Theory of Machines

References:

1- Mechanics of Machines: Elementary theory and examples. By: J. Hannah and R.C. Stephens .

2- Mechanics of Machines: Advanced theory and examples. By: J. Hannah and R.C. Stephens .

3- Theory of Machine. By: R.S. Khurmi and J. K. Gupta.

4- Kinematics and Dynamics of Machines. By: G.H. Martin.

Topics to be considered:

1. Kinematics and Kinetics of Mechanisms.
2. Balancing of Machines.
   a. For Rotating masses.
   b. For Reciprocating masses.
3. Cams and Followers.
4. Flywheel.
5. Gears and Gears trains.
6. Friction Clutches.
7. Belt drives and Band Brakes.
8. Power Screw.
9. Universal Joint.
10. Gyroscope.
11. Speed Governors.
12. Steering mechanism.

Chapter One

Kinematics and Kinetics of Mechanisms

Introduction:

Theory of Machines: may be defined as that branch of engineering science, which deals with the study of relative motion between the various parts of machine, and forces which act on them. The knowledge of this subject is very essential for an engineer in designing the various parts of a machine.

Sub- divisions of theory of Machines:

They Theory of Machines may be sub- divided into the following four branches:

1. **Kinematics:** is that branch of theory of machines which is responsible to study the motion of bodies without reference to the forces which are cause this motion, i.e it's relate the motion variables (displacement, velocity, acceleration) with the time.

2. **Kinetics:** is that branch of theory of machines which is responsible to relate the action of forces on bodies to their resulting motion.

3. **Dynamics:** is that branch of theory of machines which deals with the forces and their effects, while acting upon the machine parts in motion.

4. **Statics:** is that branch of theory of machines which deals with the forces and their effects, while the machine parts are rest.

There are some definitions which are concerned with this subject, must be known:

Mechanism: is a combination of rigid bodies which are formed and connected together by some means, so that they are moved to perform some functions, such as the crank- connecting rod mechanism of the I.C. engines, steering mechanisms of automobiles……. etc.

The analysis of mechanisms is a part of machine design which is concerned with the kinematics and kinetics of mechanisms (or the dynamics of mechanisms).

Rigid Body: is that body whose changes in shape are negligible compared with its overall dimensions or with the changes in position of the body as a whole, such as rigid link, rigid disc…..etc.
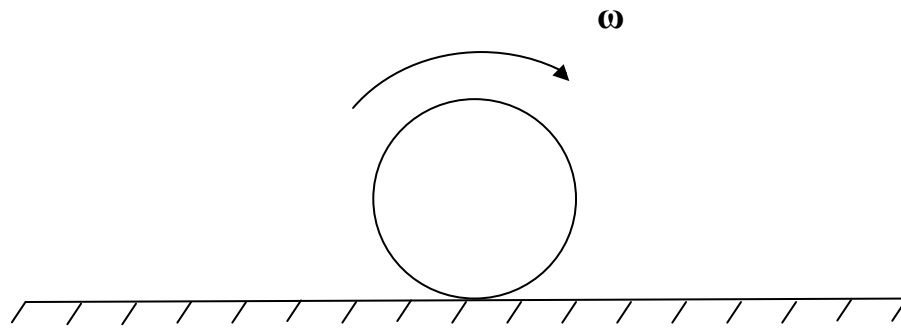
Links: are rigid bodies each having hinged holes or slot to be connected together by some means to constitute a mechanism which able to transmit motion or forces to some another locations.

Absolute motion: the motion of body in relative to another body which is at rest or to a fixed point located on this body.
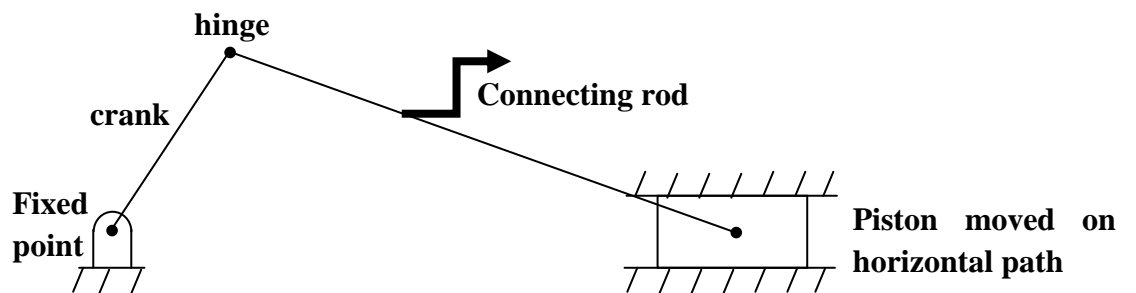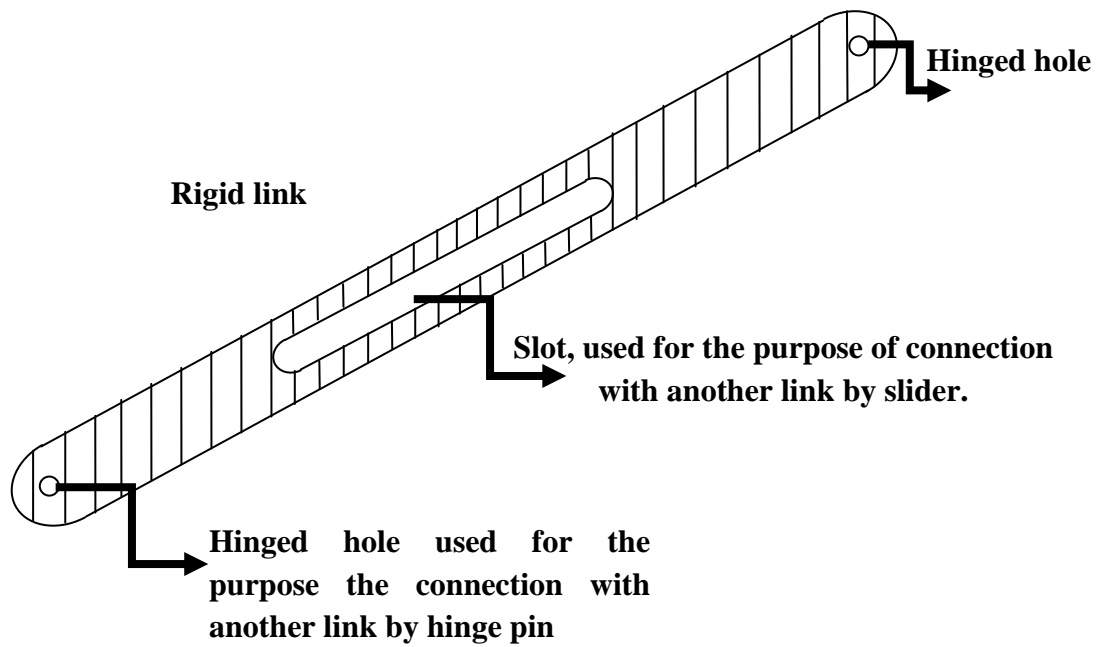
Relative motion: the motion of body in relative to another moved body.

Scalar quantities: are those quantities which have magnitude only e.g. mass, time, volume, density etc.

Vector quantities: are those quantities which have magnitude as well as direction e.g. velocity, acceleration, force etc.

ω

Disc in motion (rigid body)

Rigid link

Hinged hole

Slot, used for the purpose of connection with another link by slider.

Hinged hole used for the purpose the connection with another link by hinge pin

hinge

crank

Connecting rod

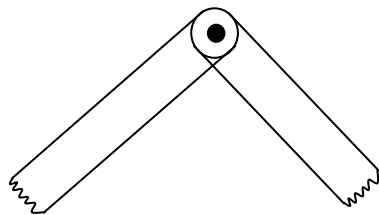**Fixed point**

Piston moved on horizontal path

Crank-Connecting rod mechanisms

Part one: Kinematics of Mechanisms:
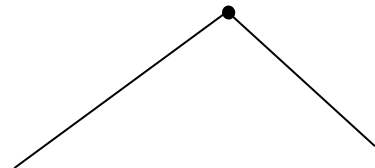
1- The connection of mechanism parts:

The mechanism is a combination of rigid bodies which are connected together using different methods:

1-1: Hinged part:

The hinge connection may be used to connect the links together or connect a link to a fixed point, piston, disc ….. etc, the connection is achieved using pin, which is pass through the hinge holes.
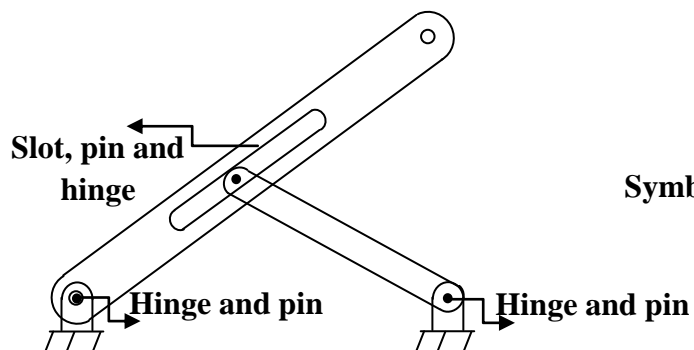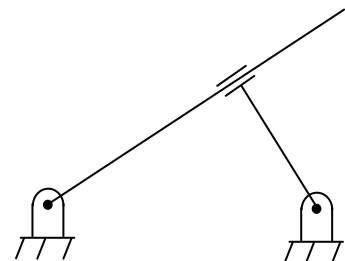

Symbled by

1-2: Sliding Parts:
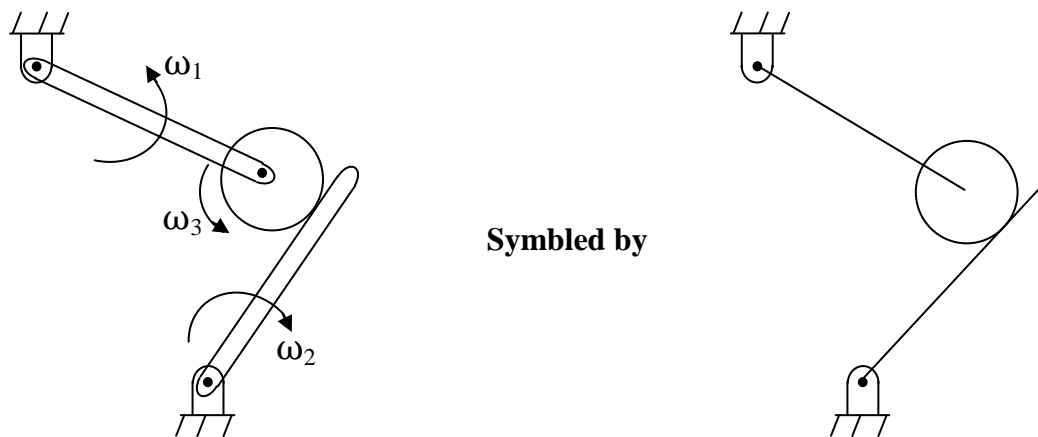
The sliding connection may be used to connect two links rotate about fixed points by means of slot, pin and hinge.

Slot, pin and hinge

Hinge and pin   **Hinge and pin**

Symbled by

1-3: <u>Rolling without slipping parts:</u>


**Symbled by**

2- Translated bodies:

There are some bodies in the mechanism which are constrained to move in translation manner, such as the piston of crank- connecting rod mechanism, the body is used to be in translation motion, if any line remain in some configuration during motion; then all the points have the same absolute velocity and acceleration.

Velocity diagram:

- ∵ the motion is absolute, then select any fixed point such as o be as a reference point (i.e point of zero velocity).
- Draw the path of translation.
- If v_B is known, select a scale factor to draw the velocity diagram (denoted by SFv)

$$SFv = \frac{draw\ value\ in\ mm}{actual\ value\ of\ velocity\ in\ (m/s)} = \frac{ob}{v_B}$$

The draw a line ob=(v_B)(SFv) in direction of v_B parallel to the path of translation.

A

ω

a_b v_b

B

D

θ

O

Path of translation of B

θ

b, d

Velocity dig.

O
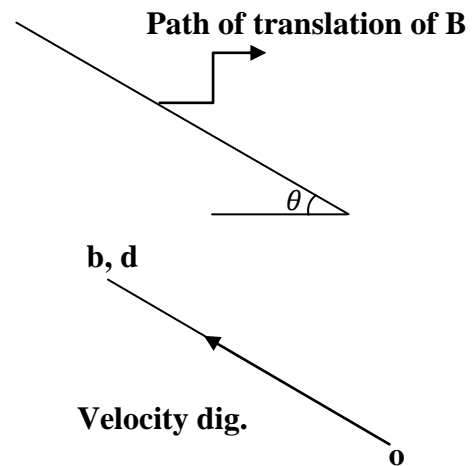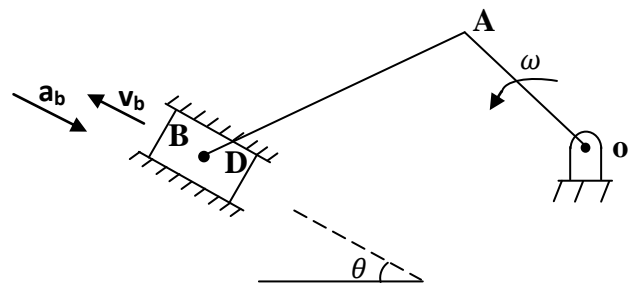
Then all points on the piston have the same velocity, such as point D, i.e on the velocity diagram, the point d coincide on the point b.

Acceleration diagram:

- ∵ the motion is absolute, then select any fixed point such as o be as a reference point (i.e point of zero acceleration).
- Draw the path of translation.
- If a_B is known, select a scale factor to draw the acceleration diagram (denoted by SFa)

$$SFa = \frac{draw\ value\ in\ mm}{actual\ value\ of\ acceleration\ in\ (m/s^2)} = \frac{ob}{a_B}$$

In which ob=(a_B)(SFa).

Then all points on the piston have the same acceleration value.

Note: the base (ref.) point o of v_o =0, a_o=0.

Path of translation of B

Acceleration dig.

b, d

o
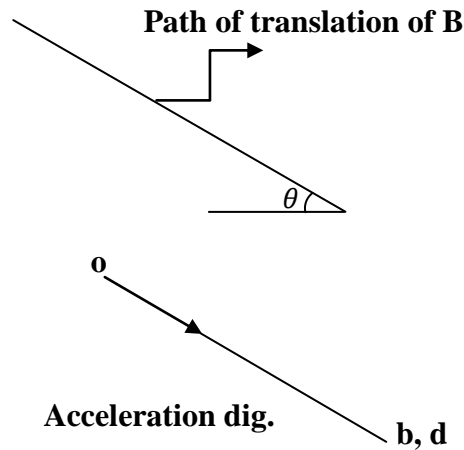
Dynamic review:

Translation motion can by treatment by the dynamics of particles i.e body B can be treatment as a particle moved on straight or curved path.

Then: $v = \dfrac{ds}{dt}$, $a = \dfrac{dv}{dt} = \dfrac{d^2 s}{dt^2}$ $\Rightarrow vdv = ads$.

Where:

s: displacement ,v: velocity , a: acceleration

$v_2 - v_1 = \int_{t_1}^{t_2} a\, dt$, if a uniform $\qquad v_2 - v_1 = a(t_2 - t_1)$.

$\dfrac{1}{2}(v_2^2 - v_1^2) = \int_{s_1}^{s_2} a\, ds$, if a uniform $\qquad v_2^2 - v_1^2 = 2a(s_2 - s_1)$.

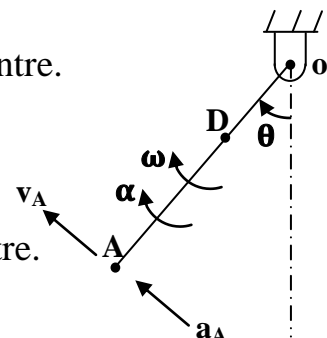$\therefore \; s_2 - s_1 = v_1(t_2 - t_1) + (t_2^2 - t_1^2)$.

3-Bodies rotate about fixed point:

Consider the link shown which is rotate about the fixed point o, the motion of this link can be analyzed using the principle of absolute motion as follow:

If θ: angular displacement about fixed rotation centre.

ω: angular velocity about fixed rotation centre.

α: angular acceleration about fixed rotation centre.

o

D

θ

ω

v_A

α

A

a_A

Then:

$$\omega = \frac{d\theta}{dt} \quad , \quad \alpha = \frac{d\omega}{dt} \quad , \quad \Rightarrow \omega d\omega = \alpha d\theta$$

$$\therefore \omega_2 - \omega_1 = \int_{t_2}^{t_1} \alpha(t) dt \quad \text{but if } \alpha \text{ is uniform} \Rightarrow$$

$$\omega_2 - \omega_1 = \alpha(t_2 - t_1)$$

and $\omega_2^2 - \omega_1^2 = \int_{\theta_1}^{\theta_2} \alpha(\theta) d\theta$ but if α is uniform $\Rightarrow$

$$\omega_2^2 - \omega_1^2 = 2\alpha(\theta_2 - \theta_1)$$

$$\therefore \theta_2 - \theta_1 = 2v_1(t_2 - t_1) + \frac{1}{2}\alpha(t_2^2 - t_1^2).$$

Velocity diagram:

In order to analyze the velocity of any point we follow with one of following methods:

1- If ω is given:-

- Draw the link by SFp (scale factor for position), SFp=$\frac{drawn\ length\ of\ link}{actual\ length\ of\ link}$

- $v_A = (\omega)(oA)$.

- Select SFv $= \frac{oa}{v_A}$, then select a reference point of zero velocity, such as o.

- Draw from o, a line of length $(oa) = v_A.SFv \perp oA$ in direction of ω.

- To find the velocity of any point located on the link, such as D, specify point d on oa such that $\frac{od}{oa} = \frac{oD}{oA}$ $\Rightarrow$ od=$\left(\frac{oD}{oA}\right)oa$.

Then:- $v_D = \frac{od}{SFv}$.

2- If v_A is given:-

- Select SFv, specify reference point of zero velocity.

- Draw oa of length (v_A)(SFv) in the same direction given.

- To find value and direction of ω:
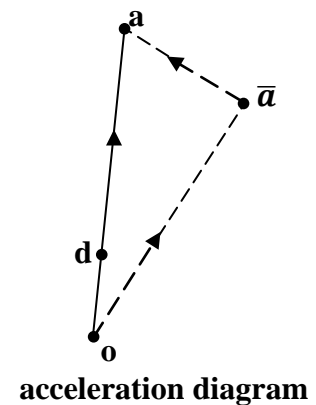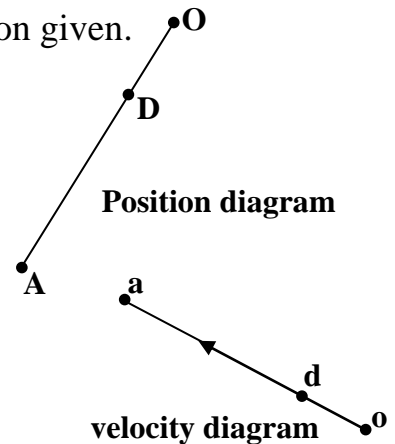
Value of $\omega = \dfrac{v_A}{oA}$ ⤵ .

Acceleration diagram:-

Also we have two method:-

1- If α is given:-

$a_{An}=(oA)\,\omega^2$ ↗ normal component of acceleration of

         A relative to   rotation centre.

$a_{At}=(oA)\,\alpha$ ↖ normal component of acceleration of

         A relative to rotation centre.

Position diagram

velocity diagram

acceleration diagram

- Select a reference point of zero acceleration (point o)

- Select SFa$= \dfrac{\text{drawn value of } a_{An} \ \text{ or } a_{At}}{\text{actual value of } a_{An} \ \text{ or } a_{At}}$. depend on which is greater a_{An} or a_{At}.

- Start from o to draw $o\bar{a} \,//$ OA directed into the rotation centre, by value of $o\bar{a} = a_{An}$. SFa.

- From point $\bar{a}$ draw $\bar{a}\,a \perp$ OA in direction of α by value $\bar{a}\,a=a_{At}$.SFa.

- Finally connect oa to represent the absolute value of acceleration of point A. $\Rightarrow a_A = a_{Ao} = \dfrac{oa}{SFa}$.

       To find the acceleration of any point located on the link, such as point D. specify d on oa such that $\dfrac{od}{oa} = \dfrac{OD}{OA}$

$$\Rightarrow od = \left(\dfrac{OD}{OA}\right) oa.$$

2- If a_A is given as a value and direction. (absolute acceleration of point A).

- Find $a_{An} = (OA)(\omega^2) = \frac{v_A^2}{OA}$ ↗.

- Select $SFa = \frac{draw\ value\ of\ a_A}{actual\ value\ of\ a_A}$, select refrence point of zero acceleration. (point O).

- Start from O, draw two lines.

❖ First line $o\bar{a} = a_{An} . SFa$ // OA directed in to point O.

❖ Second line $oa = a_A . SFa$ in direction of a_A (given).

Then connect $a\bar{a}$ to represent the drawn tangential component of acceleration of A.

$\Rightarrow a_{At} = \frac{a\bar{a}}{SFa}$

$\alpha = \frac{a_{At}}{OA}$ ↘.

acceleration diagram
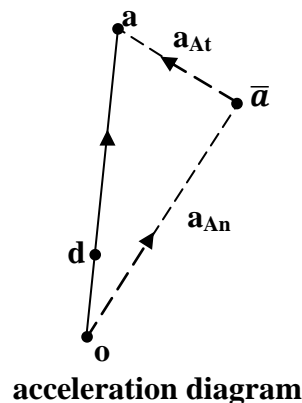
4-Bodies under general plane motion:-

If a body under general plane motion, then it's motion can be analyzed using the principle of relative motion.

The motion of any point can be discretized into translation and rotation, if consider the link shown under general plane motion, the ends , B of absolute velocities v_A, v_B, and absolute accelerations a_A, a_B then:-

$\vec{V}_A = \vec{V}_B + \vec{V}_{AB}$ or $\vec{V}_B = \vec{V}_A + \vec{V}_{BA}$

$\vec{a}_A = \vec{a}_B + \vec{a}_{AB}$ or $\vec{a}_B = \vec{a}_A + \vec{a}_{BA}$
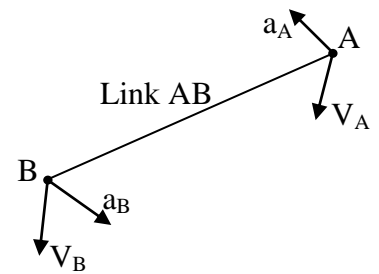
Where:-

$\vec{V}_{BA}$ is the relative velocity of B w.r.t A.

$\vec{V}_{AB}$ is the relative velocity of A w.r.t B.

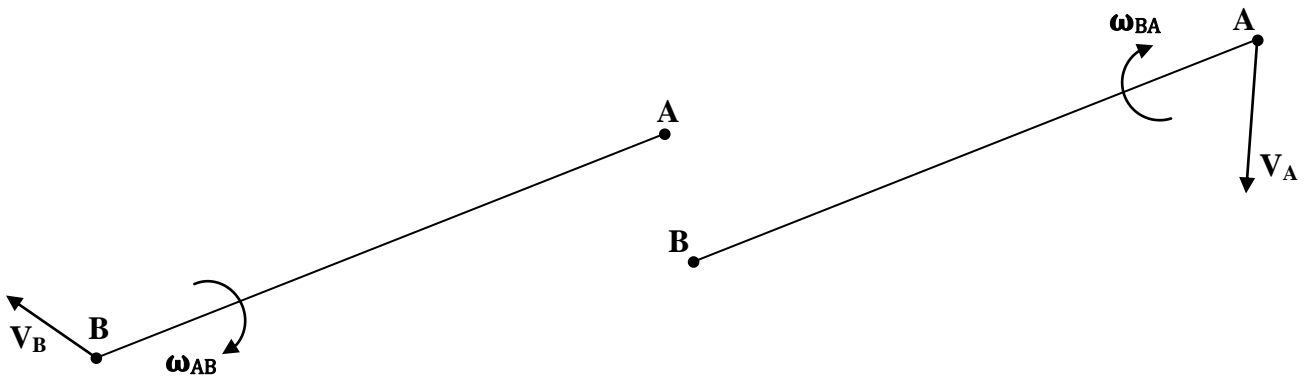$\vec{a}_{BA}$ is the relative acceleration of B w.r.t A.

$\vec{a}_{AB}$ is the relative acceleration of A w.r.t B.

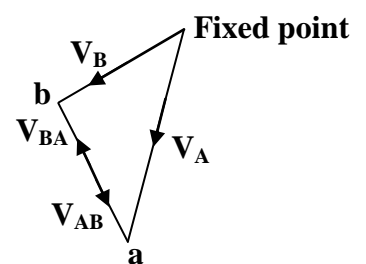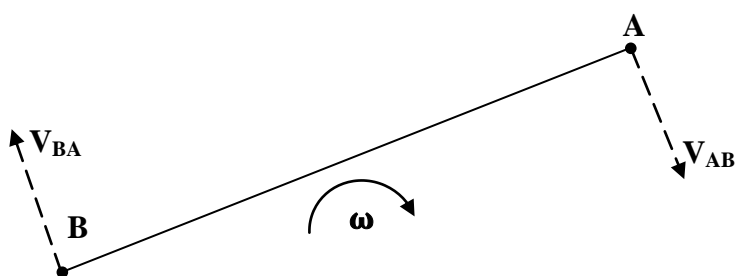i.e the state of velocity can be replaced by one of the following:-

$$\left.\begin{array}{l} \vec{V}_A = \vec{V}_B + (\vec{w}) \times (\overrightarrow{AB}) \\ \vec{V}_B = \vec{V}_A + (\vec{w}) \times (\overrightarrow{BA}) \end{array}\right\} \quad \text{vector notation.}$$
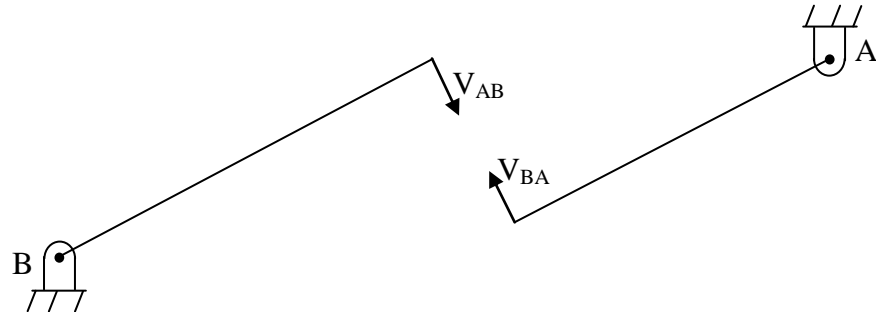
$\because V_{AB}$ and $V_{BA} \perp AB.$

$$\because \quad \omega_{AB} = \frac{V_{AB}}{AB} = \frac{V_{BA}}{AB}$$

To specify direction of ω: $\implies$

V_{AB} : mean that B is a fixed rotation a center, and A moved a round A.

V_{BA} : mean that A is a fixed rotation a center, and A moved a round B.

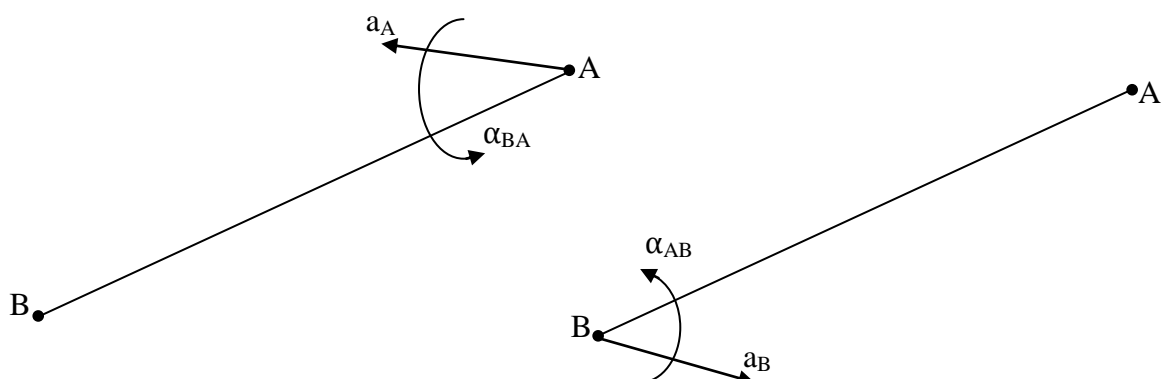And the state of acceleration can be represented by one of the following:-

$$\left.\begin{array}{l}\overrightarrow{a_A} = \overrightarrow{a_B} + \vec{\omega} \times \vec{\omega} \times \overrightarrow{AB} + \vec{\alpha} \times \overrightarrow{AB} \\ \overrightarrow{a_B} = \overrightarrow{a_A} + \vec{\omega} \times \vec{\omega} \times \overrightarrow{BA} + \vec{\alpha} \times \overrightarrow{BA}\end{array}\right\}$$ vector notation.

$\therefore a_{AB}$ contain two

comps.$\begin{cases} normal\ component = \omega^2 AB .//AB\ into B. \\ tangential\ component = \alpha\ AB . AB in\ direction\ of\ \alpha. \end{cases}$

a_{BA} contain two

comps.$\begin{cases} normal\ component = \omega^2 BA .//BA\ into A. \\ tangential\ component = \alpha\ AB . AB in\ direction\ of\ \alpha. \end{cases}$

Velocity diagram:

Consider the shown link under general plane motion, to specify the velocity of any point, it's required one of following:-
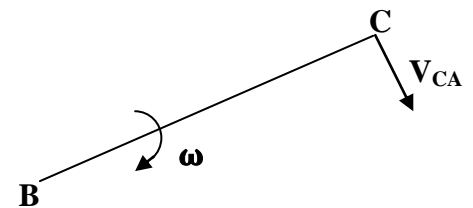
1-   * Absolute velocity of any point (value **and** direction).

*Absolute velocity of other point (value **or** direction).

2-   *Absolute velocity of any point (value **and** direction).

*Angular velocity of the link which is the same for all points.
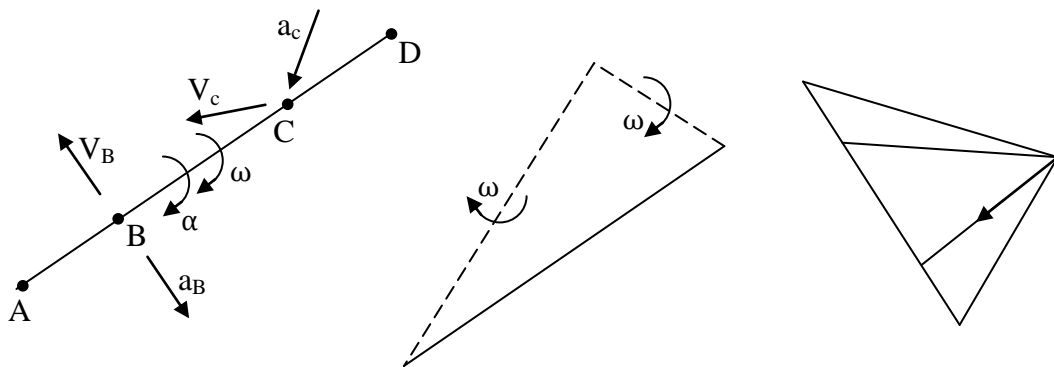
Steps:

❖ Draw the link position by scale (SFp).

❖ If V_B is known (value and direction), then select the scale factor of velocity diagram (SFv).

• Specify the point of zero velocity. (point O).

• $SFv = \dfrac{ob}{V_B}$ $\Rightarrow$ ob known in mm.

• Draw ob in direction of V_B.

- To continue we require other direction of absolute velocity of other point or ω.
- If the direction of absolute velocity of point c is known then:
  - ➢ Star from o to draw line // direction of V_c.
  - ➢ Star from b to draw line ⊥ link to be V_{bc} or V_{cb}, which is intersected with the line // direction of V_c at c.
- ❖ If ω is known:
  - $V_{BC} = \omega.(BC)$, then $bc = SFv.V_{BC}$.
  - Draw bc from b ⊥ link.
  - Draw line between o and c produce oc.
- ❖ To find V_A, V_D:-
  - Specify ba such that $\dfrac{cd}{bc} = \dfrac{CD}{BC}$.
  - Measure od $\Longrightarrow$ V_D=ad / SFv.
- ❖ To find ω value and direction, if unknown measure bc$\Longrightarrow$ $V_{BC} = \dfrac{bc}{SFv}$.

$$\Longrightarrow \omega = \frac{V_{BC}}{BC} . \quad \circlearrowleft$$

Acceleration diagram:

To draw the acceleration diagram it's required one of following:-

1- ω or V_{BC}.

❖ Absolute acceleration of any point (value **and** direction).

❖ Absolute acceleration of other point (value **or** direction).

2- ω or V_{BC}.

❖ Absolute acceleration of any point (value **and** direction).

❖ Angular acceleration of the link.

Steps:

- Find $a_{BCn} = \omega^2 BC = \dfrac{V_{BC}^2}{BC}$.

- If a_c is known (value and direction), V_B is known (direction).

- Select SFa $\Rightarrow oc = SFa.ac$, $c\grave{b} = SFa.a_{BCn}$.

- Start from point of zero acceleration such as o.

- Draw oc in direction of a_c.

- From c draw $c\grave{b}$ // link directed into point c (on the link).

- From o draw a line in direction of a_B, and from $\grave{b}$ draw a line
  $\perp$ link (to be a_{BCt}), they are intersected at b.

- If α is known (value and direction).

- Find $a_{BCt} = \alpha.BC$ then $\grave{b}b = SFa.a_{BCt}$.

- Start from $\grave{b}$ to draw $\grave{b}b \perp$ link.

- Connect ob

❖ Find a_A , a_D.

- Specify bc such that $\dfrac{ba}{cb} = \dfrac{BA}{CB}$.

- Measure oa $\Rightarrow a_A = \dfrac{oa}{SFa}$.

- Specify cd such that $\dfrac{ba}{cb} = \dfrac{BA}{CB}$.

- Measure $od \Rightarrow a_D = \dfrac{cd}{SFa}$.

- To find α (value and direction) in unknown:-

- Measure b$\grave{b}$ $\Rightarrow a_{BCt} = \dfrac{b\grave{b}}{SFa}$

- $\Rightarrow \alpha = \dfrac{a_{BCt}}{BC}$ ↺
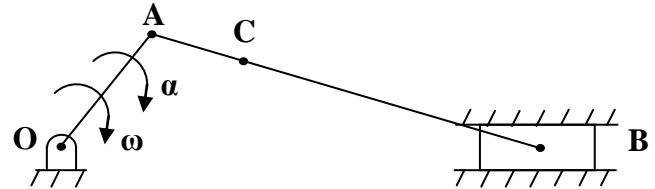

Acceleration diagram
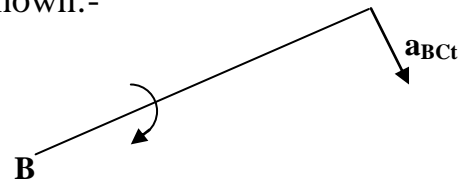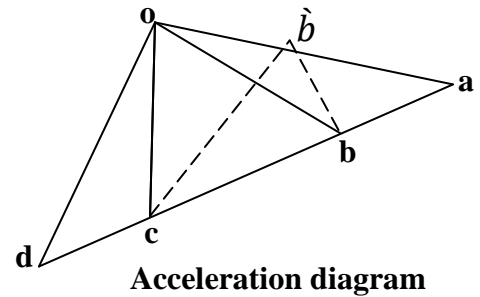
Note:- α is the same for all points of the link.

Example(1):- For the crank- connecting rod mechanism shown: OA= 10cm , AB= 30 cm, AC= 10 cm, it's single degree of freedom, coordinate is θ. If ω_{OA}=30 rad/sec, α_{OA}=100 rad/s². Find ω_{AB}, α_{AB}, V$_B$, a$_B$, a$_C$, V$_C$ at θ=30°.
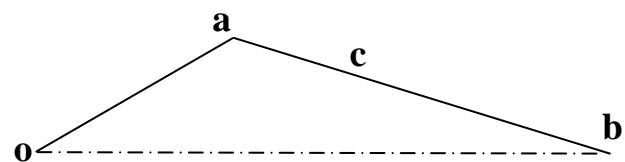
$SFP = \dfrac{2\ cm}{10\ cm} = 0.2\ cm/cm$

$oa = 2cm, ab = 6cm, ac = 2\ cm.$

to draw velocity diagram.

$V_A = (OA)(\omega_{OA}) = (0.1)(30) = 3\ m/s.$

$SFV = \dfrac{3\ cm}{3\ m/s} = 1\ cm/\ m/s$

O point of zero velocity.

$oa = (1)(3) = 3\ cm$ ↘.

Mechanism drawn by scale 0.2 cm/cm

to specify ac

$\Rightarrow ac = \left(\dfrac{AC}{AB}\right)(ab) = 8.7\ mm.$

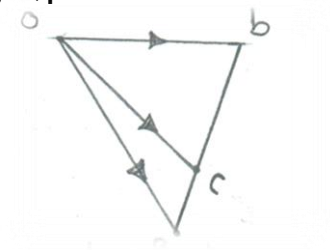$measure\ ab = 2.6\ cm \Rightarrow V_{AB} = \dfrac{2.6}{1} = 2.6\ m/sec.$

$measure\ oc = 2.45\ cm \Rightarrow V_c = 2.45\ m/s$ ↘.

$to\ find\ V_B:\ measure\ ob = 2\ cm \Rightarrow V_B = 2\ m/s$ →.

$To\ find\ \omega_{AB}:$

$\omega_{AB} = \dfrac{V_{AB}}{AB} = \dfrac{2.6}{0.3} = 8.7\ rad/s$

to draw acceleration diagram:

Velocity drawn by scale 1 cm/m/s

$a_{An} = (OA)(\omega_{OA}^2) = (0.1)(30)^2 = 90\ m/s^2$

$$a_{At} = (OA)(\alpha_{OA}) = (0.1)(100) = 10 \ m/s^2$$

$$a_{ABn} = (AB)(\omega_{AB}^2) = (0.3)(8.7)^2 = 22.7 \ m/s^2$$

$$SFa = \frac{5 \ cm}{90 m/s^2} = 0.0556 \ cm/m/s^2$$

$$\Rightarrow o\grave{a} = 5 \ cm, \ \grave{a}a = (0.0556)(10) = 0.55$$

$$a\grave{b} = (0.0556)(22.7) = 1.26 \ cm$$

$$to \ find \ a_B, measure \ ob = 4.8 \ cm \ \Rightarrow$$

acc. diag.
drawn by scale
$0.0556 \ cm/m/s^2$

$$a_B = \frac{4.8}{0.0556} = 86.3 \ m/sec^2 \leftarrow.$$

$$to \ specify \ ac \ \Rightarrow \ ac = \frac{AC}{AB} ab = \left(\frac{10}{30}\right)(32) \approx 10.7 \ mm.$$

$$measure \ oc = 4.76 \ cm \ \Rightarrow$$

$$a_C = \frac{4.76}{0.0556} = 85.611 \ m/sec^2.$$

$$to \ find \ \alpha_{AB}:$$

$$measure \ b\grave{b} = 2.9 \ cm \ \Rightarrow$$

$$a_{ABt} = \frac{2.9}{0.0556} = 52.2 \ m/sec^2 \perp AB.$$

$$\Rightarrow \alpha_{AB} = \frac{a_{ABt}}{AB} = \frac{52.2}{0.3} = 173.9 \ rad/s^2 \circlearrowleft$$

Example(2):- In the mechanism shown in Fig. below, the link AB rotates with a uniform angular velocity of 30 rad/ s. Determine the velocity and acceleration of G for the configuration shown. The length of the various links are, AB=100 mm; BC=300 mm; BD=150 mm; DE=250 mm; EF=200 mm; DG=167 mm; angle CAB=30.
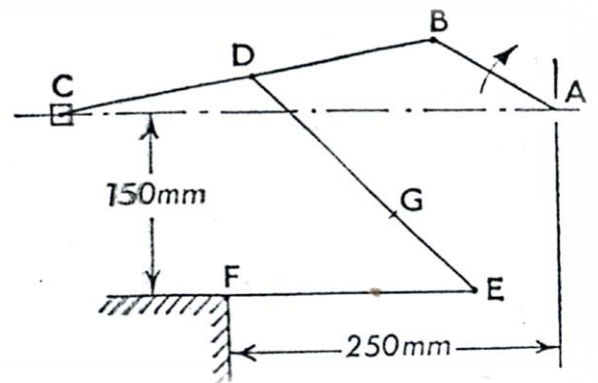
To draw velocity diagram:-

$$V_B = (30)(0.1) = 3 \ m/s$$

$$SFv = \frac{40mm}{3m/s} = 13.33 \ ^{mm}/_{m/s}$$

$$ad = 29mm \Rightarrow V_D = \frac{29}{13.33} = 2.17 \ m/s$$

$$ag = 8 \ mm \Rightarrow V_G = \frac{8}{13.33} = 0.6 \ m/s$$

$$ae = 6mm \Rightarrow V_D = \frac{6}{13.33} = 0.45 \ m/s \downarrow$$

$$bc = 35mm \Rightarrow V_D = \frac{35}{13.33} = 2.6 \ m/s \uparrow$$

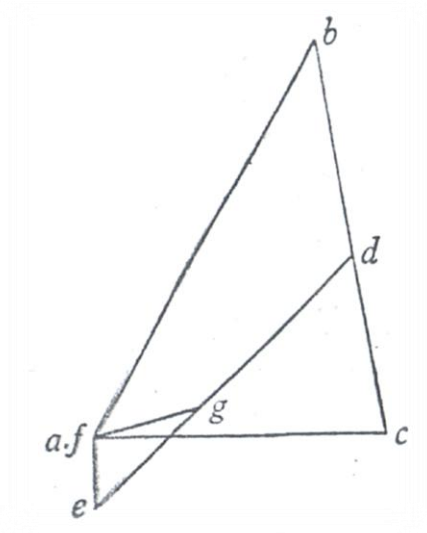$$\Rightarrow \omega_{BC} = \frac{V_{BC}}{BC} = \frac{2.6}{0.3} = 8.7 \ rad/s \ \circlearrowleft$$

$$de = 33mm \Rightarrow V_{DE} = 2.5 \ m/s \nearrow$$

$$\Rightarrow \omega_{DE} = \frac{V_{DE}}{DE} = \frac{2.5}{0.25} = 10 \ rad/s \ \circlearrowleft$$

$$\Rightarrow \omega_{EF} = \frac{V_E}{EF} = \frac{0.45}{0.2} = 2.25 \ rad/s \ \circlearrowleft$$

Mechanism drawn by scale 0.2 mm/mm

Velocity drawn by scale 13.33 $^{mm}/_{m/s}$

To draw the acceleration diagram:-

$$a_{Bn} = (AB)(\omega_{AB})^2 = (0.1)(30)^2 = 90\,{}^m/_{s^2}$$

$$a_{BCn} = (BC)(\omega_{BC})^2 = (0.3)(8.7)^2 = 22.7\,{}^m/_{s^2}$$

$$a_{DEn} = (DE)(\omega_{DE})^2 = (0.25)(10)^2 = 25\,{}^m/_{s^2}$$

$$a_{EFn} = (EF)(\omega_{AB})^2 = (0.2)(2.25)^2 = 1\,{}^m/_{s^2}$$

$$SFa = \frac{80\,mm}{90\,{}^m/_{s^2}} = 0.778\,{}^{mm}/_{m/s^2}$$

Acceleration drawn by

scale 0.778 ${}^{mm}/_{m/s^2}$

$$ac = 73mm \implies a_C = \frac{73}{0.778} = 93.9\ m/s^2 \rightarrow$$

$$ad = 68.8mm \implies a_D = \frac{68.8}{0.778} = 88.43\ m/s^2$$

$$ag = 49.5mm \implies a_G = \frac{49.5}{0.778} = 63.62\ m/s^2$$

$$ae = mm \implies a_E = \frac{57}{0.778} = 73.3\ m/s^2 \downarrow.$$

$$c\grave{c} = 32.5mm \implies a_{CBt} = 41.3\ m/s^2 \nwarrow.$$

$$\implies \alpha_{CB} = \frac{a_{CBt}}{CB} = \frac{41.8}{0.3} = 139.3\ rad/s^2\ c.w\ \circlearrowright$$

$$\grave{e}_1 e = 74.5\ mm \implies a_{EDt} = \frac{74.5}{0.778} = 95.8\,{}^m/_{s^2}\ \swarrow.$$

$$\implies \alpha_{ED} = \frac{a_{EDt}}{ED} = \frac{95.8}{0.25} = 383\ rad/s^2\ c.w\ \circlearrowright$$

$$a_{EFt} = 73.3\,{}^m/_{s^2} \downarrow\ , a_E = 73.3\,{}^m/_{s^2} \downarrow.$$

$$\implies \alpha_{EF} = \frac{a_{EFt}}{EF} = \frac{73.8}{0.2} = 367\ rad/s^2\ c.w\ \circlearrowright$$

Example(3):-Figure below shows the mechanism of a moulding press in which OA=80 mm, AB=320 mm, BC=120 mm, BD=320 mm. The vertical distance of OC is 240 mm and horizontal distance of OD is 160 mm. When the crank OA rotates at 120 r.p.m. anticlockwise, determine: the acceleration of D and angular acceleration of the link BD.

Solution:

$$SFP = \frac{6 \ cm}{32 \ cm} = 0.1875 \ cm/cm$$

$$oa = 1.5cm, ab = bd = 6cm,$$

$$bc = 2.25 \ cm, oc = 4.5 \ cm, od = 3 \ cm.$$

to draw velocity diagram.

$$\omega_{OA} = 120 * \frac{2\pi}{60} = 12.57 \frac{rad}{sec}.$$

$$V_A = (OA)(\omega_{OA}) = (0.08)(12.57) = 1 \ m/s.$$

$$SFV = \frac{3 \ cm}{1 \ m/s} = 3 \ cm/ \ m/s$$

O, C point of zero velocity.

$$oa = (1)(3) = 3 \ cm \searrow.$$
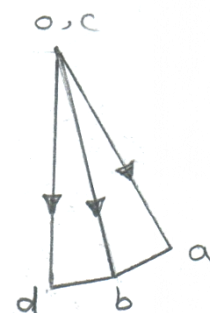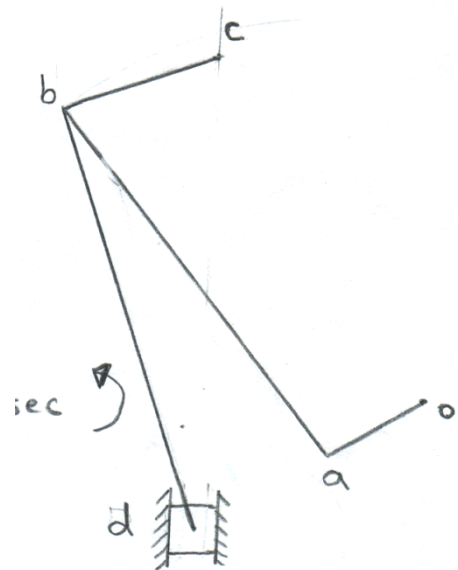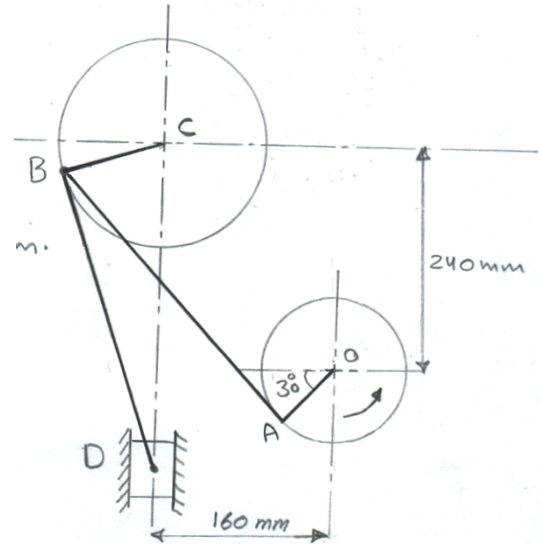
measure $cb = 3.2 \ cm \searrow$
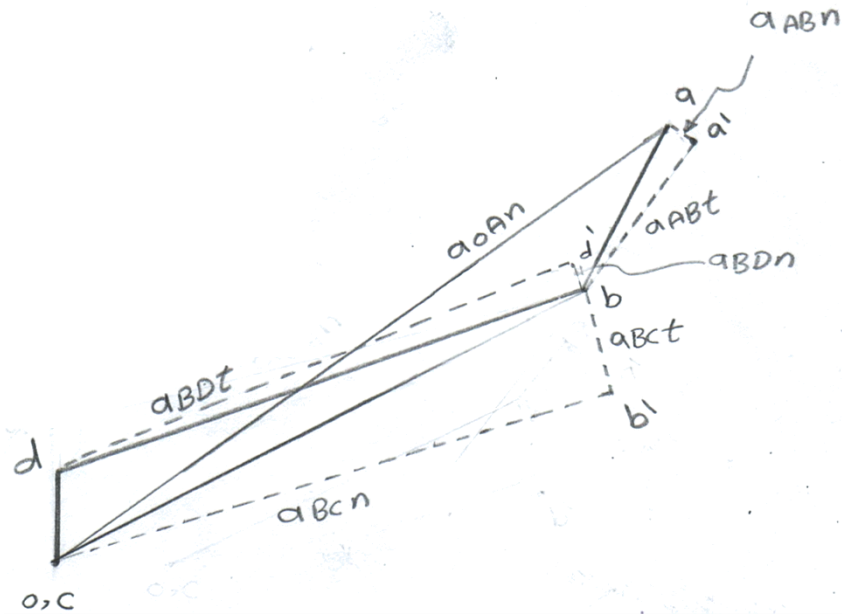
$$\Rightarrow V_b = \frac{3.2}{3} = 1.066 \ m/sec.$$

$$\omega_b = \frac{V_b}{CB} = \frac{1.066}{0.12} = 8.8833 \ rad/s \circlearrowleft$$

measure $od = 3.35 \ cm \downarrow$

$$\Rightarrow V_d = \frac{3.35}{3} = 1.116 \ m/sec \downarrow.$$

$$measure\ ab = 0.8\ cm \Rightarrow V_{ab} = \frac{0.8}{3} = 0.266\ m/sec.$$

$$\omega_{ab} = \frac{V_b}{AB} = \frac{0.266}{0.32} = 0.83125\ rad/s\ \circlearrowleft$$

$$measure\ bd = 0.93\ cm \Rightarrow V_{bd} = \frac{0.93}{3} = 0.32\ m/sec.$$

$$\omega_{bd} = \frac{V_{bd}}{BD} = \frac{0.32}{0.32} = 1\ rad/s\ \circlearrowleft$$

$$measure\ bc = 3.13\ cm \Rightarrow V_{bc} = \frac{3.13}{3} = 0.2661.08\ m/sec.$$

$$\omega_{bc} = \frac{V_{bc}}{BC} = \frac{1.08}{0.12} = 9\ rad/s\ \circlearrowleft$$

$to\ draw\ acceleration\ diagram:$

$$a_{OAn} = (OA)(\omega_{OA}^2) = (0.08)(12.57)^2 = 12.64\ m/s^2.$$

$$a_{BAn} = (AB)(\omega_{AB}^2) = (0.32)(0.83125)^2 = 0.221125\ m/s^2.$$

$$a_{BCn} = (BC)(\omega_{BC}^2) = (0.12)(9)^2 = 9.72\ m/s^2.$$

$$a_{BDn} = (BD)(\omega_{BD}^2) = (0.32)(1)^2 = 0.32\ m/s^2.$$

$$SFa = \frac{10\ cm}{12.64\ m/s^2} = 0.7911\ cm/m/s^2$$

$$to \; find \; a_D, measure \; od = 1.05 \; cm. \quad a_D = \frac{1.05}{0.7911} = 1.32 \; m/sec^2 \uparrow.$$

$$find \; \alpha_{BD}: \; measure \; b\grave{d} = 7.4 \; cm \implies a_{BDt} = \frac{7.4}{0.7911} = 9.354 \; m/sec^2.$$

$$\implies \alpha_{BD} = \frac{a_{BDt}}{BD} = \frac{9.354}{0.32} = 29.32 \; rad/s^2 \; \circlearrowright$$

Home Work:

Q1/ The diagram of a linkage is given in Fig. below. Find the velocity and acceleration of the slider D and the angular velocity of DC when the crank O_1A is in the given position and the speed of rotation is **90 rev/min** in the direction of the arrow. **O_1A= 24mm, O_2B= 60mm, CD= 96mm, AB= 72mm, CB= 48mm.**
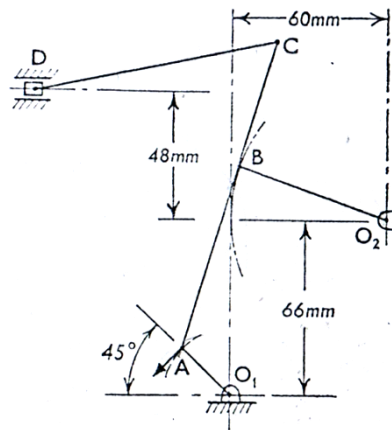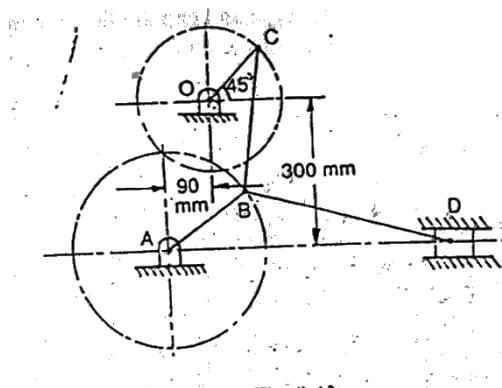

Q2/ In the mechanism shown in Fig. below the crank AOB rotates uniformly at **200 rev/min**, in clockwise direction, about the fixed centre O. Find the velocity and acceleration of slider F.
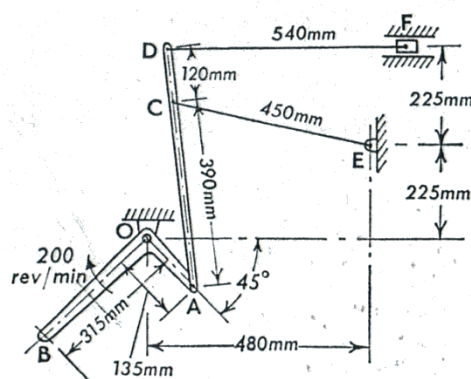
Q3/ In the toggle mechanism, as shown in Fig.below, D is constrained to move on horizontal path. The dimensions of various links are :**AB= 200 mm; BC= 300 mm; OC= 150 mm; BD= 450 mm.**

The crank OC is rotating in a counter clockwise direction at a speed of **180 r.p.m. ,** increasing at the rate of **50 rad/s^2**. Find, for the given configuration (a) velocity and acceleration of D, and (b) angular velocity and angular acceleration of BD.


Q4/ In a mechanism as shown in Fig. below , the crank OA is **100 mm** long and rotates in a clockwise direction at a speed of **100 r.p.m**. The straight rod BCD rocks on a fixed point at C. The links BC and CD are each **200 mm** long and the link AB is **300 mm** long. The slider E, which is driven by the rod DE is **250 mm** long. Find the velocity and acceleration of E.
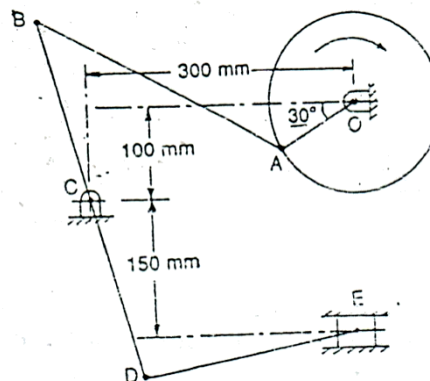
Q5/ The mechanism of a warpping machine, as shown in Fig. below, has the dimensions as follows:

O_1A= 100 mm ; AC= 700 mm ; BC=200 mm ; BD= 150 mm; O_2D= 200 mm; O_2E=400 mm; O_3C= 200 mm.
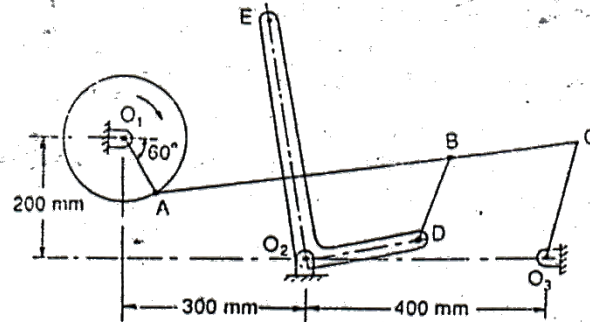
The crank O_1A rotates at a uniform speed of **100 rad/sec**. For the given configuration, determine:**1-** linear velocity of the point E on the bell crank lever, **2-** acceleration of the points E and B, and **3-** angular acceleration of the bell crank lever.

Q6/ In the mechanism shown in Fig. below , the crank AB is **75 mm** long and rotate uniformly clockwise at **8 rad/sec**. Given that **BD= DC= DE; BC= 300 mm**, draw the velocity and acceleration diagrams. State the velocity and acceleration of the pistons at C and E