} \times \text{Height} = \frac{1}{2} \times b \times \frac{\sqrt{4a^2-b^2}}{2} = \frac{1}{4} b\sqrt{4a^2-b^2}$$

$\therefore$ Area of Isoceles Triangle $= \frac{1}{4} b\sqrt{4a^2-b^2}$

Next :-

समबाहु त्रिभुज में किसी आंतरिक बिन्दु (Interior Point) से इसके तीनों भुजाओं पर डाले गए लंब की लंबाई a, b तथा c है और त्रिभुज की प्रत्येक भुजा x है



$\therefore$ Area of $\Delta AOB = \frac{1}{2} \times c \times x$

Area of $\Delta BOC = \frac{1}{2} \times a \times x$

Area of $\Delta AOC = \frac{1}{2} \times b \times x$ $\left[\because \text{Area} = \frac{1}{2} \times \text{Base} \times \text{Height}\right]$

Sum of Area

$$= \frac{1}{2} a \times x + \frac{1}{2} b \times x + \frac{1}{2} c \times x = \text{Area of } \Delta ABC$$

$$\frac{1}{2} x(a+b+c) = \frac{\sqrt{3}}{4} x^2 \qquad \left[\because \text{Area of } \underset{(\text{समबाहु})}{\Delta ABC} = \frac{\sqrt{3}}{4} a^2\right]$$

$$a+b+c = \frac{\sqrt{3}}{2} x \Rightarrow x = \frac{2}{\sqrt{3}}(a+b+c)$$

$\therefore$ Side of Triangle $= \frac{2}{\sqrt{3}}(a+b+c)$

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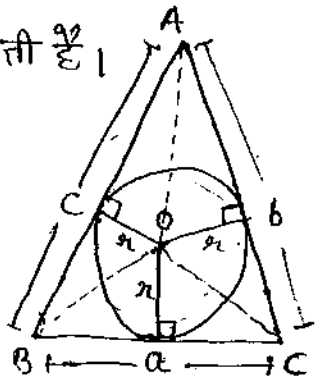
Next:→ विषमबाहु त्रिभुज की सभी भुजाएं अलग-अलग होती हैं।



$OA \perp BC$ [∵ BC is tangent]

Now Area of ΔABC
= Area of ΔAOB + Area of ΔBOC + Area of ΔAOC

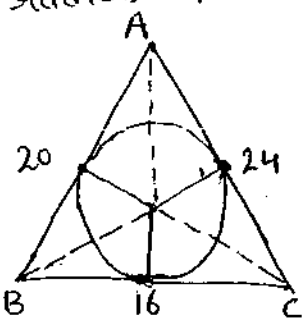
$$\Rightarrow \frac{1}{2} \times c \times r + \frac{1}{2} \times a \times r + \frac{1}{2} \times b \times r = \underbrace{\sqrt{S(S-a)(S-b)(S-c)}}_{\text{Area of } \Delta ABC}$$

$$\Rightarrow \frac{1}{2} r(a+b+c) = \text{Area of } \Delta ABC$$

$$r = \frac{\text{Area of } \Delta ABC \times 2}{a+b+c} \Rightarrow \frac{\text{Area of } \Delta ABC}{S} \quad \left[\because S = \frac{a+b+c}{2}\right]$$

$OA = OB = OC$ $\boxed{\therefore \text{Radius of Incircle} = \frac{\text{Area of } \Delta ABC}{S}}$ Very Important.

eg एक त्रिभुज की भुजाएं 20cm, 16cm, 24cm हैं, Find the radius of Incircle.



$$\text{Semiperimeter } (S) = \frac{20+16+24}{2} = \frac{60}{2} = 30 \text{ cm}$$

$$\text{Area of } \Delta ABC = \sqrt{30(30-20)(30-16)(30-24)}$$
$$= \sqrt{30 \times 10 \times 14 \times 6}$$
$$= \sqrt{3 \times 10 \times 10 \times 7 \times 2 \times 3 \times 2}$$
$$= 3 \times 10 \times 2\sqrt{7} = 60\sqrt{7} \text{ cm.}$$

Radius of Incircle

$$= \frac{\text{Area of } \Delta ABC}{S} = \frac{60\sqrt{7}}{30} = 2\sqrt{7} \text{ cm.}$$

⇒ एक त्रिभुज के अंतवृत्त की त्रिज्या 5cm है, और उसका परिमाप 60cm है त्रिभुज का क्षेत्रफल ज्ञात कीजिए।

$$\text{Radius of Incircle} = \frac{\text{Area of Triangle}}{S \rightarrow \text{semiperimeter}}$$

$$\text{Semiperimeter} = \frac{60}{2} = 30 \text{ cm.}$$

$$\text{Area of Triange} = 30 \times 5 = 150 \text{ cm}^2$$

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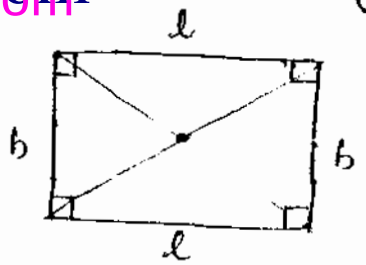
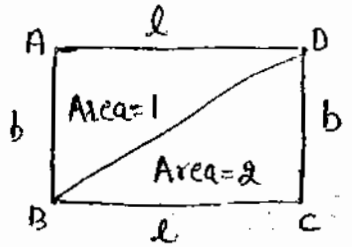
Next :→ 'Rectangle' (आयत)

* आयत के आमने-सामने की भुजाएं बराबर होती हैं।
* दोनों विकर्ण बराबर होते हैं।
* Diagnol (विकर्ण) जिस Point पर काटते हैं वहां 90° का Angle नहीं बनाते।



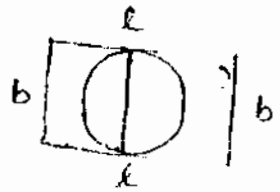
Perimeter = $2(l+b)$
Area = length × Breadth = $l \times b$

Area of ΔABD = Area of ΔBCD
Length of Diagnol = $\sqrt{l^2+b^2}$



⇒ एक आयत के अन्दर बड़े से बड़े आकार का वर्त्त का व्यास आयत के Breadth के बराबर होगा।

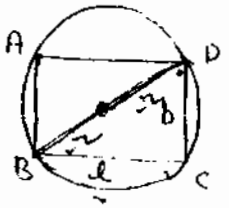
∴ Diameter of Circle = b
Radius of Circle = $b/2$



⇒ एक आयत के बाहर बड़े से बड़े आकार का वर्त्त का व्यास आयत के Diagnol के बराबर होगा।

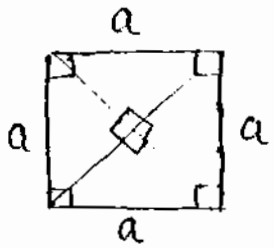
Diagnol = $\sqrt{l^2+b^2}$

∴ Diameter of Circumcircle = $\sqrt{l^2+b^2}$
Radius of Circumcircle = $\frac{1}{2}\sqrt{l^2+b^2}$



Next :→ Square (वर्ग)

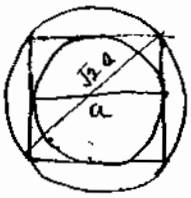
* वर्ग की सभी भुजाएं बराबर होती हैं।
* वर्ग के दोनों Diagnol बराबर होते हैं।
* विकर्ण जिस Point पर काटते हैं, वहां 90° का Angle बनाते हैं



Perimeter = $4a$
Area = $(\text{Side})^2 = a^2$
Diagnol = $\sqrt{a^2+a^2} = \sqrt{2}\,a$

* एक Square के अंदर बड़े से बड़े Circle का व्यास Square की Side के Equal होगा।

Diameter = a



* एक Square के बाहर बड़े से बड़े Circle का व्यास Square के Diagnol के Equal होगा।

Diameter = $\sqrt{2}\,a$

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In a square

$$\text{Diameter}_{\text{incircle}} : \text{Diameter}_{\text{circum}} = a : \sqrt{2}a$$
$$= 1 : \sqrt{2}$$

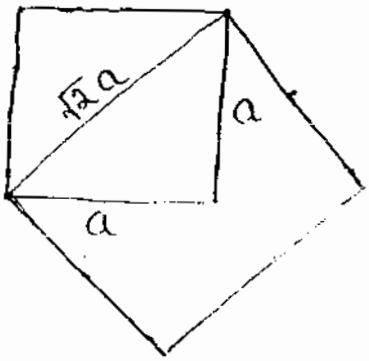
Similarly $R_{\text{incircle}} : R_{\text{circum}} = \frac{1}{2} : \frac{\sqrt{2}}{2} = 1 : \sqrt{2}$

$C_{\text{incircle}} : C_{\text{circum}} = 1 : \sqrt{2}$ [where C = Circumference]

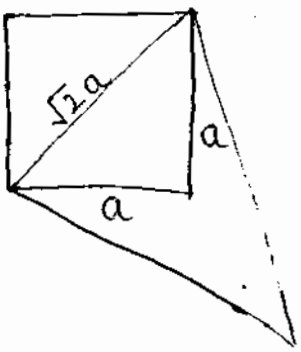
$$A_{\text{incircle}} : A_{\text{circum}} = 1^2 : \sqrt{2}^2$$
$$= 1 : 2 \quad \text{[where A = Area]}$$

Next Type :→

एक square के विकर्ण पर यदि एक और square बनाया जाता है।
दोनों square की side का Ration
$= a : \sqrt{2}a = 1 : \sqrt{2}$ होगा।
Ratio of Area
$= 1^2 : \sqrt{2}^2 \Rightarrow 1 : 2$ Ans.



Next Type :→ किसी square के विकर्ण पर एक समबाहु त्रिभुज बनाया जाता है तो उसकी side का Ratio $1 : \sqrt{2}$ होगा। Area का Ratio $2 : \sqrt{3}$ होगा।



Side of square : Side of Equilateral

$$\Rightarrow S_{\square} : S_{\Delta} = a : \sqrt{2}a$$
$$= 1 : \sqrt{2}$$

Area of square : Area of Equilateral

$$A_{\square} : A_{\Delta} = a^2 : \frac{\sqrt{3}}{4}a^2 \quad \left[\because \text{Area of Equilateral } \Delta = \frac{\sqrt{3}}{4}a^2\right]$$

$$= a^2 : \frac{\sqrt{3}}{4}(\sqrt{2}a)^2 \quad [\because \text{Side of Equilateral} = \sqrt{2}a]$$

$$= a^2 : \frac{\sqrt{3}}{\cancel{4}_2} \times \cancel{2}a^2$$

$$= 1 : \frac{\sqrt{3}}{2} \Rightarrow 2 : \sqrt{3}$$

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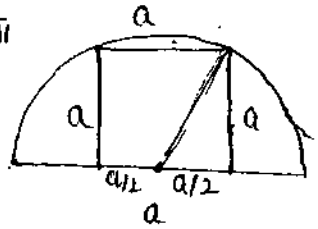
Next Type :→ यदि किसी semicircle में बड़े से बड़ा वृत्त बनाया जाता है तो



$$\text{Radius of circle (semi)} = r = \sqrt{a^2+\left(\frac{a}{2}\right)^2}$$

$$r = \sqrt{a^2+\frac{a^2}{4}} = \sqrt{\frac{5a^2}{4}}$$

$$r = \frac{\sqrt{5}a}{2} \Rightarrow \frac{r}{a} = \frac{\sqrt{5}}{2}$$

$$\boxed{r : a = \sqrt{5} : 2}$$

eg. एक अर्धवृत्त (semicircle) की त्रिज्या 20cm है, इसके अन्दर बने बड़े से बड़े square का क्षेत्रफल क्या होगा।

Using $r^2 = a^2 + \frac{a^2}{4}$

$\Rightarrow 20^2 = \frac{5a^2}{4}$

$400 \times 4 = 5a^2$

$80 \frac{400 \times 4}{5} = a^2$

$\boxed{a^2 = 320\ cm^2}$

OR

$r : a = \sqrt{5} : 2$

$\sqrt{5} \rightarrow 20$

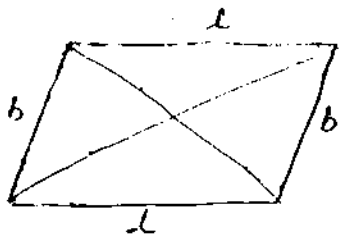
$2 \rightarrow \frac{20}{\sqrt{5}} \times 2$

$a \rightarrow 8\sqrt{5}$

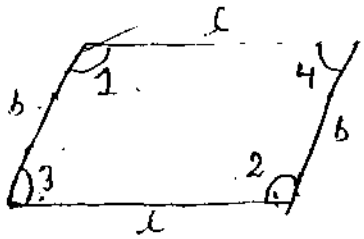
$a^2 = (8\sqrt{5})^2$

$= 320 cm^2$

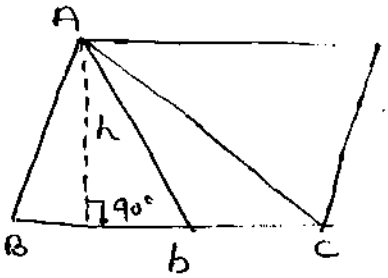
Next Type :→ Parallelogram (समांतर चतुर्भुज)



* समांतर चतुर्भुज में आमने सामने की भुजा बराबर होती है।
* Diagnol की लंबाई अलग-अलग होती है।
* $\angle 1 = \angle 2$ & $\angle 3 = \angle 4$
* $\angle 1 + \angle 3 = \angle 3 + \angle 2 = \angle 2 + \angle 4 = \angle 4 + \angle 1 = 180^\circ$



$$\boxed{\begin{array}{l}\text{Perimeter} = 2(l+b) \\ \text{Area} = \text{Base} \times \text{Height}\end{array}}$$



Area of $\Delta ABC = \frac{1}{2} \times \text{Base} \times \text{Height}$

$\therefore$ Area of whole Parallogram

$= 2 \times \frac{1}{2} \times \text{Base} \times \text{Height}$

$= \text{Base} \times \text{Height}$

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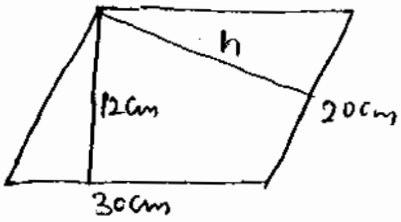
एक समांतर चतुर्भुज के आसन्न भुजा (Adjecent side) की लंबाई 30cm और 20cm है अगर बड़ी भुजा के बीच की लंबवत दूरी 12cm है तो इनके छोटी भुजा के बीच की लंबवत दूरी क्या होगी।



when Base = 30cm, Height = 12cm

Area = $30 \times 12\ cm^2$ [H×B] —I

when Base = 20cm, let Height = h cm

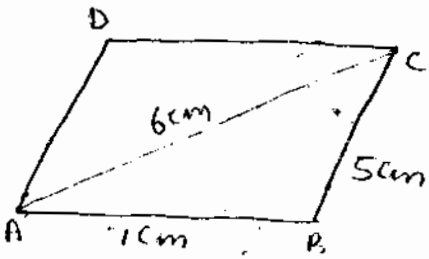
Area = $20 \times h\ cm^2$ —II

from I and II

$30 \times 12 = 20 \times h$

$h = \frac{30 \times 12}{20} = 18$ cm Ans.

⇒ एक समांतर चतुर्भुज के आसन्न भुजा की लंबाई 7cm और 5cm है अगर एक विकर्ण की लंबाई 6cm है। find Area.



ΔABC is a Scalane (विषमबाहु) Triangle

$\therefore S = \frac{a+b+c}{2} = \frac{5+6+7}{2} = 9cm$

Area = $\sqrt{S(S-a)(S-b)(S-c)}$

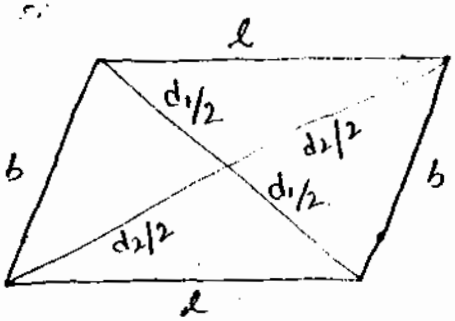
$= \sqrt{9(9-7)(9-6)(9-5)} = \sqrt{9 \cdot 2 \cdot 3 \cdot 4}$

$= 6\sqrt{6}\ cm^2$

Area of Parallelogram = 2 × Area of Δ ABC

$= 2 \times 6\sqrt{6} = 12\sqrt{6}\ cm^2$

⇒ Nom



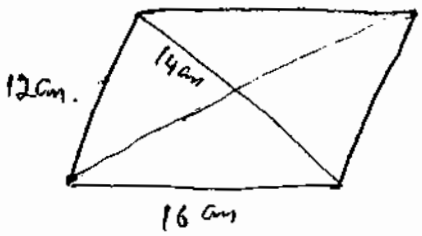
$l^2 + b^2 = 2\left(\frac{d_1}{2}\right)^2 + 2\left(\frac{d_2}{2}\right)^2$

$= 2 \times \frac{d_1^2}{4} + 2 \times \frac{d_2^2}{4}$

$\Rightarrow \frac{d_1^2}{2} + \frac{d_2^2}{2}$

$$\boxed{d_1^2 + d_2^2 = 2(l^2 + b^2)}$$

⇒ एक समांतर चतुर्भुज के आसन्न भुजा की लंबाई 16cm और 12cm है और एक विकर्ण की लंबाई 14cm है तो दूसरे विकर्ण की लंबाई क्या होगी।



Using $d_1^2 + d_2^2 = 2(l^2 + b^2)$

$\Rightarrow (14)^2 + d_2^2 = 2(12^2 + 16^2)$

$= 2(144 + 256)$

$d_2^2 = \frac{2 \times 400}{\cancel{196}} - 196$ $\quad 800 - 196 = 604$

$d_2 = \sqrt{604}$ cm.

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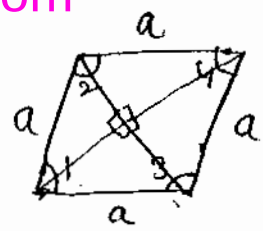
Next Type :→ Rhombus (समचतुर्भुज)

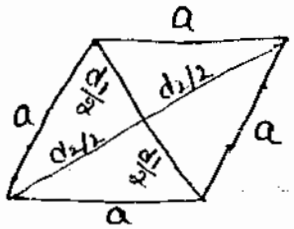


* समचतुर्भुज की सभी भुजाएं बराबर होती है।
* दोनों विकर्णों की लंबाई अलग-अलग होती है।
* दोनों विकर्ण एक दूसरे को जिस Point पर काटते हैं 90° का Angle बनाते हैं।
* $\angle 1 = \angle 4$ & $\angle 2 = \angle 3$
* $\angle 1 + \angle 2 = \angle 2 + \angle 4 = \angle 4 + \angle 3 = \angle 3 + \angle 1 = 180°$

Perimeter = 4a

Area = $\frac{1}{2} \times d_1 \times d_2$ = Base × Height

Side (a) = $\sqrt{\left(\frac{d_1}{2}\right)^2 + \left(\frac{d_2}{2}\right)^2} = \frac{\sqrt{d_1^2 + d_2^2}}{2}$



⇒ किसी समचतुर्भुज के दोनों विकर्णों की लंबाई 16cm और 12 cm है। Find the Area, Perimeter and Height.

$d_1 = 16cm$ $d_2 = 12cm$

∴ Side = a = $\frac{\sqrt{d_1^2 + d_2^2}}{2} = \frac{\sqrt{(16)^2 + (12)^2}}{2} = \frac{\sqrt{256 + 144}}{2} = \frac{20}{2} = \underline{10cm}$

Area = $\frac{1}{2} \times d_1 \times d_2 = \frac{1}{\cancel{2}} \times \overset{8}{\cancel{16}} \times 12 = 96\ cm^2$

Perimeter = $4 \times a = 4 \times 10 = 40cm$.

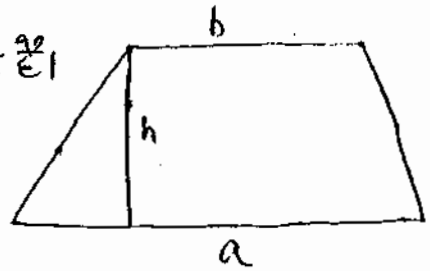
Now Area = $\frac{1}{2} \times d_1 \times d_2$ = Base × Height

⇒ 96 = 10 × Height

Height = 9.6 cm.

Next Type :→ Trapizium (समलंब चतुर्भुज)

* समलंब चतुर्भुज में दो भुजाएं Parallel होती है।

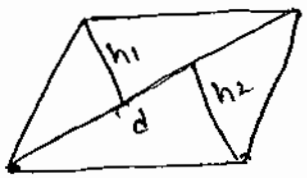


Area = $\frac{1}{2}(a+b) \times h$

= $\frac{1}{2}$(Sum of ∥ side) × h

Next Type :→ Quadilateral (चतुर्भुज)

अगर किसी भी चतुर्भुज के विकर्ण पर दो लंब डालते हैं।



Then Area = $\frac{1}{2} d h_1 + \frac{1}{2} d h_2$

= $\frac{1}{2}(h_1 + h_2) \times d$

∴ Area = $\frac{1}{2}$ (Sum of ⊥ on Diagnol) × Diagnol

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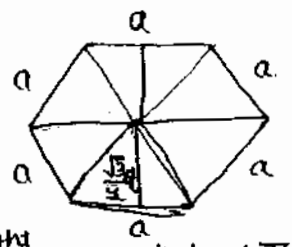
Next :→ Regular Hexagonal (सम षट्भुज)



* Regular Hexagonal में सभी भुजाएं बराबर होती हैं।

Perimeter = 6a

Height of Hexagonal = $2 \times \frac{\sqrt{3}}{2} a$ [∵ Height ~~Area~~ of Equilateral Triangle = $\frac{\sqrt{3}}{2}a$]

= $\sqrt{3}a$ [It is the ⊥ Distance Betn Two Parallal Side]

Regular Hexagonal के अंदर 6 Equilateral Triangle बनते हैं।

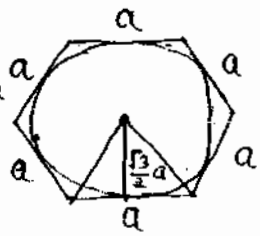
∴ Area of 1 Equilateral Traingle = $\frac{\sqrt{3}}{4}a^2$

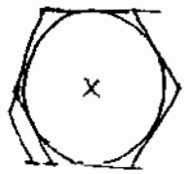
∴ = Area of Regular Hexagonal = 6 × Area of Equilateral Triangle

$= 6 \times \frac{\sqrt{3}}{4} a^2 \quad \frac{3\sqrt{3}}{2}a^2$

⇒ अगर Hexagonal के अन्दर Circle हो तो उसकी Radius वही होगी जो Equilateral Triangle की Height होगी।

$r = \frac{\sqrt{3}}{2}a$





⇒ अगर Hexagonal के बाहर कोई भी circle हो तो उसकी Radius वही होगी जो Hexagonal की Side की लंबाई होगी।

$r = a$

Now

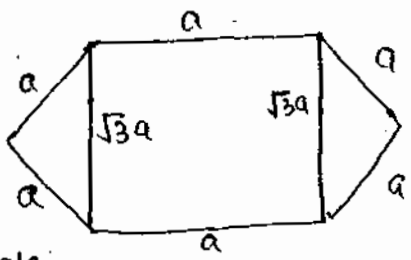
$r_{incircle} : r_{circum} = \frac{\sqrt{3}}{2}a : a$

$= \sqrt{3} : 2$

$Area_{incircle} : Area_{circum} = \pi(\sqrt{3})^2 : \pi(2)^2$

$3 : 4$

⇒ किसी भी आयत Hexagonal के अंदर A आयत Draw करते हैं। तो आयत का Area क्या होगा।



Side of Hexagonal = Length of Reactangle

= a

Height of Hexagonal = Bredth of Reactangle

= $\sqrt{3}a$

∴ Area of Reactangle = $\sqrt{3}a \times a = \sqrt{3}a^2$

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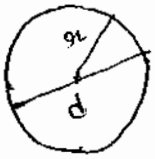
Next Type :→ Circle (वृत्त)



* Radius of Circle = r
* Diameter of Circle = d

Diameter = 2 × Radius
$d = 2r$
Circumference (परिधि) $= 2\pi r$
Area $= \pi r^2$

Where π is Irrational Number.
$\pi = \frac{22}{7}$ or 3.14

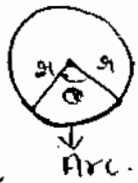
Ques :→ Choose the Irrational Number
(i) π (ii) $22/7$ (iii) 3.14 (iv) All of above
∵ π की कोई Specific Value नहीं है।

Next :→ Sector of a Circle (त्रिज्या खंड) :→ किसी भी Circle का छोटा सा Area Sector कहलाता है।

∴ जब Angle 360° होता है तब Area $= \pi r^2$
जब Angle θ° होगा तब Area $= \frac{\pi r^2}{360^\circ} \times \theta$

$$\therefore \text{Area of Sector} = \frac{\theta}{360} \times \pi r^2$$

Next :→ Arc of a Circle (चाप) :→ किसी भी Circle की छोटी सी परिधि (Circumference) Arc कहलाती है।



∴ जब Angle 360° होता है तब परिधि $= 2\pi r$
जब Angle θ° होगा तब परिधि $= \frac{2\pi r}{360} \times \theta = \frac{\theta}{180} \times \pi r$

$$\therefore \text{Length of Arc} = \frac{\theta}{180} \times \pi r$$

⇒ Relation Between Length of Arc & Area of Sector :→

$$\text{Area of Sector} = \frac{\theta}{360} \times \pi r^2 = \frac{1}{2} \times \underbrace{\frac{\theta}{180} \times \pi r}_{} \times r$$

$$= \frac{1}{2} \times \text{Length of Arc} \times r$$

$$\therefore \text{Area of Sector} = \frac{1}{2} \times \text{Length of Arc} \times r$$

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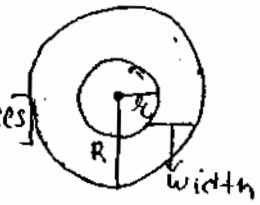
Next :→ Concentric Circle (ऐसे circle जिनका एक ही centre हो)

Find the Length of Width



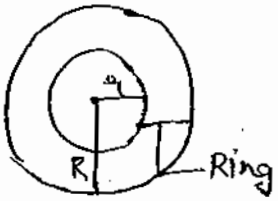
Now $2\pi R - 2\pi r = C_1 - C_2$ [C_1, C_2 are Circumferences]

$2\pi(R-r) = C_1 - C_2$

Width ⇒ $\boxed{R - r = \frac{C_1 - C_2}{2\pi}}$

⇒ Area of Ring

Area of Ring $= \pi R^2 - \pi r^2$
$= \pi(R^2 - r^2)$
$= \pi(R+r)(R-r)$



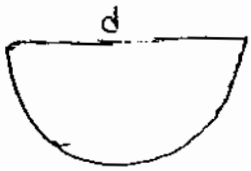
∴ $\boxed{\text{Area of Ring} = \pi(R+r)(R-r)}$

⇒ Perimeter

Perimeter = Total Length
$= 2\pi r + 2\pi R$
$= 2\pi(r+R)$

∴ $\boxed{\text{Perimeter} = 2\pi(r+R)}$

Next Type :→ Semi-Circle (अर्धवृत्त)

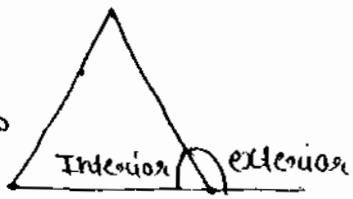


Area of Semicircle $= \frac{\pi r^2}{2}$

Perimeter $= \pi r + d = \pi r + 2r$
$= (\pi + 2) r$

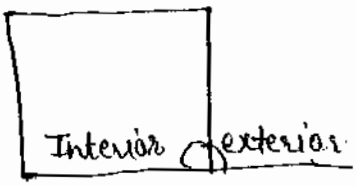
Next Type :→ Polygen (बहुभुज) :→ दो से अधिक भुजाओं की बंद आकृति Polygen कहलाती है।

In a Polygen



∠Interior Angle + ∠Exterior Angle = 180°

(आंतरिक कोण + बाह्य कोण) = 180°



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⇒ एक समबहुभुज (Polygon) के अंतःकोण (Interior Angle) उसके बाह्य कोण (Exterior Angle) का $\frac{3}{2}$ है। Find Each Interior Angle.

$$\text{Interior} : \text{Exterior} = \frac{3}{2} : 1$$
$$= 3 : 2$$

$$\text{Now} \angle \text{Interior} + \angle \text{Exterior} = 180^\circ$$
$$\therefore 3+2 \rightarrow 180^\circ$$
$$5 \rightarrow 180^\circ$$
$$\therefore 1 \rightarrow \frac{180}{5} = 36^\circ$$

$\therefore$ Interior Angle $= 3 \times 36^\circ = 108^\circ$, Exterior Angle $= 180^\circ - 108^\circ = 72^\circ$

⇒ एक समबहुभुज के प्रत्येक अंतःकोण उसके प्रत्येक बाह्य कोण के 2 गुणे से 60 अधिक है। प्रत्येक अंतःकोण का माप क्या होगा।

Let Each Exterior Angle $= x$
Each Interior Angle $= 2x+60$

$$\therefore x + 2x + 60 = 180^\circ$$
$$3x + 60 = 180^\circ$$
$$3x = 120^\circ$$
$$x = 40^\circ \text{ (Each Exterior Angle)}$$

$\therefore$ Each Interior Angle $= 2 \times 40 + 60^\circ = 140^\circ$

⇒ Sum of All Exterior Angle of a Polygen $= 360^\circ$

Each exterior angle of a Regular Polygen $= \frac{360^\circ}{n}$

$n \rightarrow$ No of Sides

No. of Side of a Regular Polygen $= \frac{360^\circ}{\text{Each Exterior Angle}}$

Sum of all Interior Angle of a polygen $= (n-2) \times 180^\circ$

Each Interior Angle of a Regular Polygen $= \frac{(n-2) \times 180^\circ}{n}$

$n \rightarrow$ No of Side

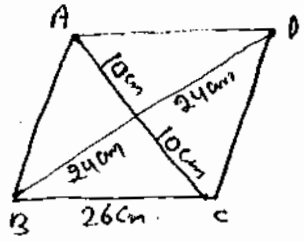
No. of sides of a Regular Polygen $= \frac{\text{Sum of all Interior Angle}}{\text{Each Interior Angle}}$

⇒ एक समचतुर्भुज के दो विकर्णों की लंबाई 48cm और 20cm है। Find the Area, Perimeter and Height.

Given $d_1 = 48\text{cm}$, $d_2 = 20\text{cm}$

$$\text{Area} = \frac{1}{2} \times \overset{24}{\cancel{48}} \times 20 = 480\text{cm}^2 \quad \left[\text{using Area} = \frac{1}{2} \times d_1 \times d_2\right]$$

$$\text{Side} = a = \sqrt{\left(\frac{d_1}{2}\right)^2 + \left(\frac{d_2}{2}\right)^2} = \sqrt{\left(\frac{20}{2}\right)^2 + \left(\frac{48}{2}\right)^2}$$
$$= \sqrt{(10)^2 + (24)^2} = \sqrt{676}$$
$$= 26\text{cm}$$



$\therefore$ Perimeter $= 4a = 4 \times 26 = 104\text{cm}$

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Now Using

Area ⇒ Base × Height = $\frac{1}{2} \times d_1 \times d_2$

$$26 \times H = 480$$

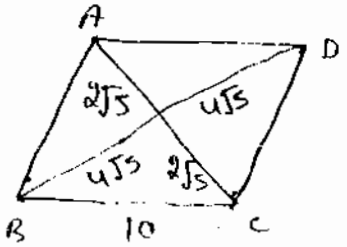
$$H = \frac{\cancel{480}\ 240}{\cancel{26}\ 13} = \frac{240}{13}\ cm.$$

⇒ एक समचतुर्भुज (Rhombus) का क्षेत्रफल $80 cm^2$ है। एक विकर्ण की लंबाई दूसरे विकर्ण की लंबाई का दो गुना है, find the Height of Rhombus.

~~Let Height of 1st~~

$$d_1 : d_2 = 2 : 1$$
$$= 2x \text{ or } x$$

[Let $d_2 = x$
then $d_1 = 2x$]



∴ Area = $\frac{1}{2} \times d_1 \times d_2 = 80$

$$\frac{1}{\cancel{2}} \times \cancel{2}x \times x = 80$$

$$x^2 = 80 \qquad x = 4\sqrt{5}\ cm.$$

∴ $d_1 = 8\sqrt{5} cm$ or $d_2 = 4\sqrt{5} cm$.

$$\text{Base} = a = \sqrt{\left(\frac{d_1}{2}\right)^2 + \left(\frac{d_2}{2}\right)^2} = \sqrt{\left(\frac{8\sqrt{5}}{2}\right)^2 + \left(\frac{4\sqrt{5}}{2}\right)^2}$$

$$= \sqrt{80 + 20} = \sqrt{100} = 10 cm.$$

Now Using

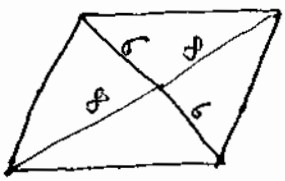
Base × Height = Area

$$10 \times H = 80$$

$$H = 8 cm$$

⇒ एक समचतुर्भुज का क्षेत्रफल $96 cm^2$ है। एक विकर्ण की लंबाई दूसरे विकर्ण की लंबाई का $\frac{4}{3}$ गुना है। find the Perimeter of Rhombus.

$$d_1 : d_2 = \frac{4}{3} : 1$$
$$\Rightarrow 4 : 3$$
$$\Rightarrow 4x, 3x$$



Area = $\frac{1}{2} \times d_1 \times d_2 = 96$

$$\frac{1}{\cancel{2}} \times \cancel{4}x \times 3x = 96$$

$$x^2 = 16$$

$$x = 4 cm$$

∴ Side = $a = \sqrt{8^2 + 6^2}$

$$= \sqrt{100}$$

$$= 10$$

∴ Length of first Diagnol = $4 \times 4 = 16 cm$
Length of 2nd Diagnol = $4 \times 3 = 12 cm$.

Perimeter = $4a$
$= 4 \times 10$
$= 40 cm$.

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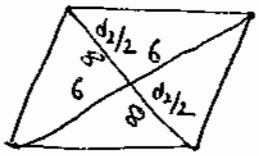
⇒ एक समचतुर्भुज का परिमाप 40cm है। उसके एक विकर्ण की लंबाई 12cm है। Find the Area. (12)

Perimeter = 4a

∴ 4a = 40

a = 10cm [a → Side of Rhombus]



Using $a^2 = \left(\frac{d_1}{2}\right)^2 + \left(\frac{d_2}{2}\right)^2$

$100 = \left(\frac{12}{2}\right)^2 + \left(\frac{d_2}{2}\right)^2$

$\left(\frac{d_2}{2}\right)^2 = 100 - 36 = 64$

∴ $\frac{d_2}{2} = 8$

$d_2 = 16cm$

OR

$\frac{d_2}{2} = \sqrt{(10)^2 - (6)^2}$

$= \sqrt{100 - 36} = \sqrt{64}$

$\frac{d_2}{2} = 8$

$d_2 = 16cm$.

∴ Area $= \frac{1}{2} \times d_1 \times d_2 = \frac{1}{2} \times 12 \times 16 = 96 cm^2$

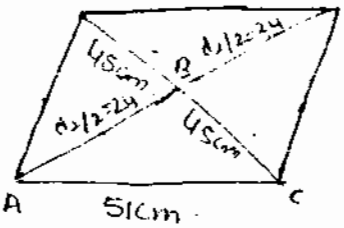
⇒ एक समचतुर्भुज का परिमाप 204cm है। एक विकर्ण की लंबाई 90cm है। Find the Height of Rhombus.

Perimeter = 4a

4a = 204

a = 51cm.

∴ $d_1 = 90cm$, $d_2 = 48cm$.



Using

Base × Height $= \frac{1}{2} \times d_1 \times d_2$

$51 \times H = \frac{1}{2} \times 90 \times 48$

$H = \frac{90 \times 48}{2 \times 51} = \frac{720}{17} cm$.

$\left(\frac{d_2}{2}\right)^2 = 51^2 - 45^2$

$\frac{d_2}{2} = \sqrt{2601 - 2025}$

$= \sqrt{576} = 24$

$d_2 = 24 \times 2 = 48cm$

⇒ एक समचतुर्भुज के दोनों विकर्णों की लंबाई x और y cm है। find the Height of Rhombus.

Using Side $= a = \frac{1}{2}\sqrt{d_1^2 + d_2^2} = \frac{1}{2}\sqrt{x^2 + y^2}$

Now Using

Base × Height $= \frac{1}{2} \times d_1 \times d_2$

$\Rightarrow \frac{1}{2}\sqrt{x^2+y^2} \times H = \frac{1}{2} \times x \times y$

$H = \frac{xy}{\sqrt{x^2+y^2}}$

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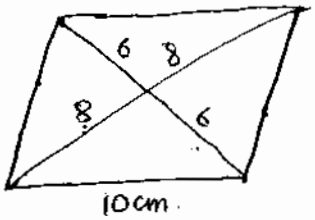
⇒ एक समचतुर्भुज का परिमाप 40cm है। और क्षेत्रफल 96cm² है, Find its Diagnol.



Detail :→ Using Area $= \frac{1}{2} \times d_1 \times d_2$

$4a = 40$

$a = 10cm$

$96 = \frac{1}{2} \times d_1 \times d_2$

$d_1 \times d_2 = 192$ — I

Side $= a = \frac{1}{2}\sqrt{d_1^2 + d_2^2}$

$10 = \frac{1}{2}\sqrt{d_1^2 + d_2^2}$

$\sqrt{d_1^2 + d_2^2} = 20$

$d_1^2 + d_2^2 = 400$ — II

$d_1 + d_2 = \sqrt{d_1^2 + d_2^2 + 2d_1d_2}$

$= \sqrt{400 + 2(192)} = \sqrt{400 + 384}$

$= \sqrt{784} = 28$ — III

Short Trick :→
We know that
$(10)^2 = 8^2 + 6^2$
$\because \frac{d_1}{2} = 8, \frac{d_2}{2} = 6$
$d_1 = 16cm$ $d_2 = 12cm$

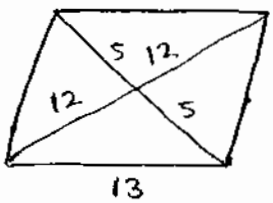
$d_1 - d_2 = \sqrt{d_1^2 + d_2^2 - 2d_1d_2} = \sqrt{400 - 384} = \sqrt{16} = 4$ — IV

Solving III and IV

$d_1 = 16$, $d_2 = 12$

Sum of Two Diagnol = 28cm Subtract of Two Diagnol = 4cm.

⇒ एक समचतुर्भुज का परिमाप 52cm है और क्षेत्रफल = 120cm² है। Find the Length of Longest Diagnol.



$4a = 52$

$a = \frac{52}{4} = 13cm$

Detail is Same as in Previous Example

$13^2 = 12^2 + 5^2$

$\therefore \frac{d_1}{2} = 12$, $\frac{d_2}{2} = 5$

$d_1 = 12 \times 2 = 24cm$ $d_2 = 5 \times 2 = 10cm$.

∴ Length of Longest Diagnol = ~~10~~ 24cm.

⇒ एक समचतुर्भुज का परिमाप 2a cm है। और क्षेत्रफल b cm है। find the sum of two Diagnol.

परिमाप = 2a cm

$\therefore$ Side (A) $= \frac{2a}{4} = \frac{a}{2}$ cm.

or $\frac{1}{2} \times d_1 \times d_2 = b$

$d_1 \times d_2 = 2b$

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Now Side of Rhombus

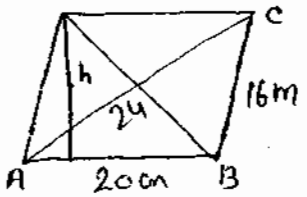
$$\frac{a}{2} = \frac{1}{2}\sqrt{d_1^2+d_2^2}$$

$$d_1^2+d_2^2 = a^2$$

Using

$$d_1+d_2 = \sqrt{d_1^2+d_2^2+2d_1d_2} = \sqrt{a^2+2\cdot 2b} = \boxed{\sqrt{a^2+4b}}$$

⇒ एक समांतर चतुर्भुज (Parallelogram) की आसन्न भुजा क्रमशः 20 और 16cm है। एक विकर्ण की लंबाई 24 cm है। Find the Area and ⊥ Distance Between the Longest side.



ΔABC is a Scalane Triangle

$$S = \frac{a+b+c}{2} = \frac{20+24+16}{2} = 30 \text{ cm}.$$

$$\text{Area of } \Delta ABC = \sqrt{S(S-a)(S-b)(S-c)}$$

$$= \sqrt{30(30-20)(30-24)(30-16)}$$

$$= 60\sqrt{7} \text{ cm}^2$$

∴ Area of Parallelogram $= 2\times 60\sqrt{7} = 120\sqrt{7} \text{ cm}^2$

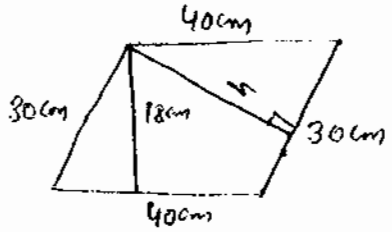
Now Base × Height = Area

$$20 \times H = 120\sqrt{7}$$

$$H = 6\sqrt{7} \text{ cm}.$$

⊥ Distance Betn Two Longest Side $= 6\sqrt{7}$cm.

⇒ एक समांतर चतुर्भुज की आसन्न भुजा (Adjecent) की लंबाई 40 और 30cm है। अगर इसके बड़ी भुजा के बीच की लंबवत दूरी 18cm है। Find the ⊥ Distance Between smallest side.



We know that Area = Base × Height

$$\therefore 18\times 40 = 30\times h$$

$$h = \frac{18\times 40}{30} = 24\text{cm}$$

∴ ⊥ Distance Between smallest side = 24cm.

⇒ एक समांतर चतुर्भुज की आसन्न भुजा की लंबाई का अनुपात 8:9 है। इसकी बड़ी भुजा और संगत ऊंचाई का अनुपात 3:2 है यदि इसकी छोटी भुजा की लंबाई 40 cm है। Find Area.

Give $l : b = 8 : 9$ [Now b is greater than l] —I

$\therefore b : h = 3 : 2$ —II

छोटी भुजा की लंबाई = 40cm

$\therefore 8 \rightarrow 40$

$9 \rightarrow \frac{40}{8}\times 9 = 45\text{cm}$ [बड़ी भुजा की लंबाई]

Acc. to II cond^n Now $3 \Rightarrow 45\text{cm}$ $2 \Rightarrow \frac{45}{3}\times 2 = 30\text{cm}$

∴ Length of Big Side (Base) = 45cm, Height = 30cm.

Area = Base × Height = $[45\times 30] = 1350 \text{ cm}^2$

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⇒ एक समांतर चतुर्भुज की आसन्न भुजा की लंबाई का अनुपात 3:2 है। इसकी बड़ी भुजा और संगत ऊँचाई का अनुपात 5:4 है यदि इसकी छोटी भुजा की लंबाई 30 cm है. find Area

$l : b = 3 : 2$ [$l > b$]

$\therefore l : h = 5 : 4$

Now 2(छोटी भुजा) → 30

$l = \because 3 \rightarrow \frac{30}{2} \times 3 = 45cm$

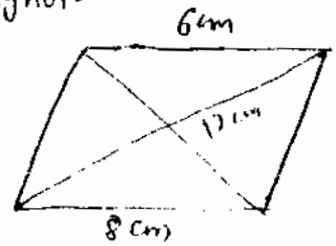
Now $l : h = 5 : 4$

$5 = 45 \quad \therefore 4 \rightarrow \frac{45}{5} \times 4 = 36cm$

$\therefore l = 45cm, \quad h = 36cm$

$Area = l \times h = 45 \times 36 = 1620cm^2$.

⇒ एक समांतर चतुर्भुज के आसन्न भुजा की लंबाई 8 और 6 cm है। एक विकर्ण की लंबाई 12 cm है find the Length of Other Diagnol.



Using $d_1^2 + d_2^2 = 2(l^2 + b^2)$

$\therefore 12^2 + d_2^2 = 2(8^2 + 6^2)$

$144 + d_2^2 = 200$

$d_2^2 = 200 - 144 = 56$

$d_2 = \sqrt{56} = 2\sqrt{14}$ cm. [Length of other Diagnol]

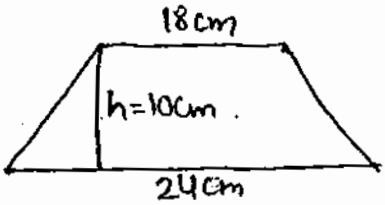
Next Type:→

⇒ एक समलंब चतुर्भुज के समांतर भुजाओं (∥side) की लंबाई 18 और 24 cm है। समलंब चतुर्भुज (Trapizium) का क्षेत्रफल 210 cm^2 है. इनके बीच की लंबाई लंबवत दूरी (Height) क्या होगी।

Using

Area of Trapizium

$= \frac{1}{2}$(sum of ∥ side) × Height



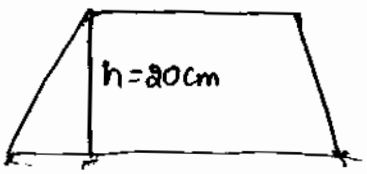
$\therefore 210 = \frac{1}{2}(18 + 24) \times h \Rightarrow 210 = \frac{42}{2} \times h$

$\Rightarrow h = 10$ cm.

∴ Length of Perpendicular Height = 10 cm

एक समलंब चतुर्भुज का क्षेत्रफल 270 cm^2 है। अगर ⊥ Distance 20 cm हो। Ratio of Length of ∥ side 4:5. Then Find the Length of Unparallel Side.

Length of ∥ side = $4x$ & $5x$



Using Area = $\frac{1}{2}$(Sum of ∥ side) × h

$270 = \frac{1}{2}(4x + 5x) \times 20$

$x = \frac{270 \times 2}{20} = \frac{270 \times 2}{4 \times 20} = 3$

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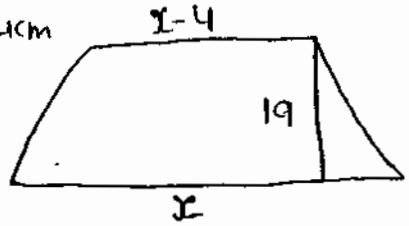
∴ Length of ll side = Resultant height = 5×3 = 15cm
or 5×3 = 15cm.

→ एक समलंब चतुर्भुज का क्षेत्रफल 475cm² है। Length of ⊥ Side (height) is 19c Difference Between Length of ll Side is 4cm. Find the Length of ll Side. Let Length of one Side = x cm, ∴ Second Side = x−4cm



Using Area

$= \frac{1}{2}(\text{sum of ll side}) \times h$

$475 = \frac{1}{2}(x + x - 4) \times 19$

$2x - 4 = \frac{475 \times 2}{19}$ ⇒ $2x - 4 = 50$

$x = 54/2 = 27cm$

∴ Length of one Side = 27cm. Second Side = 27−4 = 23cm

$475 = \frac{1}{2}(A+B) \times 19$

$A + B = \frac{475 \times 2}{19}$

$A + B = 50$

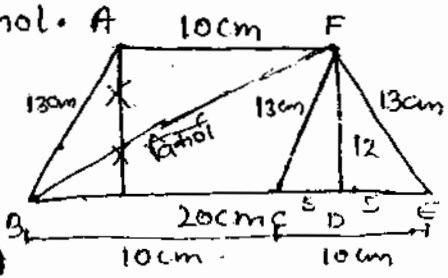
II→ $A - B = 4$

$2A = 54$ $A = 27$

∴ $B = A - 4 = 23$

NEXT TYPE :→

एक समलंब चतुर्भुज की समांतर भुजाओं की लंबाई क्रमशः 10cm और 20cm है। अन्य भुजाओं की प्रत्येक की लंबाई 13cm है, Find The Area and Length of Diagnol.



[जब किसी समलंब चतुर्भुज की असमांतर भुजाओं की लंबाई बराबर दी हो तो हम एक Side के ll side Drawn करते हैं जिससे एक समद्विबाहु त्रिभुज (Isoceles Triangle) बनाते हैं। समद्विबाहु त्रिभुज के ऊपर एक Median Drawn करते हैं तो वह तीसरी भुजा को दो समान भागों में Divide करती है।

$\text{Height} = \sqrt{13^2 - 5^2} = \sqrt{169 - 25} = \sqrt{144} = 12cm.$

∴ $\text{Area} = \frac{1}{2}(20 + 10) \times 12 = 15 \times 12 = 180cm^2$

(BF) Diagnol $= \sqrt{(BD)^2 + (ED)^2} = \sqrt{15^2 + 12^2}$ [∵ BD = BC + CD = 10 + 5 = 15cm]

$= \sqrt{225 + 144} = \sqrt{369}$ cm. [जब दोनों side (Unparallel) Equal हो तो दोनों विकर्ण भी बराबर होंगे]

OR

Using

$\text{Area} = \frac{a+b}{K}\sqrt{S(S-K)(S-C)(S-D)}$ where $K = a - b$

C, D → Unparellel side
a, b → Parellel side

$\text{Height} = \frac{2}{K}\sqrt{S(S-K)(S-C)(S-D)}$, where $S = \frac{K + C + d}{2}$

$\text{Area} = \frac{20+10}{20-10}\sqrt{18(18-10)(18-13)(18-13)}$

[∵ $S = \frac{20 - 10 + 13 + 13}{2} = 18$
$K = 20 - 10 = 10cm$]

$= \frac{30}{10}\sqrt{18 \times 8 \times 5 \times 5}$

$= 3\sqrt{2 \times 9 \times 2 \times 4 \times 5 \times 5} = 3 \times 2 \times 3 \times 2 \times 5 = 180cm^2$

$\text{Height} = \frac{2}{10}\sqrt{18 \times 8 \times 5 \times 5} = \frac{2}{10} \times 12 \times 5 = 12cm.$

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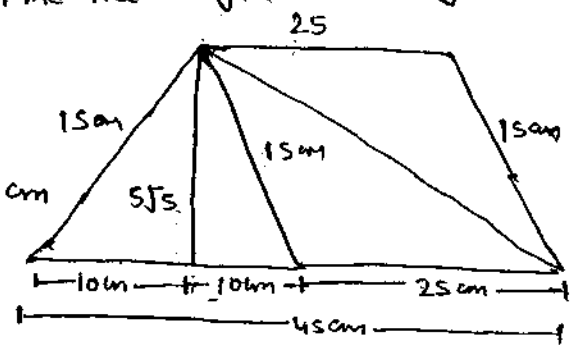
एक समलंब चतुर्भुज की समांतर भुजाओं की लम्बाई क्रमशः 25 और 45 cm है। अन्य असमांतर भुजाओं की लम्बाई प्रत्येक की 15 cm है। Find the Length and Diagnol of and Area of side.

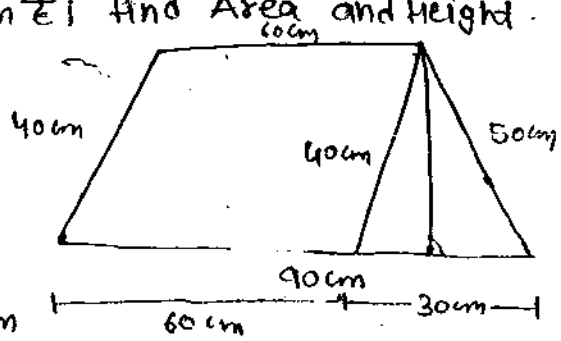


$$\text{Height} = \sqrt{15^2 - 10^2}$$
$$= \sqrt{225-100} = \sqrt{125} = 5\sqrt{5}\text{ cm}$$

$$\text{Area} = \frac{1}{2}(25+45) \times 5\sqrt{5}$$
$$= \frac{70}{2} \times 5\sqrt{5} = 175\sqrt{5}\text{ cm}^2$$

$$\text{Diagnol} = \sqrt{(25+10)^2 + (5\sqrt{5})^2} = \sqrt{35^2+125} = \sqrt{1225+125} = \sqrt{1350}\text{ cm}$$
$$= 15\sqrt{6}\text{ cm}$$

एक समलंब चतुर्भुज की समांतर भुजाओं की लंबाई क्रमशः 60cm और 90cm है। और असमांतर भुजाओं की लंबाई 40 और 50 cm है। find Area and Height.



Using Area $= \frac{a+b}{K}\sqrt{S(S-K)(S-c)(S-d)}$

$S = \frac{\overset{K}{a-b}+c+d}{2}$ [K = a−b (a/b are ∥ side) c/d are unparallel]

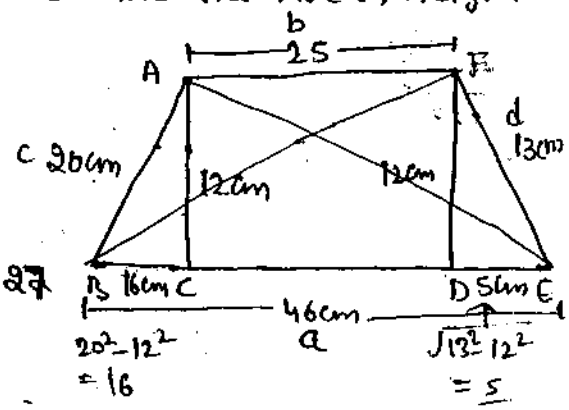
$K = 90-60 = 30\text{cm}$ $\quad S = \frac{30+40+50}{2} = 60\text{cm}$

$$\text{Area} = \frac{90+60}{30}\sqrt{60(60-30)(60-40)(60-50)}$$
$$= \frac{150}{30}\sqrt{60\times30\times20\times10} = \frac{150}{\cancel{30}} \times \overset{20}{\cancel{600}} = 3000\text{cm}^2$$

$$\text{Height} = \frac{2}{K}\sqrt{S(S-K)(S-c)(S-D)} = \frac{2}{30}\sqrt{60\times30\times20\times10} = \frac{2}{\cancel{30}} \times \overset{20}{\cancel{600}} = 40\text{cm}.$$

OR

⇒ ~~एक समांतर~~ एक समलंब चतुर्भुज के समांतर भुजा की लंबाई 25 cm और 46 cm है, और असमांतर भुजाओं की लंबाई 20 और 13 cm है find the Area, Height and Diagnol of Trapizium.



$$\text{Area} = \frac{a+b}{K}\sqrt{S(S-K)(S-b)(S-d)}$$

$K = a-b \qquad S = \frac{K+c+d}{2}$

$K = 46-25 = 21\text{cm} \qquad S = \frac{21+20+13}{2} = 27$

$$= \frac{46+25}{21}\sqrt{27(27-21)(27-20)(27-13)}$$
$$= \frac{71}{21}\times\sqrt{3\times9\times6\times7\times14} = \frac{71}{21}\times\sqrt{3\times3\times3\times3\times2\times7\times2\times7} = \frac{71}{\cancel{21}}\times3\times\cancel{3}\times2\times\cancel{7}$$
$$= 426\text{ cm}^2$$

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Height $= \frac{2}{K}\sqrt{s(s-a)(s-b)(s-c)}$

$= \frac{2}{21} \times \sqrt{27(27-21)(27-20)(27-13)}$

$= \frac{2}{21} \times 3\times3\times2\times7 = 12cm$.

(1) $\therefore DE = \sqrt{13^2-12^2} = \sqrt{169-144} = \sqrt{25} = 5cm$

$\therefore BD = BE - DE = 46 - 5 = 41 cm$.

In ΔBDF

$BF^2 = BD^2 + DF^2$

$\therefore$ Diagnol $BF = \sqrt{(41)^2+(12)^2} = \sqrt{1681+144} = \sqrt{1725} = 25\sqrt{69} cm$.

(2) $BC = \sqrt{20^2-12^2} = \sqrt{400-144} = \sqrt{256} = 16cm$.

$\because CE = BE - BC = 46 - 16 = 30 cm$.

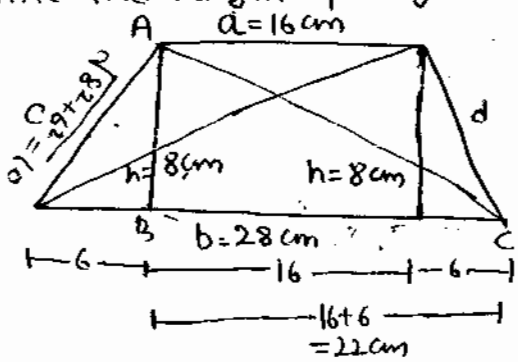
ΔACE $\therefore$ Diagnol $AE = \sqrt{AC^2+CE^2} = \sqrt{12^2+30^2} = \sqrt{144+900} = \sqrt{1044} = 6\sqrt{29} cm$

(C) एक समलंब चतुर्भुज का क्षेत्रफल $176 cm^2$ है इसमें असमांतर भुजा में से प्रत्येक की लंबाई बराबर है। Height of Trapizium is $\frac{2}{11}$ times of sum of parallel side और समांतर भुजाओं की लंबाई का अनुपात 4:7 है। find the length of Diagnol.

$a : b = 4 : 7$

$\therefore a = 4x$ or $b = 7x$

$h = \frac{2}{11}(4x+7x) = \frac{2}{11} \times 11x = 2x$



Area $= \frac{1}{2}$ (Sum of ∥ Side) $\times h$

$176 = \frac{1}{2}(4x+7x) \times 2x$

$11x^2 = 176$ $x^2 = 176/11 = 16$

$x = 4 cm$.

$\therefore a \Rightarrow 4\times4 = 16cm$ $b \Rightarrow 7\times4 = 28cm$, $h = 2\times4 = 8cm$

Now $c = d = \sqrt{8^2+6^2} = \sqrt{64+36} = 10cm$.

$\therefore$ In ΔABC

$AC^2 = AB^2 + BC^2$

Diagnol $\therefore AC = \sqrt{8^2+22^2} = \sqrt{64+484} = \sqrt{548}$ [दोनों विकर्ण बराबर होंगे।

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NEXT TYPE :→ एक सम षट्भुज (Regular Hexagonal) के प्रत्येक भुजा की लंबाई 8cm है। Find Area and ⊥ Distance Between two side.

Using Area $= \frac{\sqrt{3}}{4} a^2 \times 6 = \frac{\sqrt{3}}{4} \times 8^2 \times 6 = \frac{\sqrt{3}}{4} \times 8 \times 8 \times 6 = 96\sqrt{3}\ cm^2$

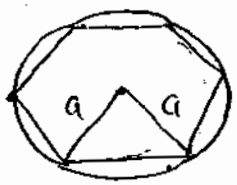
⊥ Distance (Height) $= \sqrt{3}\, a = \sqrt{3} \times 8 = 8\sqrt{3}$ cm.

⇒ एक समषट्भुज (Regular Hexagonal) की लंबाई 14 cm है find an Area of Incircle and Circumcircle.

Radius of Incircle $= \frac{\sqrt{3}}{2} a = \frac{\sqrt{3}}{2} \times 14 = 7\sqrt{3}$ cm.

Area of Incircle $= \pi r^2 = \frac{22}{7} \times 7\sqrt{3} \times 7\sqrt{3}$

$154 \times 3 = 462\ cm^2$.



Circum

Radius of CircumCircle = Side of Hexagonal

$= 14$ cm.

∴ Area of Circumcircle $= \frac{22}{7} \times 14 \times 14$

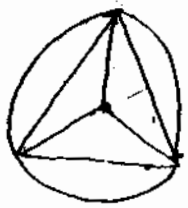
$= 616\ cm^2$.

Incircle

⇒ एक समबाहु त्रिभुज (Equilateral Triangle) के प्रत्येक भुजा की लंबाई $42\sqrt{3}$ cm है। find the Area of Incircle and Circumcircle.

Radius of Incircle $= \frac{\sqrt{3}}{2} a \times \frac{1}{3}$ [Radius of Eq. $\pi \times \frac{1}{3}$]

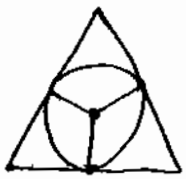
$= \frac{\sqrt{3}}{2} \times 42\sqrt{3} \times \frac{1}{3} = 21$ cm.



∴ Area $= \pi r^2 = \frac{22}{7} \times 21 \times 21 = 1386\ cm^2$

Now Radius of circum circle = 2 × Radius of Incircle.

$= 2 \times 21 = 42$ cm.



Area $= \frac{22}{7} \times 42 \times 42 = 5544\ cm^2$

⇒ एक समबाहु त्रिभुज के Incircle का Area $36\pi\ cm^2$ है Then find the Area of Equilateral Triangle.

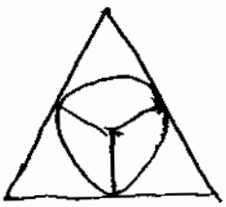
$\pi r^2 = 36\pi$

$r^2 = 36 \quad r = 6$ cm.

Now Radius of Incircle $= \frac{\sqrt{3}}{2} a \times \frac{1}{3}$

$6 = \frac{\sqrt{3}}{2} a \times \frac{1}{3}$

$a = \frac{6 \times 2 \times 3}{\sqrt{3}} = 12\sqrt{3}$ cm.



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Area of Equilateral Triangle $= \frac{\sqrt{3}}{4}a^2 = \frac{\sqrt{3}}{4} \times 12\sqrt{3} \times 12\sqrt{3}$

$= 108\sqrt{3}\ cm^2$.

⇒ एक समबाहु त्रिभुज के Circumcircle का Area $576\pi\ cm^2$ है find the Area of Equilateral Triangle.

$\pi R^2 = 576\pi$

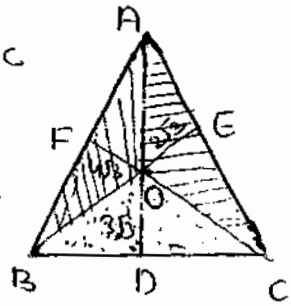
$R^2 = 576 \quad R = 24cm$

Now $R = \frac{\sqrt{3}}{2}a \times \frac{2}{3} \Rightarrow 24 = \frac{\sqrt{3}}{2}a \times \frac{2}{3}$

$\Rightarrow a = 24\sqrt{3}\ cm$.

Area $= \frac{\sqrt{3}}{4} \times 24\sqrt{3} \times 24\sqrt{3} = 432\sqrt{3}\ cm$.

⇒ एक समबाहु त्रिभुज के अन्दर किसी बिंदु से उसकी तीनों भुजाओं पर डाले गए लंब की लंबाई क्रमशः $2\sqrt{3}$, $3\sqrt{3}$ और $4\sqrt{3}\ cm$ है। त्रिभुज का परिमाप क्या होगा।



Area of ΔABC = Area of ΔAOB + Area of ΔBOC + Area of ΔAOC

$\frac{\sqrt{3}}{4}a^2 = \frac{1}{2} \times a \times 2\sqrt{3} + \frac{1}{2} \times a \times 3\sqrt{3} + \frac{1}{2} \times a \times 4\sqrt{3}$

$\frac{\sqrt{3}}{4}a^2 = \frac{1}{2}a\sqrt{3}[2+3+4]$

$\frac{\sqrt{3}}{4}a^2 = \frac{1}{2}\sqrt{3} \times a \times 9$

$\frac{a}{2} = 9 \quad a = 18\ cm$.

$\therefore$ Perimeter $= 3a = 3 \times 18 = 54cm$.

Direct $\longleftrightarrow$ $a = \frac{2}{\sqrt{3}}$ [Sum of ⊥ Height]

$= \frac{2}{\sqrt{3}}[2\sqrt{3} + 3\sqrt{3} + 4\sqrt{3}]$

$= \frac{2 \times 9\sqrt{3}}{\sqrt{3}} = 18\ cm$

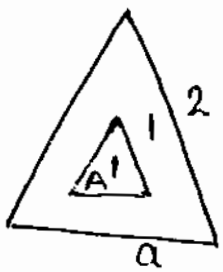
Perimeter $= 3 \times 18 = 54cm$.

⇒ एक समबाहु त्रिभुज के प्रत्येक भुजा की लंबाई $a\ cm$ है। इसके अन्दर एक और समबाहु त्रिभुज इस प्रकार बनाया जाता है कि दिया गया त्रिभुज का क्षेत्रफल दो बराबर भागों में बंट जाता है। इसकी भुजा का अनुपात क्या होगा।

Let Area of Small Triangle $(A_1) = 1$

then Area of Rest Part $= 1$

(A_2) Area of Given Triangle $= 1 + 1 = 2$



$A_1 : A_2 = 1 : 2$

$S_1 : S_2 = \sqrt{1} : \sqrt{2}$

$= 1 : \sqrt{2}$

Ans.

$\frac{\sqrt{3}}{4}A^2 = \frac{1}{2} \times \frac{\sqrt{3}}{4}a^2$

$\frac{A^2}{a^2} = \frac{1}{2}$

$\frac{A}{a} = \frac{1}{\sqrt{2}}$ $\quad A : a$

$1 : \sqrt{2}$

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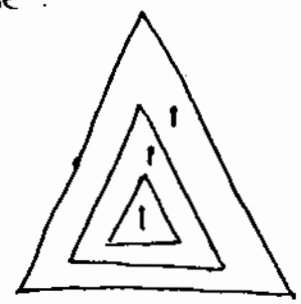
एक समबाहु त्रिभुज के प्रत्येक भुजा की लंबाई $10\sqrt{3}$ cm है इसके अन्दर दो और समबाहु त्रिभुज इस प्रकार बनाए जाते हैं कि बड़े त्रिभुज का क्षेत्रफल 3 बराबर भागों में बंट जाता है। find

(1) Ratio of Area in ascending Order.
(2) Ratio of Side in ascending order.
(3) Length of Each side of Small Equilateral Triangle.



Now Area of Smallest Triangle = 1 (A_1)
Area of Second Triangle = 1+1 = 2 (A_2)
Area of Big Triangle = 1+1+1 = 3 (A_3)

$A_1 : A_2 : A_3 = 1 : 2 : 3$

(ii) Ratio of Side.

$S_1 : S_2 : S_3 = \sqrt{1} : \sqrt{2} : \sqrt{3}$

$= 1 : \sqrt{2} : \sqrt{3}$

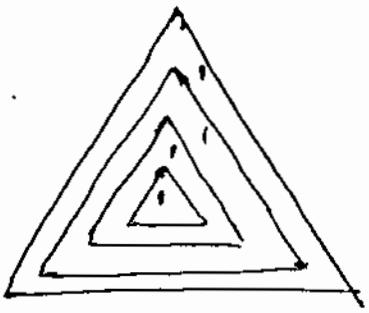
(iii) Length of Smallest side of Equilatest Triangle. $1x, \sqrt{2}x, \sqrt{3}x$

Now $\sqrt{3}x = 10\sqrt{3}$

$x = 10$ cm.

एक समबाहु त्रिभुज के प्रत्येक भुजा की लंबाई 20 है, इसके अन्दर 3 और समबाहु त्रिभुज इस प्रकार बनाए जाते हैं कि बड़े त्रिभुज का क्षेत्रफल 4 बराबर भागों में बंट जाता है। find

(i) Ratio of Area in Ascending Order.
(ii) Ratio of Side in Ascending Order.
(iii) Length of Each Side of Small Equilateral Triangle.



Let Area of Smallest Triangle (A_1) = 1
∴ Area of second Triangle (A_2) = 2
Area of Third Triangle (A_3) = 3
Area of Big Triangle (A_4) = 4.

$\therefore A_1 : A_2 : A_3 : A_4 = 1 : 2 : 3 : 4$

(ii) $S_1 : S_2 : S_3 : S_4 = \sqrt{1} : \sqrt{2} : \sqrt{3} : \sqrt{4}$

$= 1 : \sqrt{2} : \sqrt{3} : 2$

(iii) Length of Smallest side of Equilateral Traingle $x \; \sqrt{2}x \; \sqrt{3}x \; 2x$

$2x = 20$

$x = 10$ cm.

एक समबाहु त्रिभुज के प्रत्येक भुजा की लंबाई 15 cm है इसके अन्दर एक छोटा समबाहु त्रिभुज इस प्रकार बनाया जाता है कि छोटे त्रिभुज का क्षेत्रफल बड़े बचे हुए Part का $\frac{4}{5}$ है, find the Length of Each side of Smallest Triangle.

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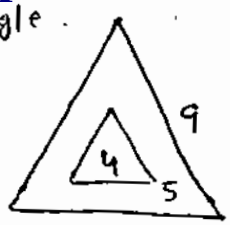
Area of Smallest Triangle = $\frac{4}{5}$ × Rest Part of Big Triangle



$\therefore$ $\frac{\text{Area of Smallest Triangle } (A_1)}{\text{Area of Rest Part } (A_{Remain})} = \frac{4}{5}$

$\therefore A_1 : A_{remain} = 4:5$

Now Area of Big Triangle (A_2) = Area of Small Triangle + Rest Part

$= 4+5 = 9$

$\therefore A_1 : A_2 = 4:9$

Ratio of side

$S_1 : S_2 = \sqrt{4} : \sqrt{9}$

$= 2:3$

Now $3 \Rightarrow 15$ $\quad \therefore 2 \Rightarrow \frac{\cancel{15}^{5}}{\cancel{3}} \times 2 = 10$

(Side) Length of Smallest Triangle = 10cm.

⇒ एक समबाहु त्रिभुज की प्रत्येक भुजा को 2cm बढ़ा दिया जाए तो त्रिभुज का क्षेत्रफल $2\sqrt{3}$ cm² बढ़ जाता है। find the length of original side of Triangle.

Let Original side of Triangle = a cm.

Area = $\frac{\sqrt{3}}{4} a^2$ cm².

Now Side of Triangle (Increase by 2) = $a+2$ cm.

Area = $\frac{\sqrt{3}}{4}(a+2)^2$

Acc. to question

$$\frac{\sqrt{3}}{4}(a+2)^2 = \frac{\sqrt{3}}{4}a^2 + 2\sqrt{3}$$

$$\cancel{\frac{\sqrt{3}}{4}a^2} + \frac{\sqrt{3}}{4}\times 4 + \frac{\sqrt{3}}{4}\times 4a = \cancel{\frac{\sqrt{3}}{4}a^2} + 2\sqrt{3}$$

$$\sqrt{3} + \sqrt{3}a = 2\sqrt{3}$$

$$\sqrt{3}a = \sqrt{3} \qquad a = 1 \text{ cm}.$$

Lenth of Original Side = 1cm.

Next Type :→ एक त्रिभुज की भुजाएं क्रमशः 9cm, 9cm और 12cm है। find the Area of Triangle

Using Heron's Formula

$$S = \frac{a+b+c}{2} = \frac{9+9+12}{2} = 15 \text{ cm}$$

$$\text{Area} = \sqrt{S(S-a)(S-b)(S-c)}$$

$$= \sqrt{15(15-9)(15-9)(15-12)}$$

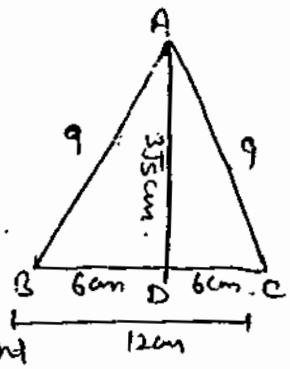
$$= \sqrt{15\times 6\times 6\times 3}$$

$$= \sqrt{3\times 5\times 3\times 2\times 3\times 2\times 3}$$

$$= 18\sqrt{5} \text{ cm}^2$$

OR

In ΔABC



$AD^2 = AC^2 - DC^2$

$AD = \sqrt{9^2 - 6^2}$

$= \sqrt{81-36}$

$= \sqrt{45} = 3\sqrt{5}$ cm.

Height = $3\sqrt{5}$ cm.

Area = $\frac{1}{2}$ × Base × Height

$= \frac{1}{\cancel{2}} \times \overset{6}{\cancel{12}} \times 3\sqrt{5}$

$= 18\sqrt{5}$ cm²

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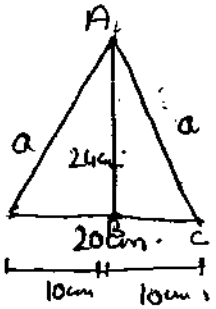
⇒ एक समद्विबाहु त्रिभुज का क्षेत्रफल $240 cm^2$ है इसके असमान भुजा की लंबाई 20cm है तो समान भुजाओं की लंबाई क्या होगी।



Using Area $= \frac{1}{2} \times$ Base $\times$ Height

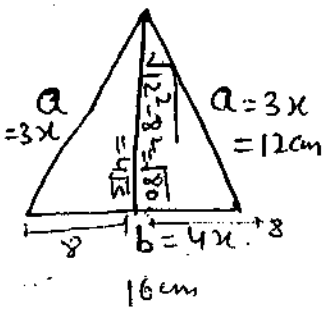
$$240 = \frac{1}{2} \times 20 \times h$$

$$h = 24 cm.$$

Now In ΔABC

$$a = \sqrt{24^2 + 10^2} = \sqrt{576+100} = \sqrt{676} = 26 cm.$$

⇒ एक समद्विबाहु त्रिभुज का परिमाप 40cm है इसकी दो बराबर भुजाओं में से एक भुजा और तीसरी भुजा का अनुपात 3:4 है find the Area of Isoceles Triangle.



$$a : b = 3 : 4$$

$$\therefore a = 3x, \; b = 4x$$

$$\therefore 3x + 3x + 4x = 40$$

$$10x = 40$$

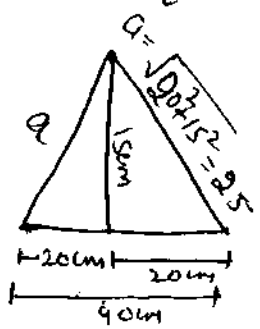
$$x = 4 cm.$$

$$\therefore a = 12 cm \quad b = 16 cm.$$

$$\therefore \text{Height} = \sqrt{12^2 - 8^2} = \sqrt{144-64} = \sqrt{80} = 4\sqrt{5} cm.$$

$$\text{Area} = \frac{1}{2} \times \text{Base} \times \text{Height} = \frac{1}{2} \times 16 \times 4\sqrt{5} = 32\sqrt{5} cm^2$$

⇒ एक समद्विबाहु त्रिभुज का क्षेत्रफल $300 cm^2$ है। Length of ⊥ on unequal side is 15cm. find the Perimeter of the Triangle.



Using Area $= \frac{1}{2} \times b \times H$

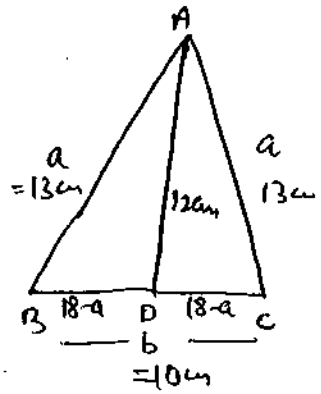
$$300 = \frac{1}{2} \times 15 \times b$$

$$b = \frac{300 \times 2}{15} = 40 cm.$$

$$\therefore a = \sqrt{20^2 + 15^2} = \sqrt{625} = 25 cm.$$

$$\text{Perimeter} = 25 + 25 + 40 = 90 cm.$$

⇒ एक समद्विबाहु त्रिभुज का परिमाप 36cm है इसके असमान भुजा पर डाले गए लंब की लंबाई 12cm है। find the Area of Triangle.



$$2a + b = 36$$

$$b = 36 - 2a$$

$$\therefore BD = BC = \frac{36-2a}{2} = 18 - a$$

In ΔADC = $AC^2 = AD^2 + DC^2$

$$a = \sqrt{12^2 + (18-a)^2}$$

$$a^2 = 144 + (18-a)^2$$

$$a^2 = 144 + 324 + a^2 - 36a, \quad 36a = 468 \quad a = \frac{468}{36} = 13$$

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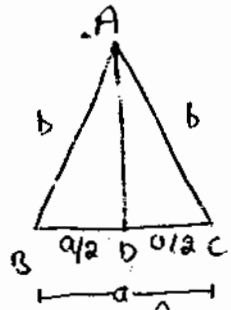
$a = 13$ cm

$b = 36 - 2a$

$= 36 - 26 = 10$ cm.

Area $= \frac{1}{2} \times \overset{5}{\cancel{10}} \times 12 = 60\ cm^2$

⇒ एक समद्विबाहु त्रिभुज के बराबर भुजाओं में से प्रत्येक की लम्बाई b cm है और असमान भुजा की लंबाई a cm है find the Area of Triangle.

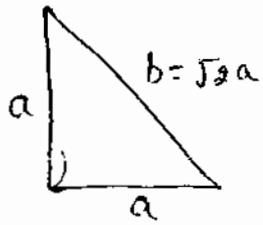


Height $AD = \sqrt{b^2 - (\frac{a}{2})^2} = \sqrt{b^2 - \frac{a^2}{4}} = \frac{\sqrt{4b^2 - a^2}}{2}$

Area $= \frac{1}{2} \times$ base $\times$ height

$= \frac{1}{2} \times a \times \frac{\sqrt{4b^2 - a^2}}{2} = \frac{1}{4} a \sqrt{4b^2 - a^2}\ cm^2$

⇒ एक समद्विबाहु समकोण त्रिभुज का परिमाप 2P है तो इसका क्षेत्रफल P की form में क्या होगा?



Now $b = \sqrt{a^2 + a^2}$
$= \sqrt{2}\, a$

Now Perimeter $= 2P$

$\therefore a + a + \sqrt{2}\, a = 2P$

$a(2 + \sqrt{2}) = 2P$

$a = \frac{2}{2 + \sqrt{2}} P = \frac{2(2 - \sqrt{2})}{2} P$

$a = (2 - \sqrt{2}) P$

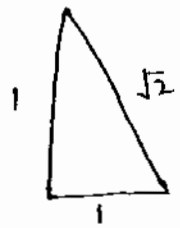
$\therefore$ Area $= \frac{1}{2} \times (2 - \sqrt{2}) P \times (2 - \sqrt{2}) P = \frac{1}{2} P^2 (4 + 2 - 4\sqrt{2})$

$= \frac{1}{2} P^2 [6 - 4\sqrt{2}] = P^2 [3 - 2\sqrt{2}]$

$\boxed{\text{Area} = P^2 [3 - 2\sqrt{2}]}$

Using this formula we can select answer with the help of Options.

Suppose $a = 1$ cm $\therefore b = \sqrt{1 + 1} = \sqrt{2}$



Area $= \frac{1}{2} \times 1 \times 1 = \frac{1}{2}\ cm^2$ — I

Perimeter $= 1 + 1 + \sqrt{2} = 2P$

$P = \frac{2 + \sqrt{2}}{2}$

Using formula $\therefore$ Area $= \left(\frac{2 + \sqrt{2}}{2}\right)^2 [3 - 2\sqrt{2}] = \frac{6 + 4\sqrt{2}}{4} (3 - 2\sqrt{2}) = \frac{(3 + 2\sqrt{2})(3 - 2\sqrt{2})}{2}$

$= \frac{9 - 8}{2} = \frac{1}{2}\ cm^2$ — II

Hence Area (both) are same.

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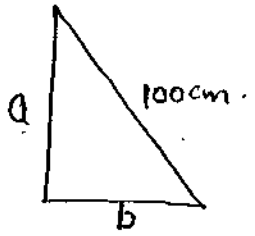
Next Type :→

एक समकोण त्रिभुज का क्षेत्रफल $2400 cm^2$ है। कर्ण की लंबाई 100 cm है। find the Difference in Length of other two side.



Detail :→ In Rightangled Triangle

$a^2+b^2=(100)^2$

$a^2+b^2=10000$ — I

Now. Area $=\frac{1}{2}\times$ Base $\times$ Height $=\frac{1}{2}\times a\times b$

$\frac{1}{2}\times a\times b=2400$ $\quad ab=4800$ — II

Now we have to find the Difference $a-b$

Using $a-b=\sqrt{a^2+b^2-2ab}$

from I or II

$a-b=\sqrt{10000-2(4800)}=\sqrt{400}=20$

∴ Difference Between two side is 20 cm.

Shortcut : $a^2+b^2=100^2$

we select two No. a and b whose square is equal 100^2

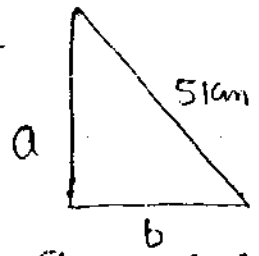
∴ $80^2+60^2=100^2$

∴ $a=80$ $b=60$

$a-b=20cm$.

∴ Area $=\frac{1}{2}\times 80\times 60=2400cm^2$

एक समकोण त्रिभुज का क्षेत्रफल $540 cm^2$ है। कर्ण की लंबाई 51cm. find the Perimeter of Triangle.



Detail :→ $a^2+b^2=51^2$

$a^2+b^2=2601$ — I

or $\frac{1}{2}\times a\times b=540$

$ab=1080$ — II

Perimeter $=a+b+51cm$.

∴ $a+b=\sqrt{a^2+b^2+2ab}$

$=\sqrt{2601+2(1080)}=\sqrt{2601+2160}=\sqrt{4761}=69cm$.

∴ Perimeter $=69+51=120cm$.

Shortcut

$(51)^2=(45)^2+(24)^2$

∴ $a=45$ $b=24$

$a+b+51$

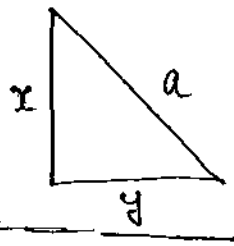
$=45+24+51$

$=120cm$.

$\frac{51}{3}=17$ or $17^2=15^2+8^2$

∴ $15\times3=45$, $8\times3=24$

एक समकोण त्रिभुज के कर्ण की लंबाई a cm है इसके समकोण बनाने वाली भुजा की लंबाई का योग b cm है find the Area of the Traingle.



Now $x+y=b$

or $x^2+y^2=a^2$

Area $=\frac{1}{2}\times x\times y$

We know that $(x+y)^2=x^2+y^2+2xy$.

∴ $2xy=\sqrt{(x+y)^2-(x^2+y^2)}$

$=\sqrt{b^2-a^2}$

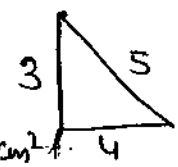
$xy=\frac{b^2-a^2}{2}$

∴ Area $=\frac{1}{2}\times\frac{b^2-a^2}{2}=\frac{b^2-a^2}{4}cm^2$

from Options.

assume side $=3,4,5$

∴ $a+b=3+4=7$



∴ Area $=\frac{1}{2}\times3\times4=6cm^2$

Now from formula $\quad b=3+4$

Area $=\frac{1}{4}\times(7^2-5^2)$

$=\frac{1}{4}\times 24=6cm^2$

Both Area are same.

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Two Equal ~~Chord~~ Circle intersect to each other at Point A and B and Passing through their Centre If the length of Common Chord is 15 cm. find the Radius of circle.

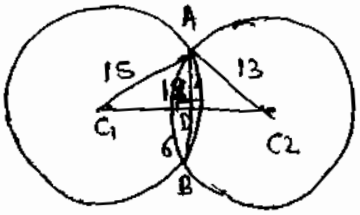
$$\text{Chord } AB = \sqrt{3}\, r$$

$$r = \frac{15}{\sqrt{3}} = 5\sqrt{3}$$

$$\therefore \boxed{\text{Radius of Circle} = 5\sqrt{3} \text{ cm}}$$

Two Circle intersect to each other at Point A and B. Their Diameter are 30 and 26 cm. If length of Common Chord is 24 cm find the Distance between their centre.

$$AD = \frac{AB}{2} = \frac{24}{2} = 12 \text{ cm}.$$



Now ΔAC_1D is a Right Angle Δ

$$\therefore C_1D = \sqrt{15^2 - 12^2} = \sqrt{81} = 9 \text{ cm}.$$

ΔAC_2D is a Right Angle Δ

$$\cancel{AC_2} \quad C_2D = \sqrt{13^2 - 12^2} = 5 \text{ cm}.$$

$$\therefore C_1C_2 = 9 + 5 = 14 \text{ cm}.$$

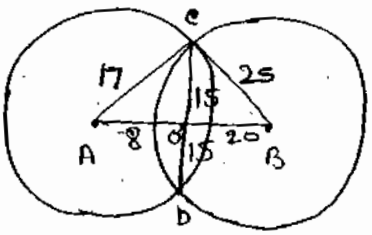
$$\boxed{C_1C_2 = 14 \text{ cm}.}$$

→ Two Circles whose centres are A and B and Intersect to each other at point C and D. The length of Common Chord is 30 cm. The line AB and CD intersect to each other at Point O if AO = 8 cm, BO = 20 cm Find the Radius of Each Circle.

$$\text{Radius } AC = \sqrt{15^2 + 8^2} = \sqrt{289} = 17$$

$$\text{Radius } BC = \sqrt{15^2 + 20^2} = \sqrt{625} = 25$$

$$\therefore \boxed{\text{Radius} = 17, 25 \text{ cm}}$$

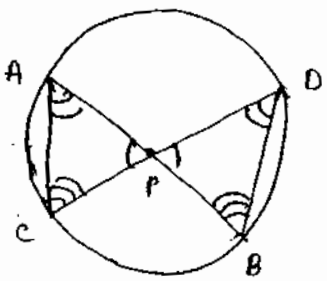


Th: → AB and CD are the two Chord of a Circle intersect to each other at Point P that $\boxed{AP \cdot BP = CP \cdot DP}$

$\Delta APC \sim \Delta DPB$ [AAA]

$$\therefore \frac{AP}{DP} = \frac{CP}{BP}$$

$$\Rightarrow \boxed{AP \cdot BP = CP \cdot DP}$$



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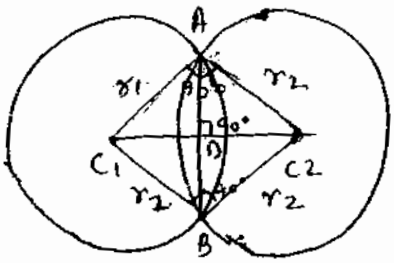
Length of Chord AB when AB is common chord (उभयनिष्ठ जीवा)

ΔABC_1 is a Isoceles Δ

C_1D is $\perp$ to AB

$\therefore$ ~~ACD is~~ AD is $\perp$ to Δ AC_1C_2



$\therefore \perp AD = \frac{r_1 r_2}{C_1C_2}$ $\left[\perp \text{Height} = \frac{Pb}{h}\right]$

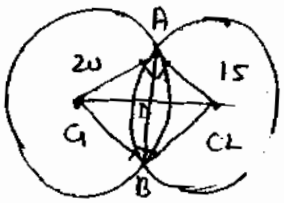
$\perp AD = \frac{r_1 r_2}{\sqrt{r_2^2+r_1^2}}$ $\left[\because C_1C_2 = \sqrt{r_1^2+r_2^2}\right.$ ΔAC_1C_2 is Right Angle

$\therefore$ $\boxed{\text{Chord } AB = 2 \cdot AD = \frac{2r_1r_2}{\sqrt{r_1^2+r_2^2}}}$

Two Circle with Centre C_1 and C_2 and Radii 20 and 15 cm intersect to eachother At Point AB such that Radii of each Become tangent of Circle of each other. find the Length of Common Chord AB.

$\perp \text{Height } AD = \frac{r_1r_2}{\sqrt{r_1^2+r_2^2}} = \frac{20\times15}{\sqrt{20^2+15^2}} = \frac{\cancel{20}^{4}\times\cancel{15}^{3}}{\cancel{25}_{5}}$

$= 12$



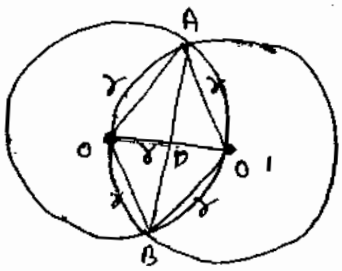
$\therefore$ $\boxed{\text{Chord } AB = 2\times12 = 24 \text{ cm}}$

→ Length of Chord AB when two Equal Circle Intersect to each other at Point A and B and Passing through their Centre.

→ Centre of Both Circle are equal

$\therefore$ Radius are also equal.

$\therefore$ AOO' is an equilateral Δ



$\therefore AD = \frac{\sqrt{3}}{2}a = \frac{\sqrt{3}}{2}r$

$\therefore AB = 2\times\frac{\sqrt{3}}{2}r \Rightarrow \sqrt{3}r$

$\boxed{\text{Chord } AB = \sqrt{3}r}$

Two equal Circle intersect to each other at Point A and B and passing throu their centre. Find the Length of common Chord AB if Radius of circ is 5cm.

(Diagram is same as above)

Using Chord $AB = \sqrt{3}r$

$= \sqrt{3}\,5 \Rightarrow \boxed{5\sqrt{3} \text{ cm}}$

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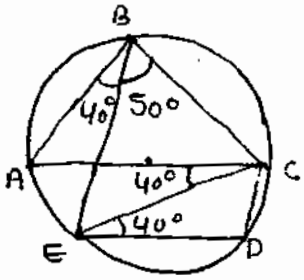


$\angle ABC = 90°$ [Diameter से Circumference पर बना Angle]

$\therefore \angle ABE = 90° - 50° = 40°$

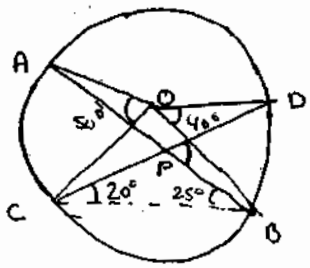
$\angle ABE = 40°$

$\therefore \angle ACE = \angle ABE = 40°$ [एक ही Chord से बने Angle]

Now $\angle DEC = \angle ACE = 40°$ [Alternate Angle]

$\boxed{\therefore \angle DEC = 40°}$

O is the centre of a circle AB and CD are the two chord Intersec to each other at Point P if $\angle AOC = 50°$, $\angle BOD = 40°$ Find $\angle BPD$.



$\angle BCP = \frac{1}{2} \times \angle BOD$ [∵ BD Chord द्वारा बना Angle]

$\angle BCP = \frac{1}{2} \times 40° = 20°$

OR $\angle ABC = \frac{1}{2} \times \angle AOC$ [∵ AC Chord द्वारा बना Angle]

$= \frac{1}{2} \times 50° = 25°$

$\angle BPD = \angle PCB + \angle PBC$ [Sum of two Interior Angle = Exterior Angle]

$= 20° + 25°$

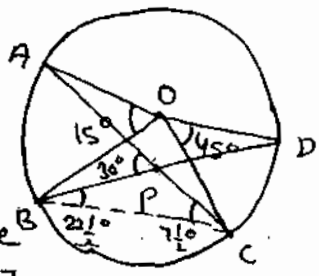
$\therefore \boxed{\angle BPD = 45°}$

[जब तक यह न कहा हो कि Chord centre के opposite लेनी है तब तक Chord को हम एक ही side में लेंगे]

OR अगर centre पर Angle दिए हो Then

$\angle BPD = \frac{50° + 40°}{2} = 45°$

O is the centre of a circle AC and BD are the two chord Intersect to each other at Point P. if $\angle AOB = 15°$, $\angle APB = 30°$ Find $\angle COD$



$\angle ACB = \frac{1}{2} \angle AOB$ [Chord AB द्वारा बने Angle]

$= \frac{1}{2} \times 15 = 7\frac{1}{2}°$

Now $\angle APB = \angle PBC + \angle PCB$ [Sum of two Interior Angle 1's = External Angle]

$30° = \angle PBC + 7\frac{1}{2}°$

$\therefore \angle PBC = 22\frac{1}{2}°$

$\therefore \angle COD = 2 \times \angle PBC = 2 \times \frac{45}{2} = 45°$ [Chord CD द्वारा बना Angle]

$\therefore \boxed{\angle COD = 45°}$

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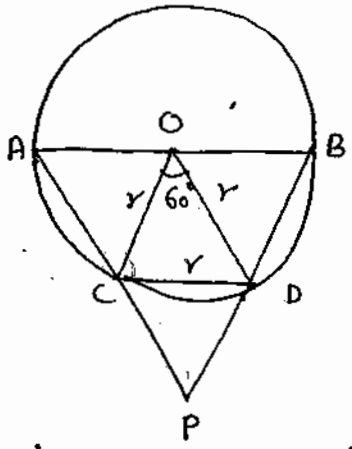
O is the Centre of a Circle. AB is the Diameter. CD is the Chord Equal to Radius of Circle. The Produce Line AC and BD intersect To each other at Point P. Then Find $\angle APB$



[C and D are the Mid Point of AP and BP
$\because CD = \frac{1}{2} AB$. $\therefore$ A line In a Δ that is Half Of the line of Δ Divides the other two line in to equal Parts.

Now ΔOCD is an Equilateral Δ

$\because OC = CD = OD = r$

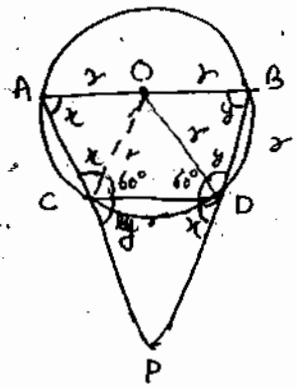
$\therefore \angle COD = 60°$

→ अगर किसी Big Δ में कोई Small Δ Equilateral हो तो वह Big Δ भी Equilateral Δ होगी.

$\therefore \Delta ABP$ is an Equilateral Δ

$\therefore \boxed{\angle APB = 60°}$

Detail



from ΔAOC $AO = OC$

$\therefore \angle OAC = \angle OCA = x$

Similarly in $\Delta OBD = OB = OD$

$\therefore \angle OBD = \angle ODB = y$

Now ABCD is a Cyclic Quadrilateral

in which $x + y + 60 = 180°$

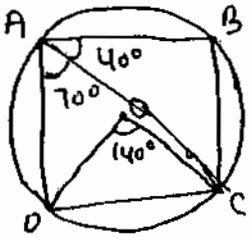
$\therefore x + y = 120°$

Now $\angle PCD = \angle OBD = y$ and $\angle PDC = \angle CAO = x$ [$\because$ Exterior Angle of ▱ ABCD]

$x + y + \angle APB = 180°$

$\boxed{\angle APB = 180° - 120° = 60°}$

ABCD is a Cyclic Quadrilateral O is the centre of circle if $\angle COD = 140°$ $\angle BAC = 40°$ Find $\angle BCD$.



$\angle CAD = \frac{1}{2} \times \angle COD$ [CD द्वारा बने Angle]

$\therefore \angle CAD = \frac{1}{2} \times 140° = 70°$

$\therefore \angle DAB = 70° + 40° = 110°$

Now $\angle DAB + \angle BCD = 180°$ [ABCD is a Cyclic Quad]

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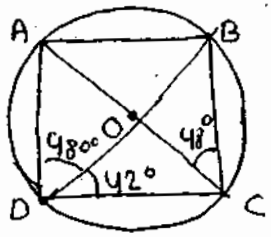
$\therefore \angle BCD = 180^\circ - 110^\circ$

$\boxed{\angle BCD = 70^\circ}$

ABCD is a cyclic Quadrilateral. AC is the Diameter of a circle if $\angle BDC = 42^\circ$ Find $\angle ACB$



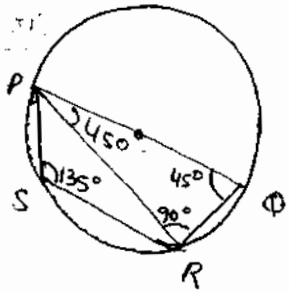
$\angle ADC = 90^\circ$ [$\because$ Diameter पर Circumference पर बना Angle]

$\therefore \angle ADB = 90^\circ - 42^\circ = 48^\circ$

Now $\angle ADB = \angle ACB = 48^\circ$ [AB Chord से बना Angle.

$\therefore \boxed{\angle ACB = 48^\circ}$

PQRS is a Cyclic Quadrilateral and PQ is the Diameter of Circle if $\angle QPR = 45^\circ$ find $\angle PSR$



$\angle QPR = 45^\circ$

$\therefore \angle PRQ = 90^\circ$ [Diameter ~~पर से~~ Circumference पर बना Angle]

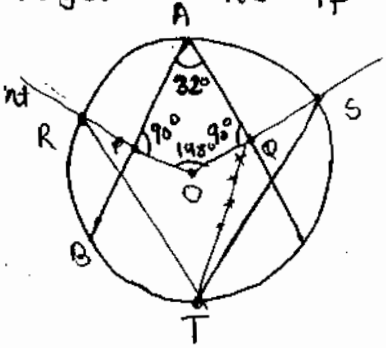
$\therefore \angle PQR = 180^\circ - (90^\circ + 45^\circ) = 45^\circ$

$\therefore \angle PQR + \angle PSR = 180^\circ$ [PQRS is a Cyclic Quadrilateral]

$\therefore \angle PSR = 180^\circ - 45^\circ = 135^\circ$

$\boxed{\angle PSR = 135^\circ}$

O is the Centre of a circle AB and AC are the two chord P and Q are the Mid Point of the Line AB and AC respectively. The produce line OP and OQ Intersec the Circle at the Point R and S. T is a point on the Major arc RS. if $\angle BAC = 32^\circ$ Find $\angle RTS$.



$\angle APO = \angle AQO = 90^\circ$ [$\because$ Centre से Chord के Mid Point पर डाली गई Line Chord पर $\perp$ होती है]

$\therefore \angle PAQ + \angle POQ = 180^\circ$

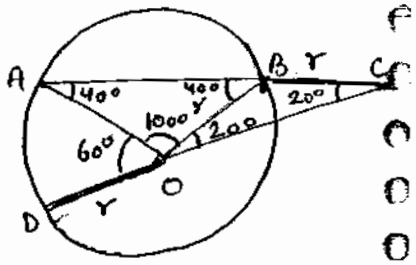
$\therefore \angle POQ = 180^\circ - 32^\circ = 148^\circ$

$\therefore \angle RTS = \frac{1}{2} \times \angle POQ$ [$\because$ RS द्वारा बने Angle]

$= \frac{1}{2} \times 148 = 74^\circ$

$\therefore \boxed{\angle RTS = 74^\circ}$

AB is a Chord of a Circle. O is the Centre of a Circle. DOC is a Line Seagment originating from the Point O on the Circle Intersect the Produce Line AB at the Point C and BC = OD if $\angle BCD = 20^\circ$ find $\angle AOD$.



Draw a Line OB

Now $OD = BC$

$\therefore$ In ΔOBC $OB = OB = BC = Y$

$\therefore \angle BOC = \angle BCO = 20^\circ$

Now $\angle OBA = 20^\circ + 20^\circ$ [Exterior Angle of ΔOBC]

$\angle OBA = 40^\circ$

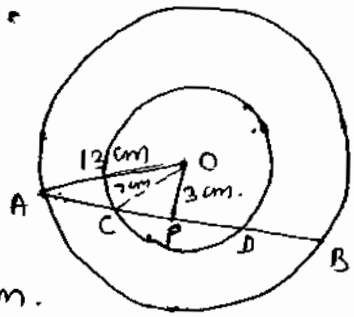
Now $OB = OA$ $\therefore \angle OAB = \angle OBA = 40^\circ$

$\therefore \angle AOB = 180^\circ - (40^\circ + 40^\circ) = 100^\circ$ [from ΔAOB]

$\therefore \angle AOD = 180^\circ - (100^\circ + 20^\circ) = 60^\circ$

$\therefore \boxed{\angle AOD = 60^\circ}$

O is the Centre of two Concentric Circle whose Radii is 13cm and 7cm AB is the Chrod of largest Circle which Intersect the Smallest Circle at Point C and D. If Length of $\perp$ from the Centre on Chord AB is 3cm find the Length of AC.



from ΔAOP

$AP = \sqrt{13^2 - 3^2} = \sqrt{169 - 9} = \sqrt{160}$

$= 4\sqrt{10}$ cm.

from ΔCOP

$CP = \sqrt{7^2 - 3^2} = \sqrt{49 - 9} = \sqrt{40} = 2\sqrt{10}$ cm.

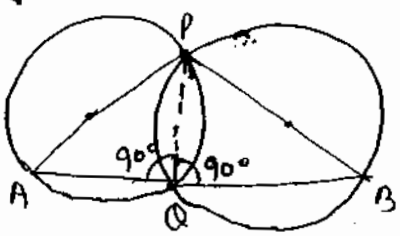
Now $AC = AP - CP = 4\sqrt{10} - 2\sqrt{10}$

$\boxed{AC = 2\sqrt{10} \text{ cm.}}$

Two circle intersect to each other at the point P and Q. PA and PB are the Diametere of two circle Respectively. Find $\angle AQB$

$\angle PQA = 90^\circ$ [Diameter से ~~Centre~~ Circum पर बना Angle]

$\angle PQB = 90^\circ$ ["]



$\therefore \angle AQB = \angle PQA + \angle PQB$

$= 180^\circ$

$\therefore \boxed{\angle AQB = 180^\circ}$

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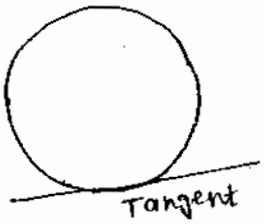
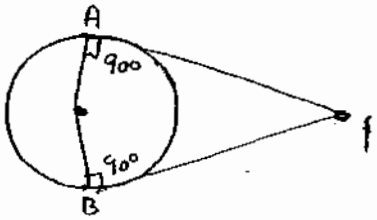
TANGENT [स्पर्श रेखा]



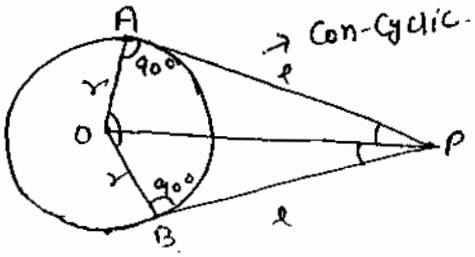
* वह Line जो Circle को Touch करते हुए जाती है।
* Circle पर एक बिंदु पर एक ही Tangent Drawn कर सकते है
* किसी Exterior Point से Circle पर दो Tangent Drawn कर सकते है और उस Point से Drawn किए गए Tangent की Length Equal होती है। PA = PB
* Radius Tangent के साथ हमेशा 90° का Angle बनाती है।



Con-Cyclic Quadrilateral [एक वृत्तीय चतुर्भुज]

* एक चतुर्भुज जिसका कुछ Part Circle के अन्दर होता है और कुछ Circle के बाहर होता है।
* $\angle AOB + \angle APB = 180°$

अगर हम Centre से P Point पर Line Draw करते है तो वह Quadrilateral को 2 equal Parts मे Divide कर देती है।



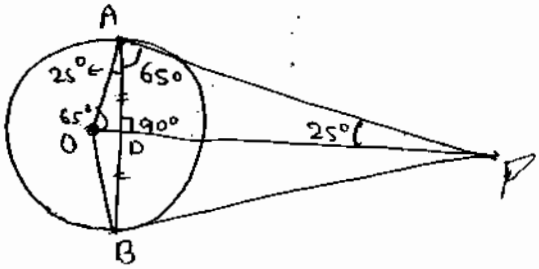
* $\angle AOP = \angle BOP$

  $\angle APO = \angle BPO$
* Area of $\triangle AOP$ = Area of $\triangle BOP$

If AB is a chord

then * $\angle AOP = \angle PAD$

* $\angle OAD = \angle APD$



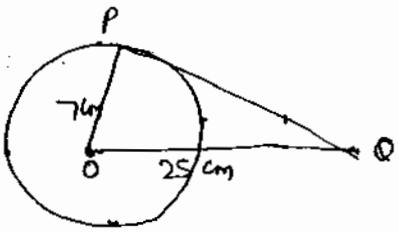
e.g → if $\angle APO = 25°$.

→ O is the Centre of a Circle. Radius of Circle is 7cm. PQ is the tangent which touch the Circle at point P. if OQ = 25cm, find the length of Tangent PQ.

$$PQ = \sqrt{OQ^2 - OP^2}$$

$$= \sqrt{25^2 - 7^2} = \sqrt{576} = 24$$

$$\therefore \boxed{PQ = 24\,cm}$$

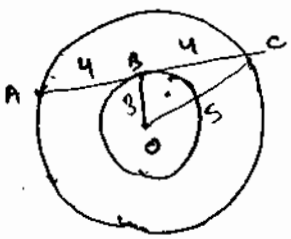


O is the Centre of two Concentric Circle whose Radii are 3 and 5 cm. find the length of Chord of Largest Circle which is the tangent of smaller

$$BC = \sqrt{5^2 - 3^2} = 4\,cm$$

$$\therefore AC = 2 \times 4 = 8\,cm$$

$$\therefore \boxed{\text{Length of Chord} = 8\,cm}$$



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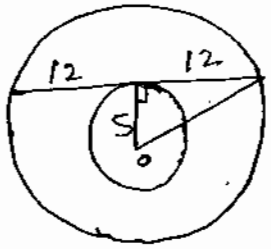
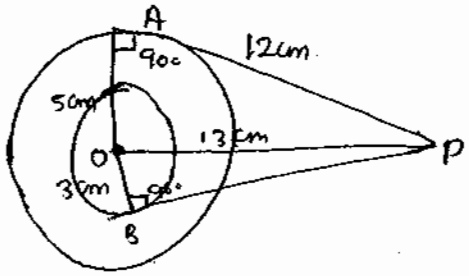
O is the Centre of the Circle. Length of Chord of Largest Circle which is the tangent of Smaller is 24cm. If the Radius of Small Circle is 5 cm Find the Radius of Largest.



$$\text{Radius of Largest Circle} = \sqrt{12^2 + 5^2}$$
$$= \sqrt{144+25}$$
$$= 13\text{cm}.$$

$$\boxed{\text{Radius of Largest Circle} = 13\text{ cm}}$$

• O is the centre of two concentric Circle of Radii 5 and 3cm PA and PB are the two tangent of Larger and Smaller Circle Respectively which touch the Circle at Point A and B the Length of PA is 12cm and Find the Length of PB



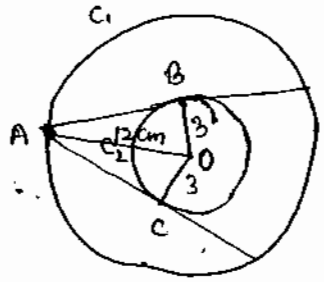
$$PO = \sqrt{PA^2 + AO^2}$$
$$= \sqrt{12^2+5^2} = \sqrt{169} = 13\text{cm}$$
$$PO = 13\text{cm}.$$

from ΔPOB

$$PB = \sqrt{PO^2 - OB^2} = \sqrt{13^2 - 3^2} = \sqrt{169-9}$$
$$= \sqrt{160} = 4\sqrt{10}\text{ cm}.$$

$$\boxed{PB = 4\sqrt{10}}$$

C_1 and C_2 are the two Concentric Circle whose centre is o and their radii are 12 and 3cm. A is a Point on the Circle C_1. AB and AC are the two tangent of Smallest Circle C_2. Find the Area of Quadrilateral ABOC



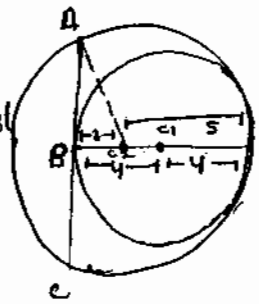
$$AB = \sqrt{12^2 - 3^2} = \sqrt{144-9}$$
$$AB = \sqrt{135} = 3\sqrt{15}\text{cm}.$$

$$\text{Area of Quad ABOC} = \frac{1}{2} \times 3\sqrt{15} \times 3 \times 2$$
$$\boxed{= 9\sqrt{15}\text{ cm}^2}$$

Two Circle touch internally find the Length of Largest Chord of Largest Circle which the tangent of Smaller if their Radii are 5 and 4 cm.

Radius of Small Circle = 4cm.

Diameter of Small circle = 8 cm



∴ Radius of Largest Circle = 5cm [∵ Centre of Circle Largest Lies in Small Circle]

$$\therefore BC_2 = 8 - 5 = 3\text{cm}$$

from ΔABC_2

$$AB = \sqrt{AC_2^2 - BC_2^2}$$

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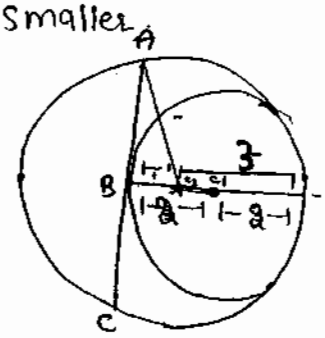
$= \sqrt{5^2 - 3^2} = \sqrt{16} = 4cm.$

A $\therefore$ $\boxed{\text{Length of Chord } AC = 2 \times 4 = 8cm}$

Two circle touch internally whose radii are 2 and 3cm. Find the Length of Largest Chord of Largest circle which is the tangent of Smaller.



$BC_2 = 6 - 5 = 1cm$

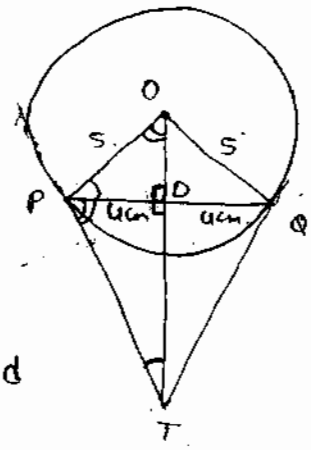
$BC_2 = 1cm$

$\therefore AB = \sqrt{AC_2^2 - BC_2^2} = \sqrt{3^2 - 1^2} = \sqrt{8} = 2\sqrt{2}$

Length of $AC = 2 \times 2\sqrt{2} = 4\sqrt{2}$ cm.

$\boxed{\text{Length of Chord} = 4\sqrt{2} \text{ cm.}}$

PQ is a Chord of Circle whose Length is 8cm. Radius of Circle is 5cm the tangent from the Point P and Q intersect to each other at Point T find the Length of PT

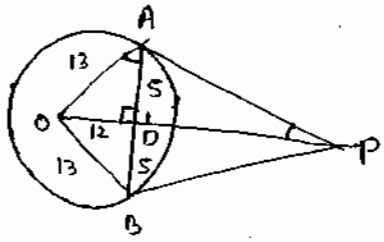


$\Delta PDT \sim \Delta PDO$

$\frac{PT}{5} = \frac{4}{3}$ $\boxed{PT = \frac{20}{3} cm}$

AB is a Chord of a Circle whose Length is 10cm and Radius of Circle is 13cm. The tangent from A and B intersect to each other at Point P find the Length of Tangent AP.

$OD = \sqrt{13^2 - 5^2} = 12cm.$



from ΔADP and ΔAOD

$\frac{AP}{AO} = \frac{AD}{OD} \Rightarrow \frac{AP}{13} = \frac{5}{12}$

$\boxed{AP = \frac{65}{12} cm}$

O is the Centre of a Circle whose Radius is 5cm. AB is a Chord whose length is 6cm. The tangent from A and B intersect to each other at Point P. the Line AB and OP intersect to each other at Point C Find the Length of PC.

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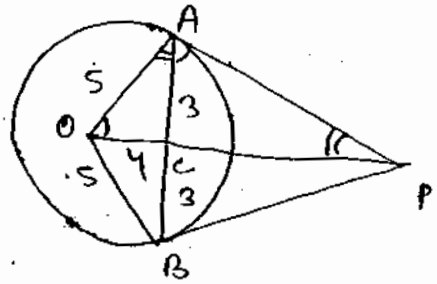
Length of $OC = \sqrt{5^2 - 3^2} = 4$



from $\Delta ACP \sim \Delta ACO$

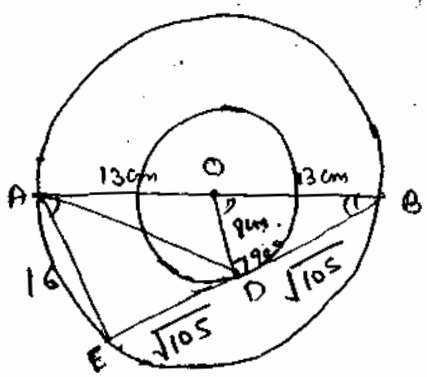
$$\frac{PC}{AC} = \frac{AC}{OC}$$

$$\frac{PC}{3} = \frac{3}{4} \qquad \boxed{PC = \frac{9}{4}\text{ cm}}$$

Radii of two concentric circle are 13 and 8 cm. AB is the Diameter of largest circle. BD is the tangent of smallest circle which touch the circle at point D. Find the length of AD..

$OD = 8\text{cm}$ [Radius of Small Circle.]

$$\therefore BD = \sqrt{13^2 - 8^2} = \sqrt{105}$$



Also $ED = \sqrt{105}$

$\Delta BOD \sim \Delta BAE$

$$\therefore \frac{OD}{AE} = \frac{OB}{AB}$$

$$\frac{8}{AE} = \frac{\cancel{13}}{\cancel{26}\,2} \qquad \boxed{AE = 16\text{cm}}$$

Now ΔAED is a Right Angle Δ.

$$\therefore AD = \sqrt{16^2 + (\sqrt{105})^2} = \sqrt{256 + 105} = \sqrt{361}$$

$$\boxed{AD = 19\text{cm}}$$

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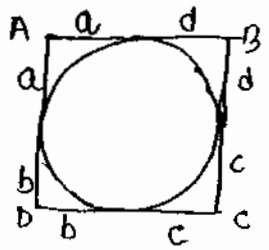
Geometry

NT If a circle touch all the side AB, BC, CD and DA of a quadrilateral ABCD then $\boxed{AB+CD = AD+BC}$

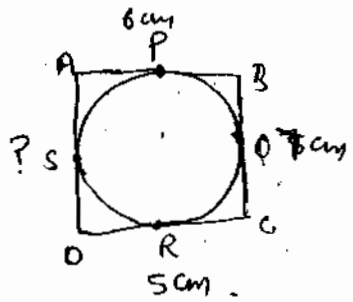


$AB+CD = a+d+c+b \Rightarrow a+b+c+d$

$AD+BC = a+b+d+c \Rightarrow a+b+c+d$

$\therefore \boxed{AB+CD = AD+BC}$

→ If a circle touch the side AB, CB, CD and DA of a quadrilateral ABCD at the point respectively P, Q, R and S if AB = 6cm, BC = 7cm, CD = 5cm find the Length of AD.

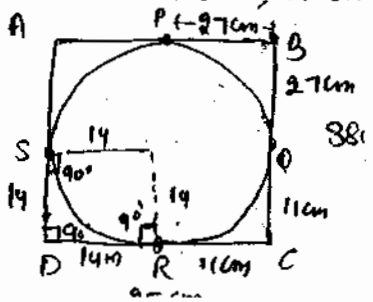


Now $AB+CD = BC+AD$

$\Rightarrow 6+5 = 7+AD$

$\boxed{AD = 4cm}$

A circle touch the side AB, BC, CD, DA of a quadrilateral ABCD at the Point respectively P, Q, R and S if $\angle D = 90°$ if BC = 38cm, CD = 25cm, BP = 27 find the Radius of circle.



$BP = 27cm$, $\therefore BQ = BP = 27cm$

Now $QC = BC - BQ = 38 - 27 = 11cm$.

$QC = RC = 11cm$

$\therefore DR = 25 - 11 = 14cm$

$\angle D = 90°$ or $\angle S = \angle R = 90°$

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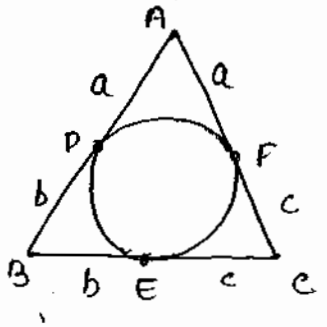
$\therefore$ ABCD is a square whose all side are 14cm

$\boxed{\therefore \text{Radius of Circle} = 14\text{cm}}$

A circle is inside a ΔABC ~~such that~~ which touch the side AB, BC, CA at the point respectively D, E, F. If AB = 12cm, BC = 8cm, CA = 10cm, Find the Length of AD, BE, CF



Now $AD = AF = a$ $\quad BD = BE = b$, $\quad CE = CF = c$

$\therefore AB = a+b = 12$ —— I

$BC = b+c = 8$ —— II

$CA = a+c = 10$ —— III

from I and II (I − II)

$a - c = 4$ —— IV

or $a + c = 10$

$a = 7\text{cm}$

$a+b = 12$

$\therefore b = 12 - 7$

$b = 5\text{ cm}$

$a + c = 10$

$c = 10 - 7$

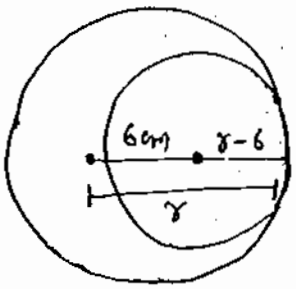
$c = 3\text{cm}$

$\boxed{\therefore AD = 7\text{cm}, \quad BE = 5\text{cm}, \quad CF = 3\text{cm}}$

Two circle touch Internally. The Sum of Area of Two Circle is $116\pi\,\text{cm}^2$ and Distance between their Centre is 6cm find the Radius of Big Circle.

Let Radius of Big circle = $r$cm

$\therefore$ Radius of Small Circle = $r - 6$ cm.



Now

$\pi r^2 + \pi(r-6)^2 = 116\pi$

$\therefore r^2 + (r-6)^2 = 116$

$r^2 + r^2 + 36 - 12r = 116$

$2r^2 - 12r - 80 = 0$

$r^2 - 6r - 40 = 0$

$r^2 - 10r + 4r - 40 = 0 \Rightarrow (r-10)(r+4) = 0$

$r = 10, \; r \neq -4$

$\therefore \boxed{\text{Radius of Big Circle} = 10\text{ cm}}$

Or from option (i) 10 (ii) 8 (iii) 6 (iv) 4

$\therefore$ Radius of Big circle = 10cm

$\therefore$ Radius of small Circle = 10 − 6 = 4cm.

Area = $\pi(10)^2 + \pi(4)^2 \Rightarrow 100\pi + 16\pi = 116\pi\,\text{cm}^2$

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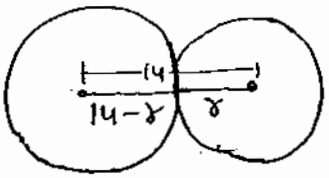
Two circle touch externally the sum of their Area is $130\pi\ m^2$. Distan between their centre is 14cm find the Radius of small circle.

Let Radius of Small Circle = r cm

∴ Radius of Big Circle = $14-r$ cm



$$\therefore \pi r^2 + \pi(14-r)^2 = 130\pi$$

$$r^2 + (14-r)^2 = 130$$

$$r^2 + 196 + r^2 - 28r = 130$$

$$2r^2 - 28r + 66 = 0$$

$$r^2 - 14r + 33 = 0$$

$$r^2 - 11r - 3r + 33 = 0 \Rightarrow (r-11)(r-3) = 0$$

$$\therefore r = 11,\ r = 3$$

∴ $\boxed{\text{Radius of Small Circle} = 3\text{ cm}}$

from option :→ (iii) 3

Let Radius of small Circle = 3cm

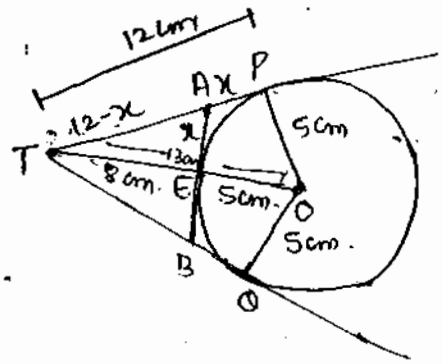
∴ Radius of Big Circle = 14-3 = 11cm

∴ Area = $\pi(3)^2 + \pi(11)^2 \Rightarrow 9\pi + 121\pi = 130\pi$

O is the Centre of a Circle whose Radius is 5cm. PT and QT are the two Tangent which touch the Circle at Point P and Q respectively. If Length of OT = 13cm, and the line OT intersect the circle at Point E. If AB is the Another tangent which touch the Circle at E and A or B are the Point on the line PT and QT respectively find the Length of Tangent AB.

∴ OT = 13 cm.

∴ ET = 13 − 5 = 8cm.



from △OTP

$$\Rightarrow PT = \sqrt{OT^2 - OP^2} = \sqrt{13^2 - 5^2}$$

$$= \sqrt{144} = 12\text{ cm}.$$

Let AE = x ∴ AE = AP = x

Now from △AET

$$x^2 = (12-x)^2 - 8^2$$

$$x^2 = 144 + x^2 - 24x - 64$$

$$24x = 80$$

$$x = \frac{80}{24} = \frac{10}{3}$$

Now $AB = 2x = 2 \times \frac{10}{3} = \frac{20}{3}$ cm

∴ Length of AB = 20/3 cm

Or △AET ~ △POT

[∵ ∠T = Common, ∠AET = ∠OPT = 90°]

$$\therefore \frac{AE}{OP} = \frac{ET}{PT}$$

$$\frac{AE}{5} = \frac{8}{12} \quad AE = \frac{10}{3}\text{ cm}$$

$$\boxed{AB = 2 \times \frac{10}{3} = \frac{20}{3}\text{ cm}}$$

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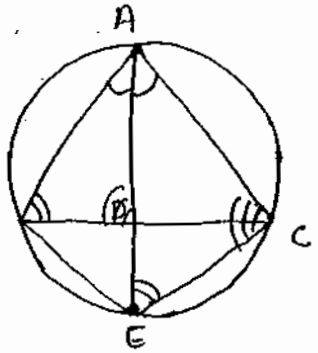
In ΔABC AD is the Interior Bisect of $\angle A$. The Produce Line AD intersect the circum circle of ΔABC at the Point E. Find.

(i) $AB \cdot AC$ (ii) $AB \cdot AC + DE \cdot AE$.



(i) from ΔABD and ΔACE

$\angle BAE = \angle CAE$ [AD is the interior Bisect of $\angle A$]

$\therefore \angle ABD = \angle AEC$ [AC Chord द्वारा बने Angle]

$\therefore \Delta ABD \sim \Delta ACE$.

$\therefore \frac{AB}{AE} = \frac{AD}{AC}$ $\therefore$ $\boxed{AB \cdot AC = AE \cdot AD}$

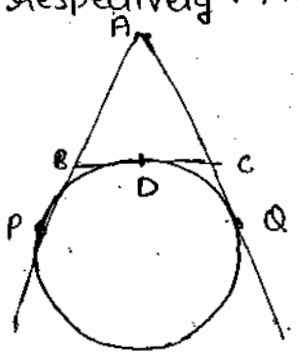
(ii) $AB \cdot AC + DE \cdot AE$

$\Rightarrow AE \cdot AD + DE \cdot AE$ [from I $AB \cdot AC = AE \cdot AD$]

$\Rightarrow AE \cdot (AD + DE) \Rightarrow AE \cdot AE = AE^2$

$\therefore$ $\boxed{AB \cdot AC + DE \cdot AE = AE^2}$

N.T A Circle touch the Side BC of a ΔABC at the Point D. The Produce Line AB and AC touch the Circle at Point P and Q respectively. Find the Length of tangent AP.



Now $AP + AQ = AB + BP + AC + CQ$

$\Rightarrow AP + AP = AB + AC + BD + CD$ [$\because AP = AQ$, $BP = BD$, $CQ = CD$]

$\therefore 2AP = AB + AC + \underbrace{BD + CD}_{BC}$

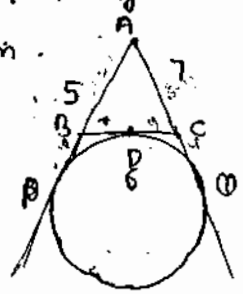
$\Rightarrow 2AP = AB + AC + BC$ [$BD + CD = BC$]

$\Rightarrow$ $\boxed{2AP = \text{Perimeter of } \Delta ABC}$

$\therefore AP = \frac{\text{Primeter of } \Delta ABC}{2}$

[$\because$ Primeter of $\Delta ABC = AB + AC + BC$]

A Circle touch the Side BC of a ΔABC at the Point D. The Produce Line AB and AC touch the Circle at Point P and Q respectively. Find the Length of tangent AP if Side of Δ are 5, 6 and 7 cm.



Using $AP = \frac{\text{Perimeter of } \Delta ABC}{2}$

$= \frac{5+6+7}{2} = \frac{18}{2} = 9cm$

$\boxed{AP = 9cm}$

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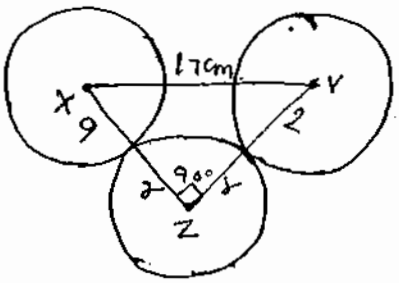
Two Circle whose Centre are x and y and Radius are 9 and 2 cm and Distance between their Centre is 17 cm. A third Circle whose centre is z and Radius r are touch the Given Circle if $\angle xzy = 90^\circ$ find r.

ΔXZY is a Right Angle Δ



in which

$$(r+9)^2 + (r+2)^2 = 17^2$$

$$r^2 + 81 + 18r + r^2 + 4 + 4r = 289$$

$$2r^2 + 22r - 204 = 0$$

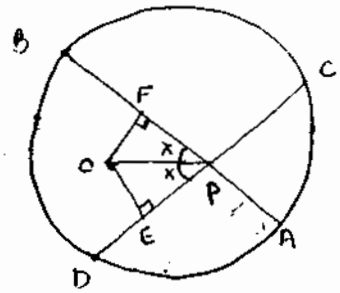
$$r^2 + 11r - 102 = 0$$

$$r^2 + 17r - 6r - 102 = 0$$

$$(r+17)(r-6) = 0 \qquad r = -17, \quad r = 6$$

$\boxed{\therefore \text{ Radius of Circle whose Centre is } z = 6 \text{ cm}}$

AB and CD are the two Chords of a Circle to Intersect to each other at the Point P and O is the Centre of Circle OP is Bisector of $\angle BPD$ if the Length of CD = 12 cm. find the Length of AB.



$\Delta EOP \sim \Delta FPO$

and OP is Common

$\therefore \Delta EOP \simeq FOP$

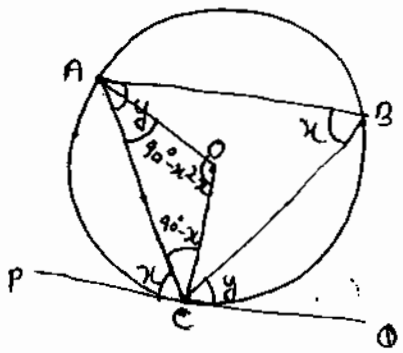
$\therefore OE = OF$.

→ Centre से जीवा पर डाला गया ⊥ यदि बराबर हो तो वे दोनो जीवा भी बराबर होती है

$\therefore \boxed{AB = CD = 12 \text{ cm}}$.

N.T.

→ ΔABC is a Triangle in Circle. whose Vertex C act as tangent of Circle then Prove



$\angle BAC = \angle BCQ$

$\angle ABC = \angle ACP$

Now $\angle OCP = 90^\circ$ [Radius Tangent पर ⊥ होती है]

$\therefore \angle OCA = 90^\circ - x$

Now $OA = OC$ (Radius of Circle

$\therefore \angle OCA = \angle OAC = 90^\circ - x$

$\therefore \angle AOC = 180^\circ - (90^\circ - x + 90^\circ - x) = 2x$

$\therefore \angle ABC = x$ [Chord AC द्वारा बना Angle]

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