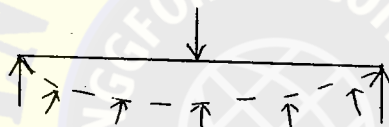# STRUCTURAL ANALYSIS

1. ## Introduction to Structures

   Stable system which offer resistance to

   > A system subjected to external loads will undergo deformation. If the internal resistance is developed and the body is able to come back to its original state, it is called structure.

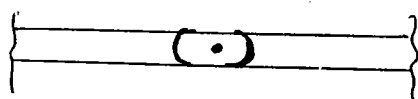   > Deflected profile of a structure is nonlinear, hence it is also called 'Elastic line'.

   > Mechanism — unstable systems are called mechanisms

   > It will have a linear deformation like rigid body.
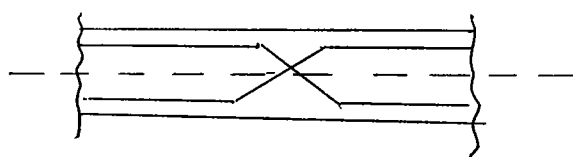

   > Practical examples for hinges.

   (i) Steel structures

   A single rivet or bolt or pinned connecting two steel plates will act like a hinge.
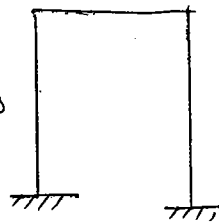
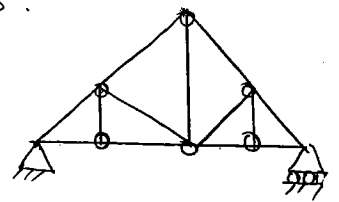

   (ii) RCC

2. Classification of Structures

(i) Skeletal Structures.

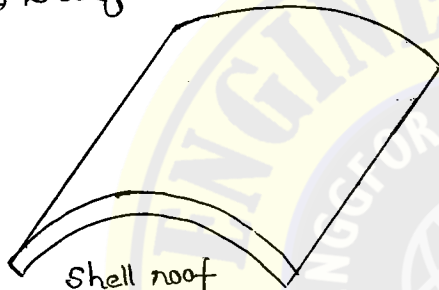A structure which has linear and non linear members as elements
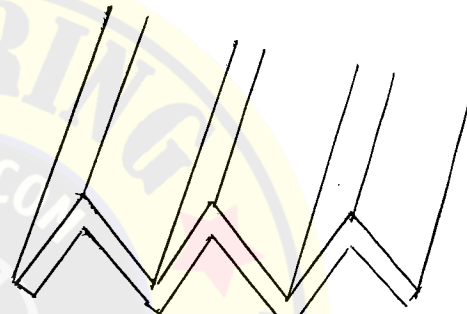
Eg: Portal frames.

Trusses.

> 2D & 3D skeletal structures are also possible.

> In skeletal structure, only 1 dimension is predominant, say Length.

(ii) Surface Structures
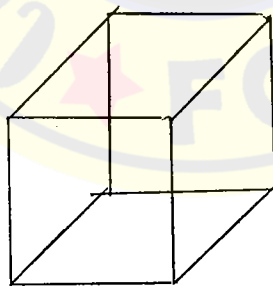
Shell roof   folded roof

> 2 Dimensions are considerable; length & width

(iii) Solid Structures.

machine foundations

> All the three dimensions are considered.

> Skeletal structures can be analysed using the traditional methods such as Force methods and displacement methods.
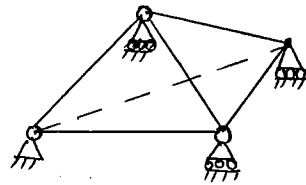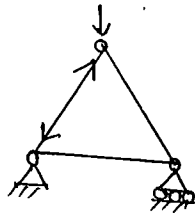
> Analysis of 2-way slabs (surface structures) is done using Johansen's 'Yield Line Theory'.

Method II : Based on type of Joints
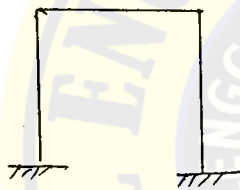
③

(i) Pin-jointed structures. (Trusses).

> Plane Trusses.   > Space Trusses.

> Load applied at a joint is transferred to other members in axial directions. ∴ axial forces are the design forces only They cannot resist BMs and shear forces.

(ii) Rigid-jointed structures

> 2D frame   > 3D frame
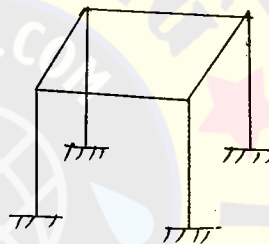
> ~~Design~~ forces are axial force, shear force and BM (because of rigidity) for 'in plane loading'.

centroidal plane

> If the centroidal plane of the structural member and plane of the loading coincide, then it is called. 'in plane loading'

W

e

W

W × e

> If a rigid jointed structural member is subjected to outplane loading, possible design forces are axial force, shear force, BM and torsional moment

■ Difference b/w BM & Torsional moment

> The moments which are within the plane of c/s.

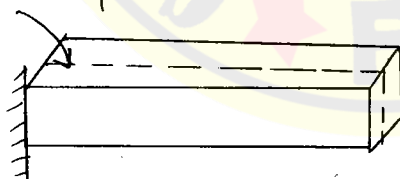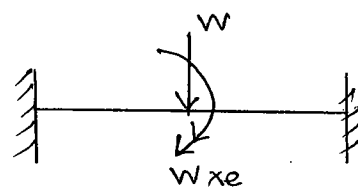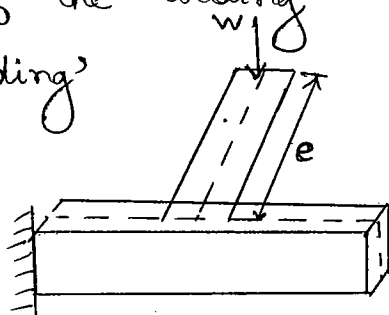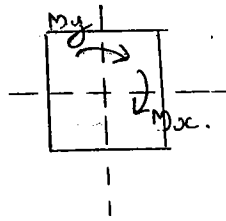For eg: M_x & M_y are moments wrt x & axces which are within the c.s
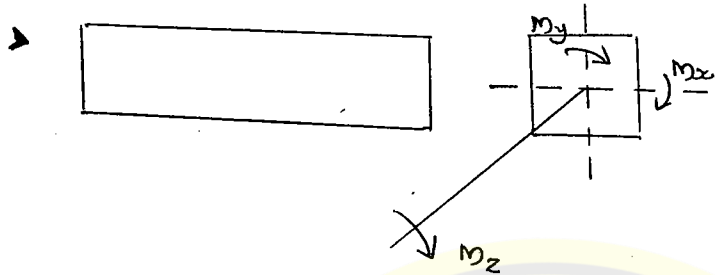
> Torsional moment / twisting moment / torque is the moment about longitudinal axis of the member M_z.

3. Equilibrium Equations.

> Deals with balancing of forces

(i) For plane frames (as a whole).

$$\Sigma V = 0, \quad \Sigma H = 0, \quad \Sigma M = 0.$$

or $\Sigma Fy = 0, \quad \Sigma Fx = 0, \quad \Sigma M = 0$

> Minimum 3 eqbm eqns for all plane frames whether pin-jointed or rigid jointed. $(\Sigma V = \Sigma H = \Sigma M = 0)$.

> Any structure in the universe shall be stable against overturning $(\Sigma M = 0$ shall be satisfied).

(ii) For space frames (as a whole for both rigid jointed & pin jointed space frame)

> $\Sigma Fx = \Sigma Fy = \Sigma Fz = \Sigma Mx = \Sigma My = \Sigma Mz = 0,$

Fx & Fy are shear forces (tangential force.

Fz → axial force.

Mx & My are BM

Mz is torsional moments.

(iii) Eqbm eqns of Joints

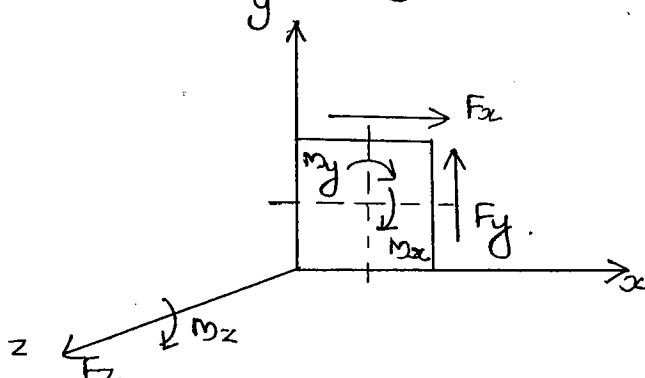a) Pin-joint of a Plane frame.
> 2 eqbm equations.
$$\Sigma V = \Sigma H = 0.$$
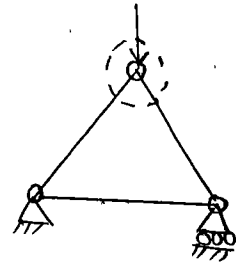
b) Pin-joint of a Space frame.
$$\Sigma F_x = \Sigma F_y = \Sigma F_z = 0.$$
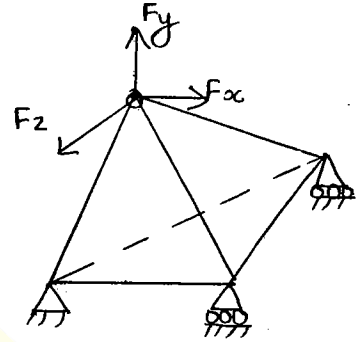
c) Rigid joint of a Plane frame.
> rigid joints can resist moments.
∴ 3 eqbm eqns.
$$\Sigma H = \Sigma V = \Sigma M = 0.$$
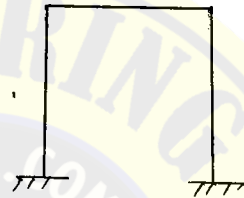
d) Rigid joint of a Space frame.
> 6 eqbm equations.
$$\Sigma F_x = \Sigma F_y = \Sigma F_z = \Sigma M_x = \Sigma M_y = \Sigma M_z = 0$$

4. Type of Support & Reaction Components.

> Reaction is the resistance against deformation.

Type of Support	Reaction Components
a) Free end	0.
b) Roller end.	1. (R_V)
c) Hinged support.	2. (R_V & R_H).

> For free end, due to vertical load shown, tip of the free end has the deformation (f_x, f_y, θ) as no resistance against the deformation. No reaction

> Roller support shown is free to move horizontally, free to rotate. Hence neither horizontal reaction nor moment reaction at roller support. However it cannot move in the vertical direction. Hence it has only one reaction, ie, vertical reaction

> Hinged support shown is free to rotate, hence no moment reaction, only two reactions R_V & R_H.

(iv) Fixed end.

$\theta = 0$ 3 (R_V, R_H & m).

(v) Shear hinge supports.

2 (R_V & m).

Horizontal shear hinge support.

R_V

> A hinge is a device which makes some force zero. Moment hinge releases moment. ($m = 0$). Shear hinge releases shear force. Reaction at a support is nothing but the shear force at the support. In the support shown, horizontal reaction, (rollers are in the horizontal plane) is zero. implies, horizontal shear force is zero. Hence it is called horizontal shear hinge support.

> Reaction at a roller support is normal to the plane of rolling.

(vi) Vertical shear hinge
supports

2 (R_H & m).

R_H

(vii) Damper.

2 (R_V & m).

↑ R_V

(viii).
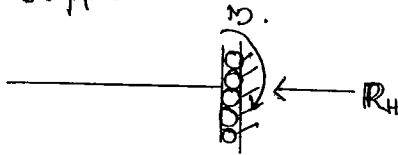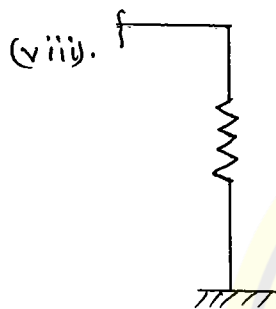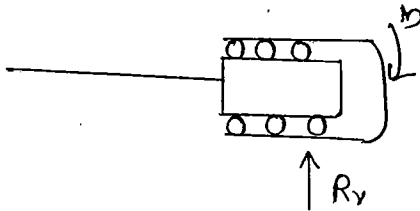
5. Compatibility Equations

> Compatibility equations deal with displacements. These equations are related to balancing of deformations.

> Elastic props can compress. Eg: springs.
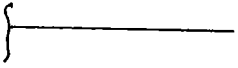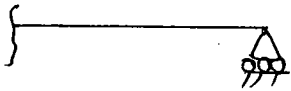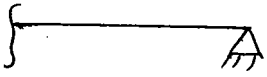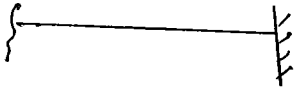
> Rigid props cannot deform.

w/m
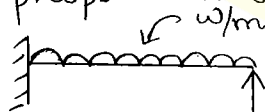
d/w deflection = u/w deflection.

∴ $\delta_V = 0$ at rigid prop.

> No: of compatibility eqns at a support shall be equal to number of reaction components.

Type of Support		Compatibility Eqns.
1. Free End		0.
2. Roller End		$\delta_V = 0$ (1).
3. Hinged end.		$\delta_V = 0$ $\delta_H = 0$ (2).
4. Fixed end.		$\delta_V = 0$ $\delta_H = 0$ (3) $\theta = 0$

⑤

Static Indeterminacy. (Ds)

> If eqbm equations are sufficient to analyse a structure completely for unknown forces, it is called statically determinate structure.

Apply $\Sigma V = 0$,

$\qquad R_v = w\sin\theta$

Apply $\Sigma H = 0$

$\qquad R_H + w\cos\theta = 0$,

$\qquad R_H = -w\cos\theta$.

-ve shows that assumed direction is wrong.

Apply $\Sigma M = 0$.

$\qquad \therefore M = wl\sin\theta$.

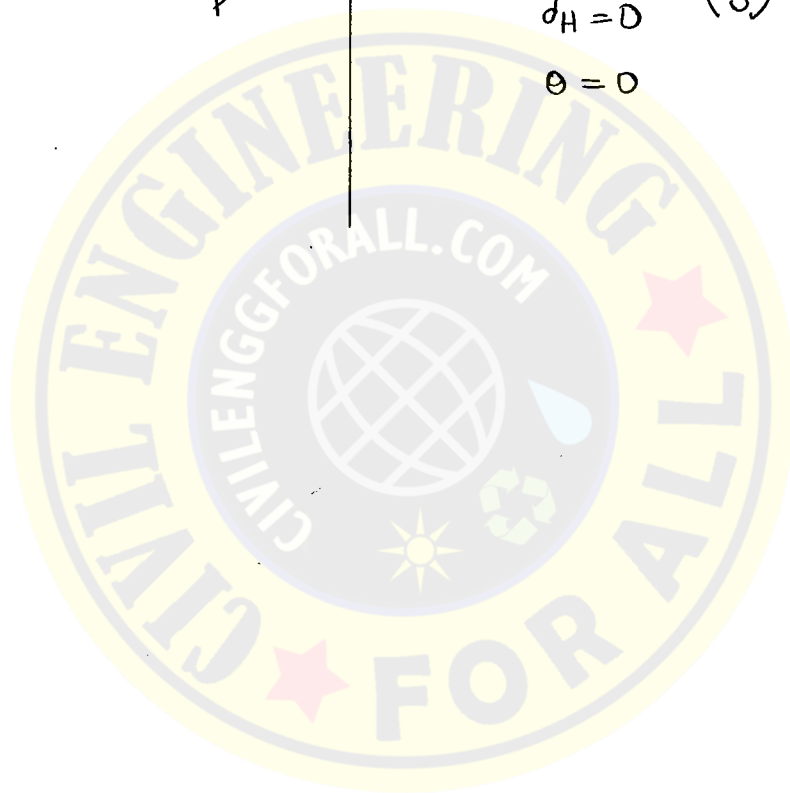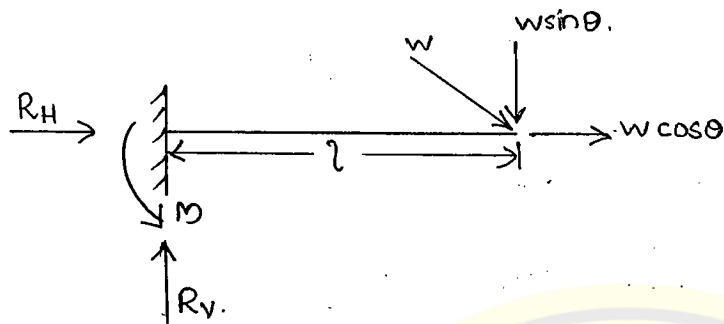As all the 3 unknown forces at fixed support are calculated using the eqbm equations alone, it is called Statically determinate structure.
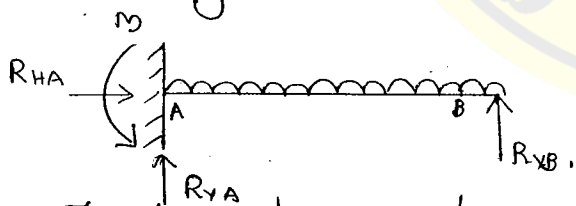
Total unknown forces = 4.

Available eqbm equations = 3.

Hence the beam cannot be analysed using eqbm eqns alone.

Hence it is statically indeterminate.

So we use compatibility equations at B. for the complete analysis

$\qquad \delta_v = 0$. at B.

$\qquad$ u/w deflection = d/w deflection

> Equations for D_S

$$D_S = D_{se} + D_{si} - \text{no. of force releases.}$$

where $D_S \to$ total static indeterminacy.

$D_{se} \to$ external indeterminacy (related to support reactions)

$D_{si} \to$ internal indeterminacy (related to type of joints & frame)

D_{si} related to internal configuration

Force releases are due to hinges

> $D_{se} = r - $ eqbm equations.

$r \to$ total reaction components of the given structure.

$D_{se} = r - 3$ for plane frames.

$\quad\;\; = r - 6$ for space frames
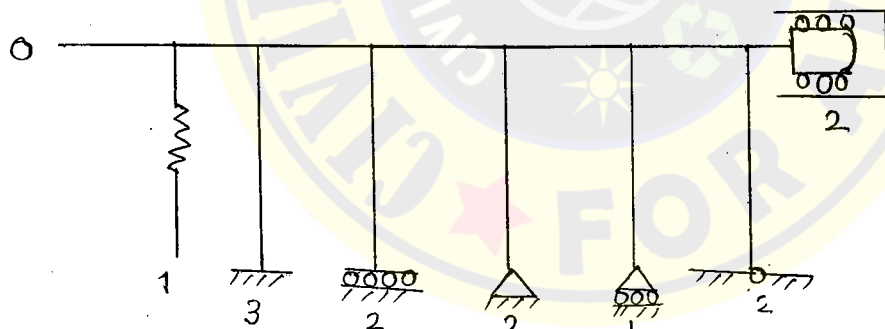
$D_{se} = 13 - 3 = \underline{\underline{10}}$

> Fixed support of a space frame has 6 eqbm reactions and 6 eqbm equations.
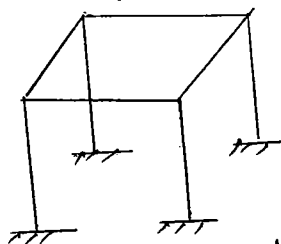
$$D_{se} = 4 \times 6 - 6$$
$$= \underline{\underline{18}}$$

> D_{si} of rigid jointed frames.

(i) $D_{si} = 3c \Rightarrow$ for plane frames

$\qquad\;\; = 6c \Rightarrow$ for space frames.

$c \to$ no. of closed boxes or no. of cuts required to convert a closed structure to open tree struct

> Open tree has no redundant forces internally
like a cantilever
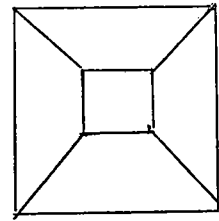
$D_{si} = 3 \times 6$
$= 3 \times 1$
$= \underline{\underline{3}}$

$D_{si} = 3 \times 6$
$= 3 \times 5$
$= \underline{\underline{15}}$

Q. Assume Asoka Dharma Chakra as rigid jointed plane frame.

$D_{si} = 3 \times 24$
$= \underline{\underline{72}}$

Q.
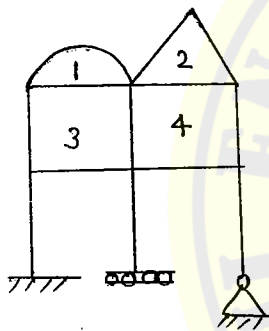
$D_{se} = (3 + 2 + 2) - 3 = 4$

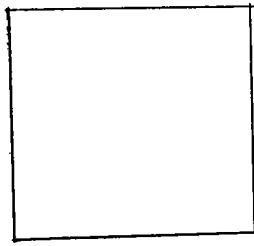$D_{si} = 3 \times 4 = 12$

$D_S = 4 + 12 = \underline{16}$

> Force releases – give additional eqbm eqns

a) Moment Hinge (internal)

$\sum M = 0$
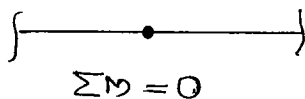
Force includes Axial, Shear, BM, TM.

Displacement includes axial deformation, shear deformation, vertical deflection, horizontal, rotation & twisting angle.

> The no. of moments released at an internal hinge
$= n - 1$; $n \rightarrow$ number of members passing through the hinge

$FR = 3 - 1$
$= 2$
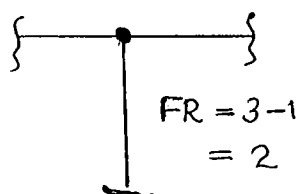
As shown in fig, ABC shall be treated as one member with common rotation. Hence for calculation purpose two members.

$$n-1 = 2-1 = 1.$$

A —————— B —————— C

D
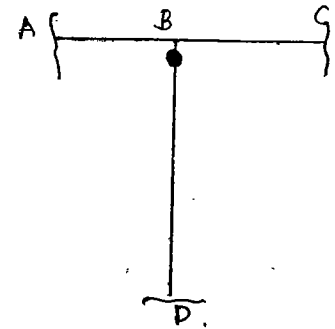
b) Shear Force Releases

(i) Horizontal SF release.
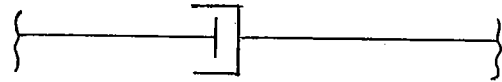
H is released, $\Sigma H = 0$.

(ii) Vertical SF release.

← lubricant oil.
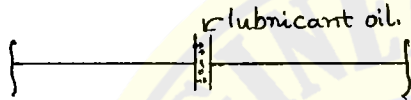
V is released, $\Sigma V = 0$

c) Link.
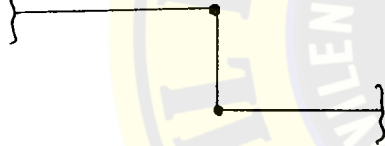
$\Sigma M = 0$
$\Sigma H = 0$

Vertical bar with hinges on either side.

∴ A link has two force releases

> Effect of Normal or Lateral loads only on Ordinary Beams
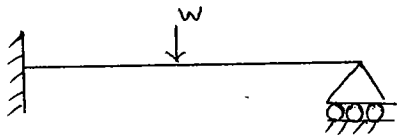
When there is only normal loads on beams,
   $\Sigma H = 0$ need not be considered.
   Also, horizontal reactions need not be considered.

Q.
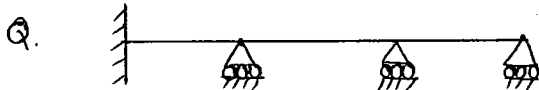
$$r = 5$$
$$e = 2$$
$$D_s = \underline{3}$$

> In the above beam, by removing 3 roller supports the beam is stable and determinate. Hence, D_s = unnecessary reactions removed.

> In ordinary beams, no closed boxes (it is a ready-made open tree structure), hence $D_{si} = 0$.

Q. A fixed beam subjected to lateral load only, $D_s =$
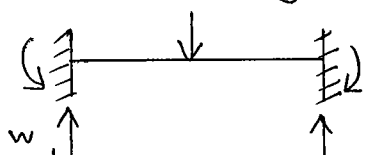
$r = 4$
$c = 2$
$D_s = 4 - 2 = \underline{\underline{2}}$
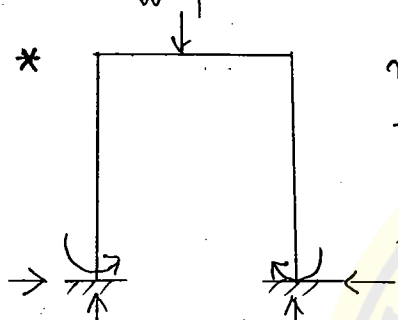
* When a vertical load is applied on a frame, it will try to spread the frame to a horizontal member. But the fixed supports will offer resistance against horizontal motion.

Even though given frame is symmetrical in all aspects, it will have a horizontal reaction at the supports (equal & opp).

Q.
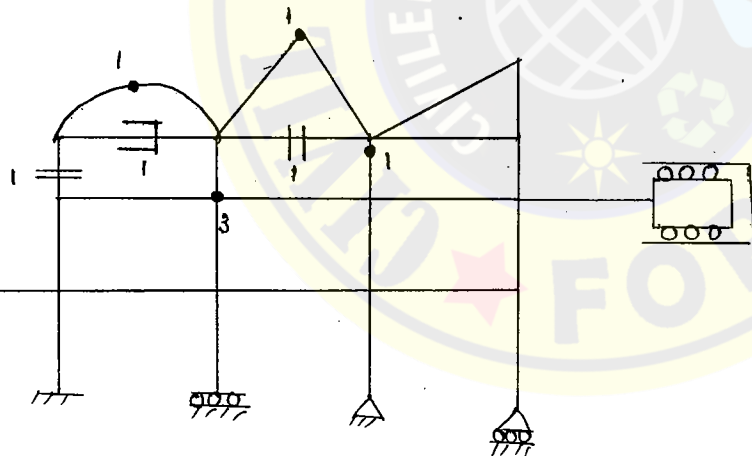
$D_{se} = r - 3$
$\quad = 11 - 3 = 8$

$D_{si} = 3c$
$\quad = 3 \times 9 = 27.$

Force release = 9.

$D_s = 27 + 8 - 9 \quad = \underline{\underline{26}}$
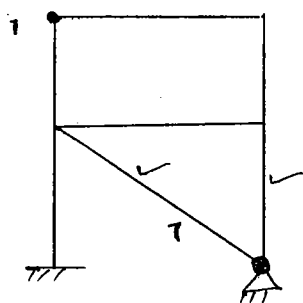
NOTE: If additional members are connected to a hinged support, additional moment releases equal to additional number of members (considering reactions at hinged support to be 2)

$D_{se} = 3 + 2 - 3 = 2.$

$D_{si} = 3 \times 2 = 6.$ $\qquad D_s = 6 + 2 - 2 = \underline{\underline{6}}$

Releases $= 2$

Q
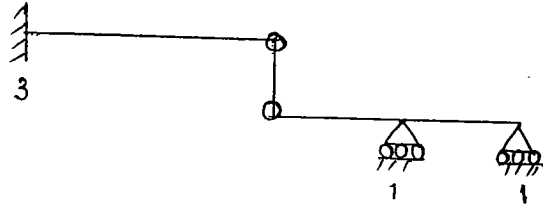
$D_{se} = 5 - 3 = 2$

Releases $= 2.$

$D_s = 2 - 2 = \underline{\underline{0}}$

If the above beam carries, vertical load only,

$D_{se} = 2 + 1 + 1 = 4 - 2 = \underline{2}$

$D_{si} = 0.$

Releases $= 1$ (no horizontal / inclined loads, $\therefore \Sigma H = 0$ not considered)

$D_s = 2 - 1 = \underline{\underline{1}}$

> D_{si} of Pinjointed Plane Trusses
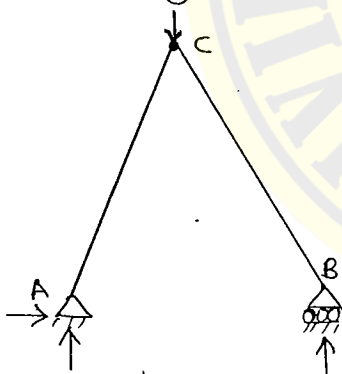
$\Rightarrow$ unstable as it can become flat because of external load
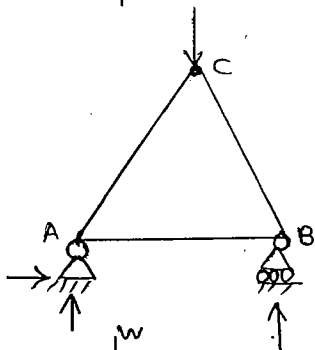
$\Rightarrow$ stable because of bottom member connected to the supports A & B. Truss shown is stable due to $\underline{\Delta^r \text{ shape}}$
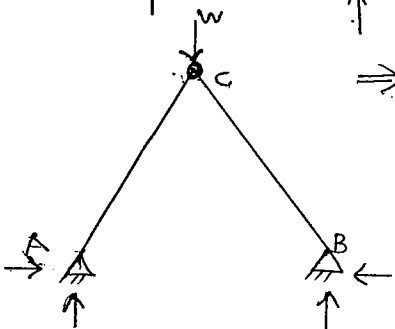
$\Rightarrow$ stable due to $\underline{\Delta^r \text{ behaviour}}$, the extra horizontal reaction at support B takes care of the deficiency of the bottom member

Conclusion:

The basic perfect frame is Δ^r, either in shape or behaviour in case of pin-jointed trusses.

> $m = 2j - 3$ is the general formula for internal determinacy of pin-jointed plane trusses

$$m = 2j - 3 \quad ; \text{ determinate (internally)}$$

$$m < 2j - 3 \quad ; \text{ deficient or unstable frame}$$

$$m > 2j - 3 \quad ; \text{ redundant frame. (extra members)}$$

(9)

Q.
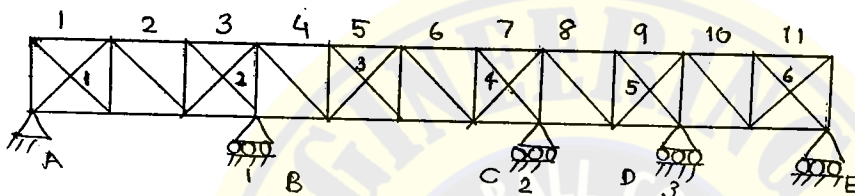
$D_{se} = 3$.
$D_{si} = 6$.
$D_s = \underline{9}$

For external stability, certain minimum reactions are sufficient. For the given frame supports at A & E are giving minimum reactions. for external stability. Hence, the three reactions at B, C, D are ext $\therefore D_{se} = 3$. Basic perfect frame is a Δ, 8 pans. 1, 3, 5, 7, 9, 11 have additional members (in total 6). $D_{si} = 6$.

Total static indeterminacy $= \underline{9}$

> Question of force releases does not arise in the case of trusses

> Static Indeterminacy of Pin-jointed frames

$$D_{se} = r - 3$$

$$D_{si} = m - (2j - 3)$$

$$D_s = m + r - 2j$$

Stability vs. Instability

```
                    Unstable
                    Structures.
        ┌──────────────┼──────────────────────┐
        ↓                                      ↓
   Externally                            Internally
   unstable                              unstable.
    ┌──────────────┐
    ↓              ↓
Statically    Geometrically
Unstable.     unstable.
```
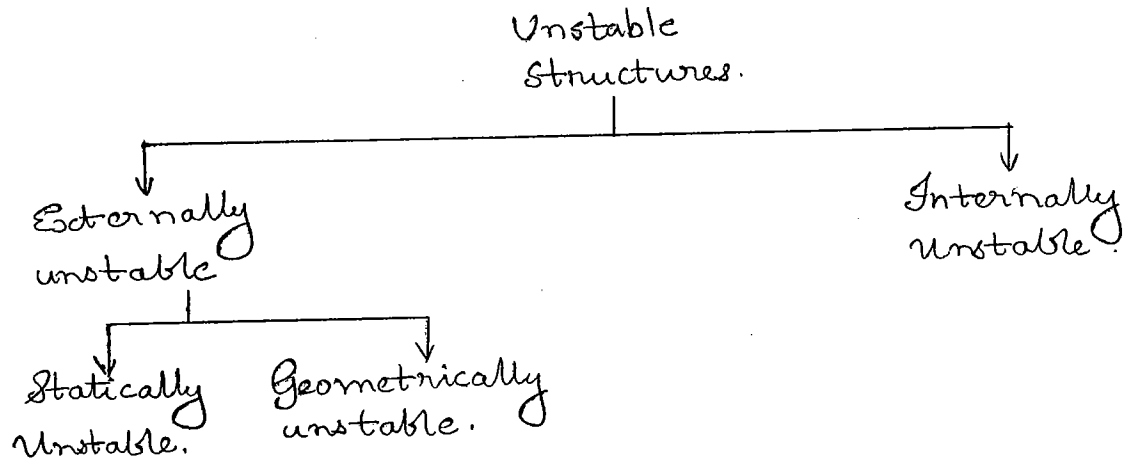
> Externally Unstable Systems.

  > Geometrically unstable structures

The beam shown has 3 reactions which is the min. reqd for external stability of the beam shown. "If the reactions develope are parallel to each other then the structure will be unstab to some load system. The beam given is unstable to the horizontal loads. Such cases are called geometrically unstable systems.

Externally unstable structure.

Q.

line of reaction

cable   cable

cable.
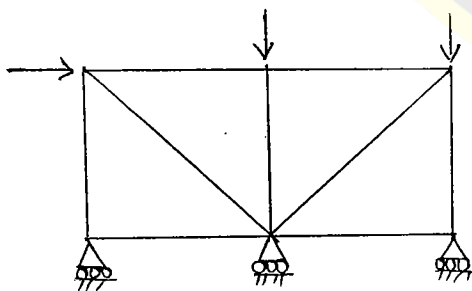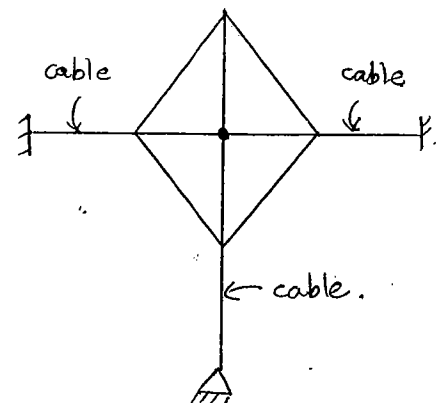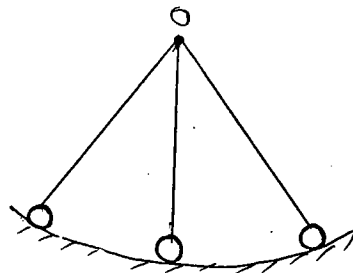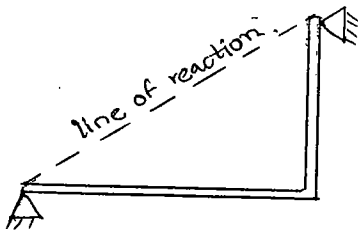
Three structures shown are unstable
If the reactions developed at supports are concurrent, structure becomes unstable and it may rotate like a rigid body

⑨
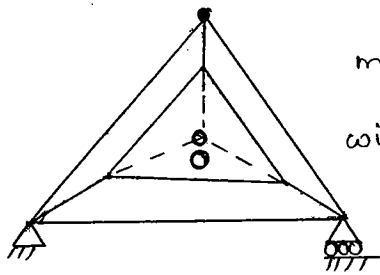
The lines of action as shown are meeting at 0. Hence the central part will rotate like a rigid body and becomes unstable to some loading.

⑩

> Statically Unstable.

In the beam shown only two reactions are developed, but min. 3 reactions is the requirement. Such unstable systems are called Statically unstable systems
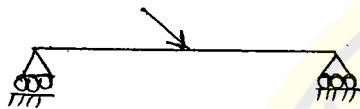
> Internally Unstable Systems.

> If $m < 2j-3$, plane trusses are internally unstable.

> A structure must be stable not only globally, but also locally
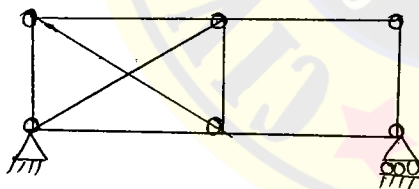
$$D_{se} = 3-3 = 0$$
$$D_{si} = m - (2j-3) = 9 - (2 \times 6 - 3) = 0$$
$$D_s = 0$$

By this mathematical relation it appears that given truss is stable however it's not true. The second panel is rectangular, not satisfying Δ^r shape or Δ^r behaviour. Hence, the 2nd panel'll have local failure. This is also called by some authors as 'Failure by Panel Shear'

> If $D_s < 0$, unstable.

> In a rigid jointed structure if force releases are constructed in an irrational manner, the structure becomes unstable.

$D_S = 3.$

$D_S = 0.$

link
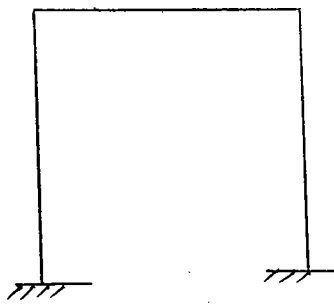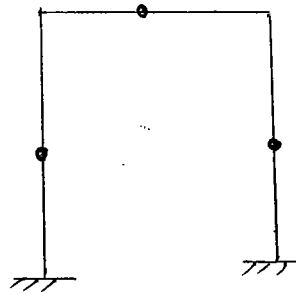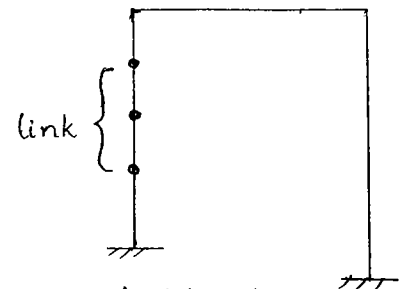
Unstable due to irrational construction of force releases.

Statically Determinate Structures.

1. Eqbm eqns are sufficient.

2. No thermal stresses

case1: temp. grad zero.

> If SSB shown is subj. to uniform temp, from top to bottom; then it will move horizontally without any resistance. No resistance means no stresses.

In determinate structures, temp change (uniform) will not cause thermal stresses, but can cause deformations.

>

$D_{se} = 2+2-3$
   $= 1$
$D_{si} = 0.$
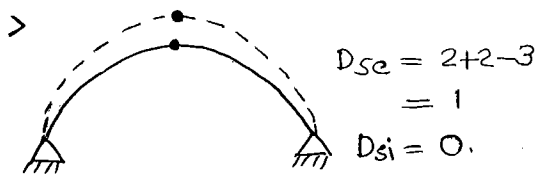
$D_s = 1 - 1 = 0$

3 hinged arch is statically determinate and hence no thermal stresses. However the level of crown changes

Statically indeterminate structures.

Eqbm eqns + compatibility eqn

Thermal stresses will develop.

case 2: $EI, l.$ T
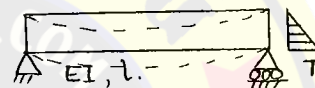
Temp changes non uniformly (temp gradient exist)

Deflection downwards as temp is more at bottom fibre.

$$\delta = \frac{\alpha (\Delta T) l^2}{8EI} \; ; \; \Delta T - \text{change in temp}$$

$\alpha \rightarrow$ coefficient of thermal expansion

$EI \rightarrow$ flexural rigidity.

In this case, though determinate, thermal stresses are developed due to temp. gradient.

Q. $l = 4m.$
   $l = 4m$ $l = 4m.$
A   B   C
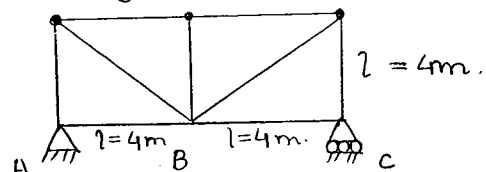
Temp. of bottom chord members increased uniformly. $\alpha = 10 \times 10^{-6}/°c$ $\Delta T = 100°c.$ Force developed in bottom chord members is ....?

The given truss being determinate no. thermal stresses are developed.

$\therefore$ force $= 0$

⑩

fixed beam.

If temp. increases, it will try to elongate. However the fixed supports will not allow. Hence it is subj. to axial compression.
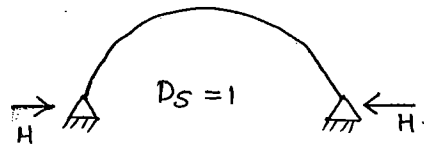
2-hinged arch

⑪

Crown isn't flexible as there is no hinge. Supports cannot move as they are hinged. Hence as temp increases, horizontal reaction are developed at supports. Hence thermal stresses are developed.
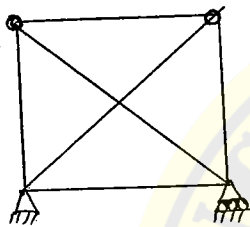
The given truss is statically indeterminate. ($D_s = 1$). Hence thermal stresses are developed

3. Lack of fit : if the length of a member is either less or more than actual length of member slightly, then it is called lack of fit.

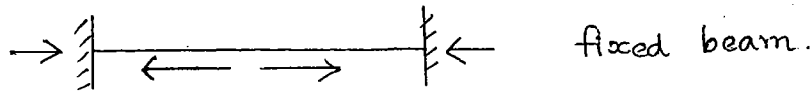> No stresses due to lack of fit in determinate structures | > Stresses will develop due to lack of fit.

Q.
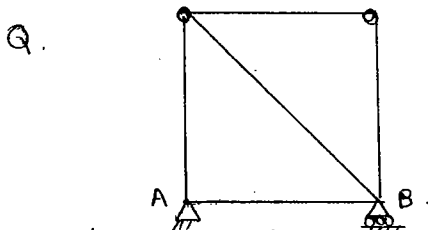
In the truss given, member AB is 5mm less than the actual length. Force developed in member AB is ____?
Zero as determinate structure.

4. No stresses due to sinking of supports in determinate structures

Q. One of the supports of SSB sinks by δ. Shear forces developed at supports are ____? Zero.

Shear force at a support is the reaction at the support.
As it determinate, no stresses, no shear forces.

Reaction = $\dfrac{Net\ moment}{l}$

$$= \left(\dfrac{6EI\delta}{l^2} + \dfrac{6EI\delta}{l^2}\right)\dfrac{1}{l}$$

$$= \underline{\dfrac{12EI\delta}{l^3}}$$
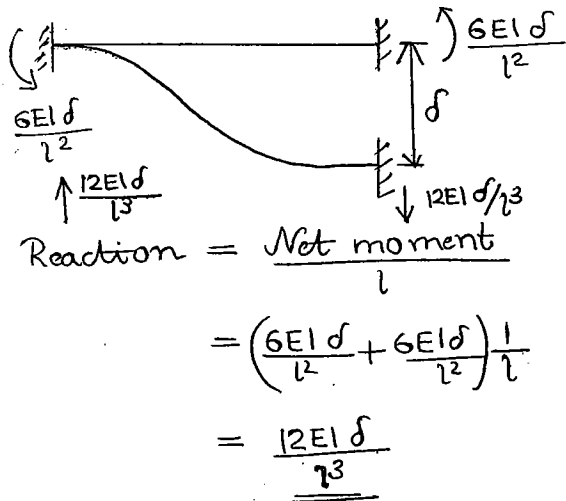
Indeterminate structures develop stresses on sinking of supports. Moments and reactions are developed at supports

5. Determinate structures are uneconomical for long spans | More economical as moment to be resisted is less.
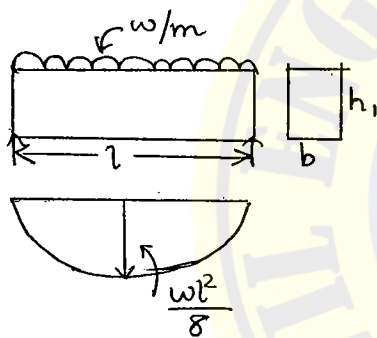
Design moment $= \dfrac{wl^2}{8}$
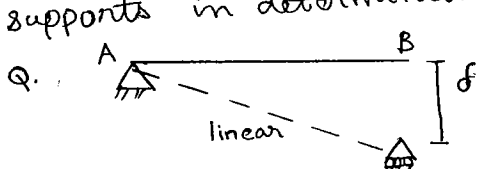
Say design depth $= h_1$
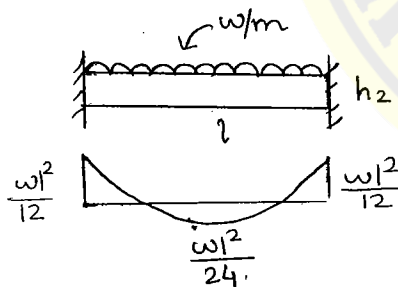
Design moment is max value, ie $\dfrac{wl^2}{12}$

Design depth, say h_2

As $M_{fixed} < M_{SSB}$ $\left(\dfrac{wl^2}{12} < \dfrac{wl^2}{8}\right)$; $h_2 < h_1$ ($M \propto d^2$).
So concrete reqd is less and hence it is economical.
In indeterminate structures, moments are redistributed all along the length of beam. Hence design moments are less. The material is properly utilised in indeterminate structures

6. No effect of material and change of c/s on forces |
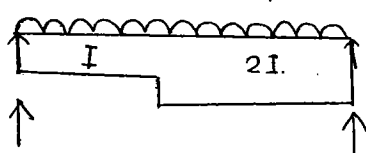
Irrespective of material & c/s, for the SSB shown, $M = \underline{\dfrac{wl^2}{8}}$

(12)
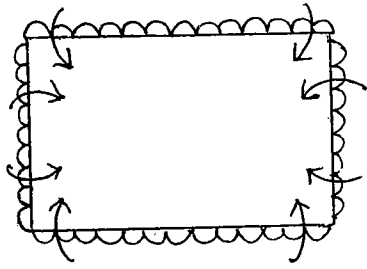
Direction of moments.
(depends on the direction in which load is acting)
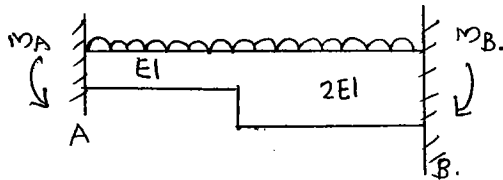
$M_A < M_B$.
(2EI stronger than EI).

Balanced Cantilever Bridge:

River beds will have weak soils causing settlements
Bridges constructed across rivers may undergo settlements. If
statically indeterminate designs are done for bridges due to
sinking of supports additional stresses may develop, making
uneconomical. If determinate structures are used, no stresses
due to sinking of supports. Balanced cantilever bridge will have
benefits of both indeterminate (economical moments) and determina
structures (no moments due to sinking of supports)
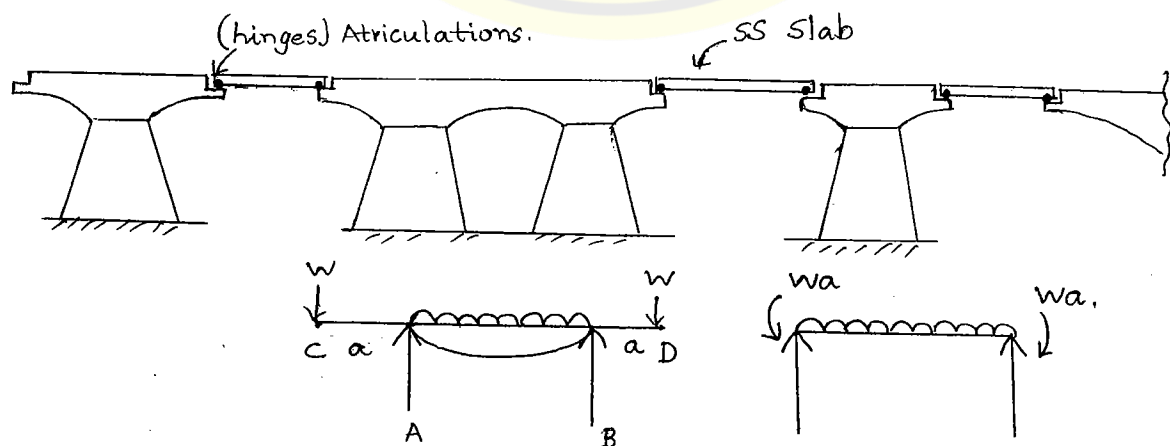
On idealisation, balanced cantilever bridge reduces to SSB
with overhangs on either side. Reactions from the suspended
SS slab will be transferred to tip of the cantilever producing

hogging moments at the supports. These hogging moments will balance or reduce the sagging moment of the main SS span.

Level 2 : Q-04.
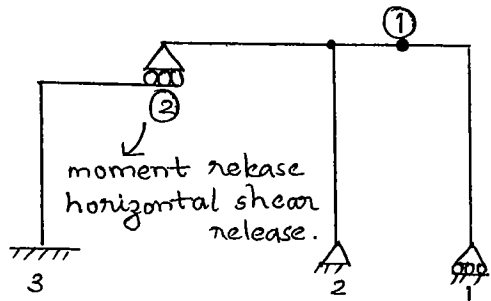
$D_{se} = 3+2+1-3 = \underline{\underline{3}}$

$D_{si} = 0$

Releases $= 2+1 = 3$.

$D_s = 3-3 = \underline{\underline{0}}$

OR

$D_s = 0$

(acts as a cantilever).
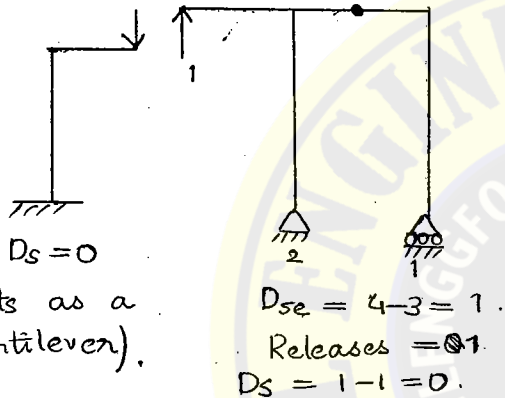
$D_{se} = 4-3 = 1$.

Releases $= 1$

$D_s = 1-1 = 0$.

Level 2 : Q-14.

a)
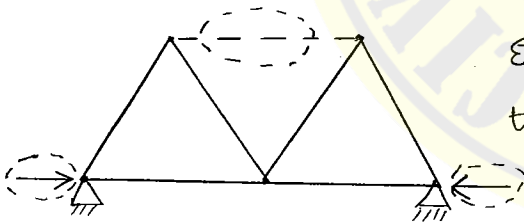
Extra horizontal reaction is not in the line of deficient member
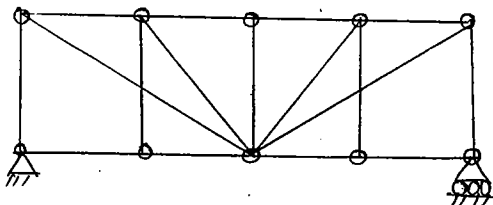
Structure is stable and determinate.
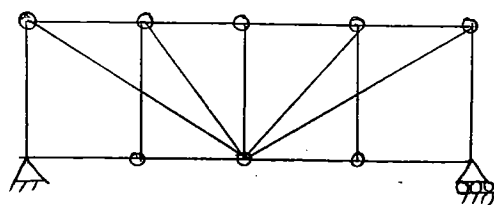
First span and last span are deficie But that deficiency is taken care by the extra member.

Level 1 : Q-04.

Cable will be subj. to axial tension

Level 1:

(13)

01. A simply supported beam having an internal hinge is a mechanism.

02. Elastic structural analysis makes use of Elastic and linear stress-strain relation.

03. A. Folded plate — Surface.

B. Shell roof — Surface.

C. Building frame — Skeletal.

04. Arch subjected to inplane loading.

05. A determinate structure requires only statical eqbm equations for its analysis.

06. A statically indeterminate structure is the one which can be analysed using equations of static and compatibility equations.

07. Plane frame $\rightarrow 3m + r - 3n$; m = number of members.

Space truss $\rightarrow m + r - 3n$.   n = number of joints

Space frame $\rightarrow 6m + r - 6n$ r = no. of reaction elements.

08.
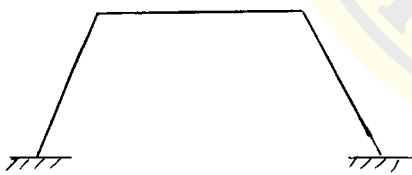
$D_S = 3 + 3 - 3 = \underline{3}$

09.
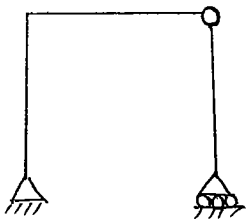
$D_{se} = 2 + 1 - 3 = 0.$

$D_{si} = 0$

$D_S = 0 - 1 = \underline{-1} < 0$; Unstable.

10.

	General Loading	Vertical Loading

(i)

General Loading:
$D_S = 3 + 1 - 3$
$= 1$

Vertical Loading:
$D_S = 2 + 1 - 2$
$= \underline{1}$

(ii)

General Loading:
$D_S = 3 + 2 - 3.$
$= \underline{2}$

Vertical Loading:
$D_S = 2 + 1 - 2$
$= \underline{1}$

(iii)

$$D_S = 3+2-3$$
$$= 2$$

$$D_S = 4-2 = 2$$

(iv)

$$D_{se} = 3+1+2-3.$$
$$= 3.$$
$$D_S = 3-2 = 1$$

$$D_{se} = 2+1+1-2 = 2.$$
$$D_S = 2-1 = 1$$

(v)

$$D_{se} = 4-3 = 1$$
$$D_S = 1-1 = 0.$$

$$D_{se} = 2+1-2 = 1$$
$$D_S = 1-1 = 0$$

11.

$$D_S = 6-3 = 3$$

12.

$$D_S = 5-3 = 2$$

13.

$$D_{se} = 6-3 = 3$$
$$D_S = 3-1 = 2$$

14.
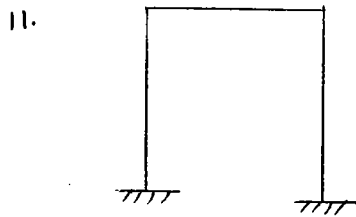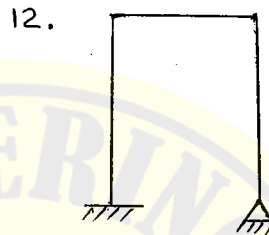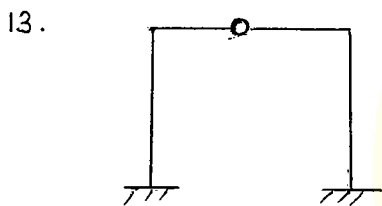
$$D_{se} = 6-3 = 3.$$
$$D_{si} = 3 \times 2 = 6.$$
$$D_S = 6+3 = 9$$

15.

$$D_{se} = 5-3 = 2$$
$$D_S = 2-1 = 1$$

16.

$$D_{se} = 3+2+2-3 = 4.$$
$$D_S = 4-1 = 3$$

17.

$$D_S = 9-3$$
$$= 6$$

18.

$$D_{se} = 12-3 = 9.$$
$$D_S = 9-4 = 5$$

19.

$$D_{se} = 6-3 = 3$$
$$D_{si} = 3 \times 1 = 3.$$
$$D_S = 6$$

20.

$$D_{se} = 4-3 = 1$$
$$D_S = 1-1 = 0.$$

∴ Stable &
Statically determine.

21.
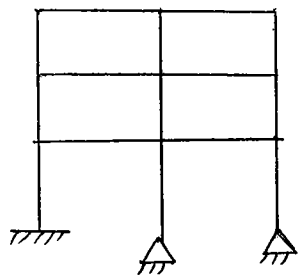
$D_{se} = 7 - 3 = 4$.

$D_{si} = 3 \times 4$
$\quad = 12$

$D_s = 12 + 4 = \underline{16}$

22.
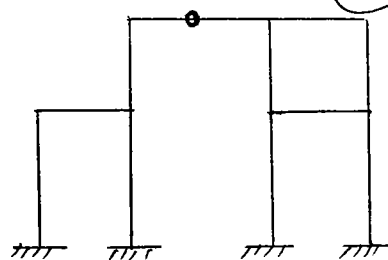
$D_{se} = 3 \times 4 =$

$D_{si} = 3$

$D_s = 12 - 3 + 3$
$\quad = \underline{11}$

Level 2:

01
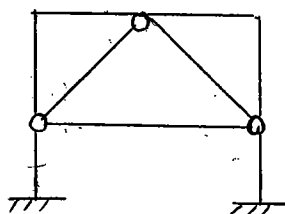
$D_{se} = 6 - 3 = \underline{3}$

$D_{si} = 3 \times 3 = \underline{9}$

$D_s = 9 + 3 - 3 \times 3$
$\quad = \underline{3}$

02.
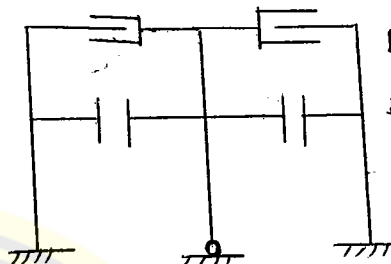
$D_{se} = 8 - 3 = 5$

$D_{si} = 3 \times 2 = 6$

$D_s = 11 - 4 =$

03.
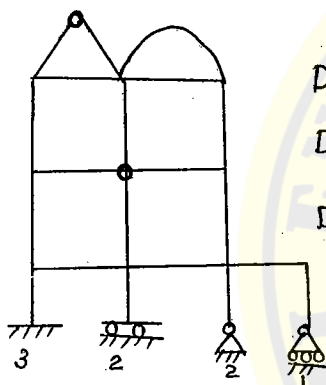
$D_{se} = 8 - 3 = 5$.

$D_{si} = 3 \times 6 = 18$.

$D_s = 23 - 3 - 1$
$\quad = \underline{19}$

3   2   2   1

04.

05

B   C   D   E   F

A   H   G
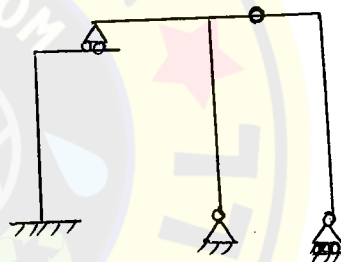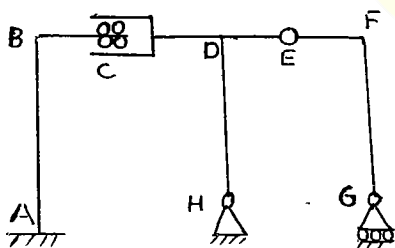
$D_{se} = 6 - 3 = 3$.

$D_s = 3 - 2 = 1$

06.
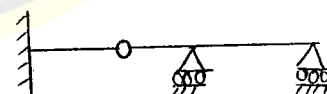
$D_{se} = 5 - 3 = 2$.

$D_s = 2 - 1 = \underline{1}$

07.
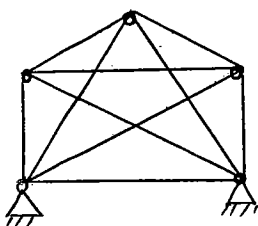
* To maintain geometry, 2 cables are required.

For n cables, static indeterminacy = n-2

4 cables

$D_s = 4-2 = 2$

stable.

stable.

stable. $3 \times C = 3 \times 1 = 3$
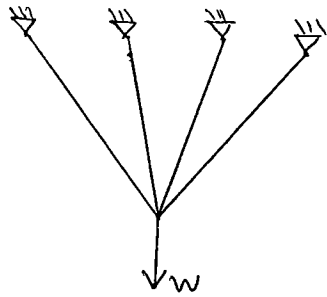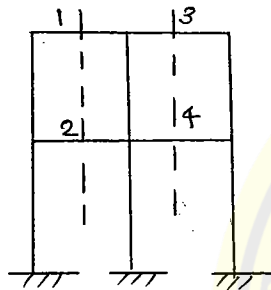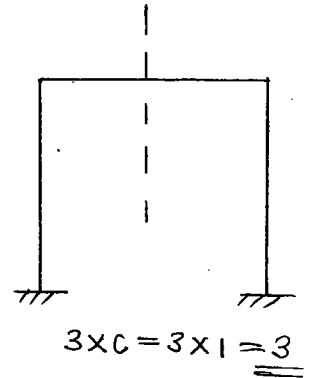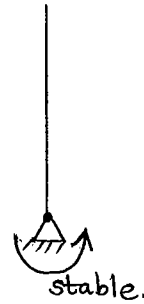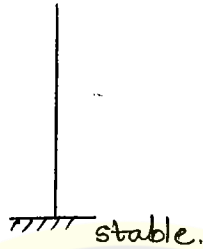
$D_s = 3c$
$= 3 \times 4 = 12$
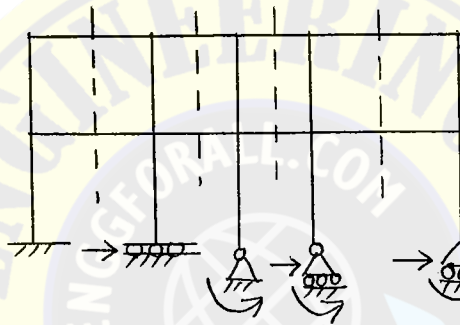
Unnecessary reactions (horizontal reactions & moments) are added so that it act like a fixed support

$D_s = 3 \times$ no. of cuts — no. of unnecessary reactions added to convert to stable cantilever.

$= 3 \times 8 - 6 = 18$

This technique is possible only perfect symmetric structures.

Q.
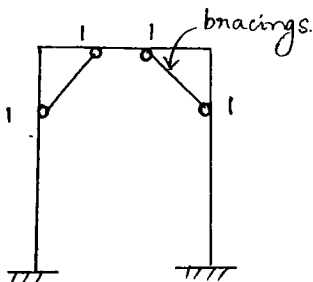
bracings.

Method 1:

Neglect bracings.

$D_{se} = 3$.

Axial deformations $= 2$

Total $D_s = 3+2 = 5$.

Method 2:

$D_{se} = 3$.

$D_{si} = 3 \times 2 = 6$.

Releases $= 4$

$D_s = 9-4 = 5$

14th July
Friday

CHAPTER - 2.

KINEMATIC INDETERMINACY.

Denoted by D_k.

Also called as degrees of freedom (DOF).

Kinematic Indeterminacy:

      The no. of unknown joint displacements is called Degrees of Freedom

Types of Joints	Degrees of Freedom.
1. Rigid joint of a plane frame	3 (δx, δy, θ).
NOTE: At a rigid joint, the included angle remains the same before and after displacement.	
2. Rigid joint of a Space frame	6 (3 rotations θ_{xy}, θ_{yz}, θ_{xz} & 3 translations δx, δy & δz)
3. Pin joint of a plane frame.	2 (δx, δy)
NOTE: As moments are not the design forces rotations are not considered in trusses	
4. Pin joint of a Space frame	3 (δx, δy, δz)
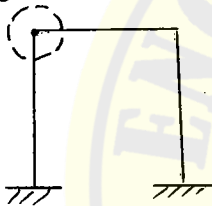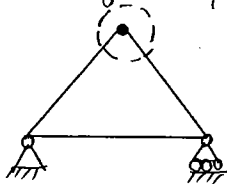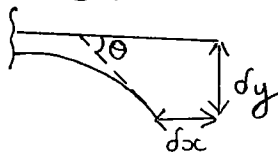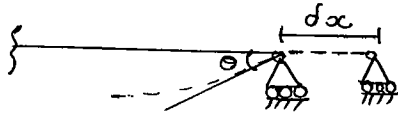
Types of Support	Degrees of freedom
1. Free end.	3 (δx, δy, θ).
2. Roller support.	2 (θ, δx).
3. Hinged / Pinned. support	1 (θ)
4. Fixed support	0
5. Vertical shear hinge	1 (δy)
6. Horizontal shear hinge,	1 (δx).
7. Damper support.	1 (δx).
8. Spring support	2 (θ, δx).
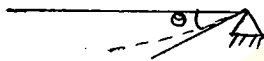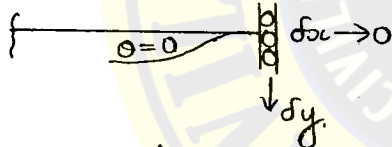
NOTE: reactions will resist displacements.

Vertical reaction, $\delta_v = 0$.

Horizontal reaction, $\delta_h = 0$

Moment reaction, $\theta = 0$

Effect of force releases on D.O.F :

(16)

1. Internal moment hinge.

4 (2 rotations — θ_1 & θ_2
2 translations — δx & δ_c
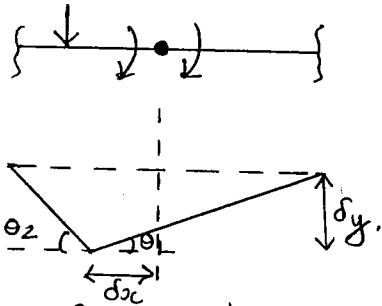
NOTE: Each member connected to a hinge can have its own rotation, in addition to δx & δy.

Eg:—
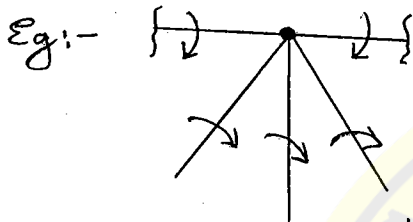
5 rotations & 2 translations.

2θ, δx, δy.

2. Horizontal Shear Release.
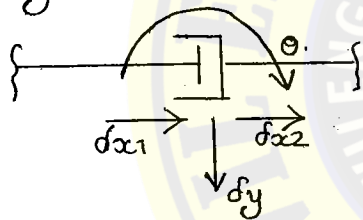
4 D.O.F
(2 horizontal transl. — δx_1 &
1 vertical trans — dy & θ.)

3. Vertical. Shear release
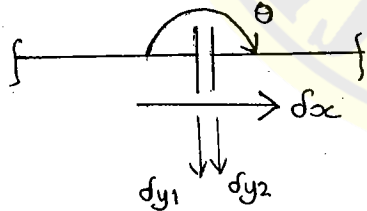
4 D.O.F
(δy_1, δy_2, δx & θ).

D_k of rigid jointed Plane Frame :
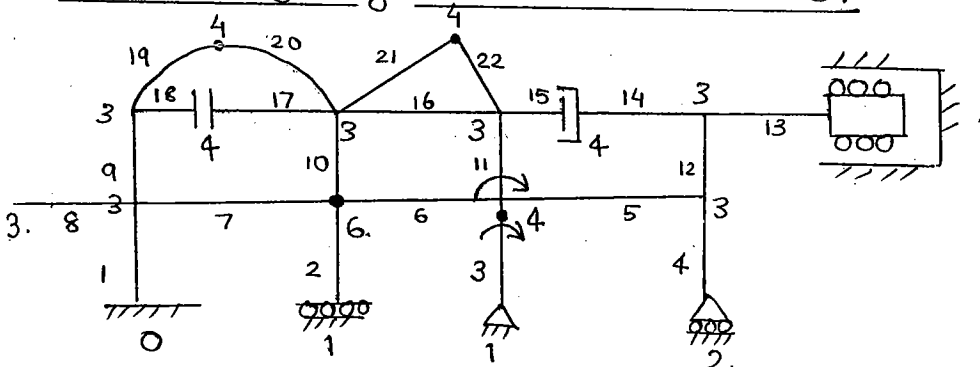
$D_k = 52$ (considering axial deformations)
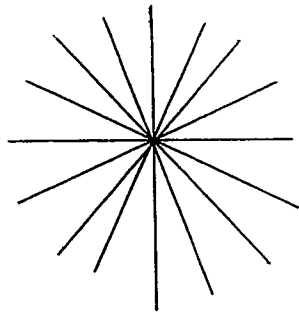
For a rigid joint with infinite members, there is only single rotation

For a hinged joint, there will be infinite rotations.

NOTE: Practically the axial deformations of members of rigid jointed structures are negligible.

Assume axial deformations of all members are neglected then $D_k = 52 - $ total no. of members.

$$= 52 - 22 = \underline{30} \text{ (neglecting axial deformations)}$$

* Axial deformations neglected or
members are rigid or
members are stiff or
members inextensible
} neglect axial deformati.

Q Find D_k when only beam is rigid
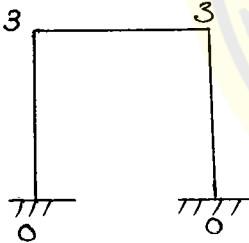
$D_k = 6 - 1$ (beam is rigid).

$= 5$.

No: of columns = 2.

$D_k = 6 - 2$ (if only beam columns are rigid).

$= 4$.

$D_k = 6 - 3$ (if all members are rigid).

$= \underline{3}$
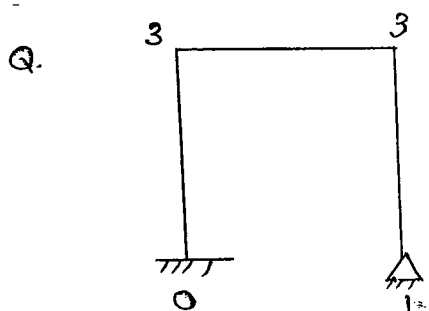
Q. $D_k = 7$ (considering axial def.)

No: of members = 3.

$D_k = 4$ (neglecting axial deformation).

$(\theta_B, \theta_C, \theta_D, \text{sway deflection } (\Delta))$

a) 2
b) 3
c) 4
d) 5.
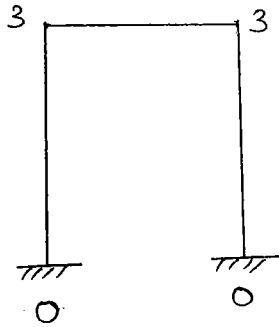
Ans : (b)
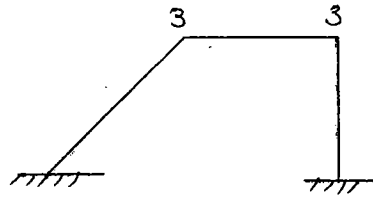
NOTE: whether vertical or inclined member, concept is same
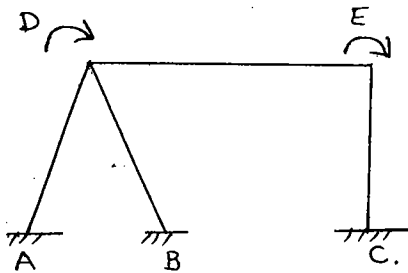
$D_k = 6 - 3$

$= \underline{\underline{3}}$

NOTE: as while working out axial deformations, consider the two inclined members as if 1.
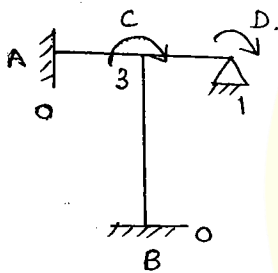
$D_k = 4$ (considering axial deformations).

$= 2$ (without A.D).
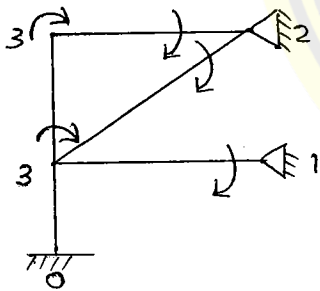
$D_k = 5$ (neglecting AD).

$= 9$ (considering AD).

D_k of Beams :-
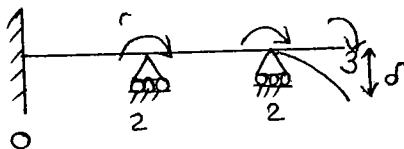
$D_k = 2 + 2 + 3 = 7$ (considering AD).

$= 7 - 3 = 4$. (neglecting AD)

Q A fixed beam is

a) Statically indeterminate

b) Kinematically determinate.   Ans : (c)

c) Both

d) None.

Q. A cantilever beam is

a) Statically determinate

b) Kinematically indeterminate

c) Both

d) None.

$(D_k = 3$ — considering AD.

$= 3-1$

$= 2$ — neglec. AD).

$\hookrightarrow \theta, \delta y$

Q.

a) $D_k = 3$

b) $D_s = 3$.

c) Both

d) None.

Ans : (c)

Q 3.

$D_s = 2$; $D_k = 2$. (neglecting AD).

Q. Dk of Pinjointed Plane Frames:

$D_k = 9$

(count the no: of members)
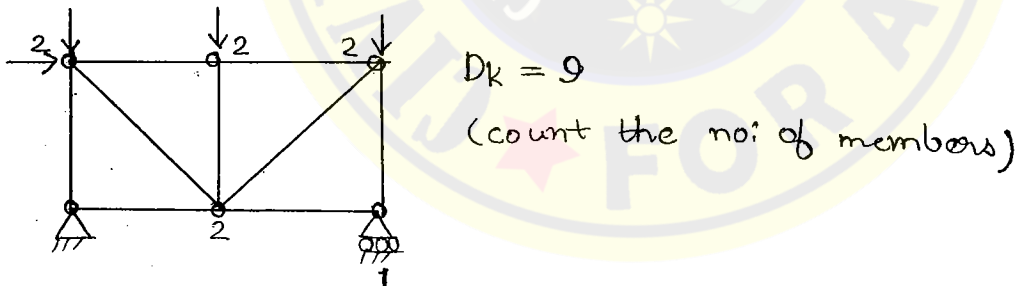
NOTE: Rotations are not considered in trusses. The only possible D.O.F in trusses are axial deformations. Hence the question of neglecting AD do not arise in pin-jointed trusses.

Formula for D_k :

$$D_k = NJ - c \quad \text{where} \quad N = \text{D.O.F at a joint.}$$

$J = $ no. of joints.

$c = $ compatibility equations

$N = 3$; rigid jointed plane frame

$N = 6$; " space frame

$N = 2$; pin jointed plane frame

$N = 3$ " space frame

(4)

(17)
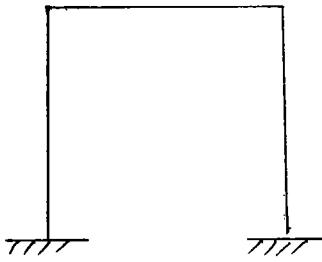
$J = 4$ (supports are also considered as joints).

C = reactions, if actual deformations considered

= $m + r$; if axial deformations are neglect

where $m \rightarrow$ no. of members.

$$D_k = 3 \times 4 - 6 = \underline{\underline{6}}$$
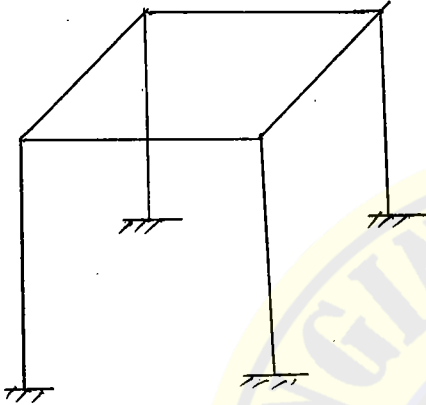
$$D_k = 6 \times 4 \text{ (joints)}$$

$$= 24 \text{ (considering AD)}$$

$$D_k = 24 - 8 \text{ (members)}.$$

$$= 16 \text{ (neglecting AD)}.$$

JULY

URDAY

Example 8:

But in actual case, 11 members will have 11 DOF.

2ⁿᵈ July
OT.

CHAPTER - 3

(18)

METHODS OF INDETERMINATE STRUCTURAL ANALYSIS.

Force method
or.

Compatibility method
or.

Flexibility Coefficient method.

Displacement method
or

Equilibrium method
or.

Stiffness Coefficient method

Displacement Method:

Displacements are unknowns.

Eg:- Slope deflection method.

$$M_{BC} = \frac{2EI}{L}\left(2\theta_B + \theta_C - \frac{3\delta}{L}\right) + \overline{M}_{FABC}$$
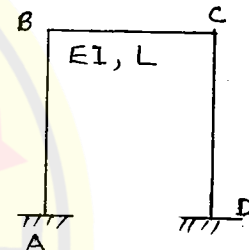
The end moment M_{BC} depends upon rotations at B & C (θ_B & θ_C) and relative sinking of B & C. These displacements are treated as unknowns. If displ. are calculated, then end moment M_{BC} can be calculated Since displ. are treated as unknowns initially, it is called Displacement method.

$$M_{BA} = \frac{2EI}{L}\left(2\theta_B + \theta_B - \frac{3\delta}{l}\right) + \overline{M}_{FBA}$$

We know $\Sigma M = 0$ at a rigid joint, say joint B.

Apply the eqbm eqn $\Sigma M = 0$ at B.

$$ie \quad M_{BA} + M_{BG} = 0$$

Similarly $M_{CB} + M_{CD} = 0$

We write such eqbm eqns and solve them to calculate unknown displacements. As eqbm eqns are used. This method is called Eqbm method.

In these methods, we use stiffness coefficients of various members. Hence called Stiffness method.

Eg :- ① Slope deflection method – GA Mani

② Moment distribution method – Hardy Cross
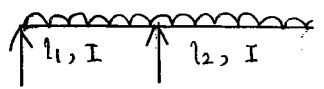
③ Kani's method (Rotation Contribution method).

④ Stiffness Matrix method

Force Method :

Forces are unknowns. (forces means redundant forces)

Eg: Theorem of Three moments – Prof. Claypeyron

$$M_A l_1 + 2M_B (l_1 + l_2) + M_C l_2 = -\frac{6a_1 \overline{x_1}}{l_1} - \frac{6a_2 \overline{x_2}}{l_2}$$

In the above eqn, M_A, M_B & M_C are unknowns. ~~Hence~~ It is called Force method. because forces are directly treated as unknowns Since in these methods, compatibility eqns are used, it is called Compatibility method.

Flexibility is the inverse of stiffness. As we use the flexibility concepts, force methods are also called Flexibility Coefficient method.

Eg :- ① Theorem of 3 moments.

② Method of consistent deformation

③ Elastic Centre method.

④ Column Analogy method.

⑤ Energy Principles (Castigliano's min. Strain energy method).

⑥ Flexibility matrix method.

Suitability of these Methods :

In displacement method, displacements are unknowns ie DOF (Dk).

In force method, redundant forces are unknowns. Hence Ds is the deciding factor.

If Dk < Ds for a structure, displacement methods are preferred

If Ds < Dk, force methods are preferred.
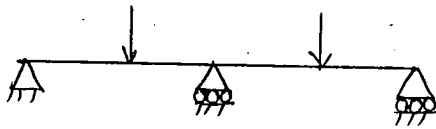
$D_S = 3 - 2 = 1$

$D_K = 1 + 1 + 1 = 3$

$D_S < D_K \Rightarrow$ force methods are recommended.

$D_S = 5 - 2 = 3$

$D_K = 1$.

$D_K < D_S \Rightarrow$ displacement methods are preferred.

NOTE: Irrespective of the values of D_S & D_K, displacement or stiffness matrix methods are more popular, compared to force or flexibility matrix methods. Generation of stiffness matrix or elements of displ. is easy.

NOTE:
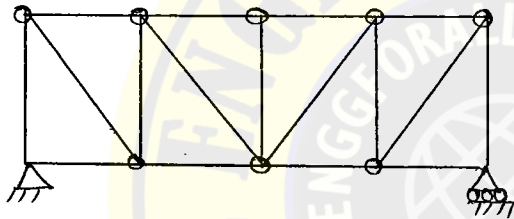
$D_S = 0$

$D_K = 17$

In the case of trusses, as shown in eg, $D_S \ll D_K$. Hence force methods are preferred for pin-jointed truss analysis.

CHAPTER - 4

DETERMINATE TRUSS ANALYSIS

20

Assumptions :

1. Members of the truss will be subj. to axial forces only. The SF & BM are neglected.

2. Members are straight.

3. Loads will act at joints only.
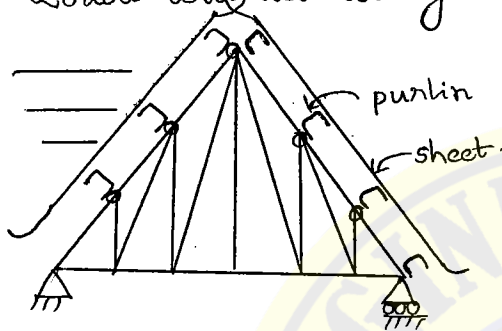
Wind forces will act on the roof sheet. Wind load is transferred to purlin as reaction. As the purlins are kept at joints, loads will be transferred to the joints.

4. Joints are frictionless hinges.

   NOTE: Friction means rotational resistance. Resistance to rotation means moment. As moments are not considered in the design of trusses, friction is neglected.

5. All the members of truss are assumed in the same plane called middle plane of truss.

   NOTE: While fabrication, different members of the truss are joined so that their centroidal axis will coincide, as eccentricity is zero, BM are zero.

   Joints are also designed so that the CG of the member and the CG of the welded joint/slash CG of rivetted joint will coincide.

Sign Convention of Forces :
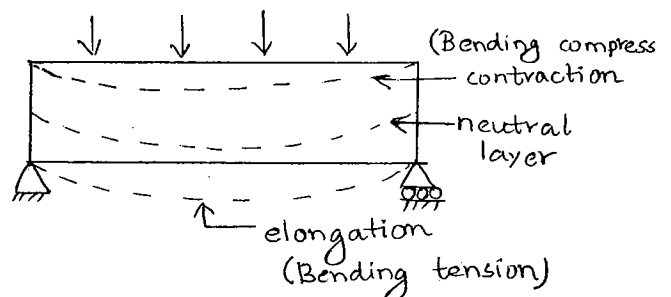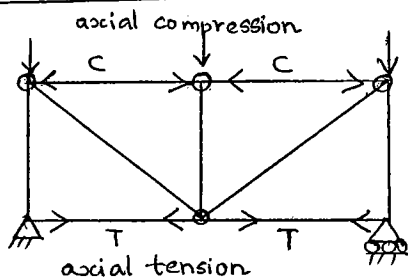
Pushing the Joints or Arrows towards the joint → Axial comp⁸

Free Body Diagram (FBD):

Part of a structure with actions and reactions is called FBD. Actions means external loads. Reactions means internal forces developed. The actual design force of a member can be calculated from FBD only.

$$\xrightarrow{\hspace{1cm}} \underleftarrow{\hspace{1.5cm} C \hspace{1.5cm}} \qquad \underrightarrow{\hspace{1.5cm} T \hspace{1.5cm}} \xleftarrow{\hspace{1cm}}$$
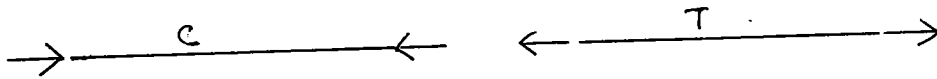
Pulling the joints or arrows away from the joint → axial tens⁸

Methods of Truss Analysis:

1. Methods of Joints
2. Methods of Sections.
3. Tension Coeffecient method.
4. Graphical method.

$y = F \sin\theta$
$x = F \cos\theta$.

$x = F \cos 90$
$= 0$

$y = F \cos 90°$
$= 0$

$\theta = 45°$ $F/\sqrt 2$

$45°$
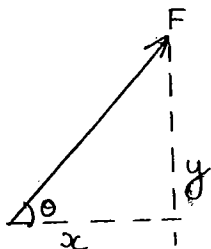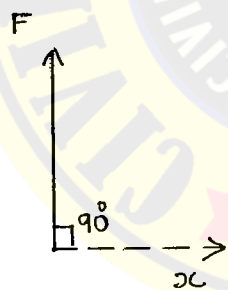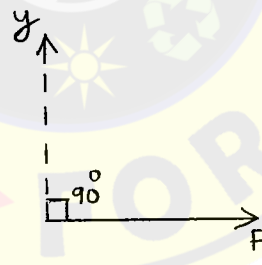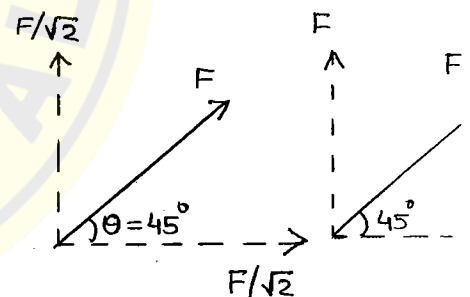
NOTE: Horizontal force cannot have vertical component, whereas the vertical force cannot have horizontal component.

Rule 1: A single force cannot exist in nature. If it exist it must be zero.

Rule 2: If two forces act at a joint and if they are not in the same line, then each force must be zero.
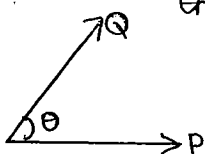
$Q \sin\theta$

Resolving vertically,

$Q \sin\theta = 0$ ($\sin\theta \neq 0$).

$\Rightarrow Q = 0$

$P + Q \cos\theta$

Corollary: If the two forces are of the same line, their ㉒ they must be equal and opposite.

Rule 3: If 3 forces act at a joint and if two of them are in the same line, then the third force must be zero

$$\Sigma V = 0$$
$$R\sin\theta = 0 \quad (\sin\theta \neq 0)$$
$$\Rightarrow R = 0$$
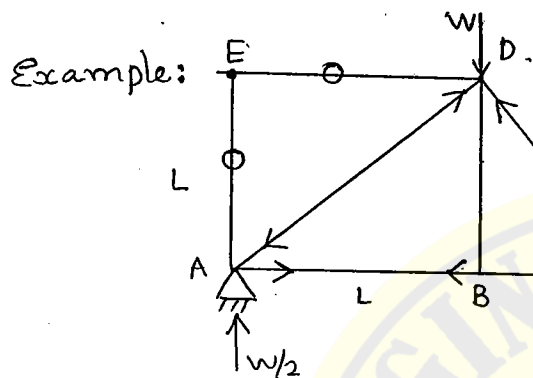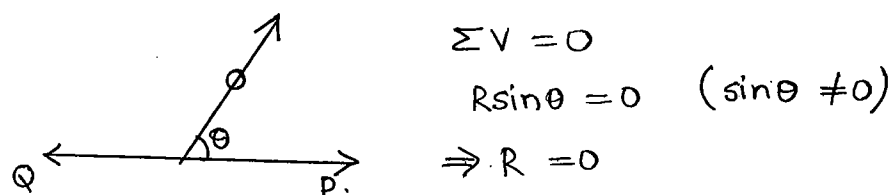
Example:

Apply Rule 2 at E,
$$F_{EA} = F_{ED} = 0$$

Apply Rule 3 at B,
BA & BC are in the same line
Third force BD must be zero.

NOTE: A member will have 2 joints. (for example BD).
Analyse at that joint where unknown forces are minim
For the member BD, joint B has min. forces

Method of Joints:

Step 1 : Calculate the reactions at the supports
$$V_A = V_C = \frac{W}{2}$$

Step 2 : Start from that joint where the unknown forces ar not more than two.

The no. of eqbm eqns at the pin-joint of a pin jointed plane frame are two.

Step 3: Then move from joint to joint till the analysis is completed.

1) Start at joint E:
Two forces EA and ED are zero.

2) Analyse at joint A:
Resolving vertically,
$$F_{AD}\sin 45 = \frac{W}{2} \quad \Rightarrow F_{AD} = \frac{W}{\sqrt{2}}$$
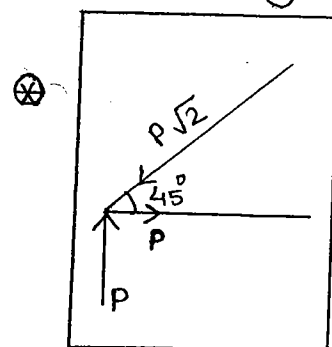
Resolve horizontally at A,

$$F_{AD} \cos 45 = F_{AB}.$$

$$\therefore \frac{W}{\sqrt{2}} \times \frac{1}{\sqrt{2}} = F_{AB}$$

$$\Rightarrow F_{AB} = \frac{W}{2} \text{ (pulling the joint, tension).}$$

3) Analyse at joint D,

$$\Sigma V = 0 \text{ at D,}$$

$$F_{DA} \cos 45 + F_{DB} = W + F_{DC} \cos 45 \rightarrow \text{①}$$

$$\Sigma H = 0 \text{ at D,}$$

$$F_{DA} \sin 45 = F_{DC} \sin 45$$

$$\Rightarrow F_{DA} = F_{DC} = \frac{W}{\sqrt{2}} \rightarrow \text{②}$$

Substituting ② in ①,

$$2 F_{DA} \cos 45 + F_{DB} = W$$

$$2 \frac{W}{\sqrt{2}} \times \frac{1}{\sqrt{2}} + F_{DB} = W.$$

$$\therefore F_{DB} = 0$$
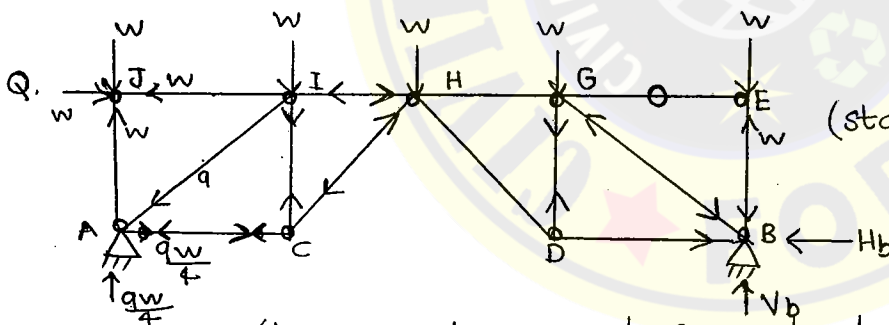
(stable : bottom chord is missing, but that is taken care of by extra horizontal reaction in the line of deficient mem be

NOTE : If supports are at same level, the reactions can be calculated similar to that of a SSB (for vertical reactions)

$$\Sigma M = 0 \text{ at A,}$$

$$V_B \times 4L = W \times 4L + W \times 3L + W \times 2L + W \times L + W \times 0 + W \times L$$

$$V_B = \frac{11W}{4}$$

$$V_A = 5W - \frac{11W}{4} = \frac{9W}{4}$$

Calculation of horizontal reaction :

Apply $\Sigma M = 0$ at H, (from right side).

$$H_b \times L + W \times 2L + W \times L = \frac{11W}{4} \times 2L$$

$\Rightarrow H_b = +\dfrac{5w}{2}$ ('+ sign' indicates assumed direction is correct)

(22)

$\therefore -H_a + H_b = w.$

$\Rightarrow H_a = 1.5 w$
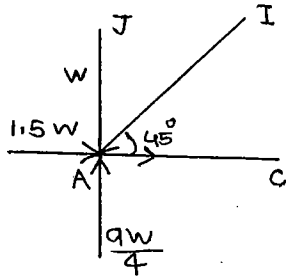
Joint A:

$\Sigma V = 0$

$w + F_{AI} \sin 45 = \dfrac{9w}{4}$

$F_{AI} \sin 45 = 1.25 w.$

$F_{AI} = \dfrac{5}{2\sqrt{2}} w$

$\Sigma H = 0.$

$1.5w + F_{AC} = F_{AI} \cos 45°.$

$= \dfrac{5}{4} w$

$F_{AC} = -\dfrac{w}{4}$ (-ve sign shows assumed direction is wrong.

Joint I:
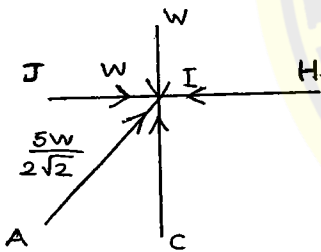
$\Sigma V = 0 \Rightarrow \dfrac{5w}{2\sqrt{2}} \times \dfrac{1}{\sqrt{2}} + F_{CI} = w.$

$F_{CI} = w - \dfrac{5w}{4} = \dfrac{-w}{4}$ (assumed direct is wrong).

$F_{CI} = \dfrac{w}{4}$ (tension).

$\Sigma H = 0 \Rightarrow F_{HI} = w + \dfrac{5w}{2\sqrt{2}} \times \dfrac{1}{\sqrt{2}} = 2.25 w$ (compression

Joint C:
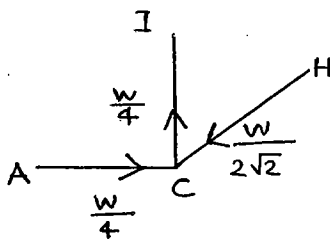
$\Sigma H = 0 \Rightarrow F_{CH} \cos 45 = \dfrac{w}{4}.$

$\therefore F_{CH} = \dfrac{w}{2\sqrt{2}}$ (compression).

Joint H:
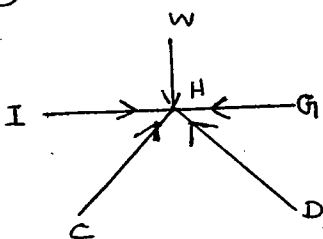
$\Sigma H = 0 \Rightarrow F_{IH} + \dfrac{w}{2\sqrt{2}} \times \dfrac{1}{\sqrt{2}} = F_{GH} + F_{DH} \times \dfrac{1}{\sqrt{2}}.$

$(2.25 + 0.25) w = F_{GH} + \dfrac{F_{DH}}{\sqrt{2}} \longrightarrow ①$

$$\Sigma V = 0 \Rightarrow \frac{w}{2\sqrt{2}} \times \frac{1}{\sqrt{2}} + \frac{F_{DH}}{\sqrt{2}} = w$$

$$F_{DH} = \left(w - \frac{w}{4}\right)\sqrt{2} = \frac{3w}{2\sqrt{2}} \quad (\text{compression}), \rightarrow ②$$

Substituting ② in ①

$$\Rightarrow F_{GH} + \frac{3w}{4} = 2.5\,w$$

$$F_{GH} = \frac{8w}{2} \, 1.75\,w \quad (\text{compression})$$

Joint G
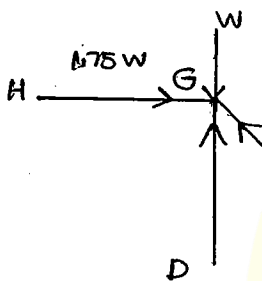
$$\Sigma H = 0 \Rightarrow F_{GH} = \frac{F_{GB}}{\sqrt{2}}$$

$$\therefore F_{GB} = \frac{7w}{4} \times \sqrt{2} = \frac{7w}{2\sqrt{2}} \quad (\text{compression})$$

$$\Sigma V = 0 \Rightarrow F_{DG} + \frac{F_{BG}}{\sqrt{2}} = w$$

$$F_{DG} = w - \frac{7w}{4} = \frac{-3w}{4}$$

$$\therefore F_{DG} = \frac{3w}{4} \quad (\text{tension})$$

Joint B :

$$\Sigma H = 0 \Rightarrow F_{BD} + \frac{7w}{4} = \frac{5w}{2}$$
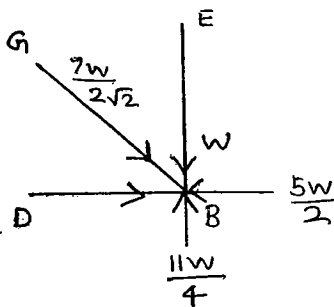
$$F_{BD} = (2.5 - 1.75)\,w$$

$$= \frac{3w}{4} \quad (\text{compression})$$

Q. Calculate magnitude of force in member AB.

(28) ④
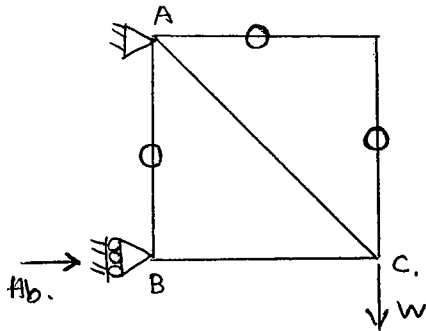
NOTE: Reaction at roller support to be B is normal to the plane of rolling At B, now we have 3 forces. 2 of them are in same line. Hence third force is zero. $\therefore F_{AB} = 0$.

Q.
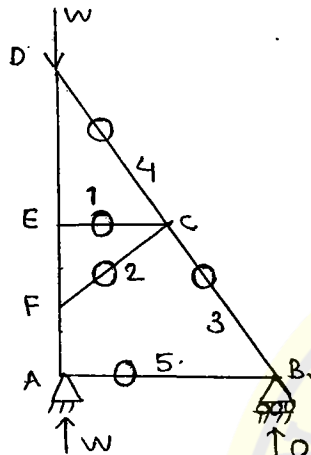
Step 1 : Calculate reactions.

Step 2 : At joint E, DE & EF in same line, $\therefore EC = 0$.

Similarly consider joint F. $\Rightarrow FC = 0$

Q.

Calculate force in AB ?
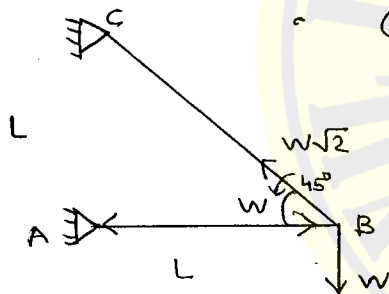
$\sum V = 0$ @ B,

$F_{BC} \sin 45 = W$

$F_{BC} = W\sqrt{2}$ (tension).

$F_{BC} \cos 45 = F_{AB}$.

$\dfrac{W\sqrt{2}}{\sqrt{2}} \times = F_{AB} \Rightarrow F_{AB} = W$ (compression).

Q.
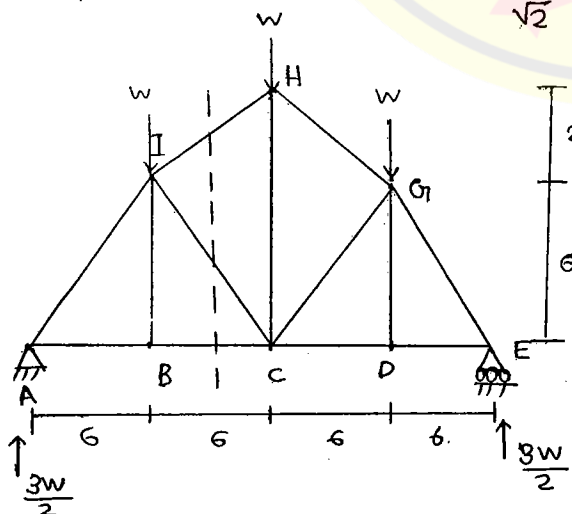
Calculate F_{BC}, F_{BI}, F_{CI}, F_{IH}, $F_{AC} =$

NOTE: In the above problem, forces of some selected members only are to be calculated. If method of joints is used, we have to proceed from one end of truss which i time consuming.

Method of Sections:

It is useful for quick soln of forces in any internal member directly. In method of sections, we choose part of a truss to one side of the section with the reaction and the external loads. Hence even if three unknowns act at a joint, the method is suitable.

NOTE: Part of the truss with the actions and reactions will behave as the truss as a whole and hence three eqbm eqns are available, ∴ even if unknowns are three, this method is useful.

Sub method 1 :

Applying $\Sigma M = 0$ concept
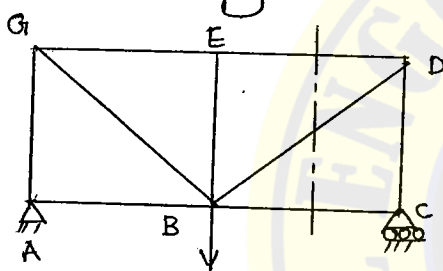
$F_{DE} = ?$

Step 1: Calculate reactions.

Step 2: Pass a section through the choosen member (DE) and two other members (BD & BC) so that the other members will pass through a common joint (say B) so that the other members will not have any moment about B.

Step 3: Consider FBD of one side of truss as shown.
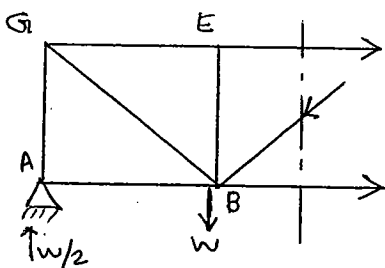
Apply $\Sigma M_B = 0$

$$F_{DE} \times L + \frac{W}{2} \times L = 0$$

$$\Rightarrow F_{DE} = -\frac{W}{2} \quad \text{(-ve indicates tension assum}$$
$$\text{for } F_{DE} \text{ is wrong. ∴ } F_{DE} = \frac{W}{2}$$
$$\text{(compression)}$$

∴ $F_{DE} = \frac{W}{2}$ (compression).

Sub method 2: $\Sigma H = 0$ Concept.

⑤
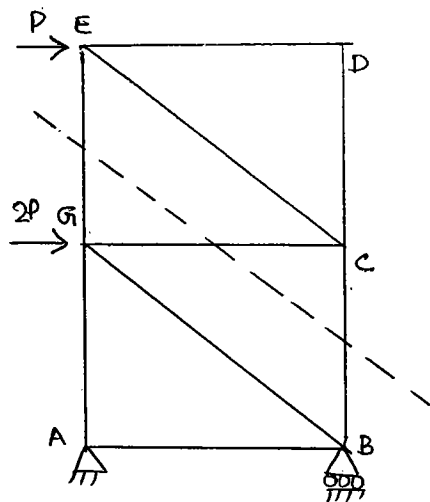
Calculate CG ?

24

Step 1: Pass a section through choosen member and other vertical members, so that these vertical members' cut will not develop any horizontal component.

Step 2: Consider 1 side of section, Apply $\Sigma H = 0$.
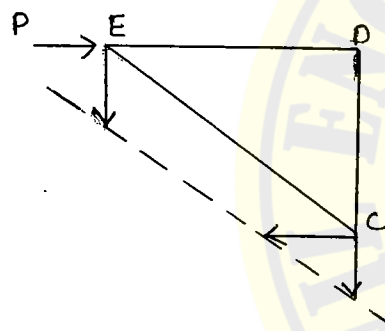
$F_{CG} = P$ (tension).

NOTE: Uncut members are already balance (don't consider ED).

Sub method 3: $\Sigma V = 0$ concept
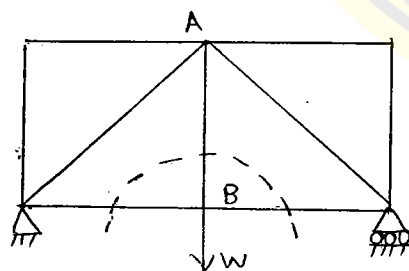
$F_{AB} = ?$

Step 1: Pass a section through the choosen vertical member and other horizontal members so that horizontal members cut will not have vertical force components.

Step 2: Apply $\Sigma V = 0$ at B.
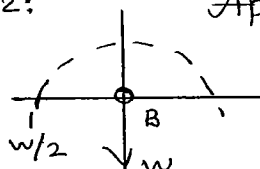
$F_{BA} = W$ (tension).

14th JULY
MONDAY

LEVEL 2 : Q - 03.

Ans : (a)
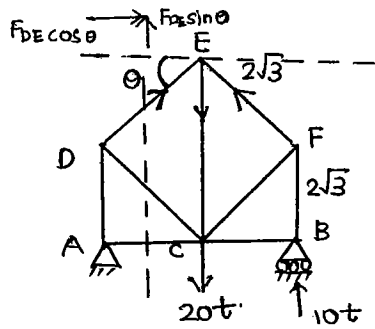
Pass a section as shown

By calculating the forces in the members DE & EF, their vertical component is the force in the member CE.

Now for the section choosen, the choosen member is DE.

Consider right side of the section

Apply $\Sigma M = 0$ at C.

$\tan\theta = \dfrac{2}{\sqrt{3}} \times \dfrac{1}{2} = \dfrac{1}{\sqrt{3}} \Rightarrow \theta = 30°$

Assume force in member DE as compressive.

Resolve the force F_{DE} in the vertical and horizontal directions.

$F_{DE} \cos 30° \times \left(\dfrac{2}{\sqrt{3}} + \dfrac{2}{\sqrt{3}} \right) = 10 \times 2$.

$$F_{DE} = \underline{10\,t} \quad (compression)$$

Now $F_{EF} = 10\,t$ (symmetry).

Apply $\Sigma V = 0$ at E,

$F_{DE} \sin 30 + F_{EF} \sin 30 = F_{CE}$.

$\dfrac{10}{2} = F_{CE}$

$\Rightarrow F_{CE} = 10\,t$ (tensile).

05.
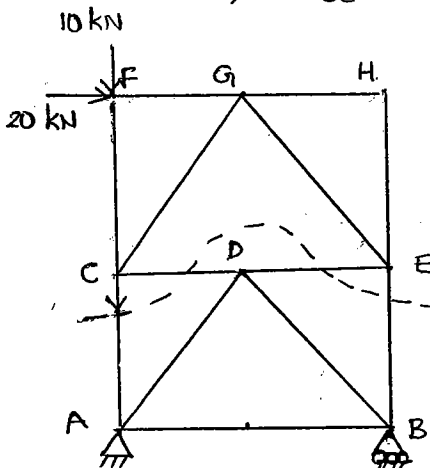
Consider section as shown. Ans : (c)

Consider upper side of the section,

By taking $\Sigma M_E = 0$.

$20 \times 3 = 10 \times 6 + F_{AC} \times 6$

$\underline{\underline{F_{AC} = 0}}$

(6)

06.
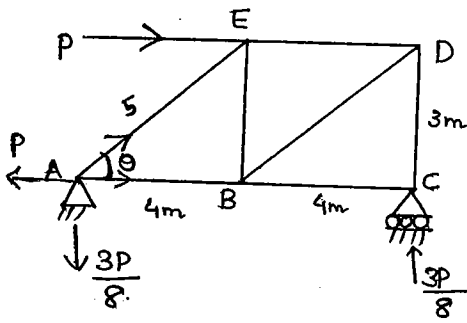
$\Sigma M_A = 0$

$8V_C = 3P.$

$V_C = \dfrac{3P}{8}$

$\sin\theta = \dfrac{3}{5}$

$\cos\theta = \dfrac{4}{5}$

Apply $\Sigma V = 0$ at A,

$$F_{AE} \sin\theta = \dfrac{3P}{8}$$

$$F_{AE} \times \dfrac{3}{5} = \dfrac{3P}{8} \implies \underline{\underline{F_{AE} = \dfrac{5P}{8}}}$$

Similarly, $F_{BE} = \dfrac{3P}{8}$

$\Sigma H = 0$ at A,

$$P = F_{AB} + \dfrac{5P}{8} \times \dfrac{4}{5}$$

$$F_{AB} = \dfrac{4P}{8}$$

NOTE: For the problem shown forces in the member AB, BE AE are proportional to distances

07.
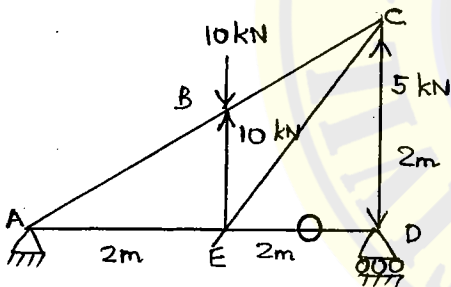
08. Pass a section as shown,

$$\tan\theta = \dfrac{2}{6} = \dfrac{1}{3}$$
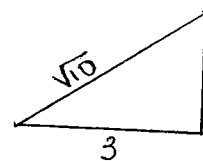
$\sin\theta = \dfrac{1}{\sqrt{10}}$

$\cos\theta = \dfrac{3}{\sqrt{10}}$

Resolve the force IH into hor. & vert. components.

It's vertical component is passing through B about which $\Sigma M = 0$

Horizontal component of $F_{HI} = F_{HI}\cos\theta.$

Apply $\Sigma M = 0$ at B,

$$6 F_{HI} \cos\theta = \dfrac{3w}{2} \times 6 = 9w$$

$$6 \times F_{HI} \times \dfrac{3}{\sqrt{10}} = 9.w$$

$$\underline{\underline{F_{HI} = \dfrac{\sqrt{10}w}{2}}} = F_{HG}$$

Calculation of force in the member BI :

Apply $\Sigma V = 0$ to the left side of the section

$$w + \frac{\sqrt{10}}{2} \sin\theta = \frac{3w}{2} + F_{BI}$$

$$F_{BI} = 0$$

* Another method to calculate force in the member BI,

Apply $\Sigma m = 0$ about imaginary point O,

$$\frac{3w}{2} \times 12 + F_{BI} \times 18 = w \times 18$$

$$\Rightarrow \underline{\underline{F_{BI} = 0}}$$

Calculation of Force in the member BH :

Pass a section as shown.
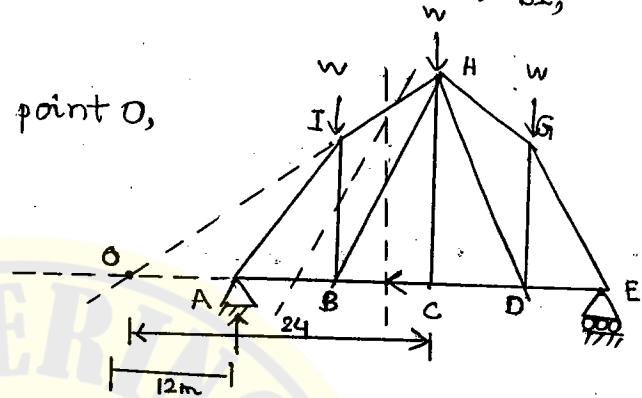
BH is chosen member. Consider left side of section.

Vertical component of BH

Calculation of Force BC :

Apply $\Sigma m = 0$ at H for the right part of the section.

$$8 F_{BC} + w \times 6 = \frac{3w}{2} \times 12$$

$$F_{BC} = \frac{3w}{2} \text{ (tensile)}$$

11.

Pass a section as shown,
Consider left side of section,
$\Sigma V = 0$,
$$w = w + F_{CF} \sin\theta$$
$$F_{CF} \sin\theta = 0 \Rightarrow F_{CF} = 0$$

(7)

(26)
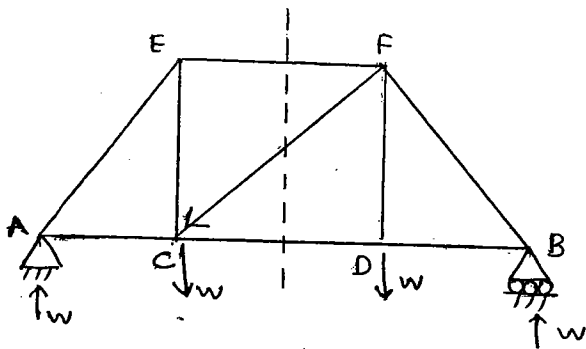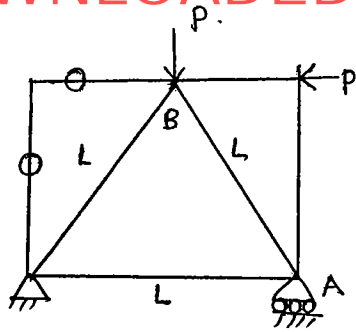
$\Sigma H = 0 \quad @ B,$

$F_{BA} \sin\theta + P = F_{BE} \sin\theta. \Rightarrow F_{BA} = F_{BE} - 2P.$

$\dfrac{F_{BA}}{2} + P = \dfrac{F_{BE}}{2}.$

$\Sigma V = 0 \quad @ B, \quad F_{BA} \cos\theta + F_{BE} \cos\theta = P.$

$\left(F_{BE} - 2P\right) \dfrac{\sqrt{3}}{2} + F_{BE} \dfrac{\sqrt{3}}{2} = P.$

CHAPTER - 05

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ENERGY PRINCIPLES

Strain Energy:

The energy stored within the recoverable part of stress-strain curve.

NOTE: It is the area under stress-strain curve.

* Area above the stress-strain curve is called 'Complementary energy' (U*)

* For a linear elastic system, $U = U^*$

Loads

Gradually Applied Loads.

$t = 0, P = 0, \delta = 0$
$t = \delta t, P = P, \delta = \delta$

Suddenly Applied Loads

$t = 0, P = P, \delta = 0$
$t = \delta t, P = P, \delta = \delta$

Impact Loads.
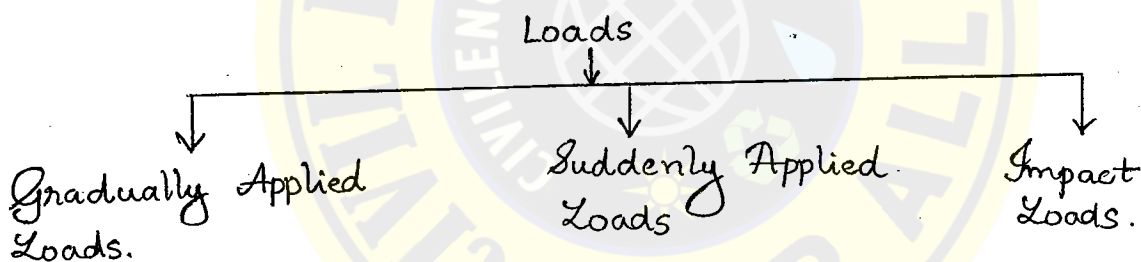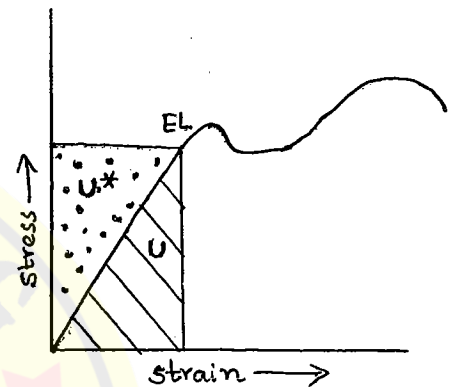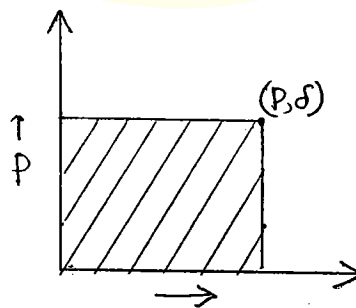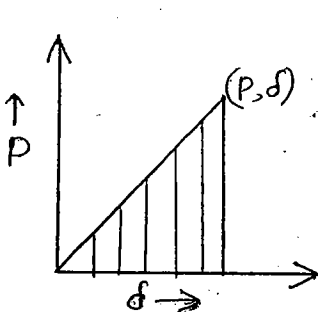
The area under load deformation curve = Work done or Strain energy.

∴ Strain energy (U) due to gradually applied load = $\frac{1}{2} P\delta$

$$U = \frac{1}{2} P \cdot \frac{Pl}{AE} = \frac{P^2 l}{2AE}$$

$$U = \frac{P^2 l}{2AE} = \frac{1}{2E} \frac{P}{A} \times \frac{P}{A} \times \frac{Al}{l}$$

$$= \frac{1}{2E} \sigma * \sigma * (volume)$$

$$= \frac{\sigma^2 * volume}{2E}$$

$$= \frac{1}{2} \times \sigma \times \frac{\sigma}{E} \times volume$$

$$\boxed{U = \frac{1}{2} \times stress \times strain \times volume}$$

$$U = \frac{\sigma e}{2} \times volume = \frac{Ee \cdot e \times volume}{2} \qquad E = \frac{\sigma}{e}$$

$$U = \frac{Ee^2 \times volume}{2}$$

Strain energy due to suddenly applied load, $U = P\Delta$.

Strain energy due to gradually applied load, $U = \frac{P^2 l}{2AE}$

The truss members which have axial forces as the design forces shall use the above formula.
where 'P' is axial force in any member.
'l' is length of member
'AE' is axial rigidity.

$$F_{AB} = 0 \implies U = 0$$

$$F_{CA} = W\sqrt{2}$$

$$U = \frac{P^2 l}{2AE} = \frac{(W\sqrt{2})^2 \times (\sqrt{2} l)}{2AE}$$

$$U_{AC} = \frac{\sqrt{2} \, W^2 l}{AE}$$

Q.
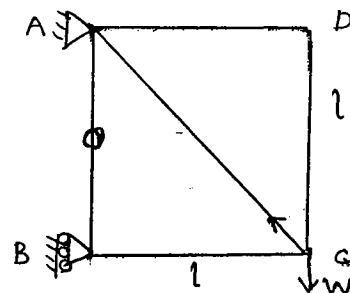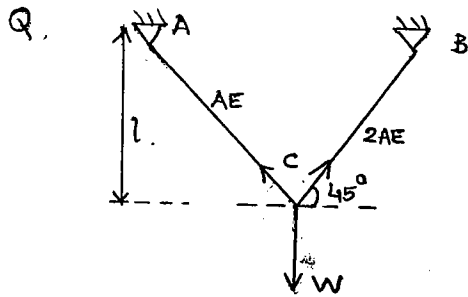
$\Sigma V = 0 \ @ \ C,$

$F_{CA} \sin 45 + F_{CB} \sin 45 = w$

$\Sigma H = 0 \ @ \ C,$

$F_{CA} = F_{CB}.$

$\Rightarrow F_{CA} = F_{CB} = \dfrac{w}{2\sin 45} = \dfrac{w}{\sqrt{2}}$

28

$F_{AC} = F_{BC} = \dfrac{w}{\sqrt{2}}$

$U_{AC} = \dfrac{P^2 l}{2AE} = \dfrac{\left(\frac{w}{\sqrt{2}}\right)^2 \times \sqrt{2}\, l}{2AE} = \dfrac{w^2 l}{2\sqrt{2}\, AE}$

$U_{BC} = \dfrac{\left(\frac{w}{\sqrt{2}}\right)^2 \times \sqrt{2}\, l}{2\times 2AE} = \dfrac{w^2 l}{4\sqrt{2}\, AE}$

$U = U_{AC} + U_{BC} = \dfrac{3w^2 l}{4\sqrt{2}\, AE}$

Deflection of Statically Determinate Truss Joints:

Castigliano's Theorem I can be used to calculate the deflection of truss joints.

Statement — In elastic structures, the partial derivative of strain energy wrt the load at any point gives the deflection or deformation in the same direction

$\dfrac{\partial U}{\partial w} = \partial v.$

$\dfrac{\partial U}{\partial w} = \delta_h.$

$\dfrac{\partial U}{\partial m} = \theta.$
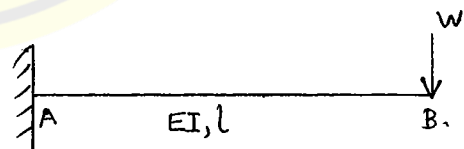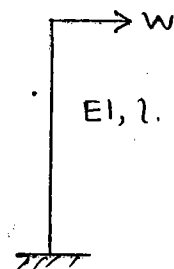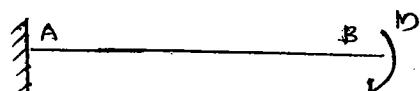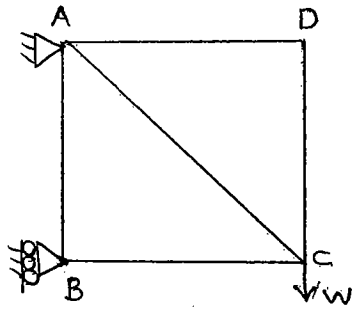
$$d_{vc} = \frac{\partial U}{\partial w}$$

$U \to$ total strain energy of truss

$d_{vc} \to$ vertical deflection at c.

Load w is acting at the point (say c) where deflection is required.

Unit Load Method :

It is the extension of Castigliano Theorem to calculate displacements in various structures.

In case of trusses, $\delta = \dfrac{\Sigma PKl}{AE}$

where $P \to$ force in a member due to the given external load system.

$K \to$ force in a member by applying unit load in the direction at the point where deflection is desire after removing the given external loads.

$l \to$ length of the concerned member

$AE \to$ axial rigidity of the member

Calculate horizontal and vertical deflections at c.
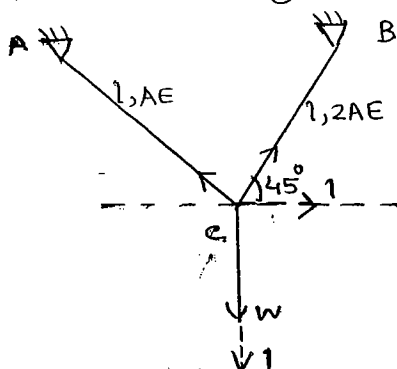
Sign Convention :

tension/elongation — (-ve)

compression — (+ve)

Member	P	K	K'	l	AE
AC	$-\dfrac{W}{\sqrt{2}}$	$\dfrac{-1}{\sqrt{2}}$	$-\dfrac{1}{\sqrt{2}}$	l	AE
BC	$-\dfrac{W}{\sqrt{2}}$	$\dfrac{-1}{\sqrt{2}}$	$\dfrac{1}{\sqrt{2}}$	l	2AE

Apply a unit vertical load as shown

For vertical deflection,

$$\delta_V = \sum \frac{PKl}{AE} = \frac{wl}{2AE} + \frac{wl}{4AE} = \underline{\frac{3wl}{4AE}}$$

Calculation of Horizontal deflection, at c :—

Apply a unit horizontal load as shown, and analyse the truss

$\sum V = 0 \Rightarrow F_{CA} = -F_{CB}$.

$\sum H = 0 \Rightarrow F_{CA} \cos 45 = 1 + F_{CB} \cos 45$.

$$\frac{2 F_{CB}}{\sqrt{2}} = -1.$$

$$F_{CB} = \frac{-1}{\sqrt{2}}$$ (-ve indicates that tension assumed in F_{CB} is wrong. It is compression)

K' represents values due to unit horizontal load.

$$\bar{\delta}_H = \sum \frac{PK'l}{AE} = \frac{wl}{2AE} - \frac{wl}{4AE} = + \frac{wl}{4AE}$$ (+ve sign indicates the horizontal deflection assumed towards right si is correct)

* $\delta = \sum \dfrac{Pkl}{AE} = \sum k\delta'$; δ' represents the actual deformation in a member may be due some external load system or temperature changes or lack of fit

Q.
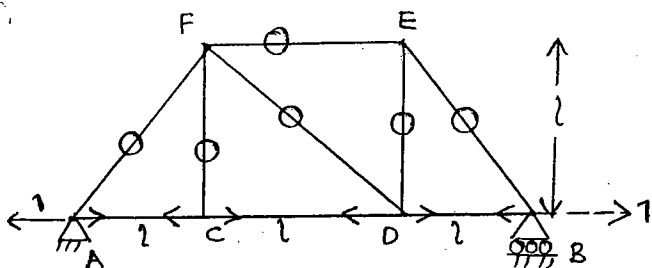
The temp of bottom chord members increased by $5°c$ each. Assume $\alpha = 10 \times 10^{-6}/°c$, $l = 4m$. Calculate the horizontal movement of support B.

Change in length, $\delta\ell = \alpha(\Delta T)\,l$

$$= 10 \times 10^{-6} \times 50 \times \underline{4000}$$

$$= \underline{2\ mm}$$

To calculate horizontal deflection at roller support B, apply unit horizontal force

Analyse the truss for the given unit horizontal load,

NOTE: No vertical reactions.

All the bottom chords have unit force

$$\delta_h = \Sigma k\delta'$$

$$= 3(1 \times 2) = \underline{6\ mm}$$

∴ Support B will move 6mm towards right.
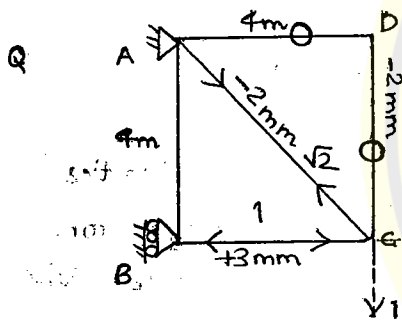
The lack of fit of various members of a truss are shown in fig. '+' indicates short fall of length; '−' indicates excess length Calculate the vertical and horizontal deflections at C.

Calculation of Vertical Deflection at C:

Apply unit vertical load at C, calculate the values of K.

$$\delta_v = (-\sqrt{2}) \times (-2) + 1 \times 3 + = \underline{3 + 2\sqrt{2}} = \underline{5.828}\ mm$$

'+ve' indicates downward deflection assumed is correct

Calculation of Horizontal Deflection at C:

$$\delta_{hc} = -1 \times 3 = \underline{-3\ mm}$$

'−ve' indicates assumed direction is wrong
Deflection is towards inside.
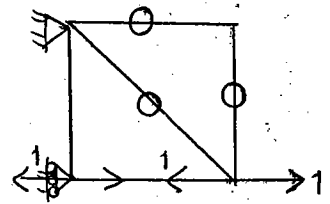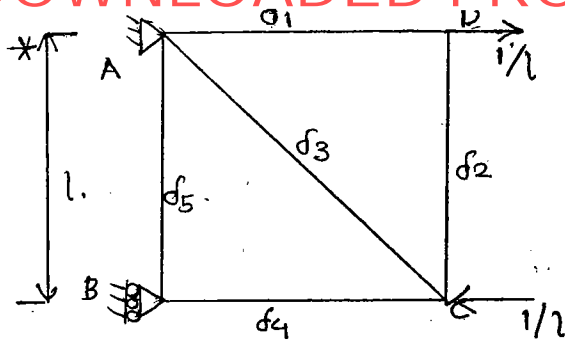
d's are individual axial deformations (4)
of member..

(36)

Calculate Rotation of member CD.

Rotation corresponds to moment couple. (unit couple).

Unit couple is created by applying a couple of $1/l$ at C & D.

Analyse the given truss ~~for the~~ forces applied.

$$\theta_{CD} = \Sigma k \delta'$$

Strain Energy due to Bending:

$$U = \int_0^l \frac{M_x^2 \, dx}{2EI_x}$$

$M_x \rightarrow$ BM @ a section 'x'.

$I_x \rightarrow$ MI @ the section x.
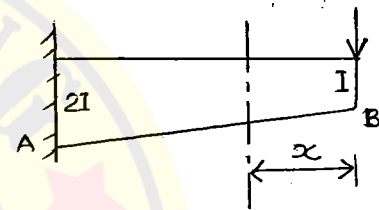
Q. Cantilever subj. to point load at free end.
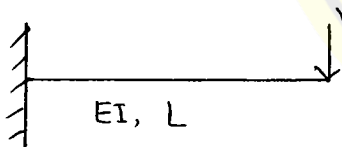
$$M_x = -wx \text{ (hogging)}$$

$$U = \int_0^l \frac{M_x^2 \, dx}{2EI}$$

$$= \frac{\omega^2}{2EI} \int_0^l x^2 \, dx = \frac{\omega^2 l^3}{6EI} \qquad \Rightarrow \qquad \boxed{U = \frac{\omega^2 l^3}{6EI}}$$

NOTE: Strain energy is always +ve (as square M_x is used).

Q.

$$M_x = -\left(\frac{\omega}{l}\right) x \cdot \frac{x}{2} = -\frac{\omega x^2}{2l}.$$
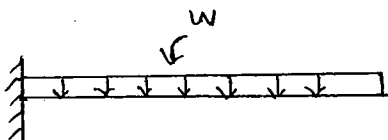
$$U = \frac{\omega^2}{8EI} \int_0^l (x^2)^2 = \frac{\omega^2 l^3}{40EI}$$

$$\boxed{U = \frac{W^2 l^3}{40EI}} = \frac{\omega^2 l^5}{40EI}$$

$W \rightarrow$ total load

$\omega \rightarrow$ load/m.

NOTE : The ratio of strain energies of a cantilever with the point load at free end and udl of same magnitude

$$\frac{U_{Pl}}{U_{udl}} = \frac{w^2 l^3}{6EI} \times \frac{40EI}{w^2 l^3} = \frac{20}{3} > 1$$

NOTE : Strain energy due to point loads more than that of distributed loads. Energy is nothing but work dor work is proportional to deflection. The deflections due to point loads are more than that of distribute loads

Q.

$M_x = M.$

$$U = \int_0^l \frac{M_x^2 dx}{2EI} = \frac{M^2}{2EI} \int_0^l dx$$

$$\boxed{U = \frac{M^2 l}{2EI}}$$

Q
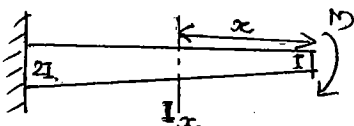
$$U = \int_0^l \frac{M^2 dx \times l}{2EI(1+x)}$$

$(1+x)^{-1}$

$l \longrightarrow$ Change in $I = 2I - I$
$= I.$

$$= \frac{M^2 l}{2EI} \Big[\log_e (1+x) \Big]_0^l$$

$$x = \frac{I}{l} x.$$

$$= \frac{M^2 l}{2EI} \log_e 2$$

$$\Rightarrow I_x = I + \frac{Ix}{l} = \frac{I(1+x)}{l}$$
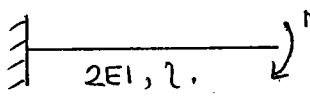
$$= 0.346 \frac{M^2 l}{EI} = \frac{M^2 l}{2.88 EI}$$

(5)

$$U = \frac{M^2 l}{2EI}$$

$$U = \frac{M^2 l}{4EI}$$
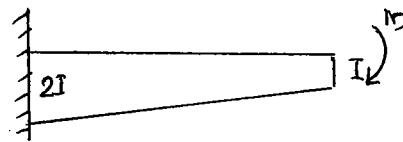
(3)

As the given beam has I varying from I to 2I, the numerical in the denominator shall be b/w 2 & 4.

Q.
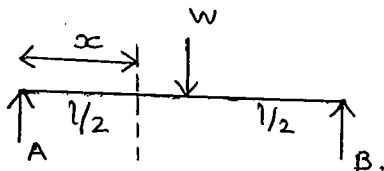
$$M_x = \frac{w}{2} x - w$$

$$U = 2 \int_0^{l/2} \frac{M_x^2 \, dx}{2EI} = \frac{w^2}{4EI} \left(\frac{x^3}{3}\right)_0^{l/2}$$

$$U = \frac{w^2 l^3}{96 EI}$$

Q.
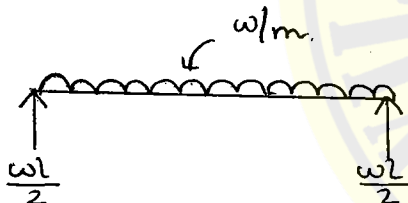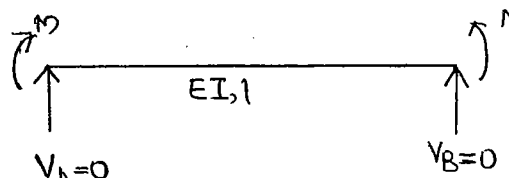
$$M_x = \frac{wl}{2} x - \frac{wx^2}{2}$$

$$U = \int_0^l \frac{M_x^2 \, dx}{2EI}$$

$$= \frac{\left(\frac{w}{2}\right)^2}{2EI} \int_0^l (lx - x^2)^2 = \frac{w^2}{8EI} \int_0^l l^2 x^2 + x^4 - 2l x^3$$

$$= \frac{w^2}{8EI} \left(l^2 \frac{x^3}{3} + \frac{x^5}{5} - 2l \frac{x^4}{4}\right)_0^l$$

$$= \frac{w^2 l^5}{240 EI} = \frac{w^2 l^3}{240 EI}$$

Q.

(case of pure bending)

$V_A = 0$ $V_B = 0$
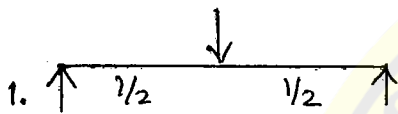
SFD — no shear
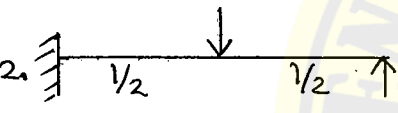
BMD — only mom M.
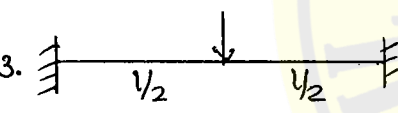
$$Reaction = \frac{Net\ moment}{l} = \frac{0}{l} = 0$$
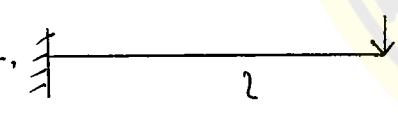
$$U = 2 \int_0^{l/2} \frac{M^2 dx}{2EI}$$

$$= \frac{M^2}{EI} \cdot \frac{l}{2} = \frac{M^2 l}{2EI}$$

Q.

1. ½ ½

2. ½ ½

3. ½ ½

4. l
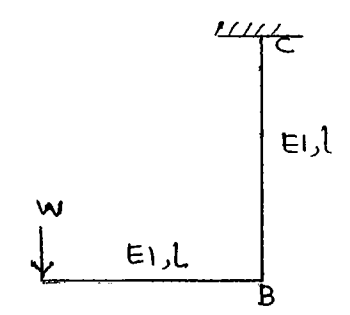
Arrange the 4 beams in the ↑ing order of strain energies

Fixed beam has least deflection and cantilever has max deflection.

NOTE: Strain energy α deflections

$$3 < 2 < 1 < 4.$$

Q. Calculate the strain energy stored of the bracket shown
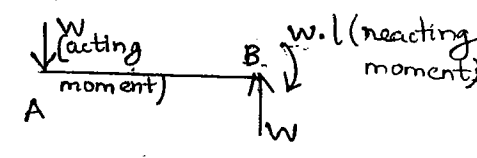
$$U_{AB} = \frac{\omega^2 l^3}{6EI}$$

$$U_{BC} = \int_0^l \frac{M_y^2 \, dy}{2EI} = \frac{1}{2EI} \int_0^l \omega^2 l^2 \, dl$$
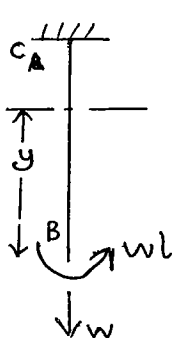
$$= \frac{\omega^2 l^2}{2EI} \cdot l = \frac{\omega^2 l^3}{2EI}$$

$$U_{bracket} = U_{AB} + U_{BC}$$

$$= \frac{4\omega^2 l^3}{6EI} = \frac{2}{3} \frac{\omega^2 l^3}{6EI}$$

FBD of AB.

$$M_y = \int_0^l \frac{M_y^2 \, dy}{2EI} - \omega l$$

In the above problem, calculate the vertical deflection at A. (6)

$$\delta_{V_A} = \frac{\partial U}{\partial w} \quad (\text{as per Castigliano's Theorem}).$$

$$= \frac{\partial}{\partial w}\left(\frac{2}{3}\frac{w^2 l^3}{EI}\right) = \frac{2}{3}\frac{2wl^3}{EI} = \frac{4wl^3}{3EI}$$

(32)

Q.
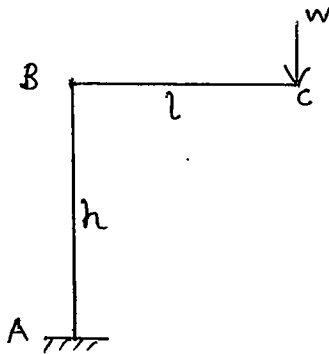
Calculate the horizontal and vertical deflections at C.

$$U = \int_0^l \frac{M_x^2 \, dx}{2EI}.$$

$$\partial_V = \frac{\partial U}{\partial w} = \frac{\partial}{\partial w}\int_0^l \frac{M_x^2 \, dx}{2EI}$$

NOTE: According to mathematics, $\delta\int = \int\delta$. However first diff. then integration is difficult compared to other.

$\delta\int$ (difficult) $\int\delta$ (easy)

$M_x = -w.$

$\frac{\partial M_x}{\partial w} = -x$

$m_x = -x$

$$\partial_V = \frac{1}{2EI}\int_0^l 2M_x \cdot \frac{\partial M_x}{\partial w}\, dx.$$
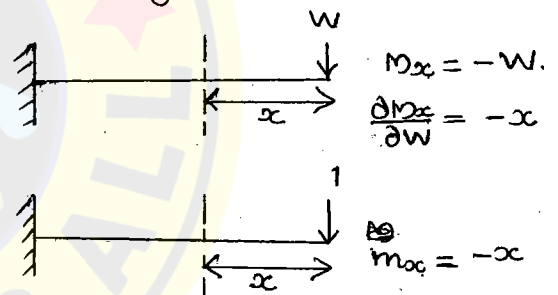
According to Unit Load method, $\frac{\partial M_x}{\partial w} = m_x$

$$\boxed{\partial_V = \frac{1}{EI}\int_0^l M_x \cdot m_x \cdot dx}$$

where $M_x \rightarrow$ moment due to external load.

$m_x \rightarrow$ moment due to unit load applied at the point where deflection is desired.

Calculation of δ_V:

Contribution of BC

(acts as cantilever)
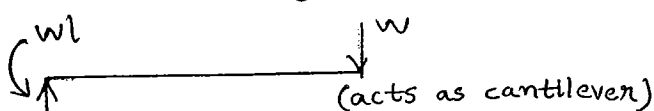
$$\delta_{VBC} = \frac{1}{EI}\int_0^l M_x \cdot \frac{\partial M_x}{\partial w}\, dx$$

$$= \frac{1}{EI}\int_0^l (-wx)(-x)\, dx$$

$$\delta_{VB} = \frac{wl^3}{3EI} \quad \left(\theta_B = \frac{wl^2}{2EI}\right)$$

$$= \frac{wl^3}{3EI}$$

Contribution of AB:
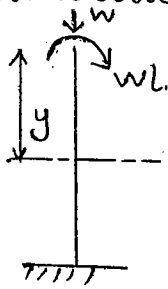
$$m_y = -wl \quad, \quad \frac{\partial m_y}{\partial w} = -l$$

$$\partial_{VAB} = \frac{1}{EI} \int_0^h (-wl)(-l) \, dy.$$

$$= \frac{wl^2 h}{EI}$$

$$\therefore \delta_{VC} = \delta_{VBC} + \delta_{VAB} = \frac{wl^3}{3EI} + \frac{wl^2 h}{EI}$$

$$= \frac{wl^2}{3EI} \left(l + 3h \right)$$

shortcut :- If $h \Rightarrow 0$, the given bracket becomes an ordinary horizontal cantilever of length l, for which vertical deflection at free end is $\frac{wl^3}{3EI}$

Calculation of horizontal deflection :

As there is no horizontal load at C, apply an imaginary load as shown.
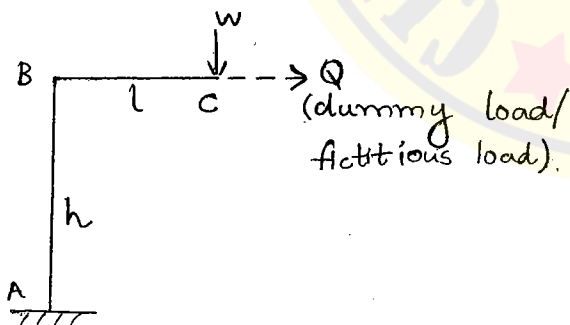
Contribution of BC:

$$\delta_{h_{BC}} = 0$$

Axial deformations of rigid jointed structures are neglected. Hence, the contribution of BC for horizontal deflection at C is zero.

$$m_x = -wx$$

$$\frac{\partial m_x}{\partial Q} = 0.$$

$$\partial_{h_{BC}} = \frac{1}{EI} \int_0^l m_x \cdot \frac{\partial m_x}{\partial Q} \, dx = 0.$$

Contribution of AB:

$$m_y = -wl - Qy \quad; \quad \frac{\partial m}{\partial Q} = -y$$

(7)

33
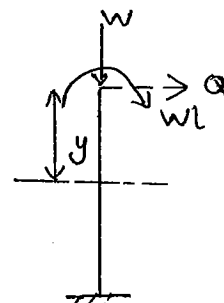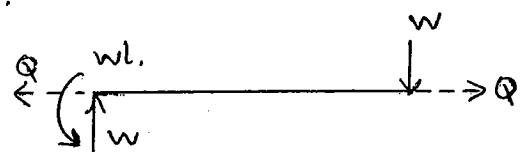
$$\partial h_{AB} = \frac{1}{EI} \int_0^{} My \cdot \frac{\partial My}{\partial Q} \cdot dy$$

$$= \frac{1}{EI} \int_0^h (-wl - Qy)(-y) \, dy \qquad Q \text{ is dummy load.}$$

NOTE: The purpose of introducing dummy load Q is to have partial differentiation wrt Q. Once the partial differential is over, substitute Q = 0.

$$\partial h_{AB} = \frac{1}{EI} \int_0^h (-wl)(-y) \, dy = \frac{wlh^2}{2EI}$$

$$\delta = \frac{Ml^2}{2EI}$$

$$\theta = \frac{Ml}{EI}$$

Shortcut :–
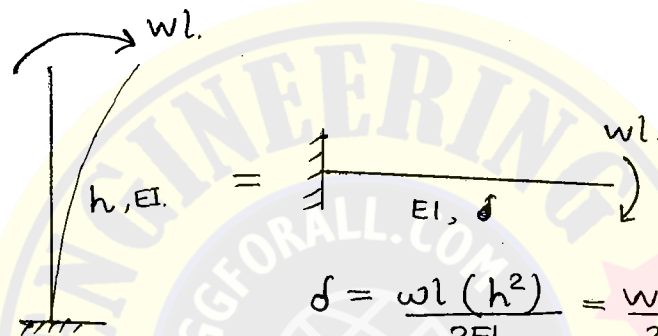
$$\delta = \frac{wl(h^2)}{2EI} = \frac{wlh^2}{2EI}$$

What is the horizontal deflection at
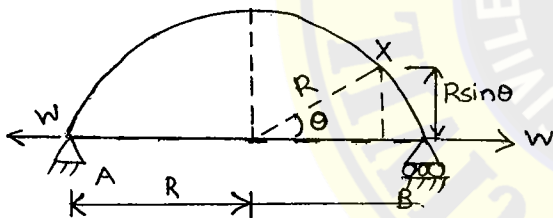
Consider a section X whose radial vector makes an angle of θ with horizontal as shown.

$$M_x = wR\sin\theta$$

$$\frac{\partial M_x}{\partial w} = R\sin\theta.$$

$$\delta_h = \frac{2}{EI} \int_0^{\pi/2} M_x \cdot \frac{\partial M_x}{\partial w} \cdot ds \qquad ds = \text{along the curve}$$
$$= Rd\theta.$$

$$= 2 \int_0^{\pi/2} wR^2\sin^2\theta \cdot R \, d\theta.$$

$$= \frac{2wR^3}{EI} \int_0^{\pi/2} \sin^2\theta \, d\theta. \qquad \theta - \frac{\sin 2\theta}{2}$$
$$\pi/2 -$$

$$= \frac{2wR^3}{EI} \times \frac{\pi}{4} = \frac{\pi wR^3}{2EI} \qquad * \int_0^{\pi/2} \sin^2\theta \, d\theta = \frac{\pi}{4}$$

Maxwell's Law of Reciprocal Deflection:
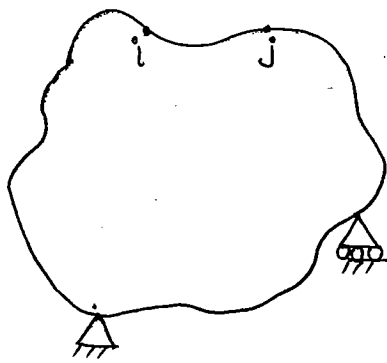
"In any elastic structure, the displacement at point j due to un load at i is equal to deflection at i due to unit load at j"

$$\delta_{ji} = \delta_{ij}$$

@ ← due to

NOTE: Maxwell's Law is valid for both prismatic and non prismatic structures

Maxwell's Law is Independent of cross sections

$\delta_{BC} = \Delta$ $\qquad$ $\delta_{CB} = \Delta$. $\qquad$ $\delta_{CB} = \Delta/2$
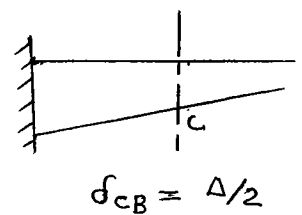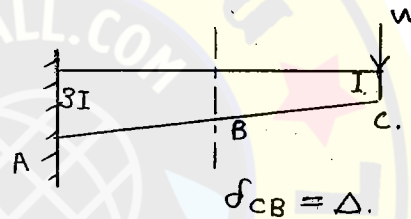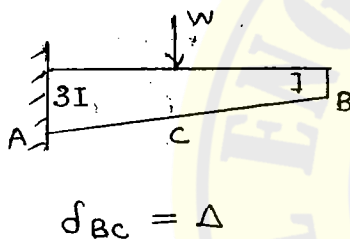
30th JULY
WEDNESDAY

WORK

Work.

├── Real work
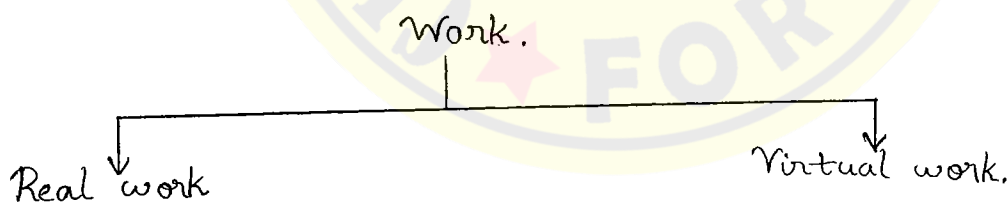
└── Virtual work.

Work done by w_1 in the direction of itself and deflection in same direction
$$= w_1 \delta_{11}$$
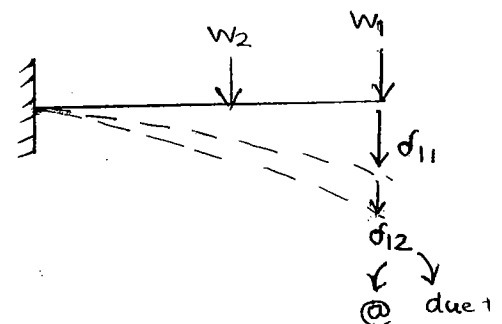
Virtual Work:

Work done by a load due to the deflection caused by some other load is called Virtual work or imaginary work

$$\text{Virtual work} = w_2 \times \delta_{12}.$$

In virtual work either force is small or displacement (8) is small. In solid mechanics, displacements are small (virtual) In fluid mechanics, forces are virtual.

③④

Maxwell's Betti Theorem:

Virtual work done by P system of loads due to. the displacements caused by Q system

$$= P_1 \delta_{1Q} + P_2 \delta_{2Q}.$$
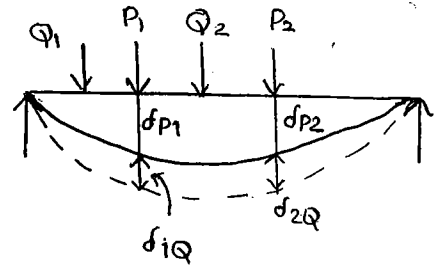
Virtual work done by Q system

$$= Q_1 \Delta_{1P} + Q_2 \Delta_{2P}.$$
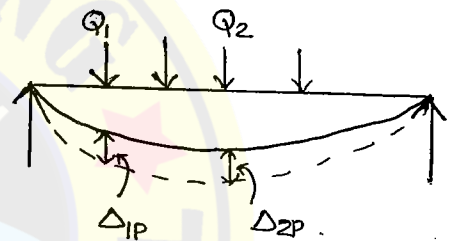
According to Maxwell—Betti, these two virtual works are equal.

Castigliano's Minimum Strain Energy Theorem.

Every system in this universe will reach its stable eqbm, if it has minimum energy. This is the basis for min. strain energy theory.

In any and every system of statical indetermination wherein a number of different values of redundant forces satisfy the conditions of statical eqbm, their actual values are those that render the strain energy stored to a minimum.

$$\frac{\partial U}{\partial R} = 0 \quad ; \quad \frac{\partial^2 U}{\partial R^2} = +ve$$

In two hinged arches, $\frac{\partial U}{\partial H} = 0$

or total strain energy stored is minimum
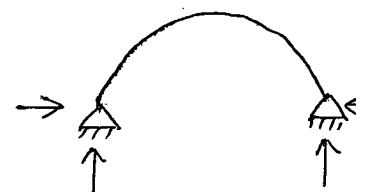
or $\dfrac{\partial^2 U}{\partial R^2} = +ve$; R represents redundant force.

Q. A load of 1000 KN applied at a point A. as shown in fig, produces vertical deflection of $\Delta b = 5$ mm. at B and $\Delta c = 2$ mm at C. Calculate the deflection at A, if the loads of 400 KN. and 200 KN act at B & C respectively.

Virtual work done
by 1000 kN
$\quad = 1000 \; \delta_A.$
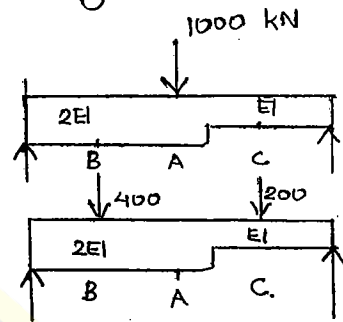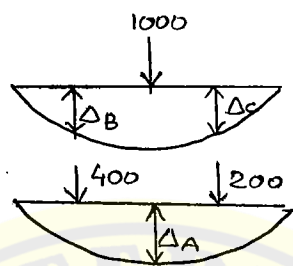
Virtual work done by
other loads $= 400 \times 5 + 200 \times 2$
$\quad = 2400 \; \text{kN}$

$\therefore \; \delta_A = \dfrac{2400}{1000} = 2.4 \, \text{mm}.$ (Virtual works equal as per Betti's Theorem)

NOTE: EI change has no significance.

㉟

6. MOMENT DISTRIBUTION METHOD
(Prof. Hardy Cross).

⊙ Displacement Method.

Equilibrium method.

Stiffness Coeffecient method.

Successive Approximation method.

NOTE : ⊙ If in any beam or frame the trial according to moment distribution doesn't end, practically we can stop when the moment at a joint to be balanced is less than 10%.

⊙ This method is also called as —
  a) Iteration method. (Kani's method deserves this name the most).
  b) Trial and Error method.

NOTE : ⊙ Prof. Hardy Cross has given Column Analogy method also

Validity of the Method :

(i) Rigid jointed indeterminate beams / frames
Invalid if a structure has internal hinges. Valid for both

(ii) Prismatic and non prismatic structures

Absolute Stiffness of a Member: (K)
The moment required to produce unit rotation at near end of a member is called 'Absolute stiffness of member' (without translation, ie without sinking)

Case 1 : Far End Fixed.

$$K = \frac{4EI\theta}{L} = \frac{4EI}{l} \quad \text{(unit rotation, } \theta = 1\text{)}$$
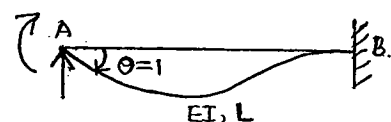
Case 2: Far End Hinged.

$$K = \frac{3EI}{L}$$

$$K = \frac{3EI\theta}{L} = \frac{3EI}{L}$$
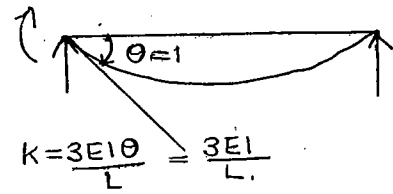
Case 3: Far End Free.

$$K = 0$$
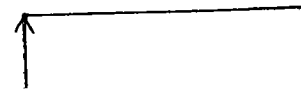
Stiffness of a Joint:

The stiffness of a joint (ΣK) is the sum of stiffnesses of all the members meeting at that joint.

$$K_{OA} = \frac{4EI}{L} \qquad K_{OC} = 4EI/L \text{ (fixed far end)}.$$

$$K_{OB} = 0 \qquad K_{OD} = 3EI/L$$

$$\Sigma K \text{ at } O = \frac{11EI}{L}$$
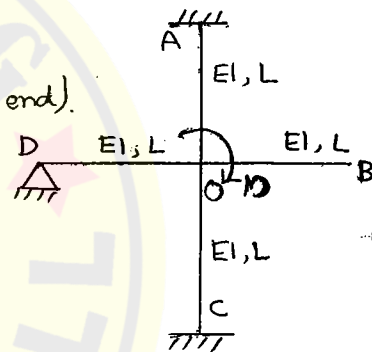
Stiffness, $\Sigma K = \dfrac{M}{\theta}$.

Rotation at joint O, $\theta = \dfrac{ML}{11EI}$

Carry Over:

When a moment M is applied to have unit rotation at near end, the moment developed at far end is called 'Carry Over Moment' (C.O.M)

Case 1: Far end fixed.

C.O.M = $\frac{1}{2}$ (moment applied at near end) and is of same sense or direction

(2)
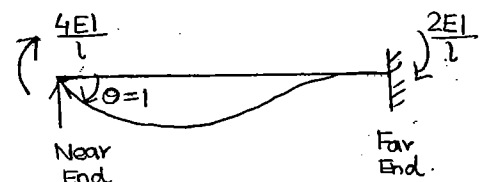
Case 2: Far End Hinged.

COM = 0 (Hinged support cannot take moment)

$$\frac{3EI}{L}$$

36
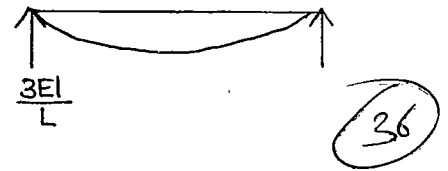
Carry Over Factor: (C.O.F).

$$COF = \frac{\text{Moment induced at far end}}{\text{Moment applied at near end}}$$

Case 1: Far end fixed.

$$COF = \frac{2EI/L}{4EI/L} = \frac{1}{2}$$

Case 2: Far end Hinged.

$$COF = \frac{0}{3EI/L} = 0$$

Q. Find C.O.M

Moment developed. (resisting moment)

at B = M (clockwise) = COM
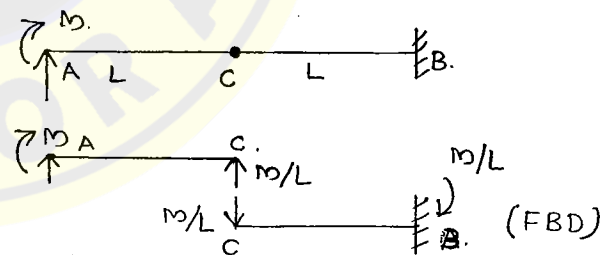
In this problem, calculate COF

$$COF = \frac{M}{M} = 1$$ (both in same direction).

couple :- two equal & opposite forces with some distan b/w them.

Q. A moment M is applied at A. The carry over factor from A to B.

a) ½   b) < ½   c) > ½   d) 1.

In the above problem, COF from B to A ?
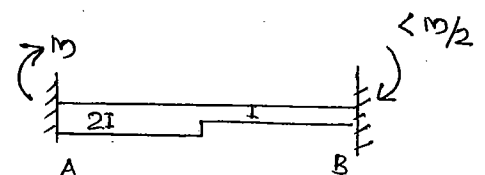
From A→B, , < ½

From B→A, > ½

Relative Stiffness of a Member:

Q. $K_{OA} : K_{OB} : K_{OC} : K_{OD} : K_{OE}$

$= \dfrac{4EI}{}$
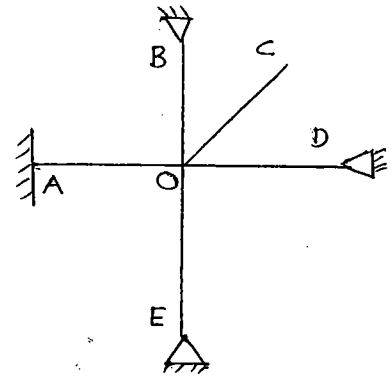
If I & L values are different,

$$\dfrac{4EI_1}{l_1} : \dfrac{3EI_2}{l_2} : 0 : \dfrac{3EI_4}{l_4} : \dfrac{3EI_5}{l_5}$$

$$= \dfrac{4I_1}{l_1} : \dfrac{3I_2}{l_2} : 0 : \dfrac{3I_4}{l_4} : \dfrac{3I_5}{l_5}$$

$\div$ing by 4,

$$= \dfrac{I_1}{L_1} : \dfrac{3}{4}\dfrac{I_2}{L_2} : 0 : \dfrac{3}{4}\dfrac{I_4}{L_4} : \dfrac{3}{4}\dfrac{I_5}{L_5}$$

Relative stiffness of a member if far end fixed is $\dfrac{I}{L}$.

If far end hinged $= \dfrac{3}{4}\dfrac{I}{L}$.

Q. Ratio of relative stiffness of far end fixed to far end hinged.

$K_{BA} : K_{BC} = 4 : 3$

The stiffness of far end fixed hinged [member with] is 75% of that of far end fixed, assuming same values of I & L.

Distribution Factors: (D.F)

D.F of a member meeting at a joint $= \dfrac{K}{\Sigma K}$

where $K \rightarrow$ stiffness of the member selected.

$\Sigma K \rightarrow$ sum of the stiffnesses of all members meeting at the joint.

$K_{BA} = \dfrac{4EI}{4} = EI$

$K_{BC} = \dfrac{3EI}{3} = EI$
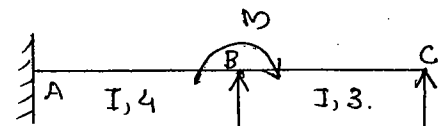
DF of BA $= \dfrac{K_{BA}}{K_{BA} + K_{BC}} = \dfrac{EI}{EI + EI} = \dfrac{1}{2}$

Sly, DF of BC $= \dfrac{1}{2}$

⊙ Sum of the distribution factors at a joint = 1. ㊲
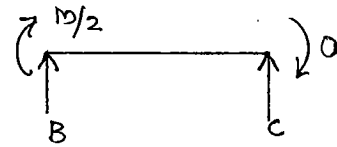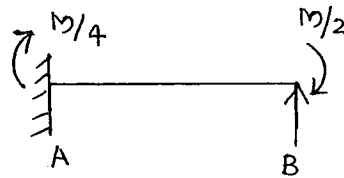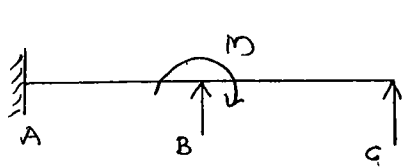
com at A.
$= \frac{1}{2}\left(\frac{M}{2}\right) = \frac{M}{4}$ ↷

com at C (hinged)
$= 0$

$$\frac{R_B L^3}{3EI} = \frac{ML^2}{4EI.}$$

$$R_B = \frac{3M}{4L.}$$

$$M_A = \frac{-3M}{4L} + \frac{M}{2}$$

$$= -\frac{M}{4} \text{ (developed moment).}$$

Resisting moment at A $= \frac{M}{4}$ (clockwise).
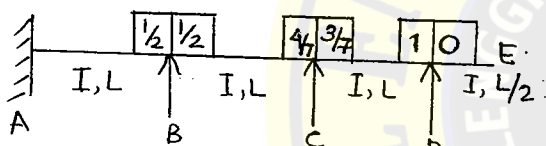
Calculate DF at B, C, D?

$$K_{BA} = \frac{4EI}{L}$$

NOTE: Though C is a hinged support, to the right side of C another support is existing. Hence, for the purpose of analysis C shall be treated as rigid or fixed support.

$$\therefore K_{BC} = \frac{4EI}{L}$$

$$DF = \frac{1}{2} \quad (K_{BA} = K_{BC}).$$

At C,

$$K_{CB} = \frac{4EI}{L} \qquad K_{CD} = \frac{3EI}{L.}$$

(beyond D, only overhang exists and no support to the right side hence D will act like hinged support)

$$(DF)_{CB} = \frac{4}{7}$$

$$(DF)_{CD} = \frac{3}{7}$$

At D,

$$K_{DC} = \frac{4EI}{L} , \quad K_{DE} = 0 \implies (DF)_{DC} = 1 \qquad (DF)_{DE} = 0$$

Aug,
ATURDAY

Q Calculate DF @ C:
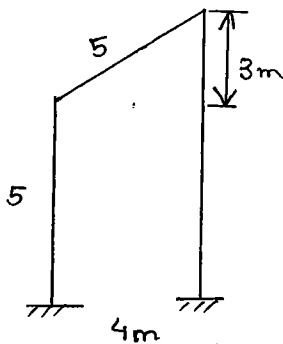
$$K_{CA} = \frac{4EI}{L} = \frac{4EI}{5}$$

$$K_{CD} = \frac{4EI}{5}$$

$$DF = 0.5 \ \& \ 0.5.$$

Calculation of Rotation of Rigid Joints.:

$$\text{Stiffness} = \frac{M}{\theta}.$$

To calculate the rotation of a joint, sum of the stiffnesses of all the members meeting at the joint is required.
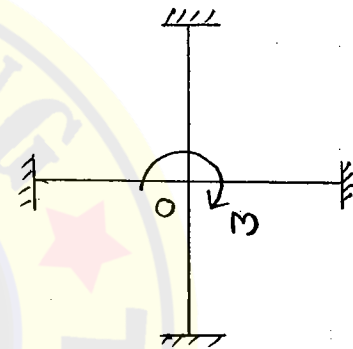
-fig.

NOTE:

Sum of the stiffnesses of all members meeting at a joint is called 'Rotational Stiffness of Joint' (ΣK)

In the above fig,

$$\Sigma K = \frac{16 EI}{L}$$

$$\theta = \frac{M}{\Sigma K} \quad ; \quad M \rightarrow \text{moment acting at the join}$$

$$= \frac{ML}{16 EI}$$

Q.
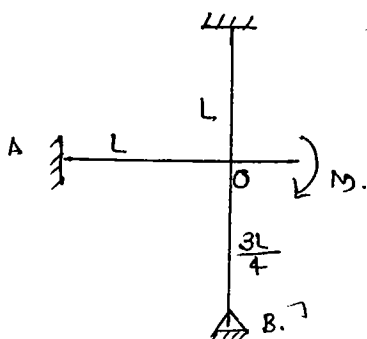
$$K_{OA} = \frac{4EI}{L} \ ; \ K_{OB} = \frac{3EI}{3/4 \, L} = \frac{4EI}{L} \ ; \ K_{OC} = \frac{4EI}{L}.$$

$$(DF)_{OA} = (DF)_{OB} = (DF)_{OC} = \frac{1}{3}.$$

$$M_{AO} = \frac{1}{3} \times \left(\frac{1}{2} \times M_{OA}\right) = \frac{M}{6}$$

$$M_B = 0 \quad (\text{com at hinged support})$$

(4)

(38)

Procedure of MOMENT DISTRIBUTION METHOD.

Step 1: Idealise the structure.

Step 2: Calculate DF at the joints.

Step 3: Calculate initial moments assuming a restrained. structure.

NOTE: If one of the end support is simple support, assume it as. fixed support.

Moments on AB:

(i) due to couple ($\curvearrowright$) = $\dfrac{M}{4} = \dfrac{20}{4}$

= +5 kNm.

$\therefore M_{FAB} = +5$ kNm.

$M_{FBA} = +5$ kNm.

(ii) due to central point load:

$M_{FAB} = -\dfrac{wl}{8} = \dfrac{-40 \times 4}{8} = -20$ kNm.

$M_{FBA} = +\dfrac{wl}{8} = +20$ kNm.

Moments on BC:

$M_{FAB} = -\dfrac{wl^2}{12} = \dfrac{-20 \times 3^2}{12} = -15$ kNm

$M_{FBA} = +15$ kNm

			1/2	1/2		1
			−15	+25	−15	+15
Bal			—	—	—	−15
C.O					−7.5	
Modified Moments			−15	+25	−22.5	0
Bal			—	−1.25	−1.25	—
C.O			−0.62			0
Final Support Moments			−15.63	+23.75	−23.75	0

eccentric load = direct load + moment.
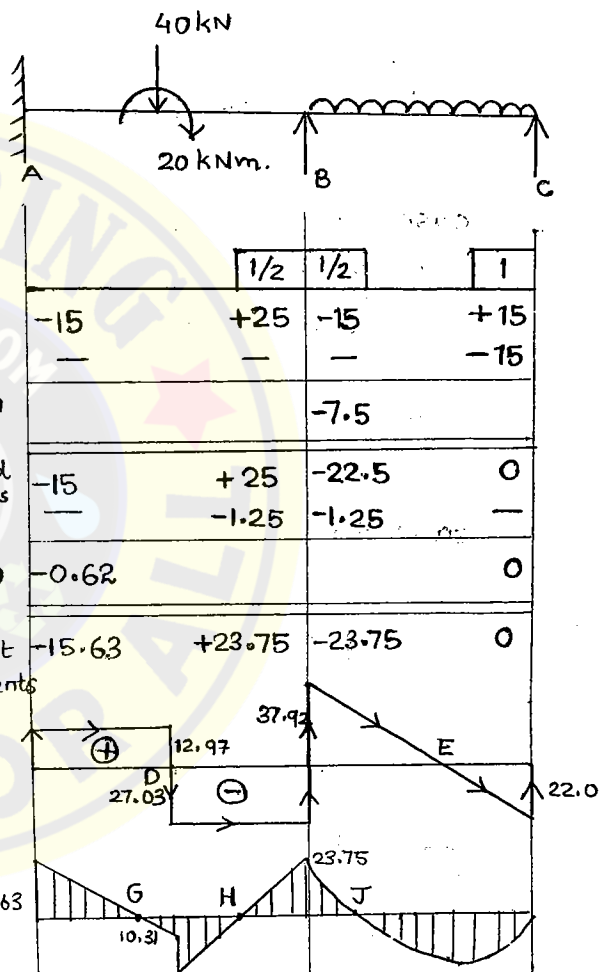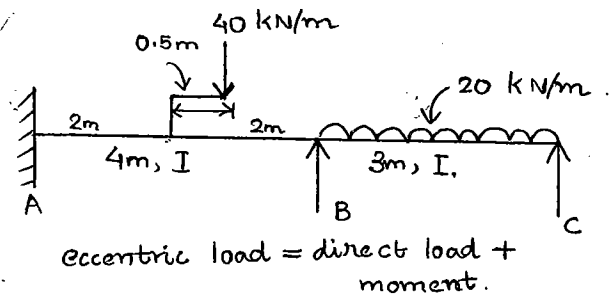
Step 4: Release or Unlock the end moment at simple supports at the end.

Step 5: Apply carry over to the nearer adjoining supports.

Step 6: Calculate modified initial moments after the end simple suppor is balanced and necessary c.o is made.

Step 7: Balance the internal support moments to satisfy momen eqbm equation. The unbalanced internal moment is distributed in the ratio of D.Fs. Apply C.O.

Step 8: Balance, C.O, Balance, C.O.., (Process of balancing can b restricted in a trial where the value of moment is ≈ < 10% initial mome

■ Standard Cases of Initial Moments for Fixed Beams.

case 1:

Sign convention:

Clock wise moment $= +ve$

Anti-clockwise moment $= -ve$.

$$M_{FAB} = -\frac{WL}{8} \qquad ; \qquad M_{FBA} = +\frac{WL}{8}$$

case 2:
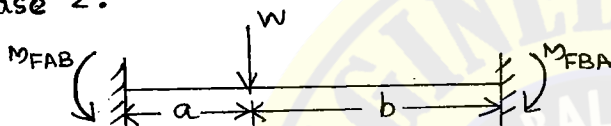
$$M_{FAB} = -\frac{Wab^2}{l^2} \qquad ; \qquad M_{FBA} = +\frac{Wba^2}{l^2}$$

case 3:
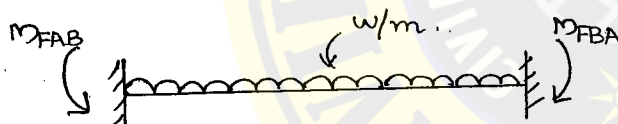
$$M_{FAB} = -\frac{wl^2}{12} \qquad ; \qquad M_{FBA} = +\frac{wl^2}{12}$$
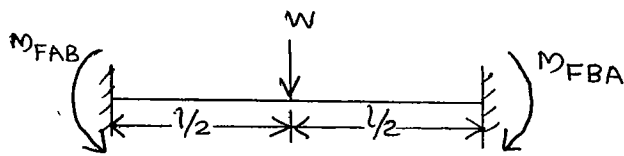
case 4:
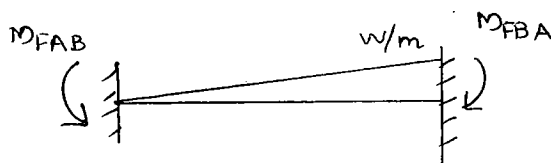
$$M_{FAB} = -\frac{\omega l^2}{30} \qquad ; \qquad M_{FBA} = +\frac{wl^2}{20}$$

case 5:

$$M_{FAB} = +\frac{Mb(2a-b)}{L^2} \qquad ; \qquad M_{FBA} = +\frac{Ma(2b-a)}{L^2}$$
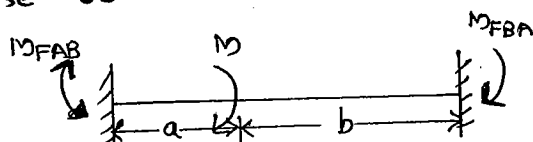
NOTE:

Initially, assume direction of support moments due to a moment couple same as that of applied couple.

⑤

㊵

- If $a = b = l/2$;

$$M_{FAB} = M_{FBA} = +\frac{M}{4}$$

- If $a = 1/3$ and $b = \frac{2l}{3}$;

$$M_{FAB} = 0 ; \quad M_{FBA} = +\frac{M}{3}.$$

- If $a = 1/4$ and $b = 3l/4$;

$$M_{FAB} = -\frac{3M}{16} \quad (\text{assumed direction is wrong}).$$

$$= \frac{3M}{16} \quad (\text{anti-clockwise}).$$

NOTE: The last step should be 'carry over'.

Procedure to draw SFD:

(i) FBD of AB :—
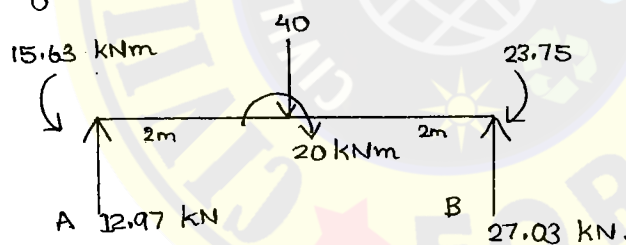
Apply $\Sigma M_A = 0$,

$$R_B \times 4 + 15.63 = 23.75 + 20 + 40 \times 2$$

$$\therefore R_B = 27.03$$

(ii) Start from left. Move in the direction of normal force

SIGN CONVENTION for Shear force:

$$\uparrow \downarrow \quad - \quad +ve \quad (\curvearrowright \text{shear} +ve)$$

(iii) FBD of BC :—
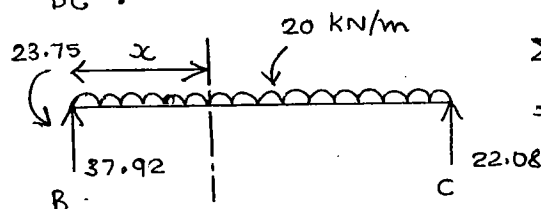
$$\Sigma M_B = 0$$

$$\Rightarrow R_C \times 3 + 23.75 = 20 \times 3 \times 3/2$$

$$R_C = 22.08 \text{ kNm}.$$

NOTE:

- Wherever udl is present, SF varies linearly,

$$V_x = R_B - 20x.$$
$$= 37.92 - 20x$$

- Wherever SF changes, sign, <u>span moment</u> is maximum.

Procedure to draw BMD:

(i)
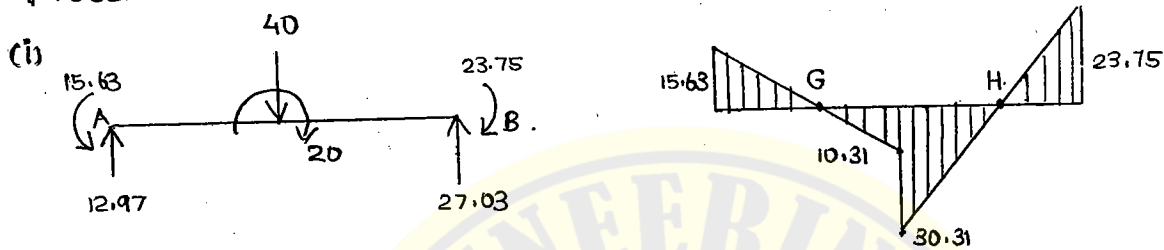
NOTE: ◉ Sign Convention for BM:- (while calculating initial support moments).

Clockwise   +ve

Anti-clockwise   −ve

◉ While drawing BMD,

Sagging — +ve

Hogging — −ve

Hogging (−ve)   Sagging (+ve)

◉ International sign convention to draw BMD:

Draw BM on tension side

(ii) BM just to the left of point load,

$$M_D = 12.97 \times 2 - 15.63 = 10.31.$$

BM just to the right of couple,

$$M_D = 27.03 \times 2 - 23.75$$
$$= 30.31$$

Points of Contraflexure (G & H) are the points where BM changes sign <u>and zero.</u>

(iii)

40

NOTE:

At any joint without couple, BMD will be as follows.

Left top — Right top

Left bottom — Right bottom.

Coloumn out — Beam out.

Coloumn in — Beam in.
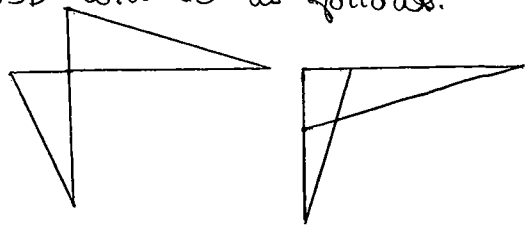
Location of Points of Contraflexure in Span AB:

At point of contraflexure, $\Sigma M = 0$

Consider a section at a distance x from left support,

$M_x = 0$

$12.97x - 15.63 = 0$

$\underline{x = 1.205\ m}$

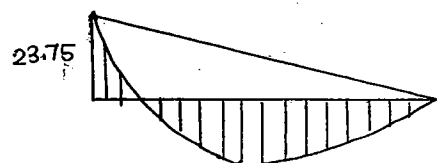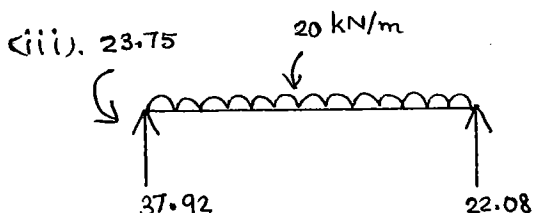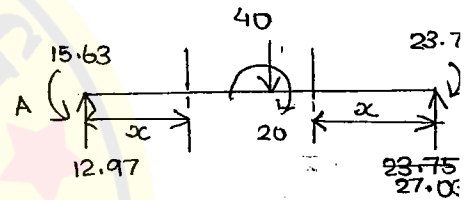To locate point of conflexure H, apply $\Sigma M = 0$ from right side of

$M_x = 27.03x - 23.75 = 0$

$\Rightarrow \underline{x = 0.88\ m}$

Max. span moment in AB = 30.31 kN m
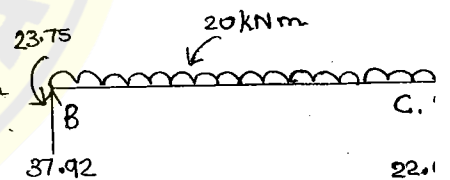
Location of point of contraflexure J,

$\Sigma M_J = 0$ from left side of C,

$22.08x - 20\dfrac{x^2}{2} = 0$

$\underline{x = 2.208\ m}$

Max. span moment in BC occurs where SF changes sign or numerically zero.

Say $V_x = 0$,

$22.08 - 20x = 0$.

$\underline{\underline{x = 1.104\ m}}$

Max. span moment in BC = occurs at a distance of 1.104 m f

$= 22.08 \times 1.104 - 20 \times \dfrac{1.104^2}{2} = \underline{12.19\ kNm}$

Q. Calculate moment at A.
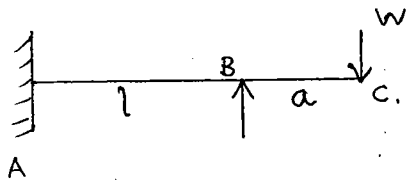
$$\therefore M_A = \frac{wa}{2} \text{ (sagging)}.$$

	1	0
Initial	0	$-wa$
Bal	wa	0
c.o	$\frac{wa}{2}$	
	$+\frac{wa}{2}$	

(+ve − sagging).

Q.
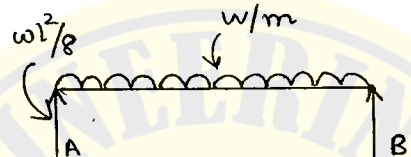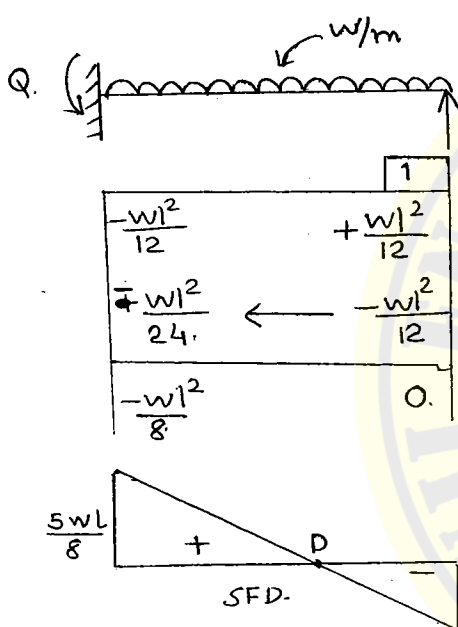
$$\Sigma M_A = 0$$

$$R_B \cdot l + \frac{wl^2}{8} = wl \times \frac{l}{2}$$

$$R_B = \frac{3wl}{8} \quad \& \quad R_A = \frac{5wl}{8}$$

Max. span moment occurs at D where SF changes sign and is zero.

$$\therefore V_x = 0.$$

$$R_B - wx = 0$$

$$\frac{3wl}{8} - wx = 0, \quad \therefore \underline{x = \frac{3l}{8}}$$

$$M_{max} = R_B x - \frac{wx^2}{2}.$$

$$= \frac{3wl}{8} \times \frac{3l}{8} - \frac{w}{2}\left(\frac{3l}{8}\right)^2.$$

$$= \underline{\frac{9wl^2}{128}}$$

SFD.

Q. Two span symmetrical continuous beams with simple supports ④

$$\frac{3wl}{8} \quad \frac{10wl}{8} \quad \frac{3wl}{8}$$

3 : 10 : 3
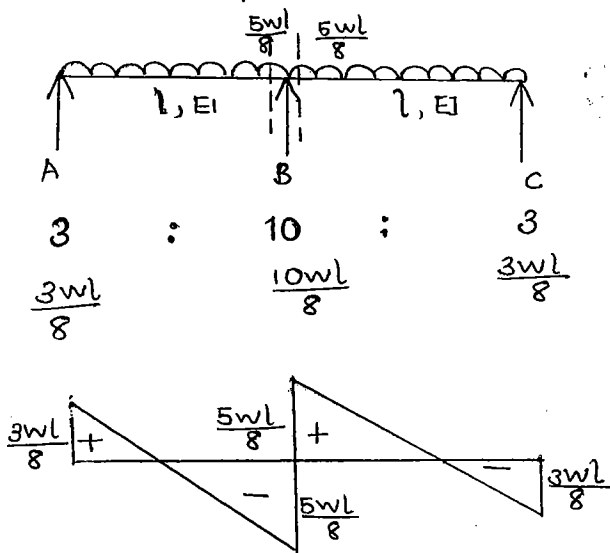
NOTE:

The column at B shall be designed for an axial load of $\frac{10w}{8}$ and moment of zero.

$$\frac{wl^2}{8} \quad \frac{10wl}{8} \quad \frac{10w}{8}$$
$$\frac{w^2}{8}$$

B

Q.
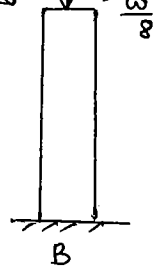
$$\sum M_A = 0$$

$$R_B \times l + \frac{3wl}{16} = \frac{wl}{2}$$

$$R_B = \frac{5wl}{16} \quad \& \quad R_A = \frac{11wl}{16}$$

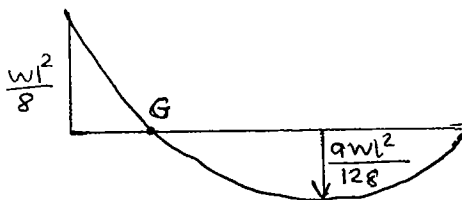| $\frac{-wl}{8}$		$\frac{wl}{8}$
$\frac{wl}{16}$		$\frac{-wl}{8}$
$\frac{-3wl}{16}$		

$$M_B = \frac{3wl}{16}.$$

⊛

A   B   C
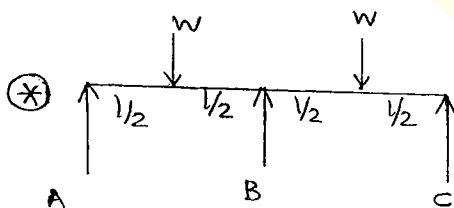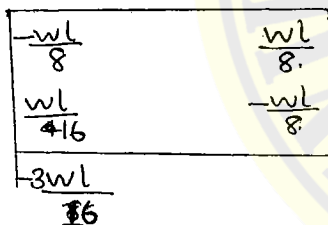
Q. A SSB of span l and uniform section is subj. to udl throughout. Suddenly a rigid prop placed at the centre, the BM at the rigid support
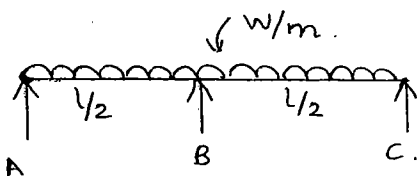
$$M_B = \frac{w}{8}\left(\frac{l}{2}\right)^2 \qquad R_B = \frac{10}{16} \times \text{total load.}$$

$$= \frac{wl^2}{32} \qquad\qquad = \frac{10}{16} \times wl = \frac{5wl}{8}$$

Sinking of Supports :

$\dfrac{6EI\delta}{l^2}$

$\dfrac{6EI\delta}{l^2}$

A

δ
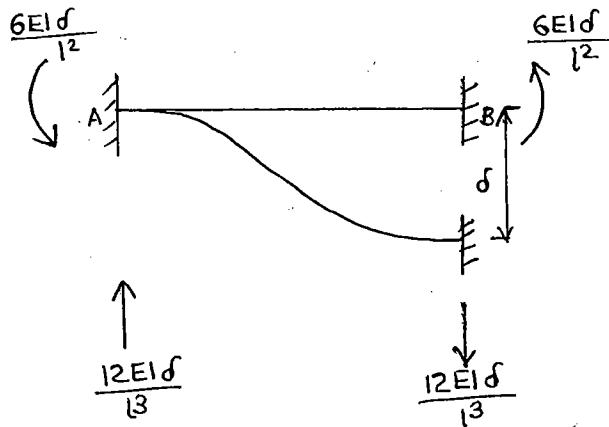
$\dfrac{12EI\delta}{l^3}$

$\dfrac{12EI\delta}{l^3}$

NOTE :

Reaction due to couple

on simply supported ends

$= \dfrac{Net\ moment}{l_1}$

$= \dfrac{12EI\delta/l^2}{l} = \dfrac{12EI\delta}{l^3}$

$\dfrac{3EI\delta}{l^2}$

O

$\dfrac{3EI\delta}{l^3}$

$\dfrac{3EI\delta}{l^3}$

$\dfrac{3EI\delta}{l^3}$

$\dfrac{3EI\delta}{l^3}$

$\dfrac{3EI\delta}{l^2}$

O

Rotation of Supports :

$\dfrac{4EI\theta_A}{l}$

θ_A

$\dfrac{2EI\theta_A}{l}$

$\dfrac{3EI\theta_A}{l}$

θ_A

O

$\dfrac{6EI\theta_A}{l^2}$

$\dfrac{6EI\theta_A}{l^2}$

$\dfrac{3EI\theta_A}{l^2}$

$\dfrac{3EI\theta_A}{l^2}$

(8)

Q.
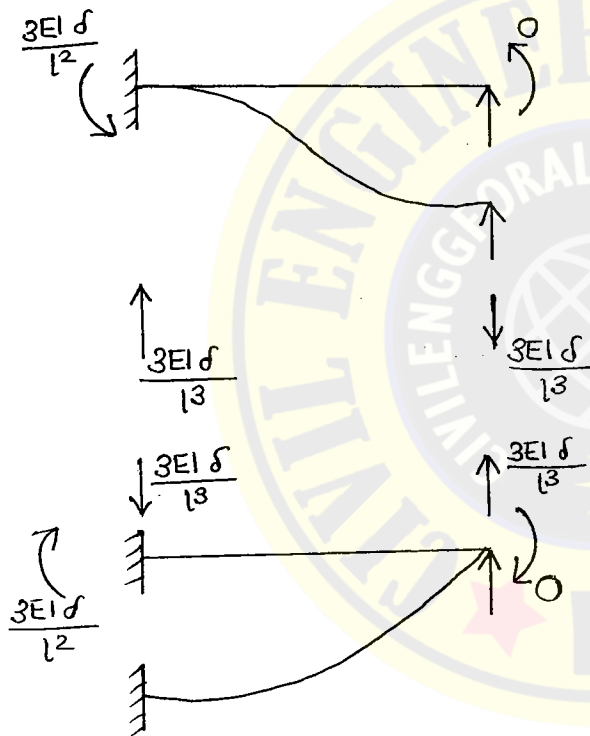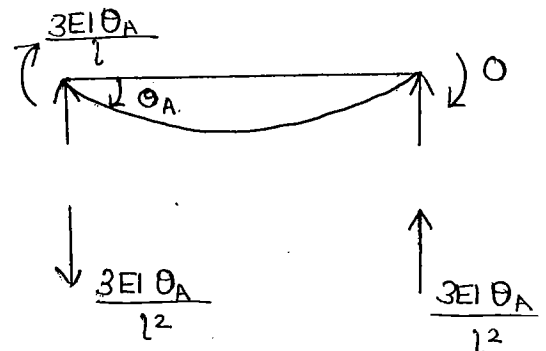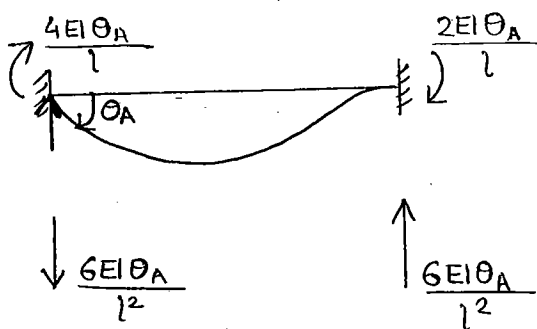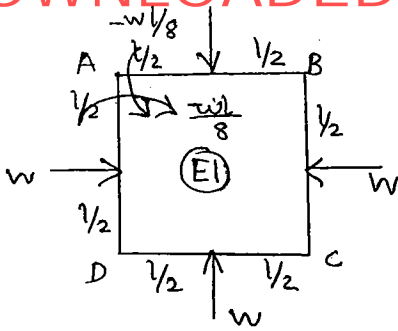
The design moment for the frame ___ ?

(12)

The structure is symmetrical. Hence DF are equal to 0.5 & 0.5 at every joint.

Say at joint A, $M_{FAB} = -\frac{wl}{8}$ & $M_{FAD} = \frac{wl}{8}$.

∴ Joint A is already balanced. Similar is the case with other joints. ∴ Design moment $= \frac{wl}{8}$

Q
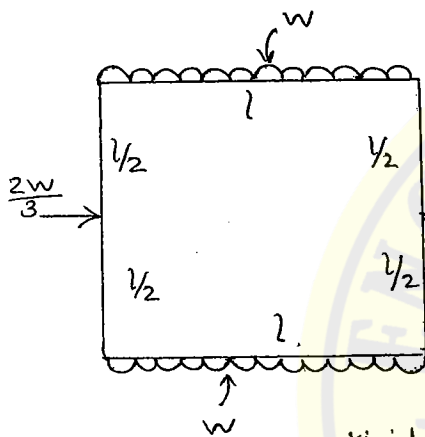
$M_{FAB} = -\frac{wl}{12}$

$wl = W$

$M_{FAD} = +\left(\frac{2w}{3}\right)\cdot\frac{l}{8} = +\frac{wl}{12}$

Joints are already balanced.

∴ Design moment $= \frac{wl}{12}$

Q.
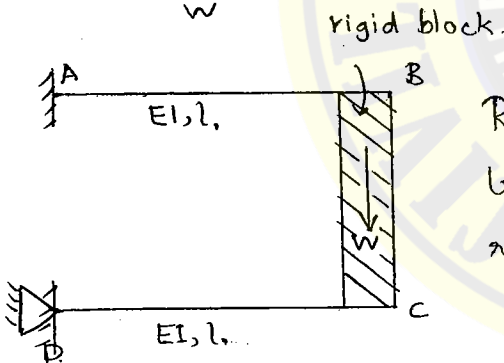
Rigid block W shown is supported by two beams AB & CD. Deflection of the rigid block is ___ ?

Joint B and joint C of rigid block will act like fixed supports with sinking.
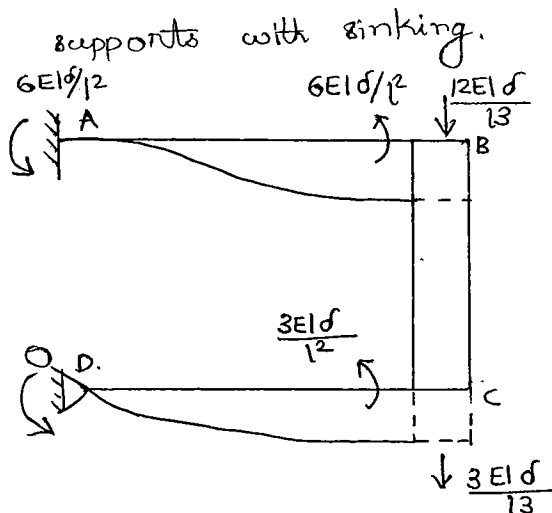
$\frac{15EI\delta}{l^3} = W$

$\delta = \frac{Wl^3}{15EI}$

⊙ If D is fixed,

$\frac{12EI\delta}{l^3} + \frac{12EI\delta}{l^3} = W$

$\Rightarrow \delta = \frac{Wl^3}{24EI}$

Frames

    (i) Non – sway Frames

    (ii) Sway Frames

→ Non-sway Frames

A frame will not sway, if its symmetrical in all aspects.

1. Symmetry of Supports.
2. Symmetry of height of column.
3. Symmetry of C.S of column.
4. Symmetry of material of column.
5. Symmetry of beam section.
6. Symmetry of load.
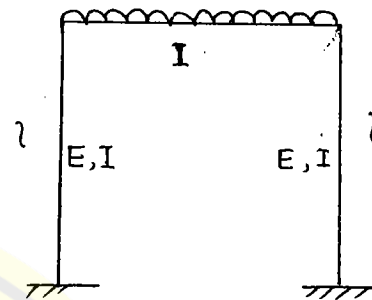
Even if a frame is unsymmetrical but if its clamped from one side, it will not sway.
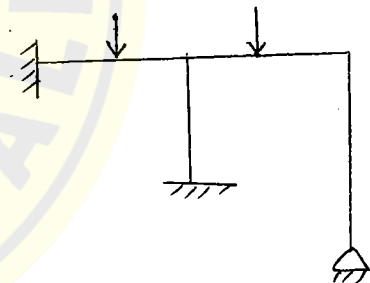
→ Sway Frames

1. Unsymmetry of load.
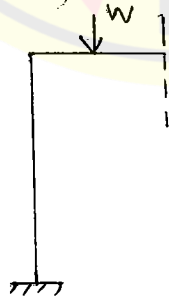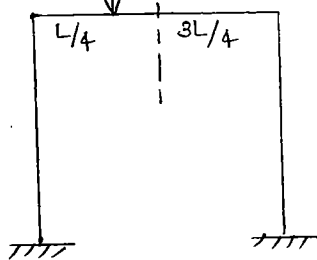
● Frame will sway towards least reaction side.
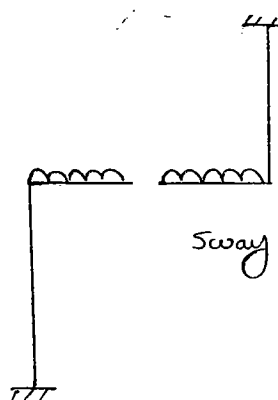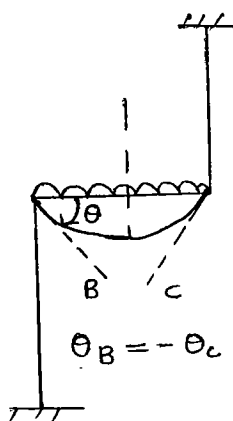
Sway towards right.

$\theta_B = -\theta_c$

2. Unsymmetry of supports.   ⑨

                                                        ⓜ₃

3. Unsymmetry of height of columns.

I, L   I, 1.5L

4.

I, L

5.

6.

I, L

20I, 2L.

$$\frac{6EI\delta}{l^2}$$

$$\frac{12EI\delta}{l^3}$$

$$\frac{6EI\delta}{l^2}$$

$$\frac{6 \times 20EI\delta}{4l^2} = \frac{30EI\delta}{l^2}$$

$$\frac{60EI\delta}{l^2 \times 2l} = \frac{30EI\delta}{l^3}$$

$$\frac{30EI\delta}{l^2}$$

P-42

10.

$$\delta_A = \frac{P(3L)^3}{3EI} - \frac{2PL(3L)^2}{2EI} = 0$$

14.

Apply unit couple at BC to get rotation of member BC.

$$\theta_{BC} = \Sigma k\delta'$$

$$= \left(-\frac{1}{4} \times -4 + \frac{1}{4} \times 2.\right)10^{-3}$$

$$= 1.5 \times 10^{-3} \text{ radians}$$

P-55

07.

$$\Sigma M_G = 0.$$

$$\frac{P}{2} \times \frac{l}{2} = M_A \qquad \therefore M_A = \frac{Pl}{4}$$

12

(10)

(44)

18.
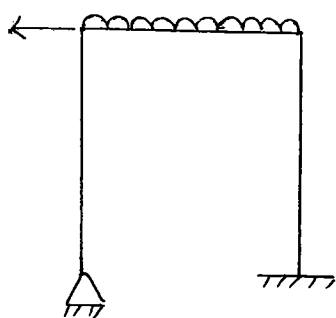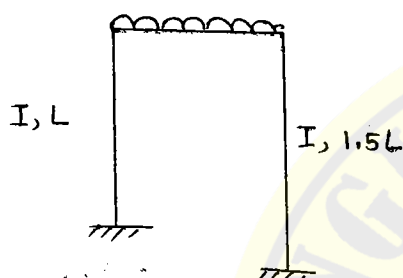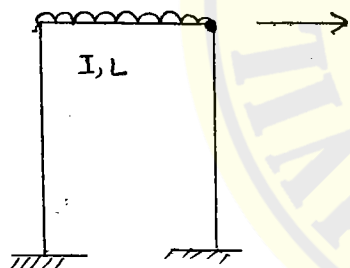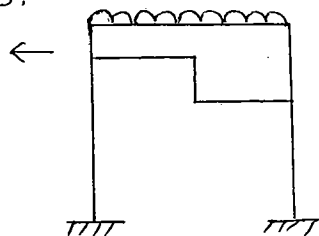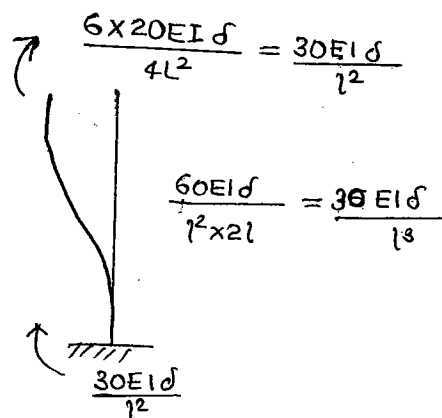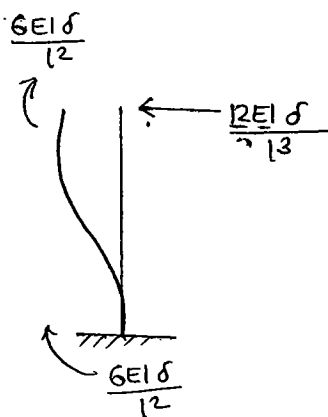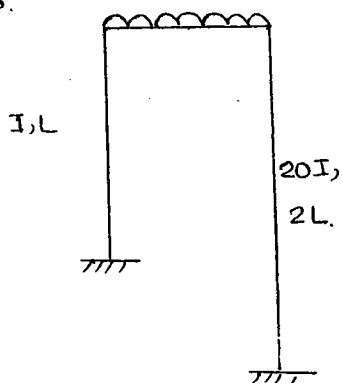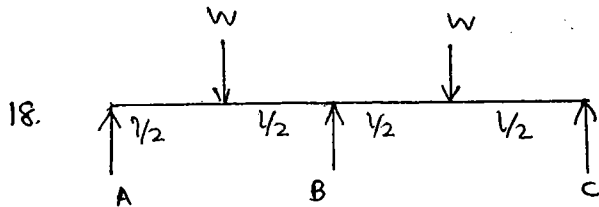
Free moment at mid-span of $AB = \dfrac{w}{2} \times \dfrac{l}{2} = \dfrac{wl}{4}$

Moment at $B = \dfrac{3wl}{16} = 0.75 \times \dfrac{wl}{4}$

19.
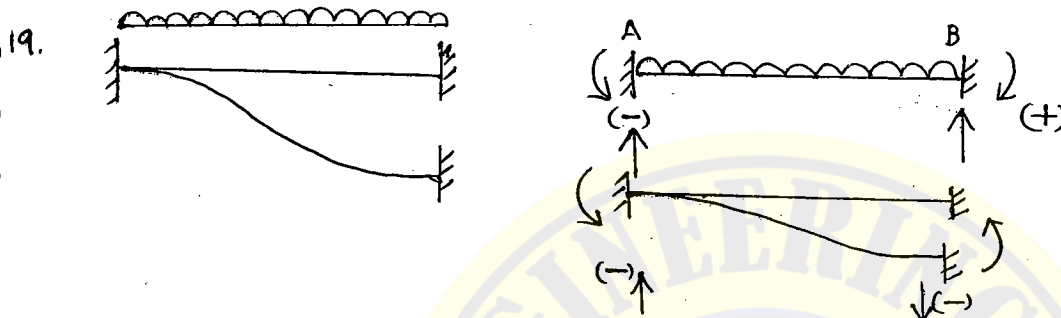
In case of fixed beam, moments at other support increases. Moment decreases due to sinking at support.

The net reaction at the sinking support decreases. The net reaction at other support increases.

25

26
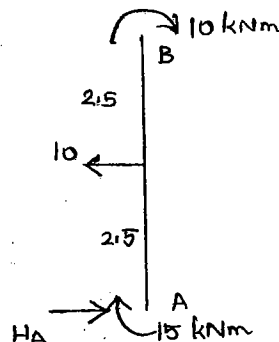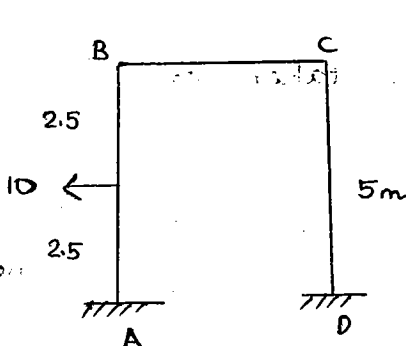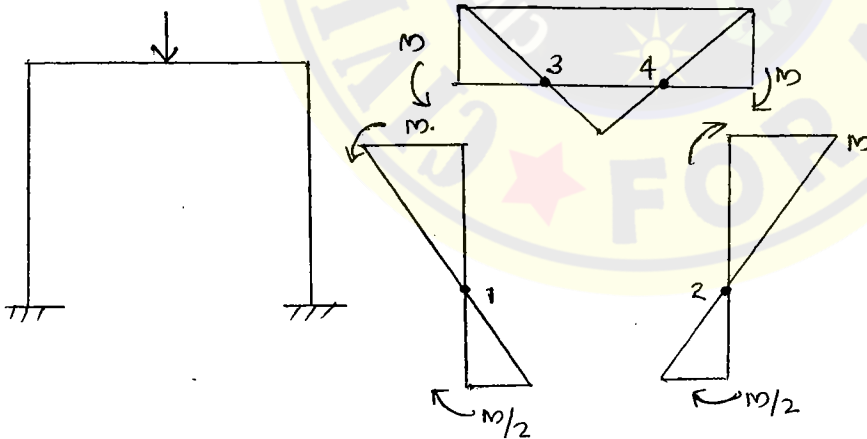
Apply $\Sigma M = 0$ at B,

$15 + 10 + 10 \times 2.5 = 5 H_A$

$H_A = 10$ kN

$5 H_D = M_{CD} + 20$

$H_D = \dfrac{M_{CD}}{5} + 4$
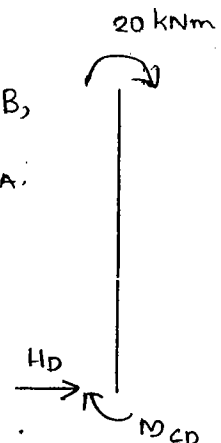
For horizontal eqbm,

$$H_A + H_D = 10 \text{ kN.}$$

$$H_D = 10 - H_A = 0.$$

$$\therefore 0.2 \, M_{CD} + 4 = 0.$$

$$M_{CD} = -20 \text{ kNm}$$

27. Since the given frame has roller support at right side, it cannot have horizontal reaction. Further as there are no external horizontal loads, no horizontal reaction at left support

subjected to axial force only

BC will act like a SSB (no mom

$$\delta = \frac{5 w l^4}{384 \, EI} = 1 \text{ mm}$$

28.
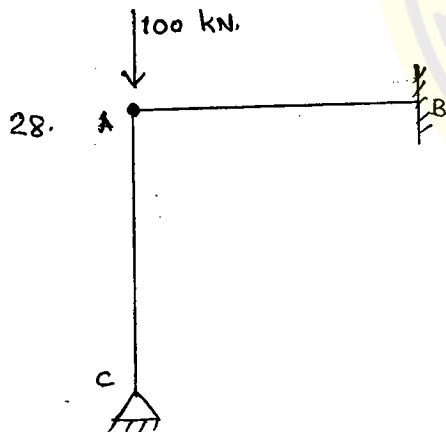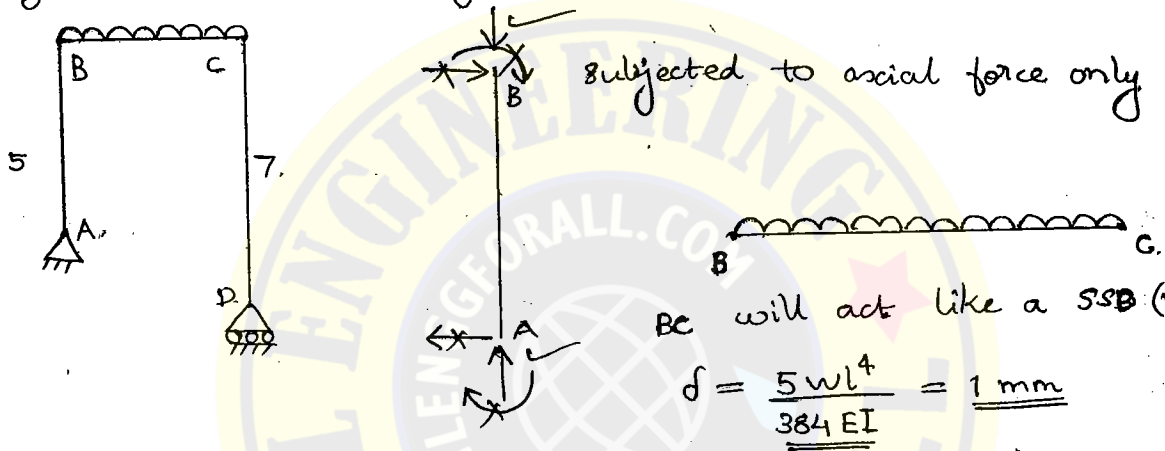
$$A = 1,50,000 \text{ mm}^2$$

$$1 = 3.125 \times 10^9 \text{ mm}^4.$$

In this problem, axial deformations are considered.

Consider FBD's
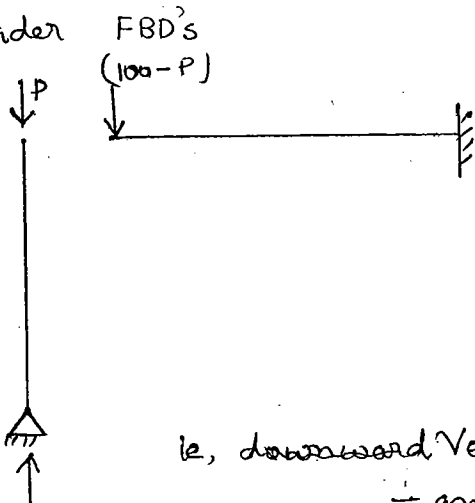
Load P will be taken care of by column.

Since member AB and AC are joined together from compatibility condition they'll have same deflection.

i.e, ~~downward~~ Vertical deflection of A of beam AB = axial deformation of column

Axial deformation of column $= \dfrac{Pl}{AE}$

$$= \dfrac{P \times 1000}{1,50,000 \, E}$$

Deflection of cantilever tip, $A =$

$$\dfrac{(100-P) \, 1000^3}{3 \times E \times 3.125 \times 10^9}$$
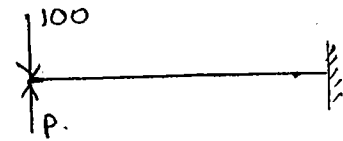
$$\Rightarrow \dfrac{1000 \, P}{15000 \, E} = \dfrac{(100-P) \, 1000^3}{3 \times E \times 3.125 \times 10^9}$$

$$P = \underline{5.9} \text{ kN}$$

30.
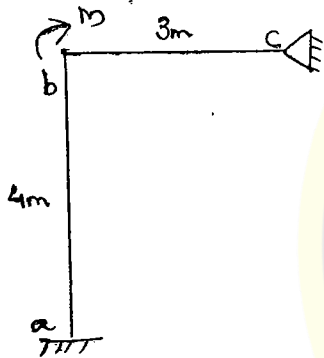
DF $= \dfrac{1}{2}$ & $\dfrac{1}{2}$

$M_{ba} = \dfrac{M}{2}$

$M_{ab} = \dfrac{M}{4}$
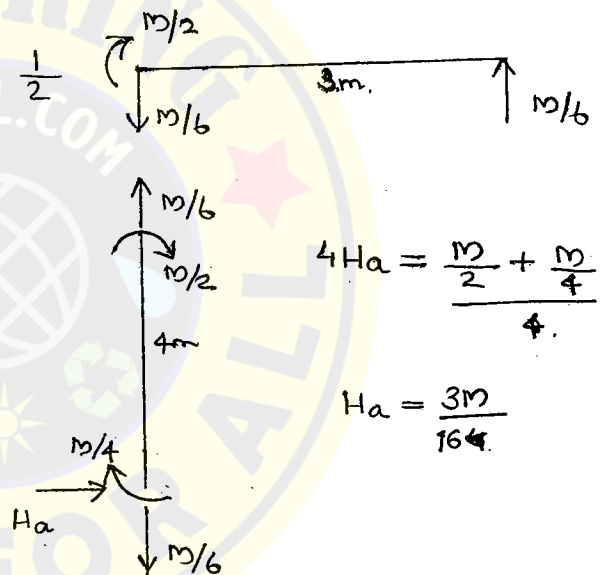
$4 H_a = \dfrac{\dfrac{M}{2} + \dfrac{M}{4}}{4}$

$H_a = \dfrac{3M}{16 \times 4}$

$$R_a = \sqrt{V_a^2 + H_a^2}$$

$$= \sqrt{\left(\dfrac{M}{6}\right)^2 + \left(\dfrac{3M}{16\Theta}\right)^2}$$

$$= \underline{\underline{\dfrac{M}{4}}}$$

54.

18.
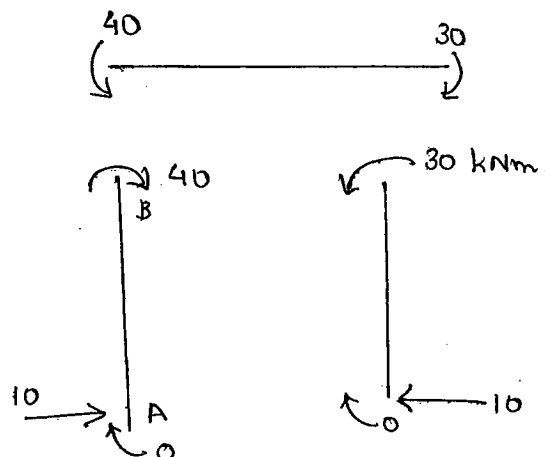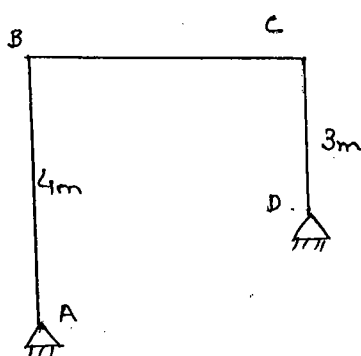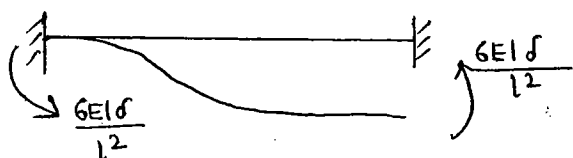
40 30

40 B

30 kNm.

$10 \rightarrow$ A
O

$\leftarrow 10$
O

Q3

$$\downarrow \frac{6EI\delta}{l^2}$$

$$\uparrow \frac{6EI\delta}{l^2}$$

1.

SLOPE DEFLECTION METHOD

⊙ Displacement Method
    Equilibrium method.
    Stiffness method.

⊙ This is the Father of all methods. given by Maney

Validity:
  — useful for analysis of rigid jointed frames/beams.
  — it cannot be use

Basic Steps in Slope Deflection Method:

Step 1: Calculate the unknown joint displacements.

2 — θ_B, Δ 1 — θ_B 4 — $\theta_B, \theta_C, \theta_D, .$
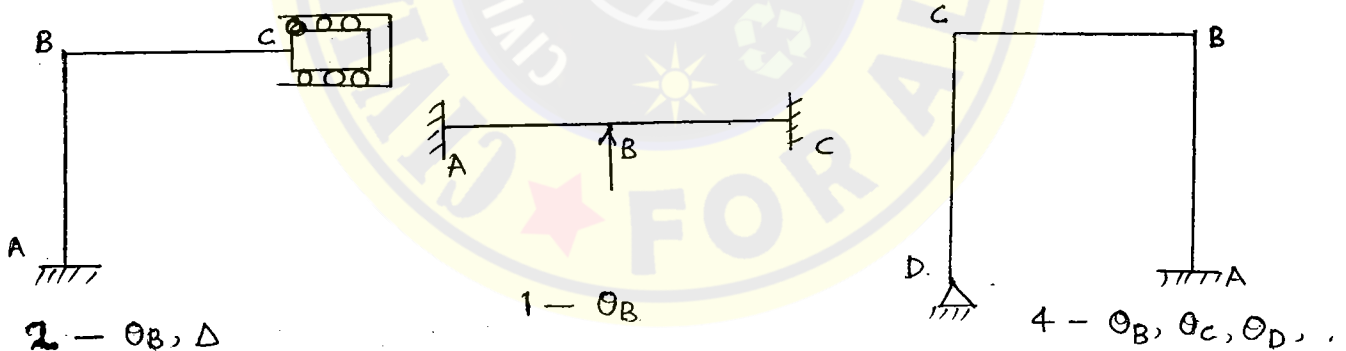
Step 2: Assume restrained structure and calculate initial support
        moments.

⊗ Stiffness methods starts with restraining the structure, ie,
formulation of compatibility equations.

Step 3: Write down Slope-Deflection equations.

$$\frac{4EI\theta_A}{L} \qquad \frac{2EI\theta_A}{L}$$

θ_A

A   EI, L   B
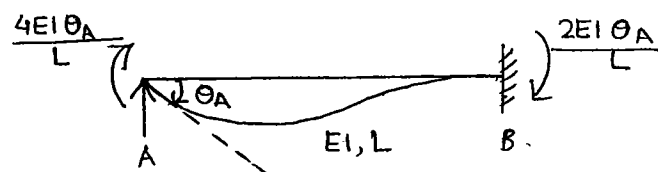
a) Allowing rotation, θ_A (clockwise) at near end A.

b) Allow rotation θ_B (clockwise) at B.

$$\frac{2EI\,\theta_B}{L}$$

$$\frac{4EI\,\theta_B}{L}$$
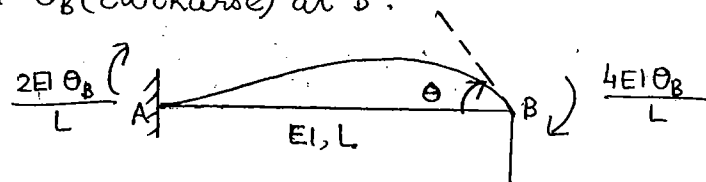
c) Assume <u>right support sinks</u> by Δ
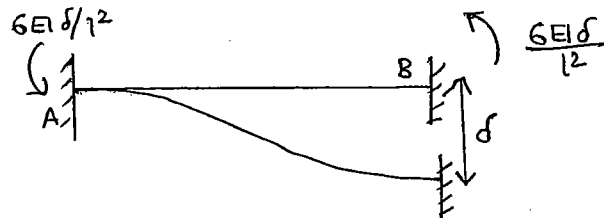
$$\frac{6EI\,\delta/l^2}{}$$

$$\frac{6EI\,\delta}{l^2}$$

d) Assume initial support moments by clockwise.

$$\overline{M}_{FAB}$$ $$\overline{M}_{FBA}$$

NOTE: $\delta \to$ relative settlement of A & B. Assume right support sinks more than that of left.

$M_{AB} \to$ final end moment @ A of beam AB.

$$M_{AB} = \frac{4EI\,\theta_A}{L} + \frac{2EI\,\theta_B}{L} - \frac{6EI\,\delta}{L^2} + \overline{M}_{FAB}$$

$$= \frac{2EI}{L}\left(2\theta_A + \theta_B - \frac{3\delta}{L}\right) + \overline{M}_{FAB}.$$

$$\boxed{M_{AB} = \frac{2EI}{L}\left(2\theta_A + \theta_B - \frac{3\delta}{L}\right) + \overline{M}_{FAB}}$$

Similarly,

$$M_{BA} = \frac{2EI}{L}\left(2\theta_B + \theta_A - \frac{3\delta}{L}\right) + \overline{M}_{FBA}.$$

NOTE:
⊙ Slope-deflection equation is the relation b/w the end moment and displacements of the corresponding beam.
⊙ The slopes and deflections (displacements) considered in the slope deflection method are due to <u>flexure only</u>. The displacement due to axial force & shear force are negligible and hence not conside

47

$$M = EI \frac{d^2y}{dx^2} \quad \text{(Moment Curvature Relationship)}$$

First integration of this gives slope. Second integration gives deflection.

⊙ The displacements due to shear force can be worked out using <u>Energy Principles.</u>

Step 4: Write down equilibrium equations equal to no number of unknown joint displacements
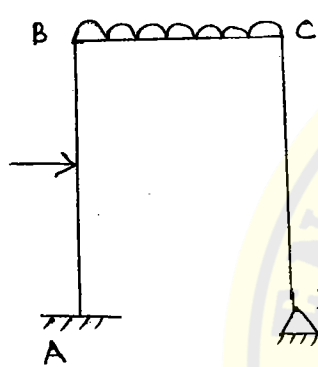
Unknown displacements $= 4 \, (\theta_B, \theta_C, \theta_D, \Delta)$

(i) $M_{BA} + M_{BC} = 0$

(ii) $M_{CB} + M_{CD} = 0$

(iii) $M_{DC} = 0$

(iv) $\Sigma H = 0$ (Horizontal Shear equation,

* Assume joint moments as clockwise

Eg:-
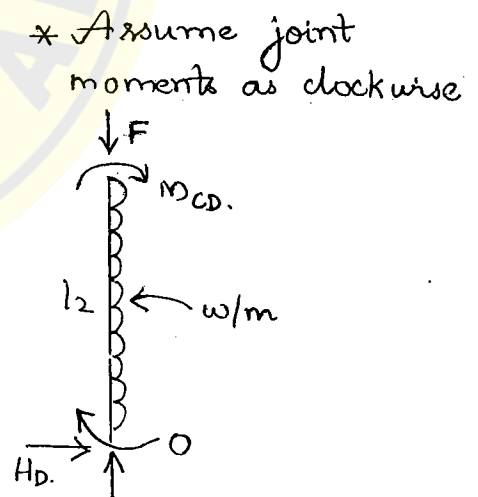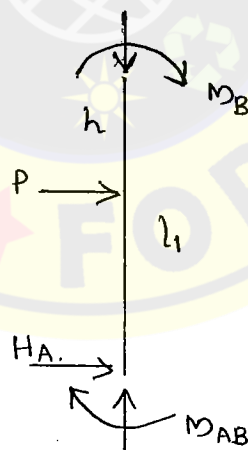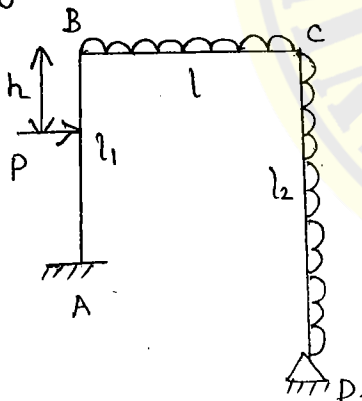
$\Sigma M_B = 0$

$\Rightarrow M_{AB} + M_{BA} - Ph - H_A \cdot l_1 = 0.$

$$H_A = \frac{(M_{AB} + M_{BA} - Ph)}{l_1}$$

$\Sigma M_C = 0,$

$$H_D l_2 = M_{CD} + w \cdot l_2 \cdot \frac{l_2}{2}$$

$$H_D = \frac{\left(M_{CD} + \frac{wl_2^2}{2}\right)}{l_2}$$

$\Sigma H = 0$,

$$H_A + H_D + P - wl_2 = 0.$$

$$\left(\frac{M_{AB} + M_{BA} - Ph}{l_1}\right) + \frac{M_{CD} + \frac{wl_2^2}{2}}{l_2} + P - wl_2 = 0$$

Q.
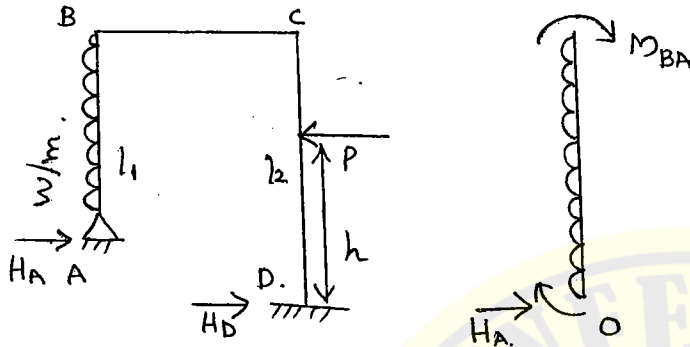
$$H_A + H_D + wl_1 - P = 0.$$

$$\frac{M_{BA} - \frac{wl_1^2}{2}}{l_1} + \frac{M_{CD} + M_{DC} + P(l_2 - h)}{l_2} + wl_1 - P = 0.$$

Step 5 : Solve the equilibrium equations to calculate unknown joint displacements and substitute them in the slope deflection equations to calculate the end moments.

8. PLASTIC THEORY

Centroid:

It is the point through which if any axis is drawn (centroidal axis), moment of intertia is the least.

$$I_{xx} = I_G + A\bar{x}^2$$

$y_b > y_t$

$f_b > f_t$
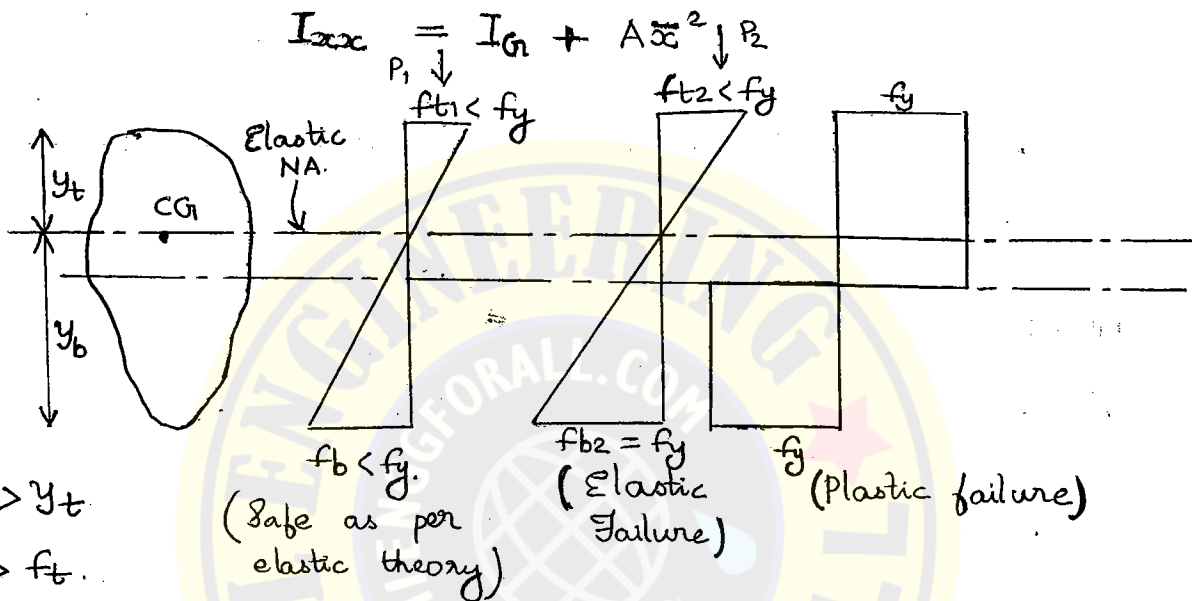
NOTE:

⊙ According to Elastic theory, a section is said to have failed even if one of the extreme fibre reaches the yield stress.

⊙ All other ~~interior~~ fibres are understressed (less than f_y). It means the material of the entire section is not utilised fully. Hence designs as per elastic theory are uneconomical.

⊙ In plastic theory, material of the entire c/s is utilised fully and hence economical.

⊙ Redistribution of stresses beyond elastic zone is the basis for plastic theory. Plastic theory is widely used in the design of statically indeterminate structures (steel structures) where high tensile / high carbon steels are not used
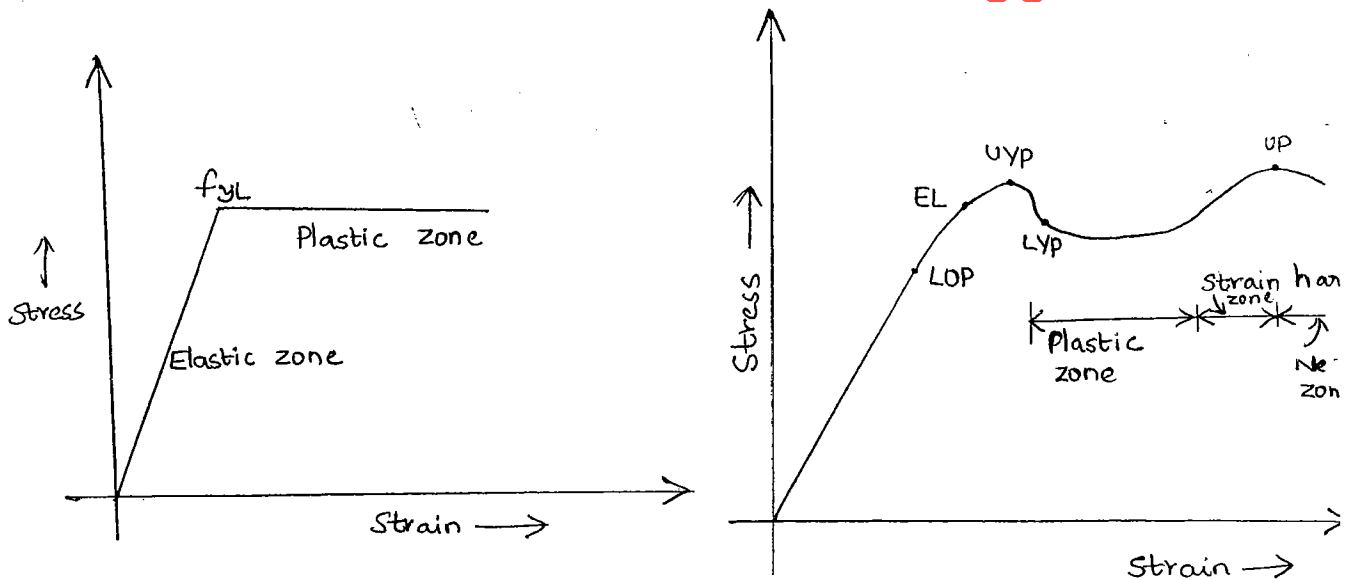
The 'idealised' or bi-linear elasto plastic stress strain curve' of mild steel is the basis for plastic theory.

NOTE:

⊙ Strain hardening & necking zones are neglected. It means some margin of safety is available.

⊙ Upper yield point is neglected as it is unreliable (shape sensitive). Lower yield point is the considered as the yield stress of the material.

⊙ Within the elastic zone, Hooke's Law is valid. (Stress ∝ Strain It means as strain increases, stress increases.

⊙ Hooke's Law is not valid in the plastic zone. As seen in the fig; in the plastic zone stress is constant irrespective of the strain.
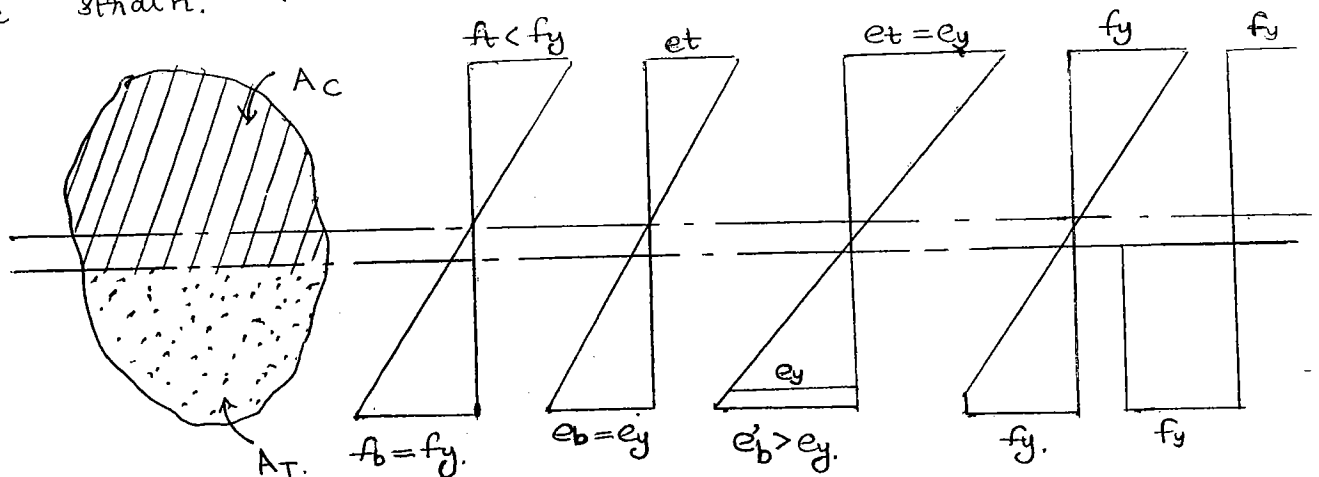
If the loads are more than elastic failure load, the ②49

strain of some of fibres will be more than yield strain, ey. Wherever strain is more than ey, stress becomes fy. At a particular load, strain of all the fibres becomes greater than ε and hence all fibres reach yield stress, fy. The failure when there is total redistribution of stresses (all fibres reaching fy) is called Plastic failure.

The section will have <u>infinite rotation</u> (infinite curvature) at plastic failure, hence it's called '<u>Plastic Hinge</u>'

* Reason for Plastic NA <u>may not</u> coincide with Elastic NA

Any theory has to satisfy eqbm equations.

Say $\Sigma H = 0$.

Compressive force, C = Tensile force, T.

$$f_{yc} \cdot A_c = f_{yT} \cdot A_T.$$

If it's assumed that $f_{yc} = f_{yT}$, then $A_c = A_T$. It means the plastic NA <u>shall</u> divide the total area into two equal parts. Hence plastic NA is also called '<u>Equal Area Axis</u>' (assuming $f_{yc} = f_{yT}$).

Elastic NA will pass through the centroid of the section. The centroidal axis <u>need not</u> divide the total area into two equal parts.

Hence, elastic NA and plastic NA <u>need not</u> coincide.

Exceptions:

(i) In Sections which are symmetrical about bending axis, the elastic NA & the plastic NA will coincide.
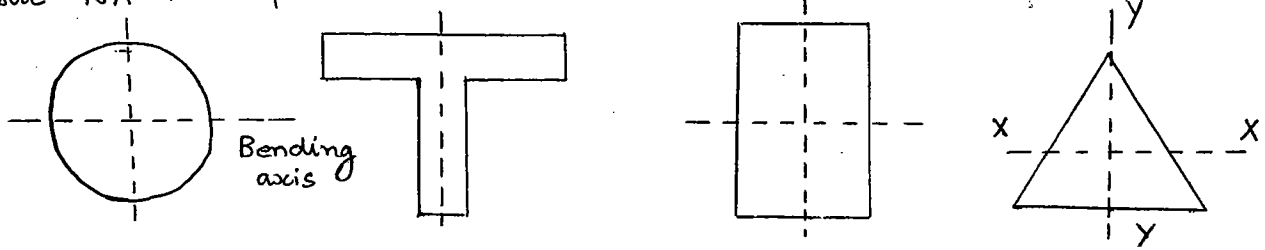
Bending axis

For the equilateral triangle shown, for bending about y-ax plastic and elastic neutral axis will coincide. For bending abou x-axis of triangle, they do not coincide.

Validity of Plastic Theory:

This theory is valid for ductile materials like mild steel for which low carbon steels are used. If carbon content increases, ductility decreases at the cost of plastic zone. It is not useful for brittle materials like plain concrete, high carbon steel, brass, bronze, masonry etc.
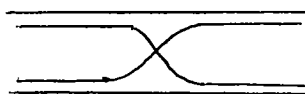
NOTE:

⊙ It can be used in RCC but in a different way compared to mild steel.

⊙ It can be used with copper and aluminium.

⊙ Structures subj. to impact and vibration shall not be designed using plastic theory. Eg: Bridges, parking lots in upstairs.

Aug, WEDNESDAY

Difference b/w Structural Hinge & Plastic Hinge:

Structural Hinge	Plastic Hinge
(i) $M = 0$	(i) $M = M_p$
(ii) Finite rotation.	(ii) Infinite rotation
(iii) Artificial.	(iii) It is the internal response at the time of plastic failure at which all the fibres reach the max. yield stress.

RCC
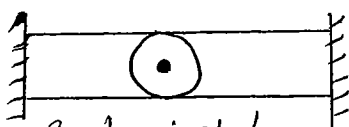
Steel Structures

Single rivet / bolt / pin.

(3)

* Plastic Modulus (z_p)

z_p = First moment of area about the plastic or equal area axis

$$z_p = A_c \overline{y_c} + A_t \overline{y_t}$$

But $A_c = A_t$,

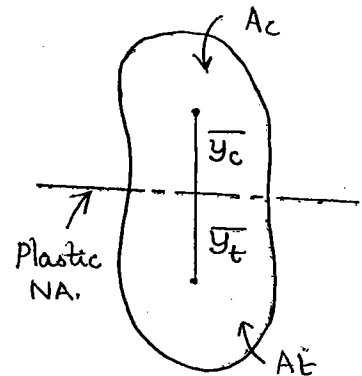$$\therefore \quad z_p = \frac{A}{2}(\overline{y_c} + \overline{y_t}).$$

* Plastic Moment of Resistance, M_p

Plastic moment, $M_p = f_y z_p.$ $\quad (= M_u)$ Ultimate moment)

Elastic moment, $M_e = f_y z.$ $\quad (= $ yield moment$).$
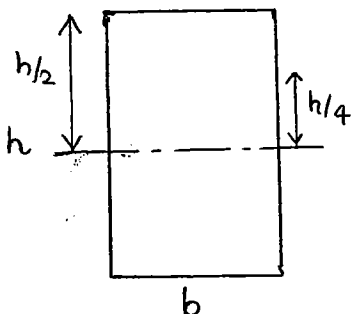
* Shape Factor.

$$\text{Shape factor,} = \frac{M_p}{M_e} = \frac{z_p}{z_e} = \frac{\text{Plastic modulus}}{\text{Section modulus}}$$

* Load Factor

$$Q = \frac{\text{collapse load}}{\text{working load (service load)}} = \frac{W_c \text{ or } W_u}{W_e}.$$

* Conceptual Formula for Load Factor.

$$Q = \frac{(\text{FOS as used in elastic theory})}{1 + \% \text{ additional stresses which can be allowe}} * \text{Shape factor}$$
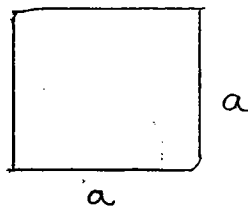with wind forces.

$$z_p = \frac{A}{2}(\overline{y_c} + \overline{y_t}).$$
$$= \frac{bh}{2}\left(\frac{h}{4} + \frac{h}{4}\right) = \frac{bh^2}{4}$$
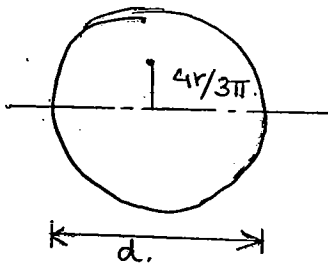$$z = \frac{I}{y_{max}} = \frac{bh^3/12}{y/2} = \frac{bh^2}{6}$$

Shape factor $= \dfrac{z_p}{Z} = \dfrac{bh^2/4}{bh^2/6} = \underline{1.5}$

$z_p = \dfrac{a^3}{4}$

Shape factor $=1.5$

$Z = \dfrac{a^3}{6}$

$z_p = \dfrac{A}{2}\left(\overline{y_c} + \overline{y_t}\right)$

$= \dfrac{\pi d^2}{4 \times 2}\left(\dfrac{2d}{3\pi} + \dfrac{2d}{3\pi}\right) = \dfrac{d^3}{6}$

$z_p = \dfrac{\pi d^4/64}{d/2} = \pi d^3/32$

Shape Factor $= \dfrac{d^3/6}{\pi d^3/32} = 1.7$
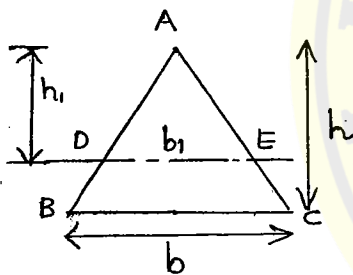
Area of ADE $= \dfrac{1}{2} \times$ area of ABC (plastic NA divides total area into two equal areas).

$\dfrac{1}{2} b_1 h_1 = \dfrac{1}{2} \times \dfrac{1}{2} bh = \dfrac{bh}{4}$
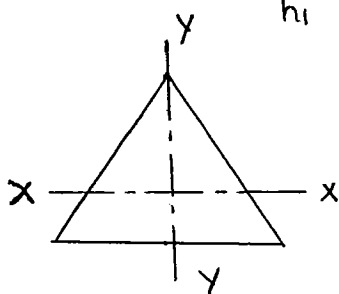
$\therefore h_1 = \dfrac{bh}{2b_1} \longrightarrow ①$

From similar triangles,

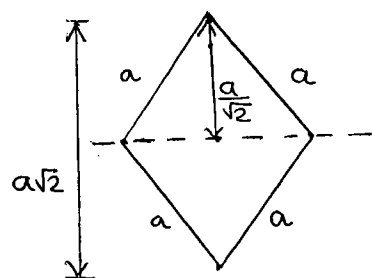$\dfrac{h_1}{b_1} = \dfrac{h}{b} \Rightarrow h_1 = \dfrac{b_1 h}{b}$ or $b_1 = b\dfrac{h_1}{h} \longrightarrow ②$

Substitute ② in ①,

$h_1 = \dfrac{bh \times h}{2bh_1}$ or $h_1 = \dfrac{h}{\sqrt{2}}$ & $b_1 = \dfrac{b}{\sqrt{2}}$

Shape factor along X-X axis $= 2.34$

Shape factor along Y-Y axis $= 2$

④

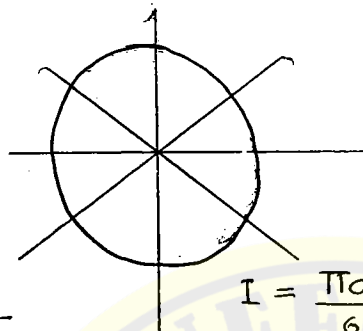$$Z_p = \frac{A}{2}(\overline{y_c} + \overline{y_t})$$

$$= \frac{a^2}{2}\left(\frac{1}{3}\frac{a}{\sqrt{2}} + \frac{1}{3}\frac{a}{\sqrt{2}}\right)$$

$$= a^3/3\sqrt{2}$$
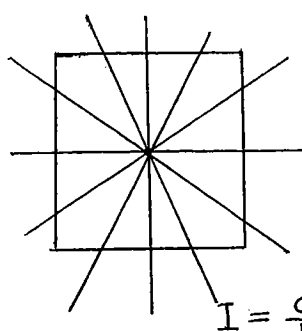
$$I = \frac{a^4}{12} \qquad\qquad I = \frac{\pi d^4}{64}$$

Moment of inertia wrt any axis passing withing the plane and centroid of a:

(i) Square $= \dfrac{a^4}{12}$

(ii) Circle $= \dfrac{\pi d^4}{64}$

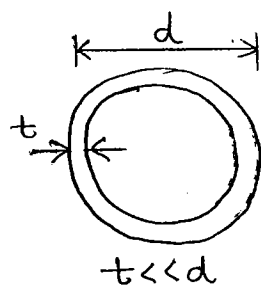$$Z = \frac{a^4/12}{a/\sqrt{2}} = \frac{a^3}{6\sqrt{2}}$$

$$\overline{h} = \frac{I_{G_1}}{A\overline{x}} + \overline{x} = \frac{I_0}{A\overline{x}}$$

$$\text{Shape Factor} = \frac{a^3/3\sqrt{2}}{a^3/6\sqrt{2}} = \underline{\underline{2}}$$

$$I_0 = \frac{\frac{\pi d^4}{64}}{2} = \frac{\pi d^4}{128}$$

$$I_0 = \text{MI about free surface}$$

$t \ll d$
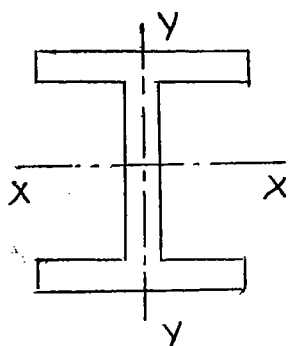
Shape Factor $= 1.27$.

$$S_{yy} \approx 1.55$$
$$S_{xx} \approx 1.11 \text{ to } 1.15 = 1.12.$$

$$Z_{xx} > Z_{yy}$$

$I_{xx} > I_{yy}$ (more area thrown away NA).

$$I = Ar^2$$

$$\boxed{\begin{array}{l} M = f \cdot z \\ z \propto I \\ I \propto A \end{array}}$$

More stronger in bending along x-x direction

Q. Statement 1: Shape factor depends upon shape of c/s $\left(S = \dfrac{Z_p}{Z_e}\right)$

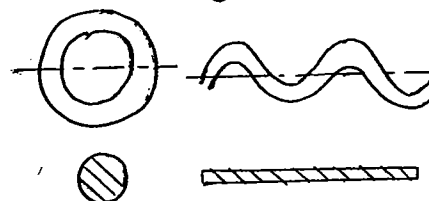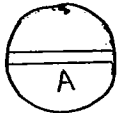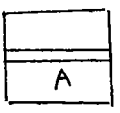Statement 2: Shape factor indicates the additional strength a section can have beyond elastic failure. $\left(S = \dfrac{M_P}{M_e}\right)$

Statement 3: Sections with bulk mass near the centroidal axis will have more shape factor compared to sections with less area at NA.

$M_p = 170$
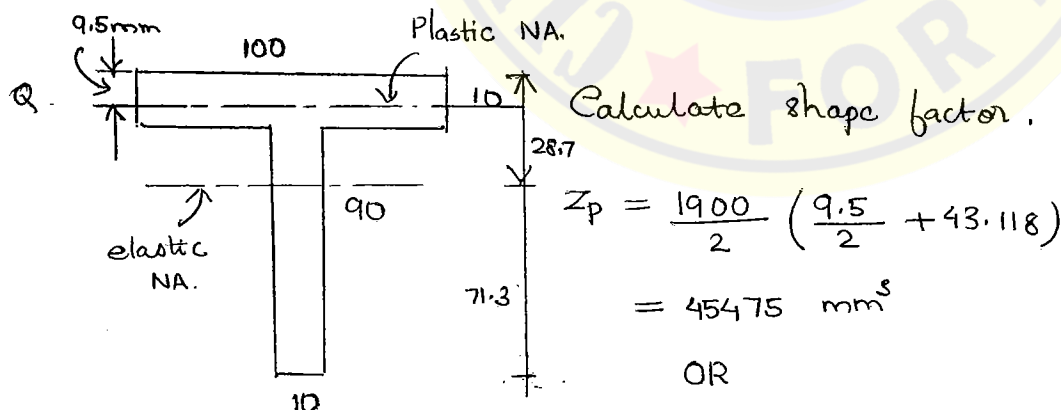$M_e = 100$

$S = 1.7$

$M_p = 170$
$M_e = 113.5$

$S = 1.5$

In simple bending, square & rectangular sections are more effective than circular section

Statement 4: An I-section is designed in bending wrt stronger axis (x-axis) and weaker axis (y axis) using both elastic and plastic designs. In fact, elastic design is uneconom compared to plastic design. In the given context, design wrt stronger axis elastically is less uneconomical compared to the design wrt y-axis.

Extra strength wrt x bending = 12% $(S_{xx} = 1.12)$

Extra strength wrt y bending = 55% $(S_{yy} = 1.55)$

Q.

9.5mm

100

Plastic NA.

10

28.7

90

elastic NA.

71.3

10

$\dfrac{A}{2} = 950$ mm²

Calculate shape factor.

$Z_p = \dfrac{1900}{2}\left(\dfrac{9.5}{2} + 43.118\right)$

$= 45475$ mm³

OR

$Z_p = 950 \times \dfrac{9.5}{2} + 0.5 \times 100 \times \dfrac{0.5}{2}$

$+ 90 \times 10 \times 45.5$
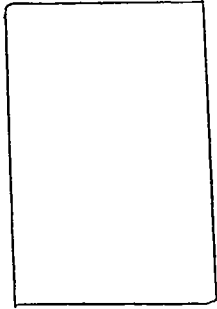
$= 45475$ mm³

100

0.5

90

10
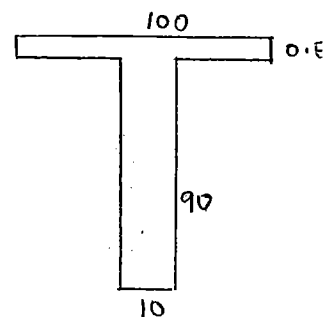
$\overline{y_t} = 0.5 \times 100 \times \dfrac{0.5}{2} +$

$\dfrac{90 \times 10 \times 45.5}{950}$

$= 43.118$

Location of elastic NA from TOP:

$\overline{y} = \dfrac{A_1 y_1 + A_2 y_2}{A_1 + A_2} = \dfrac{100 \times 10 \times 5 + 90 \times 10 \times 55}{1900} = 28.68$ mm.

(52)

$$I_G = \frac{bh^3}{12}$$

$$I_{base} = \frac{bh^3}{3}$$

$$I_{GG} = \left(\frac{100 \times 10^3}{12} + 100 \times 10 \times 23.7^2\right) + \frac{10 \times 18.7^3}{3} + \frac{10 \times 71.3^3}{3} = 1.79 \times 10^6$$

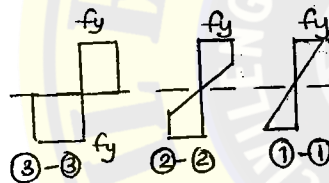Section modulus, $Z = \dfrac{I}{y_{max}} = \dfrac{1.79 \times 10^6}{71.3} = 25238 \text{ mm}^3$

Shape Factor, $S = \dfrac{Z_p}{Z} = \dfrac{45475}{25238} = \underline{\underline{1.8}}$

For T sections, $\boxed{1.79 < S < 1.81}$

5th Aug
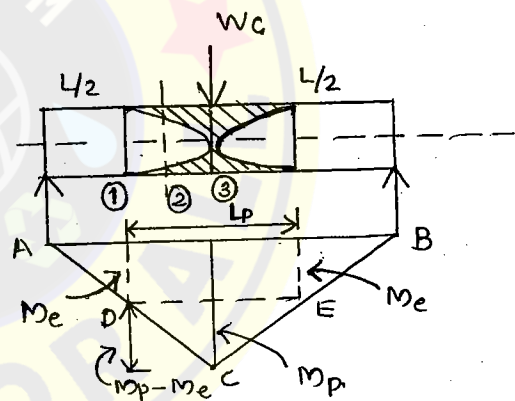TUESDAY → Length of Plastic Hinge:

(i)

①-① : Elastic
②-② : Elasto-plastic
③-③ : Plastic.

The length of the beam within which the section is either partially or fully plastified. (In the fig. shown, the hatched portion).

Using similar Δ^{le} concept for ΔABC & ΔDEC,

L_p → length of plastic zone,

$$\frac{L_p}{L} = \frac{M_p - M_e}{M_p}$$

$$L_p = L\left(1 - \frac{M_e}{M_p}\right) = L\left(1 - \frac{1}{S}\right)$$

$$\boxed{L_p = L\left(1 - \frac{1}{S}\right)} \quad ; \quad S \rightarrow \text{shape factor}$$

- If the beam is of rectangular c/s, $S = 1.5$.

$$L_p = \left(1 - \frac{2}{3}\right) L = \frac{L}{3}$$

$$\therefore \quad L_p = \frac{L}{3}$$

- If the beam is of solid circular section, $S = 1.7$.

$$L_p = L \left(1 - \frac{1}{1.7}\right)$$

$$\therefore \quad L_p = \frac{L}{2.43}$$

- If diamond section, $S = 2$.

$$L_p = L \left(1 - \frac{1}{2}\right)$$

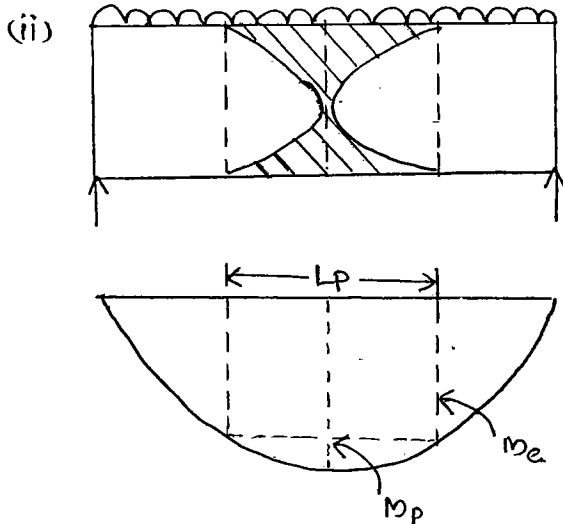$$\therefore \quad L_p = \frac{L}{2}$$

- If tubular section, $S = 1.27$.

$$L_p = L \left(1 - \frac{1}{1.27}\right)$$

$$\therefore L_p = \frac{L}{4.7}$$

- For an I-section, $S_{xx} \approx 1.12$

$$L_p = L \left(1 - \frac{1}{1.12}\right)$$

$$\therefore \quad L_p = \frac{L}{9.33}$$

(ii)

Similar parabola concept,

$$\frac{x^2}{y} = \text{const.}$$

$$\frac{L_p^2}{L^2} = \frac{M_p - M_e}{M_p}$$
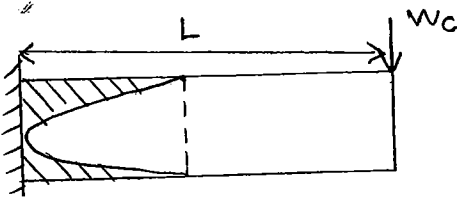
$$L_p = L \sqrt{1 - \frac{1}{S}}$$
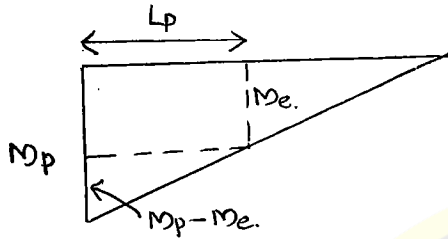
⊙ If rectangular c/s, S = 1.5.

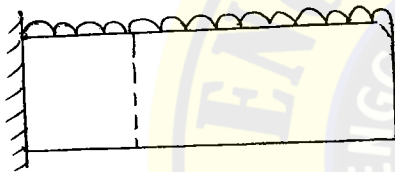$$\therefore L_P = \frac{L}{\sqrt{3}}.$$

(iii)

Using similar triangles,

$$\frac{L_P}{L} = \frac{M_p - M_e}{M_p}$$

$$L_P = L\left(1 - \frac{1}{S}\right).$$

(iv)

Using parabola concept,

$$\frac{x^2}{y} = C.$$

$$\frac{L_P^2}{L^2} = \frac{M_p - M_e}{M_p}.$$

$$L_P = L\sqrt{1 - \frac{1}{S}}$$

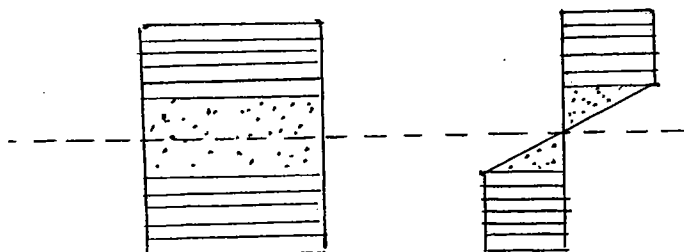→ Moment of Resistance of Elasto Plastic Section
(Plastified Partially)

Q. A rectangular section of b×h is plastified to one ~~four~~ third depth from top & bottom. Calculate moment of resistance

M_{ep} = moment of resistance of elasto plastic section

$$= M_{e_1} + M_{P_1} \; ; \quad \text{where } M_{e_1} \rightarrow \text{MR of elastic par}$$

$$M_{P_1} \rightarrow \text{MR of plastified part}$$

$$M_{e1} = f_y \, Z_{e1} = f_y \times \frac{b}{6} \left(\frac{h}{3}\right)^2 = f_y \, \frac{bh^2}{54}$$

$$M_{p1} = f_y \, Z_{p1} = 2 \times f_y \cdot \frac{bh}{3} \left(\frac{h}{6} + \frac{h}{6}\right) \dots$$ Z_{p1} : moment of plastic area wrt plastic

$$= 2 \, f_y \, \frac{bh^2}{9}.$$

$$M_e = f_y \, \frac{bh^2}{54} + 2 f_y \, \frac{bh^2}{9} = \frac{13}{54} \, f_y \, bh^2 = \frac{f_y \, bh^2}{4.154}$$

shortcut :

If the rectangle is fully plastified,

$$Z_p = \frac{bh^2}{4}$$

$$M_p = f_y \, \frac{bh^2}{4}.$$

If the rectangle is fully elastic,

$$M_e = f_y \, Z = f_y \, \frac{bh^2}{6}.$$

If the rectangle is partially plastified, denominator should be b/w 4 & 6.

$$M_e < M_{ep} < M_p.$$

$$f_y \, \frac{bh^2}{6} < M_{ep} < f_y \, \frac{bh^2}{4}$$

$\rightarrow$ Location of Possible Plastic Hinges :

1. At the points of max. moment, plastic hinges'll form

For eg:— a) at fixed supports.

b) at rigid joints.
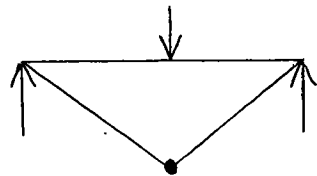
c) change of material and c/s

d) under point loads. with supports on either side.
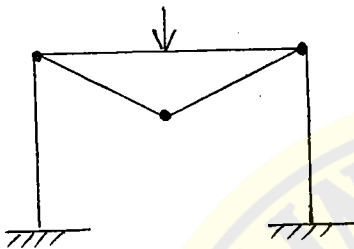
→ Types of Mechanisms (Failure modes)

⑤④

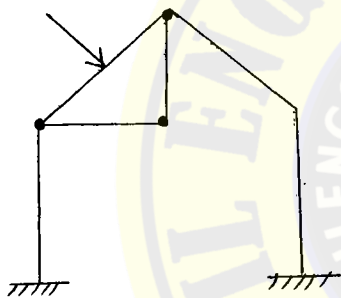A structure'll convert into a mechanism if no. of plastic hinges = $D_S + 1$; D_S → static indeterminacy.
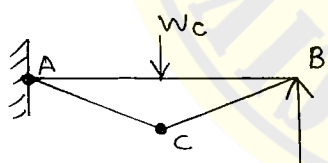
1. Beam Mechanism.

$$n = D_S + 1$$
$$= 0 + 1 = \underline{1}$$

$$n = D_S + 1$$
$$= 2 + 1 = \underline{3}$$

$$n = D_S + 1$$
$$= 2 + 1 = \underline{3}$$
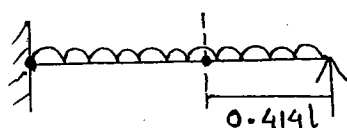
$$M_A = \frac{3wl}{16}$$

$$M_C = \frac{5wl}{16 \times 2} = \frac{5wl}{32}$$

As $M_A > M_C$, first plastic hinge'll be formed at A. (fixed support)

Then it will act as SSB; redistribution of moment occur

After sometime, second plastic hinge'll be formed under point load. Total collapse occurs for the structure.

0.414l

Beam is weaker towards right (propped support)

27th Aug,
WEDNESDAY

→ Basic Theorems of Plastic Analysis.

1. Static Theorem or Lower Bound Theorem.

In this method, BMD of the given structure is used in the analysis, hence called Static Theorem. This theorem has to satisfy the relations:
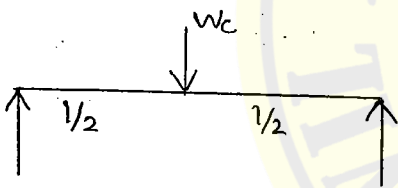
$$M \not> M_p \quad \& \quad W \leq W_c$$

since the load according to this method has a chance of less than collapse load, it is called Lower Bound Theorem. It is also called a method on 'Safer Side'.
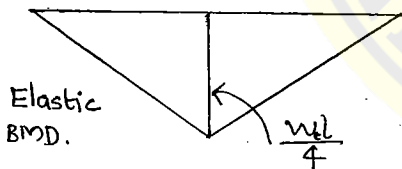
Static Theorem has to satisfy:

(i) Equilibrium conditions.

(ii) Plastic moment condition or Yield condition.

Q.
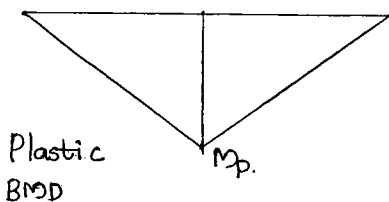

Elastic BMD.

$\dfrac{w_c l}{4}$

Plastic BMD

M_p

SSB subjected to Central Point Load.

Step 1: Draw BMD.

Step 2: At the point of max. BM, plastic hinge will develop and hence moment = M_p.
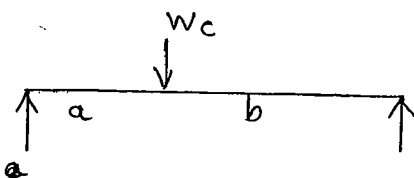
Step 3: Apply Equilibrium equations.

$$\frac{W_c l}{4} = M_p$$
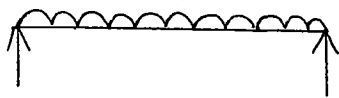
$$\Rightarrow \boxed{W_c = \frac{4 M_p}{l}}$$

Q.

$$\frac{W_c \, ab}{l} = M_p$$

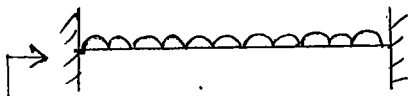$$\boxed{W_c = \frac{M_p \, l}{ab}}$$
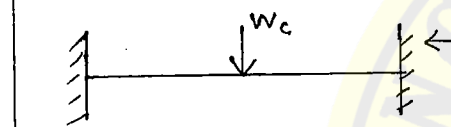
(4)

S6

$$\frac{W_c\, l^2}{8} = M_p$$

$$\boxed{W_c = \frac{8 M_p}{l^2}}$$

$W_c \to$ load/m.

$$\frac{W_c\, l}{4} = 2 M_p.$$

$$\boxed{W_c = \frac{8 M_p}{l}}$$

M_p

M_p

W_c

$$\frac{W_c\, l^2}{8} = 2 M_p$$

$$\boxed{W_c = \frac{16 M_p}{l^2}}$$

M_p

$\frac{wl^2}{8}$

M_p

W_c

$$\frac{W_c\, l}{4} = M_p + \frac{M_p}{2}$$

$$\boxed{W_c = \frac{6 M_p}{l}}$$

M_p $M_p/2$

M_p.

W_c

$$W_c \times l = M_p$$

$$\boxed{W_c = \frac{M_p}{l}}$$

M_p
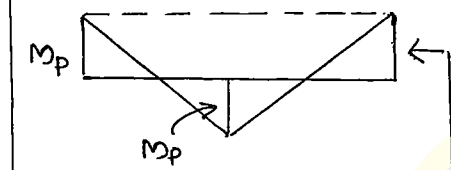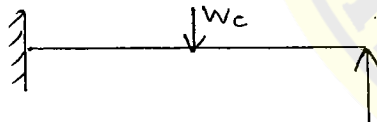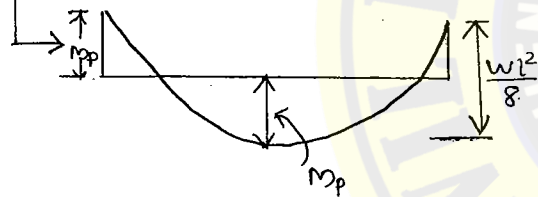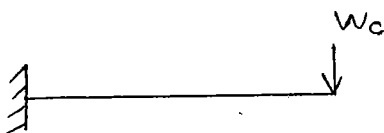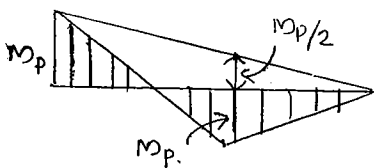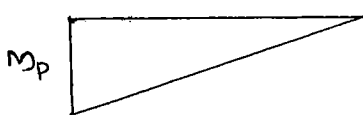
NOTE: Static theorem is useful only for simple cases as discussed above. For beams with change c/s and continuous beams, it is complicated. Kinematic Theorem is the best method for

complex problems.

2. Kinematic Theorem or Upper Bound Theorem or Mechanism method or Virtual Work Method.

It is a method on unsafer side if proper care not taken in the analysis.

* Characteristics:

$$W \geq W_c$$

$$M \not> M_p$$

Work = F × δ

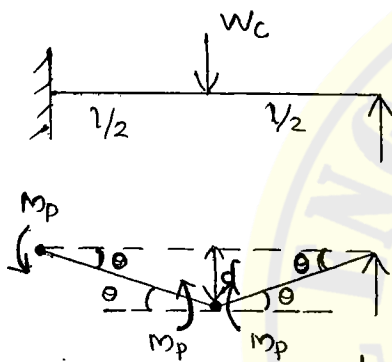External work,

$$W_e = W_c \times δ$$
$$= W_c \times θ \times \frac{l}{2}$$

Internal work,

$$W_i = M_p θ + M_p θ + M_p θ$$
$$= 3 M_p θ$$

$$W_e = W_i$$

$$W_c \times θ \times \frac{l}{2} = 3 M_p θ$$

$$\boxed{W_c = \frac{6 M_p}{l}}$$

$$\tan θ = \frac{δ}{l/2}$$

In this method, failure modes are mechanisms are considered in the analysis. Hence called 'mechanism method'

No: of independent mechanisms, $I = N - D$

$$I = 2 - 1 = 1$$

For the given beam, $D_s = 1$.

∴ No: of plastic hinges for collapse, $n = D_s + 1 = \underline{2}$

In this method, the slopes and deflection at failure, which are nothing but kinematic parameters are considered. Hence called 'Kinematic Theorem'

In this method, displacements are assumed to be small or virtual. Further external work is equated to internal energy. Hence it is called 'Virtual Work method'

According to this method, $W \geq W_c$. As the load has a chance of exceeding the collapse load, it is a method on 'unsafer side'.

⊙ Plastic moment will develop at a point if the following two conditions are simultaneously satisfied : (57)

(i) Plastic hinge formed.

(ii) Change of slope. of the member. occured.

Design M_p to be considered wherever change of c/s occur.

Q.
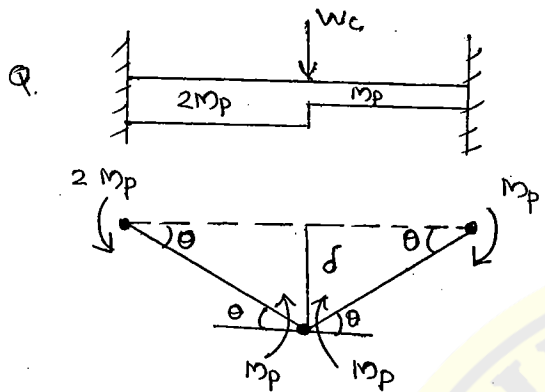
For the non-prismatic beam shown, third plastic hinge formed at the change of c/s, We know the failure at a joint occurs because of weaker member (side). At joint c on the beam, part cB has M_p. Hence both sides of c, least value M_p shall be considered.
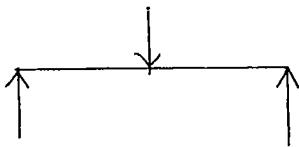
$W_e = W_c \times \delta$

$\quad = W_c \times \theta \times \dfrac{l}{2}$

$W_i = 2M_p\theta + M_p\theta + M_p\theta$

$\qquad + M_p\theta$.

$\quad = 5M_p\theta$.

$\Rightarrow W_c \times \theta \times \dfrac{l}{2} = 5M_p\theta$.

$\qquad W_c = \dfrac{10\,M_p}{l}$.
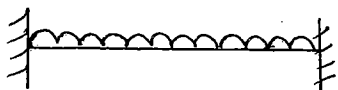

$W_e = W_c\,\delta = W\theta \times \dfrac{l}{2}$

$W_i = 2M_p\theta$
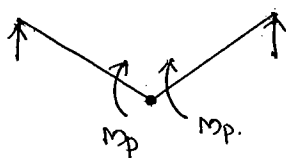
$W_c = \dfrac{4M_p}{l}$.

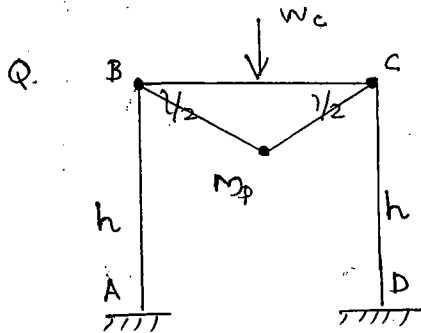$W_e = W_c\, l\left(\dfrac{\theta + \delta}{2}\right) = \dfrac{W_c\, l}{2} \times \dfrac{l}{2}\theta$.

$\qquad = W_c\left(\dfrac{l\delta}{2}\right)$

$\qquad = $ intensity of udl × area of mechanism under udl.

Internal work $= 4 M_p \theta$.

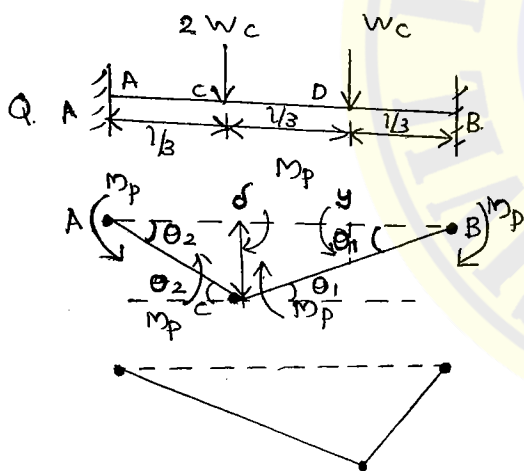$$\frac{W_c \, l^2 \theta}{4} = 4 M_p \theta$$

$$W_c = \frac{16 M_p}{l^2}$$

Q.

The given frame will not sway because of symmetry in all aspects. It can have only beam mechanism. As B & C are intern rigid joints, plastic moment can develop. Hence only beam mechanism possible like fixed beam with central point load.

$$W_c = \frac{8 M_p}{l}$$

Q.

Two different mechanisms are possible $(I = 2)$.

Mechanism I: A C B (plastic hinges at A, B & C)

Longer side — less slope $(\theta_1 < \theta_2)$.

⊙ Slopes are inversely proportional to distances

$$\delta = a \theta_2 \quad \& \quad \delta = b \theta_1$$

$$\theta_2 = \theta_1 \left(\frac{b}{a}\right)$$

$$= \theta \left(\frac{2l}{3} \times \frac{3}{l}\right) = 2\theta.$$

$$I = N - D_s$$

$$= 4 - 2 = \underline{\underline{2}}$$

$$\theta_2 = \frac{\delta}{l/3} \Rightarrow \delta = \theta_2 \times \frac{l}{3}.$$

$$\theta_1 = \frac{\delta}{2l/3} \Rightarrow \delta = \theta_1 \times \frac{2l}{3}$$

External work $W_e = 2 W_c \times \delta + W_c \times y.$

$$= 2 W_c \times \delta + W_c \frac{\delta}{2} = 2.5 \, W_c \, \delta$$

Internal work, $W_i = M_p \theta_2 + M_p \theta_2 + M_p \theta_1 + M_p \theta_1$

$$= 6 M_p \theta.$$

(11)

(S8)

$$2.5 \, w_c \times d = 6 \, M_p \, \theta.$$

$$2.5 \, w_c \times \frac{l}{3} \times 2\theta = 6 \, M_p \, \theta$$

$$w_c = \frac{18 \, M_p}{5l}$$

Mechanism II: ADB

External work done $= w_c \times d + 2 w_c \dfrac{d}{2}$
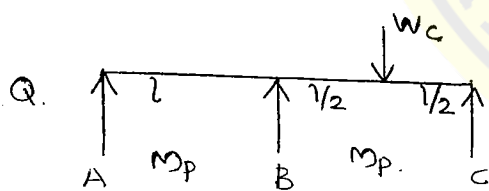
$$= 2 \, w_c \, d.$$

Internal work done $= 6 \, M_p \, \theta.$

$$2 \, w_c \times \frac{l}{3} \times 2\theta = 6 \, M_p \, \theta.$$

$$w_c = \frac{9 \, M_p}{2l}$$

The collapse load of a structure is the least value of various mechanisms. ∴ for the given beam, $w_c = \dfrac{18 \, M_p}{5l}$.

⊙ In objective paper, we can find w_c with mechanism I as $2w_c$ gives critical condition compared to w_c.
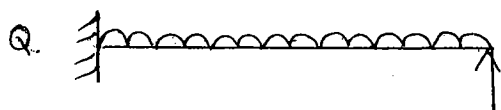
Q.

No load on AB.

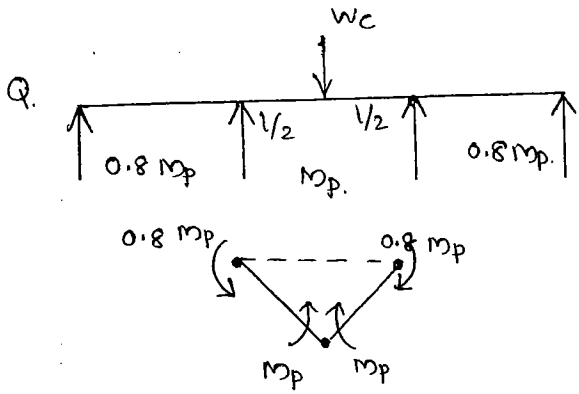B being internal joint, BC'U behave like propped cantilever

$$\therefore \quad w_c = \frac{6 \, M_p}{l}$$

Q.

$$w_c = \frac{11.66 \, M_p}{l^2}$$

For SSB with udl, $w_c = \dfrac{8 M_p}{l^2}$

For fixed beam with udl, $w_c = \dfrac{16 M_p}{l^2}$
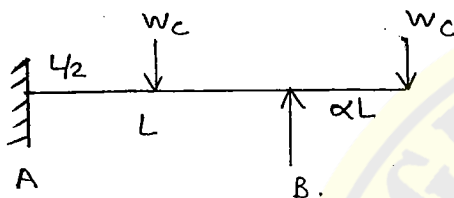
Q.

$$W_e = W_c \frac{l}{2} \theta$$

$$W_i = 0.8 M_p \theta + M_p \theta + M_p \theta + 0.8 M_p \theta$$

$$= 3.6 M_p \theta$$

$$W_c = \frac{7.2 M_p}{l}$$

8th Aug,
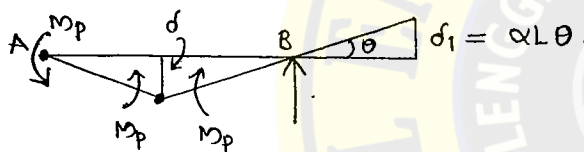THURSDAY

Q. Calculate value of α for economical design.

$$I = N - D_S$$
$$= 3 - 1 = 2$$

WD by load at c
$= - W_c \, d_1$
as W_c & d_1 are
opposite direction

Mechanism I:

$d_1 = \alpha L \theta$.

$$W_e = W_c \, d - W_c \, d_1$$
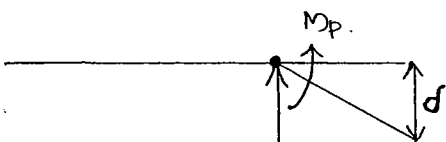$$= W_c \times \theta \times \frac{L}{2} - W_c \times \alpha L \theta = \frac{W_c L \theta}{2} (1 - 2\alpha)$$

$$W_i = 3 M_p \theta$$

$$W_e = W_i$$

$$\frac{W_c L \theta}{2} (1 - 2\alpha) = 3 M_p \theta$$

$$W_c = \frac{6 M_p}{(1 - 2\alpha) L} \longrightarrow ①$$

Mechanism II:

$$W_e = W_c \, d = W_c \times \alpha L \theta$$
$$W_i = M_p \theta$$

Towards left of hinge, there is no change of slope. So no M_p to the left.

$$W_c \, \alpha L \theta = M_p \theta$$
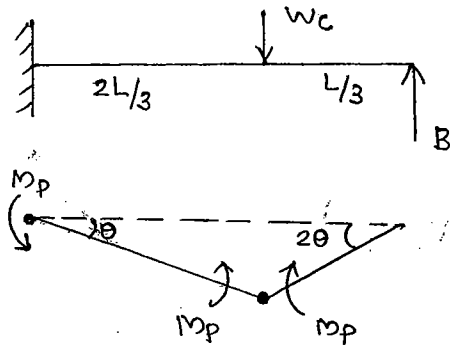$$W_c = \frac{M_p}{\alpha L} \longrightarrow ②$$

■ Design is economical if the collapse load of various mechanisms are equal.

$$\Rightarrow \frac{6 M_p}{(1 - 2\alpha) L} = \frac{M_p}{\alpha L} \qquad \Rightarrow 1 - 2\alpha = 6\alpha$$

$$\therefore \alpha = \frac{1}{8}$$

(12)

(59)

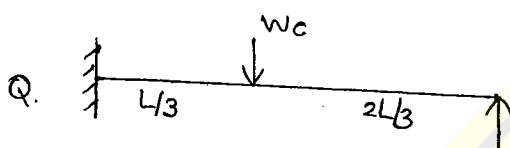**Q** Calculate value of collapse load.

$$We = Wc \times \delta$$
$$= Wc \times \frac{2L}{3} \times \theta$$
$$Wi = 4 M_p \theta$$
$$Wc \times \frac{2L}{3} \theta = \underline{4 M_p \theta}$$
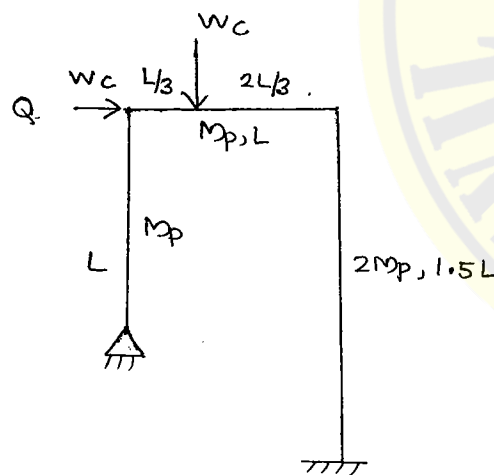
$$Wc = \underline{\frac{6 M_p}{L}}$$

Q.

$$We = Wc \times \frac{L}{3} \times 2\theta$$
$$Wi = M_p (2\theta + 2\theta + \theta) = 5 M_p \theta$$
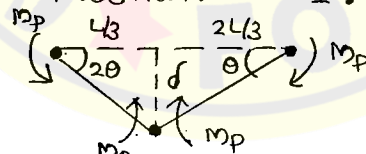$$Wc \times \frac{L}{3} \times 2\theta = 5 M_p \theta$$
$$\Rightarrow Wc = \underline{\frac{7.5 M_p}{L}}$$

Q.

$$I = N - DS$$
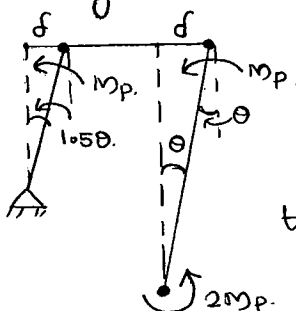$$= 4 - 2 = \underline{2}$$

Mechanism I: Beam mechanism.

$$We = Wc \times \frac{2L}{3} \times \theta$$
$$Wi = M_p (2\theta + 2\theta + \theta + \theta) = 6 M_p \theta$$
$$Wc \times \frac{2L}{3} \theta = 6 M_p \theta$$

Mechanism II:
Sway mechanism.

$$Wc = \frac{9 M_p}{L}$$

(∵ load is eccentric, it will be more than $\frac{8n}{L}$

Longer side → smaller angle.
No change of slope in beam. ∴ no M_p.
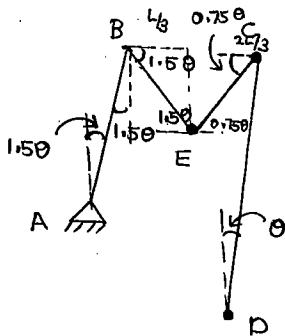◻ In this case, the horizontal load only can do the work.

$$We = Wc \delta = Wc \times L \times 1.5\theta$$
$$Wi = 2 M_p \theta + M_p \theta + 1.5 M_p \theta = 4.5 \ M_p \theta$$

$W_e = W_i$

$\Rightarrow \underline{\underline{W_c = \dfrac{3M_p}{L}}}$

Mechanism 3: Combined Mechanism.

$W_e = W_c \times 1.5\theta \times \dfrac{L}{3} + W_c \times 1.5\theta \times L$.

$= 2 W_c L\theta$.

$W_i = 2M_p\theta + M_p\theta + M_p \times 0.75\theta + 2M_p(0.75\theta + 1.5)$

$= 7.25 \, M_p\theta$.

$2 W_c L\theta = 7.25 \, M_p\theta$.

$\underline{\underline{W_c = 2.3 \dfrac{M_p}{L}}}$

Collapse load is the least of various possible mechanisms.

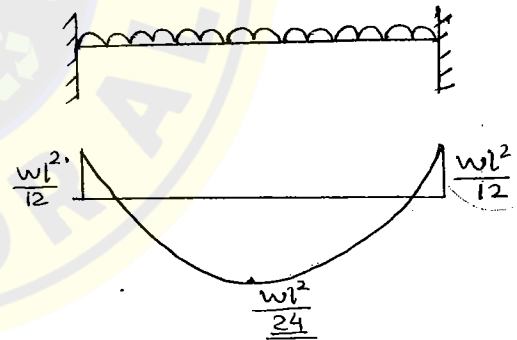$\therefore \underline{\underline{W_c = \dfrac{3M_p}{L}}}$

$\rightarrow$ Redistribution of Moments & Reserve Strength.

Design moment according to elastic theory, $M_e = \dfrac{W_e L^2}{12}$

$W_e = \dfrac{12 \, M_e}{L^2}$

$W_c = \dfrac{16 \, M_p}{L^2}$

$RS = \dfrac{W_c}{W_e} = \dfrac{16 M_p}{L^2} \times \dfrac{L^2}{12 M_e}$

$= \dfrac{4}{3} \dfrac{M_p}{M_e} = \dfrac{4}{3} S$.

Reserve strength is obtained by redistribution of moments from elastic failure to plastic failure.

■ Reserve of strength means the additional load a structure can take according to plastic design compared to elastic design.

(13)

60

For a fixed beam of rectangular c/s with udl,

$$\text{Reserve strength} = \frac{w_c}{w_e} = \frac{4}{3} \times S$$

$$= \frac{4}{3} \times \frac{3}{2} = 2.$$

ie; if elastic udl = 100 N, then plastic udl = 200 N.

■ Most of the industrial steel structures are built-up I-section for which shape factor ≈ 1.12 to 1.15.

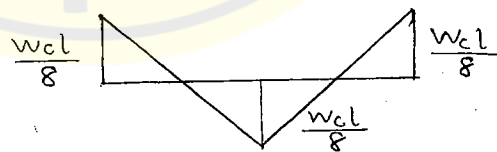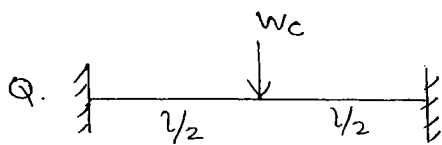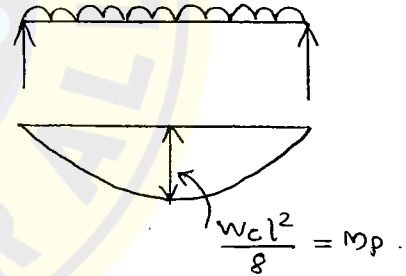Q If the fixed beam discussed above is an I-section, plastic failure load is ___ ?

$$\text{Plastic failure load}; \frac{w_c}{w_e} = \frac{4}{3} \times S = \frac{4}{3} \times 1.12 = 1.5$$

$$\therefore \quad w_c = 1.5 \, w_e = \underline{150 \, N}$$

Q
$$M_e = \frac{w_e l^2}{8}$$

$$M_p = \frac{w_c l^2}{8}$$

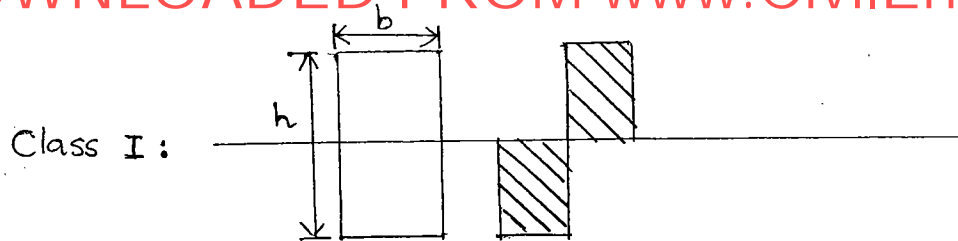$$\frac{w_c}{w_e} = \frac{8 M_p / l^2}{8 M_e / l^2} = \underline{\underline{S}}$$

$$\frac{w_c l^2}{8} = M_p$$

$$\frac{w_c l}{8} \qquad \frac{w_c l}{8}$$

$$\frac{w_c l}{8}$$

Q.
$$\frac{l}{2} \quad w_c \quad \frac{l}{2}$$

$$M_e = \frac{w_e \times L}{8} \Rightarrow w_e = \frac{8 M_p}{L}$$
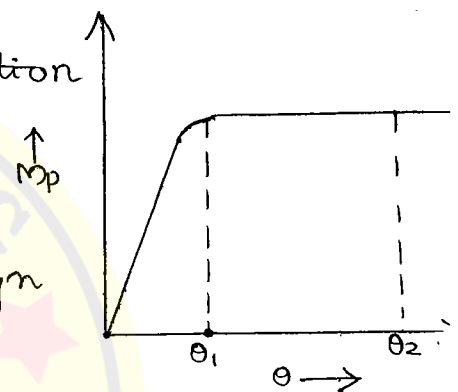
$$M_p = \frac{8 M_p}{L}$$

$$\frac{w_c}{w_e} = \underline{\underline{S}}$$

Class I :

If max stress reached in all fibres and if $\frac{\theta_2}{\theta_1} > 6$, such c/s are only to be used in the plastic design.

$\theta_1 \rightarrow$ Rotation at the elastic failure or at the beginning of plastic deformation

$\theta_2 \rightarrow$ Rotation at the end of plastic deformation

Class II : Compact Sections

Compact sections are used for the design of general structural elements.

$$\frac{\theta_2}{\theta_1} < 6.$$

Class III : Semi compact Sections :

Local buckling of the c/s shall not occur. Local buckling can be avoided by maintaining suitable height to thickness (d/t) ratio; ie slenderness ratio not exceeding certain value. In A semi-compact section, some fibres may reach yield stress, but not all the fibres.
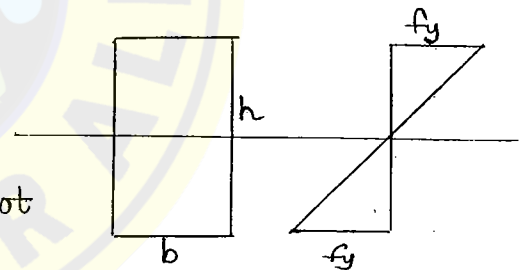
Class IV : Slender

If local buckling occurs the extreme fibres may not reach the yield stress. Such sections are called slender. These sections are used in cold formed members (cold rolling)

Q.38 A section shall be designed for max. moment of resistance

$$M_p = \frac{w_c L}{6}$$

$$= f_z \, z_p$$

$$\therefore z_p = \frac{w_c L}{6 f_y}$$

Section modulus, $z = \frac{z_p}{S} = \frac{w_c L}{6 f_y \times 1.5} = \frac{w_c L}{9 f_y}$
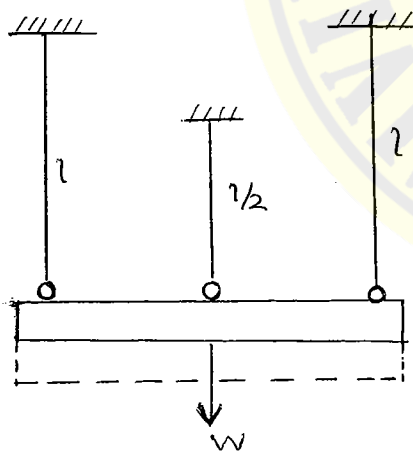
P-85

Q.09 Deformation just observed means it is elastic failure.

$$\frac{w_c}{w_e} = \frac{4}{3} S$$

$$= \frac{4}{3} \times \frac{3}{2} = 2$$

$$\therefore w_c = 2 \times 10 = 20 \text{ kN/m}$$

Q.10

$\because$ the load at the frame is symmetric in all aspects, all the 3 wires have same elongation.

$$dl = \frac{Pl}{AE}$$

$$E = \frac{f_y}{e} \implies f_y = eE$$

$$= \boxed{\frac{dl}{l}} E$$

Strain, $e = \frac{dl}{l}$; As the length of middle wire is less, it'll have max strain. According to Hooke's Law,

Stress α Strain (within elastic limit)

As middle wire has max strain, it will reach yield stress first.

Load taken by middle wire at the time of elastic failure

$$= f_y A$$

As strain is inversely proportional to length $(e = \frac{dl}{l})$ end long wires which are twice the length of middle wires will have half of the strain of middle wire. Hence stress in end wire $= \frac{f_y}{2}$.

∴ Load taken by end wires at the time of elastic failure of middle wire $= \frac{f_y}{2} \times A + \frac{f_y}{2} \times A$.

∴ Total elastic load, $W_e = 2 f_y A$.

At collapse condition the stress in the ~~middle~~ end wires all becomes f_y. ∴ collapse load, $W_c = f_y A + f_y A + f_y A$

$$= 3 f_y A.$$

$$\frac{W_c}{W_e} = \frac{3 f_y A}{2 f_y A} = \underline{3 : 2}$$

∗ Calculation of elongation :

Elastic deformation of the rigid block —

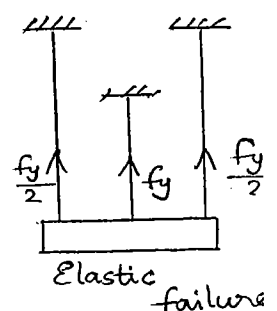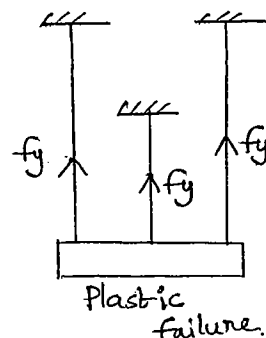It can be calculated for any of the three wires as all the 3 elongate uniformly.

$$\frac{f_y}{E} = e = \frac{dl}{l/2}$$

$$dl_1 = \frac{f_y l}{2E}$$

Additional deformation when the end wires also yield after elastic failure $= \frac{f_y}{2}$

$$dl_2 = \frac{f_y/2 \ l}{E}$$

$$= \frac{f_y l}{2E}.$$

Total deformation $dl = dl_1 + dl_2 = \underline{\frac{f_y l}{E}}$

Plastic failure.   Elastic failure

15 The frame shown is similar to propped cantilever.

No: of plastic hinges required for collapse of given frame
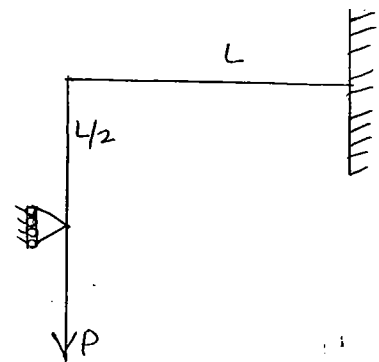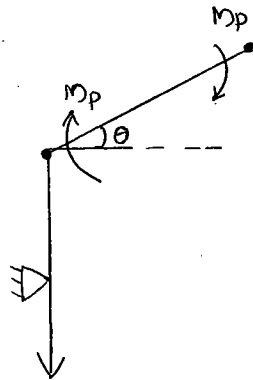
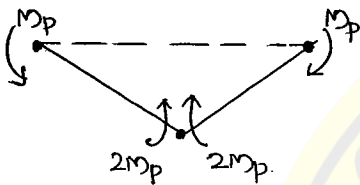$$= D_S + 1 = 1 + 1 = \underline{2}$$

$$W_e = W\delta$$
$$= W \times L\theta.$$

$$W_i = 2M_p\theta$$
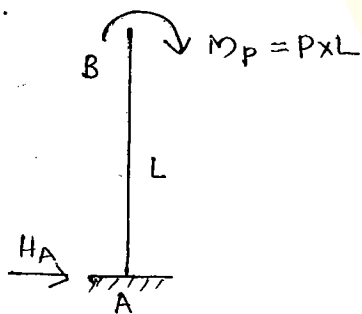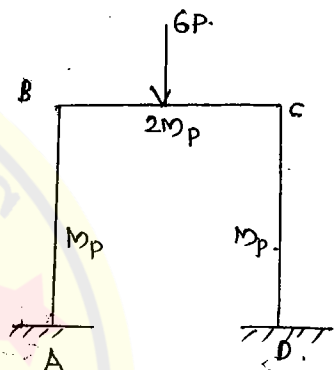
$$W = \frac{2M_p}{L}$$

18.

$$W_e = 6P\delta = 6PL\theta.$$

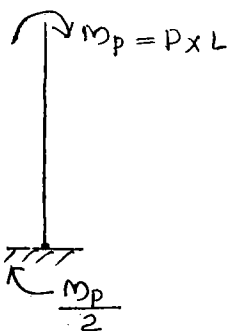$$W_i = 6M_p\theta.$$

$$P = \frac{M_p}{L} \Rightarrow M_p = PL$$

$$\Sigma M_B = 0$$

$$H_A \times L = M_p = P \times L$$

$$\underline{H_A = P} \quad (\text{wrong}; M_p \text{ gets carried over to support}).$$

$$H_A \times L = M_p + \frac{M_p}{2} = 1.5 M_p = 1.5 PL$$

$$\Rightarrow \underline{H_A = 1.5P}$$

Structure will have only partial collapse; no total collapse.
Moment M_p of the beam will have a carry over moment of $(M_p/2)$ at column support.

30th Aug,
SATURDAY

9. INFLUENCE LINES

The graphical presentation of various force parameter like reaction, shear force, and bending moment at a section as a unit force moves from one side of the beam to the other side is called Influence Line Diagram.

NOTE:
ILD's are very important in the analysis and design of bridges which are subjected to rolling or moving loads from one side of bridge to other side.

$\rightarrow$ ILD for reaction at the support of a SSB.

(i) ILD for reaction at A, R_A
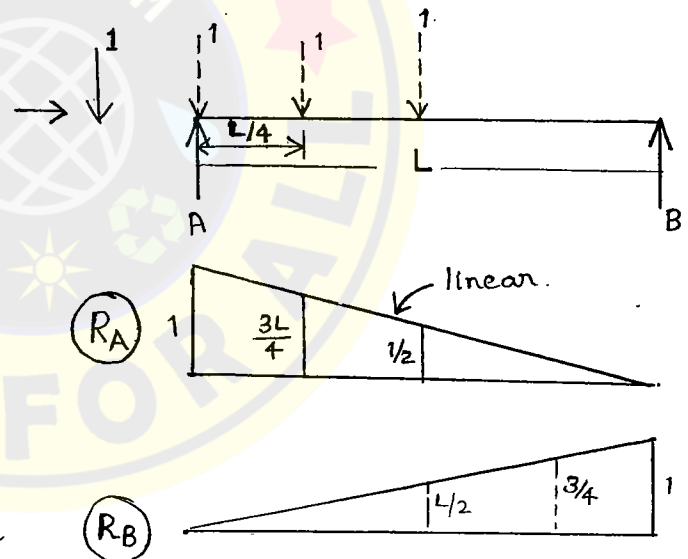
Assume the unit load is exactly at support A.

$$R_A = 1.$$

Assume load is at L/4 from support A,
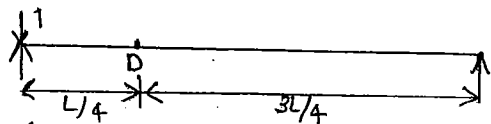
$$R_A = \frac{3L}{4} \times 1/L = \frac{3}{4}$$

Assume load is at L/2 from A, $R_A = \frac{L}{2} \times 1/L = \frac{1}{2}$

The ILDs of determinate structures are linear.

(63) ②

Draw ILD for SF at D.

Q

Assume unit load is just to the
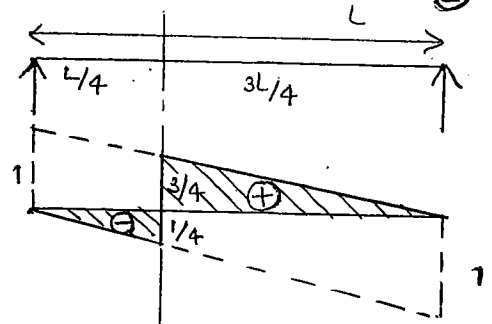left of section D. $\Rightarrow V_D = R_B (-ve)$.
Analyse from right side.
Draw ILD for R_B. As the unit loads
are to the left of D only, consider
part of ILD of R_B from A to D only
Assume unit load is right of B. But
analyse from left side.
$\therefore V_D = R_A$.

Draw ILD for R_A

$\dfrac{3}{4} + \dfrac{1}{4} = 1$

If $\dfrac{L}{4} = a$ & $\dfrac{3L}{4} =$
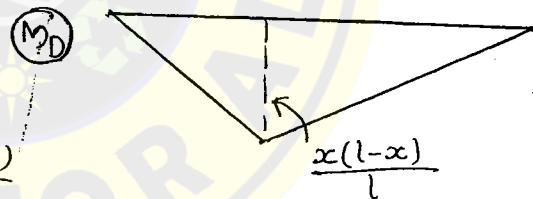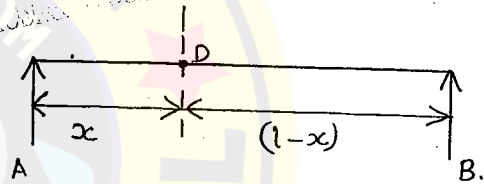
$\dfrac{a}{L} + \dfrac{b}{L} = \dfrac{L}{L} = 1$

If load is at A, $M_D = 0$

If load is at B, $M_D = 0$
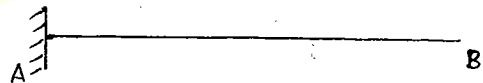
If load is at D, $M_D = \dfrac{wab}{l}$ (M_D)

$= \dfrac{1 \times x (1-x)}{l}$

$\dfrac{x(1-x)}{l}$

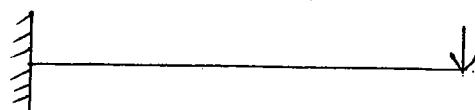When load is at B, $R_A = 1$
When load is at any distance
from B, $R_A = 1$.

(R_A)

When load is at B,
$M_A = 1 \times l = 1$
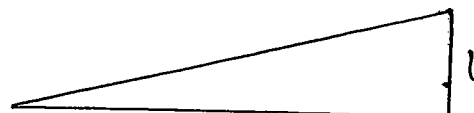
When load is at A,
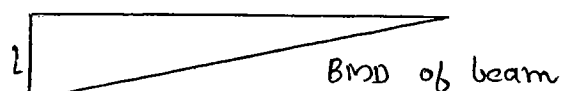$M_A = 0 \times l = 0$

(M_A)

ILD shows value of a force
parameter of a given section (section is
fixed)

BMD of beam

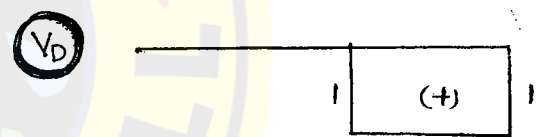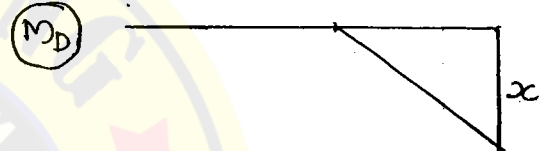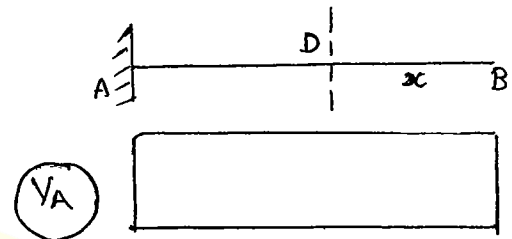for different positions of rolling loads (dynamic or moving loads)

BMD of a beam represents the values of force parameter (such as BM) for a given fixed load system. (static load system)

SF at a support = reaction at the support

when load is at B,

$$M_D = x \times 1 = \underline{\underline{x}}$$

When load is at left side of D, there won't be any load to the right side of D. $\therefore M_D = 0$
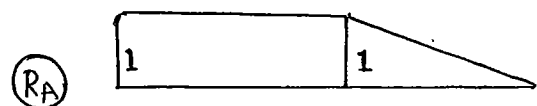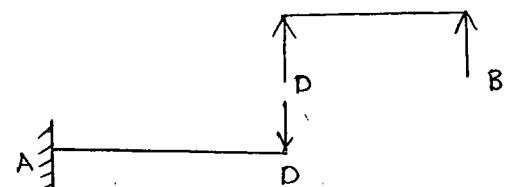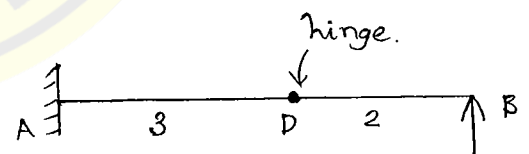

Compound Beams:

These are the statically determinate continuous beams with internal hinges.

- Break the beam into 2 parts.

BD → 'secondary beam / Child beam'

AD → 'primary beam / Parent beam'

● The loads on secondary beam may be transferred to the primary beam, whereas the loads on primary beam will not be transferred to the $2°$ beam
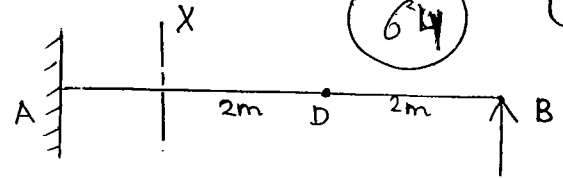
If the load is at B, $R_B = 1$

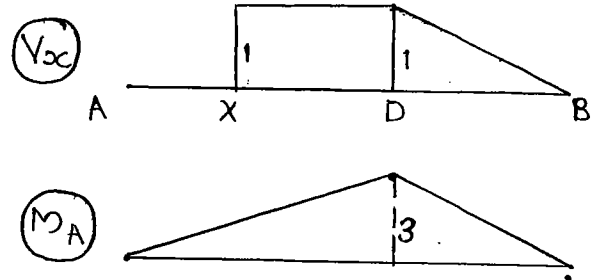If load is at D, $R_B = 0$. If loads are on main (1° beam) beam it won't be transferred to 2° beam.

When load is at B, $V_x = 0$.

When load is at D, $V_x = 1$

When load is at B, $M_A = 0$.

When load is at D, $M_A = 1 \times 3 = 3$.

→ Muller Breslau Principle:

- It is based on Maxwell-Betti's theorem.

- It considers deflection profile of a structure as the ILD. to some scale, by releasing the force parameter by a unit value to which ILD is to be drawn.

- MB Principle gives both qualitative and quantitative diagrams for determinate structures.

- MB Principle gives qualitative diagrams only for indeterminate structures.

- ILD are linear for determinate structures.

- ILDs are non linear for statically indeterminate structures

*Statement of MB Principle :-

The ILD for any force parameter is given by the deflected profile on releasing the force constraint by a unit value to some scale.

Determinate structures having linear ILDs, they give both qualitative (shape) and quantitative (values of ordinates at various sections).
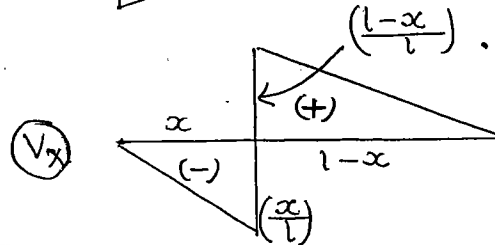
The ILDs based on MB Principle give qualitative diagrams only (shapes only) and not the values of ordinates as they are non linear for indeterminate structures
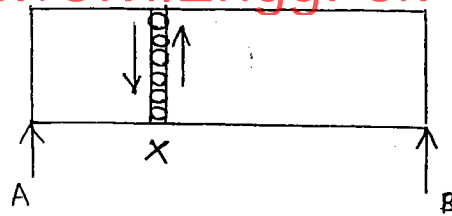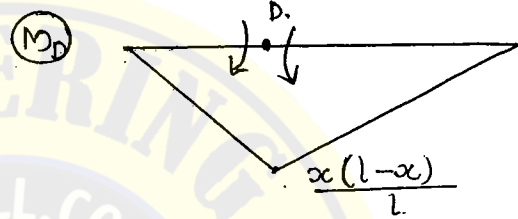
To draw ILD of R_A, release the reaction at A. Draw the deflected profile.
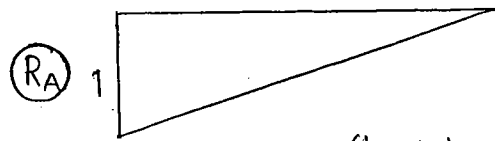
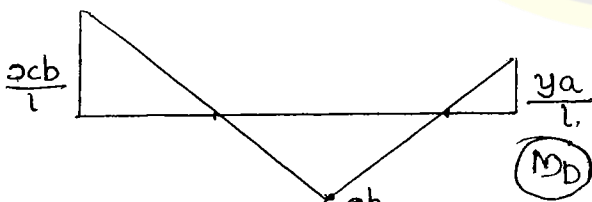To draw ILD of V_x, introduce a shear hinge at X to release SF.

To draw ILD of M_D, introduce a momentum hinge at D to release moment force.

$\rightarrow$ SSB with overhangs.

In the overhang portions, extend the ILD of simply supported portion with the same slope.
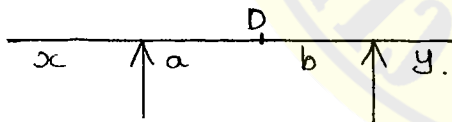
$$a \rightarrow \frac{ab}{l}$$
$$x \rightarrow \frac{xb}{l}$$

$$b \rightarrow \frac{ab}{l}$$
$$y \rightarrow \frac{ya}{l}$$

$\rightarrow$ Compound Beams

To draw ILD for reaction at C, release the reaction at C by a unit value. As to the left of hinge D, there is a continuivity with resistance, left side of D will not defo

Hinge is flexible, hence it undergoes deformation when support B is released. However support C remains in its own position
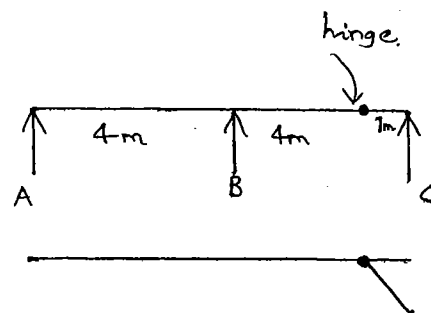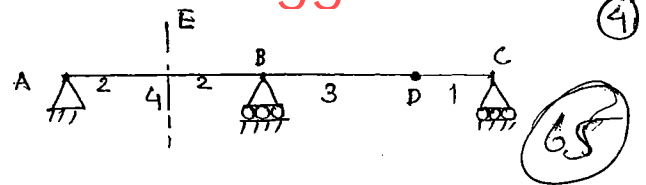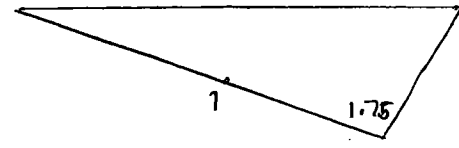
If the loads are on AB, R_A is +ve. If loads are on BC, R_A is −ve.
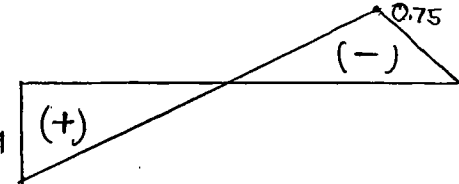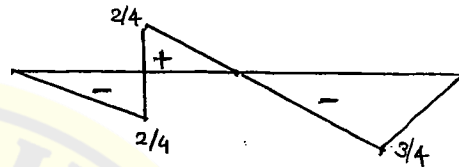
Q. Draw ILD for SF just to left of B.
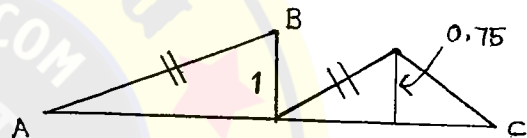
Q.9 A beam PQRS has hinges b/w PQ and RS. Beam may be subjected to a moving distributed vertical load of max intensity of 4 kN/m of any length and anywhere on the beam. The max absolute value of SF that can occur due to this loading just to the right of Q will be.

To have the maximum absolute +ve value of SF, place the live load from P to R only.
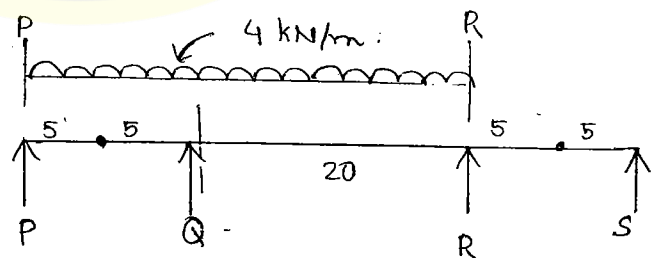
Magnitude of SF

$$= \Sigma wy \; ; \; (\text{point loads})$$

$20 \rightarrow 1$
$5 \rightarrow 1/4$
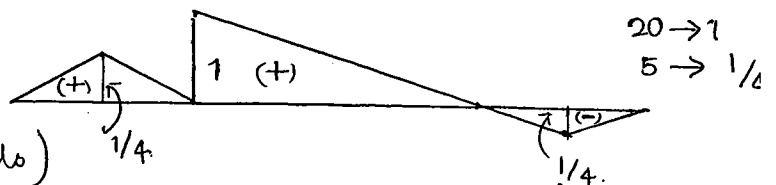
Unit of SF is kN. W has the unit of kN.

∴ y has no unit. ⇒ ordinates of ILDs of reaction and SF have no unit. for a rolling point load

Magnitude of SF = Intensity of udl * Area of ILD under u-

$$\text{Max SF} = 4\left(\frac{1}{2} \times 10 \times \frac{1}{4} + \frac{1}{2} \times 20 \times 1\right)$$

(just to right of)

$$= \underline{\underline{45 \text{ kN}}}$$

st Sept,

OONDAY

Reaction at A is removed
to cause unit vertical deflection
But moment causes zero rotation.

Moment is released. $\therefore \theta_A = 1$.

R_A $\theta_A = 0$

M_A $\theta_A = 1$

R_B

R_A

R_B $\theta = 0$

V_x

M_D

$M_{L/2}$

M_A
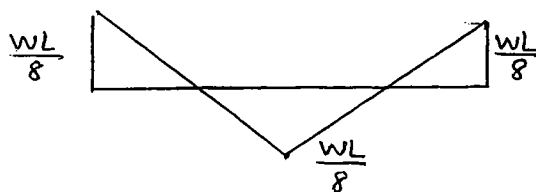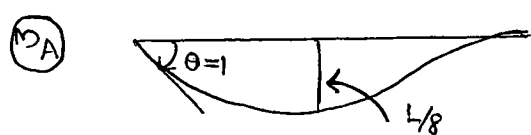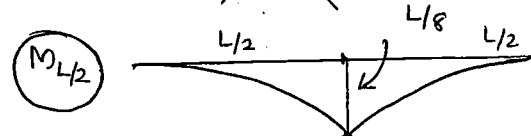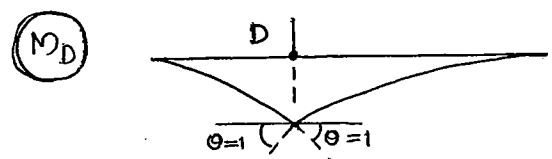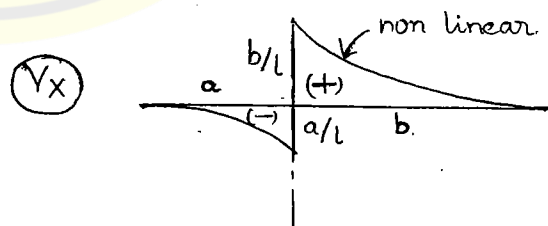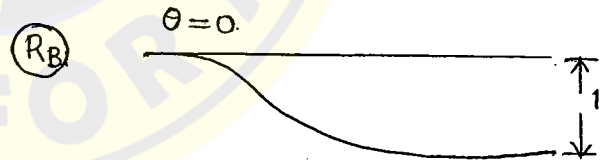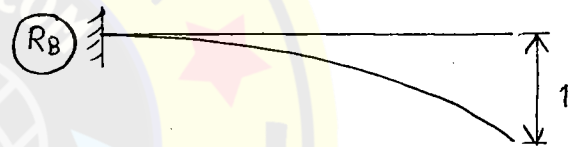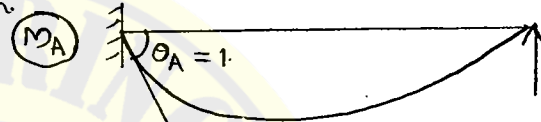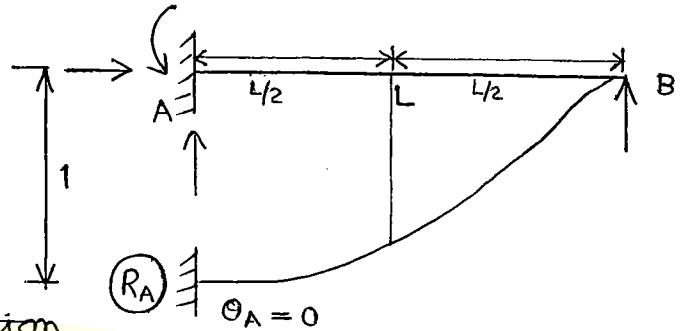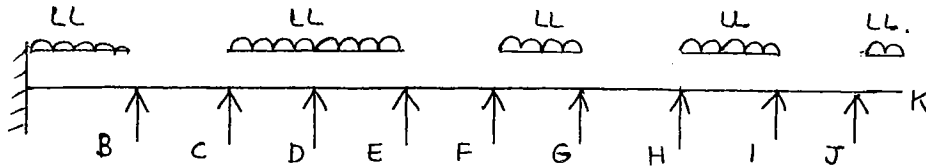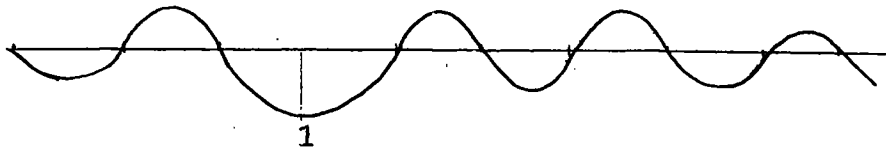
To have the max. vertical reaction at D, <u>live load</u> shall be placed on the adjoining spans CD & DE and alternate spans (BA, FG, HI, JK)

NOTE: Dead load will be on all the spans as it's a fixed load.

As DL is a fixed load existing on all spans the net forces (reaction, SF or BM) at any point will be less due to that of live load, as live load can be placed in a flexible manner to have the critical forces. Hence IS 456 gives SF and BM coefficients due to LL more than that of DL.
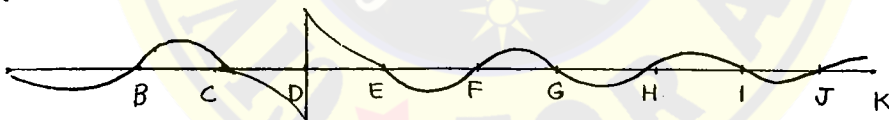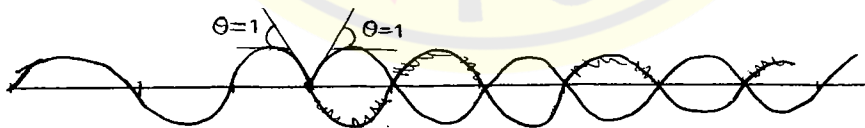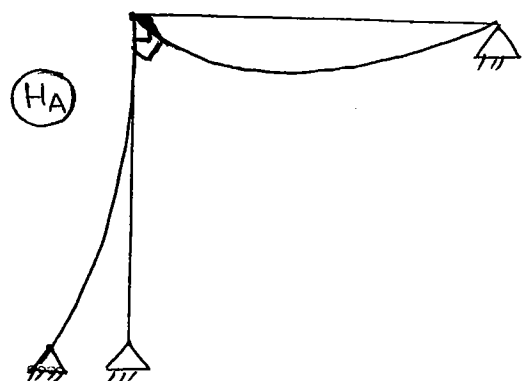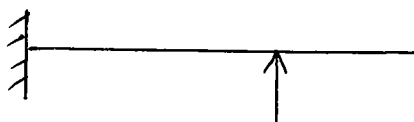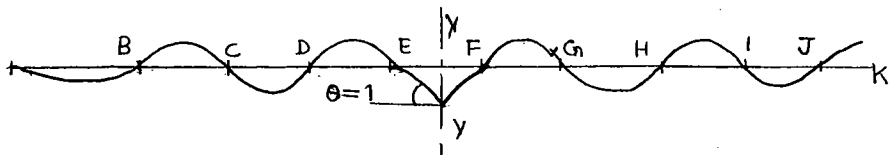
→ Conditions for Maximum BM Values :

Case (i) : Max BM ~~under~~ at a choosen ~~load~~ section

P-99
Q.06.

The condition for max BM at a choosen section for the given beam :

Average load on CA =

Average load on CB.

In fact, this condition can be satisfied for udl easily.

Eg: Assume a udl of w per m and is of 8 m length. To have max BM at C, udl can be placed as shown below.

A $\quad$ x_1 $\quad$ C $\quad$ x_2 $\quad$ B.
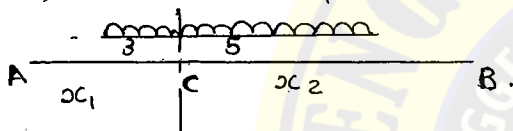
$$x_1 : x_2 = 15 : 25$$
$$= 3 : 5 .$$

It means that the given udl is divided into 8 parts, 3 parts shall be placed on CA and 5 parts shall be placed on CB. In the present problem, condition of equating averages is not possible for point loads. Hence, an alternate method is applied.

M_c = intensity of udl × area under udl

$$= 20 \left(\frac{7.5 + 9.375}{2} \right) (3 + 5)$$

$$= \underline{\underline{1350 \ kNm}}$$
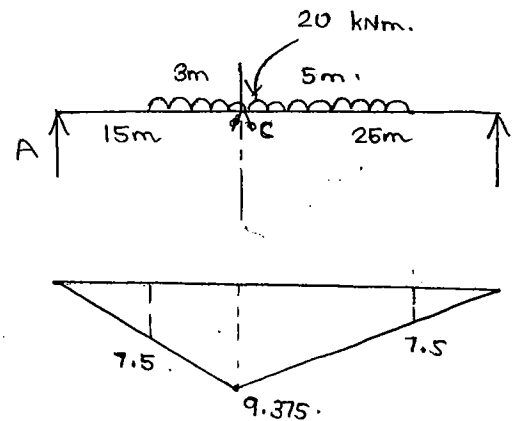
* Method for Point Loads :

Avg on AC	Avg on BC	Avg(AC) − Avg(BC)	Load rolling beyond section choosen
$\frac{360}{15}$	$\frac{40}{25}$	+	40 →
$\frac{240}{15}$	$\frac{160}{25}$	+	120 →
$\frac{140}{15}$	$\frac{260}{25}$	−	100 →

Place that load by shifting of which beyond the section choosen, the algebraic difference of averages changes sign. In the present problem, by shifting 100 kN beyond C, the sign of averages of AC &

changes sign. Hence, place 100 KN at the section choosen.

In this problem, max BM at c is ___ kNm.

(67)

$$M_c = \Sigma Wy$$

$$= 80 \times 5.6 + 60 \times 7.5 +$$

$$100 \times 9.375 + 120 \times 8.25 +$$

$$40 \times 7.125$$

$$= \underline{3110.5}$$

80 60 100 120 40

5.6

7.5

8.25

7.125

$\dfrac{15 \times 25}{40} = 9.375$
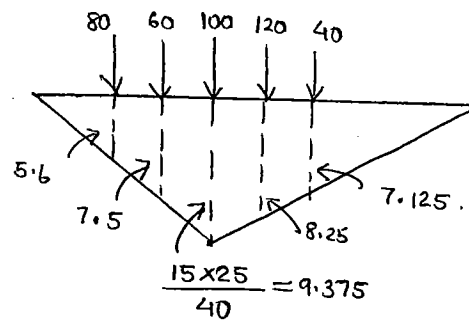
Case (ii) Max. BM under a choosen load.

Q.   Calculate the max. BM under 120 kN load

Ans: Generally BM will be maximum near the centre of a SSB. Hence the load 120 kN shall be placed near to centre.

Calculate the CG of the load system given. Assume $\bar{x}$, the location of CG of resultant load, from the 80 kN load.

$$\bar{x} = \frac{40 \times 12 + 120 \times 9 + 100 \times 6 + 60 \times 3 + 80 \times 0}{40 + 120 + 100 + 60 + 80}$$

$$= 5.85 m$$

Resultant is b/w 60 kN & 100 kN.

Calculate distance b/w choosen 120 kN and the resultant.
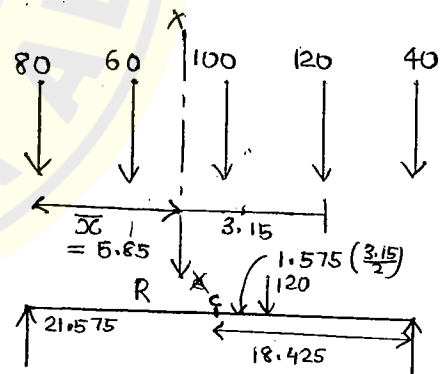
Place the choosen load and the resultant at equal distances on either side of centre.

Max BM below 120 kN =

80   60   100   120   40

$\bar{x} = 5.85$ 3.15

R   1.575 $\left(\frac{3.15}{2}\right)$ 120

21.575   18.425

(5.79)

$\dfrac{12.537}{21.575} \times 9.937$ $\dfrac{15.425}{18.425} \times 9.0$

7.17   8.56

$(21.575 \times 18.425)/40$

$= 9.937$
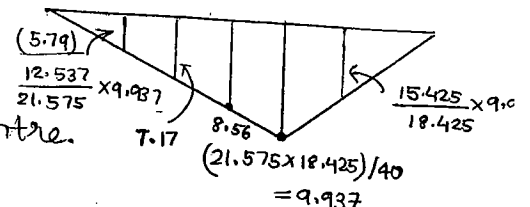
Case (iii) : Absolute maximum B.M

Generally BM will be max near centre. Hence by inspection choose the load by shifting of which from one side to the other side, the algebraic difference of averages changes sign.

40 →	340	40	+
120 →	240	160	+
100 →	40	260	—

From the above analysis, 100 kN load'll give the absolute max BM. Locate the CG of load system. (same as above)

The distance b/w 100 kN selected load and the resultant is (6 - 5.85) 0.15 m.

Place the selected 100 kN load at the resultant at equal distances on either side of centre.

Max

Absolute max BM

$$= \Sigma Wy$$

$$= 80 \times 7 + 60 \times 8.5 + 100 \times 9.99$$
$$+ 120 \times 8.5 + 40 \times 7 = \underline{3367} \; kNm$$

$$\frac{20.075 \times 19.925}{40} = 9.999$$
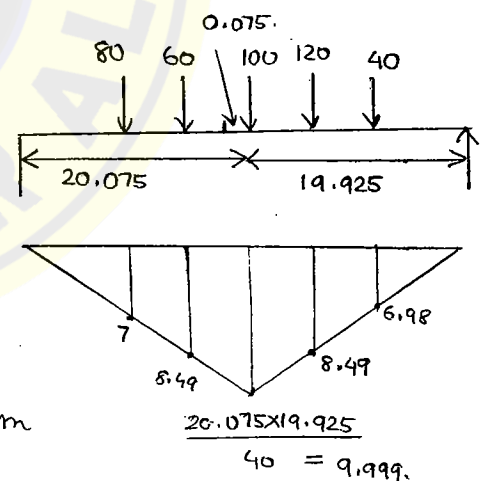
For a bridge, it shall be designed for a BM of absolute max BM

→ ILD for Trusses.

   – In a <u>deck type truss,</u> wheel loads are transferred to the top chord joints.
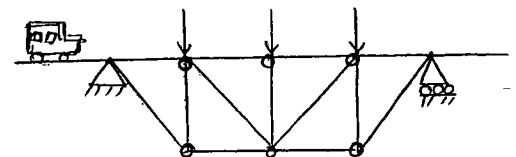
— In through type truss, loads are transferred at bottom joints.
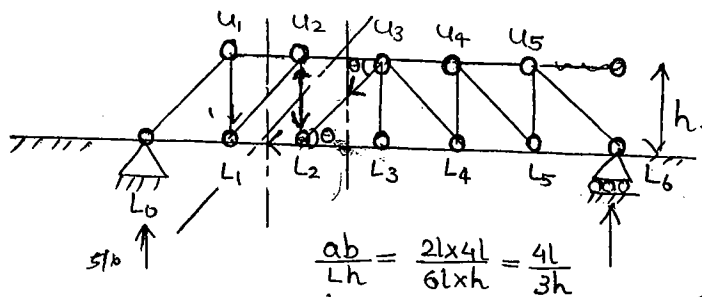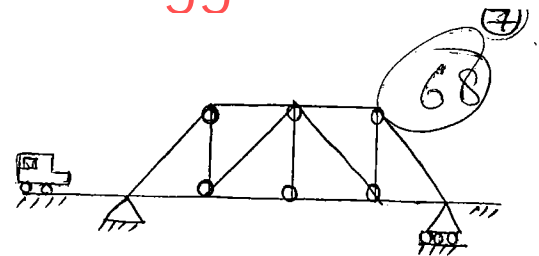
$$\frac{ab}{Lh} = \frac{2l \times 4l}{6l \times h} = \frac{4l}{3h}$$

$\boxed{L_1 L_2}$
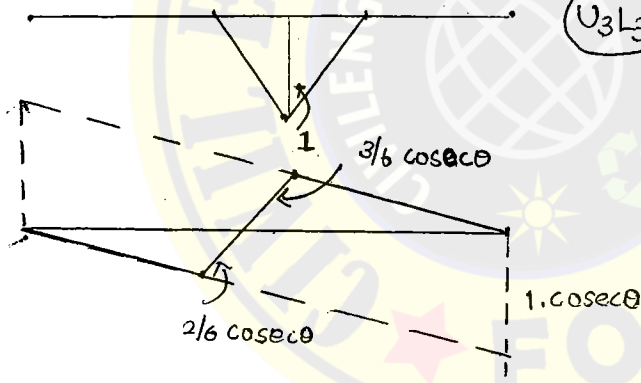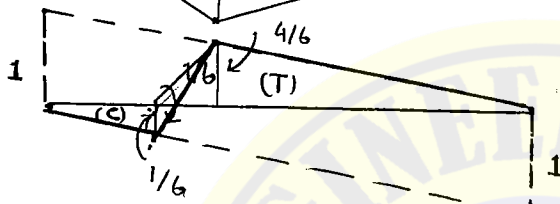
$$\sin\theta = \frac{h}{\sqrt{l^2 + h^2}}$$

$$\csc\theta = \frac{\sqrt{l^2 + h^2}}{h}$$

$\boxed{U_3 L_3}$

ILD for chord members:

* $L_1 L_2$

    — Pass a section as shown.

    — Assume loads are left side of the section, but analyse from right side. Apply $\Sigma M = 0$ about the opposite point U_2 from right side.

$$R_B \times 4L = F_{L_1 L_2} \times h.$$

$$\boxed{F_{L_1 L_2} = \frac{R_B \times 4L}{h} = \frac{\text{moment about opp. joint}}{\text{height of truss}}}$$

ILD for vertical members:

* U_2L_2

    — Pass a section as shown

    — As the truss shown is through type, loads will be acting at bottom joints.

    — Assume loads are to the left side of section. But analyse from right side.

    — Apply $\Sigma V = 0$ to the right side of section.

$$F_{L_2U_2} = R_B \text{ (compression)}$$

    — Assume loads are to the right of section. But analyse from left side.

$$F_{L_2U_2} = R_A \text{ (tension)}$$

Focal Length: Length of truss within which force changes sign (span L_1L_2) is called Focal length. The focal length in any ILD indicates that the particular member shall be designed for both compression and tension. Hence in the above case the vertical member L_2U_2 shall be designed for both compression and tension

* $U_2L_3 . L_2U_3$

    — Pass a section as shown

    — Assume loads are to the left side of section, but analyse from right side.

    — Apply $\Sigma V = 0$ to the right side of section

$$F_{U_3L_2} \sin\theta = R_B \text{ (T)}$$

$$\boxed{F_{U_3L_2} = R_B \, cosec\,\theta}$$

Q12. The question is with respect to max. qty diagram for BM

The max. quantity diagram is the locus of a force parameter at different sections.

← parabola.

17 DL = 20 kN/m
    LL = 10 kN/m

↙ 10 kN/m
8m

4m   ②   4m
   ①
4m   4m   4m   4m
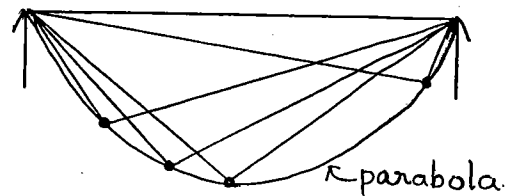
← 10 kN/m

1m
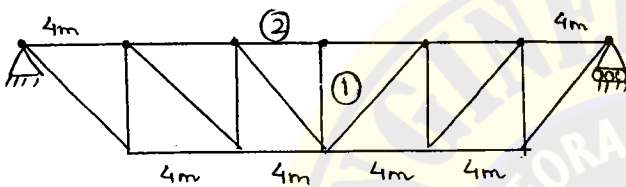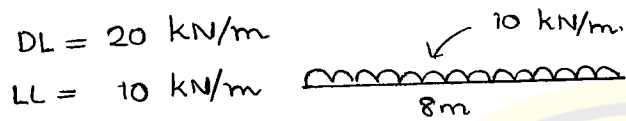
$F_1 = (20+10)\left(\frac{1}{2} \times 8 \times 1\right)$

$= \underline{120}$ kN (compression).

← 20

← 10

18.

$\frac{M}{h} = \frac{12 \times 12}{24 \times 3} = \underline{2}$

$F_2 = 20\left(\frac{1}{2} \times 24 \times 2\right) +$

$10\left(\frac{1}{2} \times 8 \times \frac{4}{3}\right) \times 2 = \underline{613}$ kN

LEVEL 2

03.   1kNm.

D
A   L
     D.   B
↓ 1/L   ↑ 1/L
R$_A$ R$_B$

L   a

(—)   1/L

L   a

(—) 1/L R$_A$

(+) 1/L R$_B$

(—) 1/L V$_D$
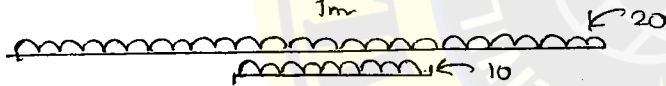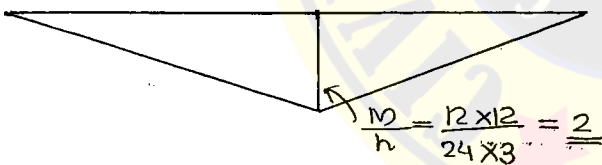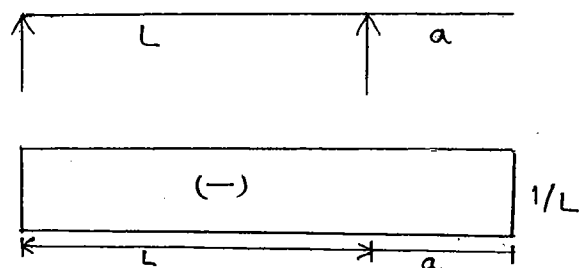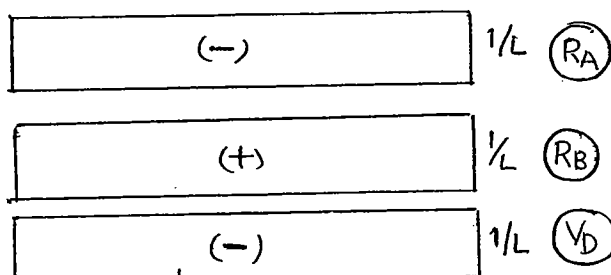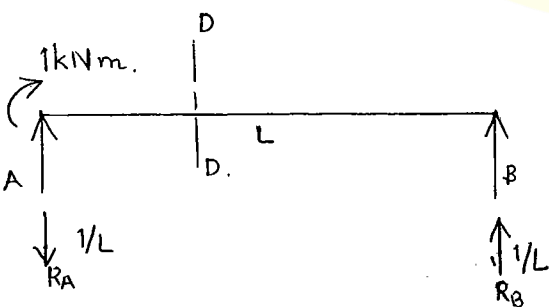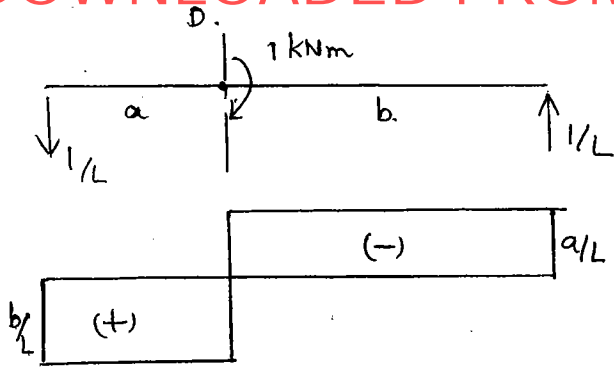
When 1 kNm couple is acting to the left of D,

$$M_D = \frac{1}{L} \times b = \frac{b}{L} \; (\text{sagging})$$

When 1 kNm is acting to the right,

$$M_D = \frac{a}{L} \; (\text{hogging}).$$

If at any section, vertical ordinate is changing sign, design force at that section depends upon max. ordinate.

3. To have absolute max. BM, place the couple at one of the supports.
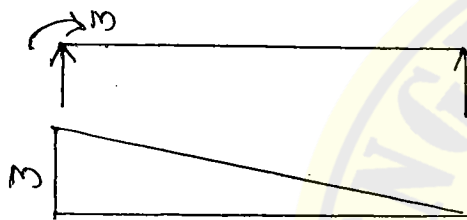
10. ARCHES & CABLES

An Arch is a curved beam in vertical plane

→ Design forces in an Arch:

P_n : normal thrust or
       axial compression.

S : Radial shear force.

M : Bending moment.
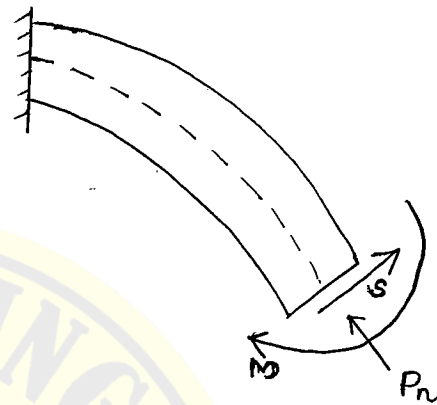
→ Ring Beam of a Water Tank

Ring beam shall be designed for SF, BM and torsion.
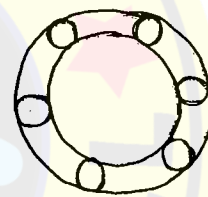
→ Advantages of Arches compared to SSB.

For a SSB:

$$M_x = R_A x - wx \cdot \frac{x}{2}$$

$$M_{beam} = R_A x - \frac{wx^2}{2}$$

For an Arch:
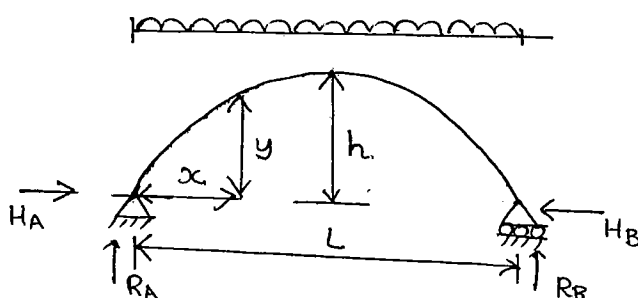
$$M_x = M_{arch} = \left(R_A \cdot x - \frac{wx^2}{2} \right) - H_A y = M_{beam} - H_A y$$

$$M_{arch} = M_{beam} - H_{moment}.$$

(i) An arch is economical for <u>long spans</u> compared to SSB

(ii) The horizontal reaction developed of the support of an arch will reduce the net moment compared to that of SSB.

NOTE :

Arches are primarily subj. to axial compression. Hence stone which strong in axial compression were used in olden days for construction of arches.

→ Classification of Arches:

1. Based on Shape.

   a) Parabolic

   b) Semi-circular.

   c) Segmental.

2. Based on number of hinges (or D_s) :

   a) Fixed arches ($D_s = 3$, $D_k = 0$)

   b) Two hinged arches ($D_s = 1$, $D_k = 2$).

   c). Three hinged arches ($D_s = 0$, $D_k = 6$ considering AD.
                                  = 4 neglecting AD)

→ Parabolic Arches

$$y = \frac{4h}{l^2} x \cdot (1-x)$$

    (one of the support as origin)

$$\tan\theta = \frac{dy}{dx} = \frac{4h}{l^2}(1-2x)$$

$$\frac{x^2}{y} = const. \text{ (crown as origin)}$$
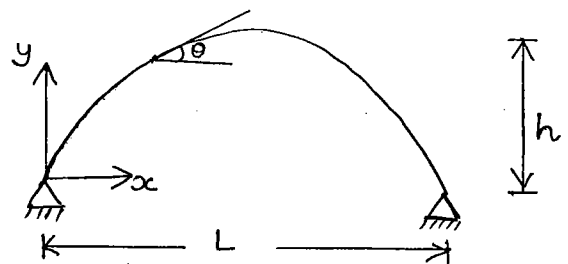
* Calculation of Reactions at Support of Arches

a) Supports are at Same Level

②

⑦

To calculate vertical reactions, if the supports are at same level, analysis is similar to that of a SSB.

$\Sigma M_A = 0$

$10 \, V_B = 100 \times 1$

$V_B = 10 \, kN$ & $V_A = 90 \, kN.$
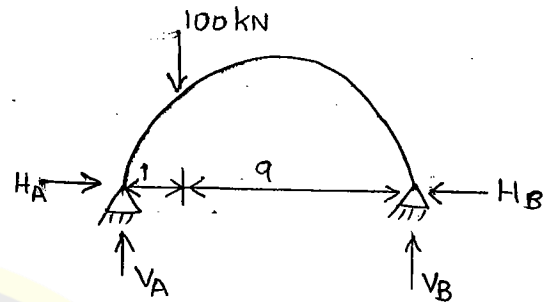
Horizontal reaction is not influencing the vertical reaction as their line of action passes through the supp

$\Sigma M_A = 0.$

$\therefore \quad 16 \, V_B = 20 \times 4 \times 2$

$V_B = 10 \, kN. \, (\uparrow)$

$V_A = 10 \, kN \, (\downarrow)$
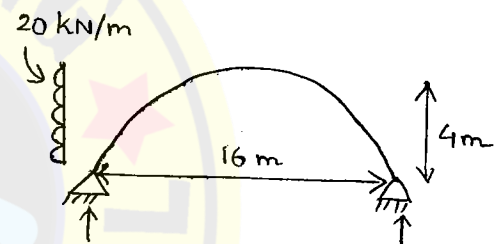
* Calculation of Horizontal Reactions

(i) Three Hinged Arches.

— parabolic arch subj. to udl throughout.

Apply $\Sigma M_C = 0$ (from right)

$\Rightarrow \quad H_B \times h + \dfrac{wl}{2} \times \dfrac{l}{4} = \dfrac{wl}{2} \times \dfrac{l}{2}.$

$$\boxed{H_B = \dfrac{wl^2}{8h}}$$

$H_A = H_B = H$ (no other horizontal forces)
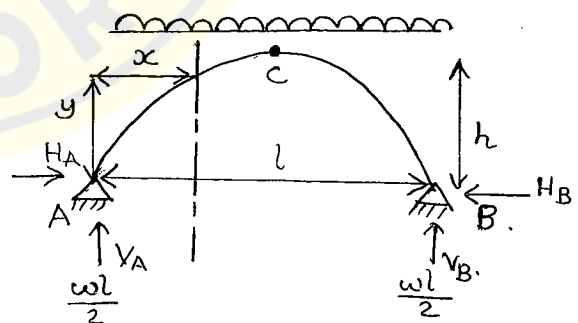
$M_x = \dfrac{wl}{2} \times x - \dfrac{wl^2}{8h} \times y - \dfrac{wx^2}{2}$

$\quad = \dfrac{wlx}{2} - \dfrac{wl^2}{8h} \left(\dfrac{4h}{l^2} x \cdot (l-x) \right) - \dfrac{wx^2}{2} = 0$

$$\boxed{M_x = 0}$$

◉ If the shape of the structure is similar to that of BMD of the external load on a SSB, then **BM and SF** at every section are zero. The arch is subjected to axial force only.
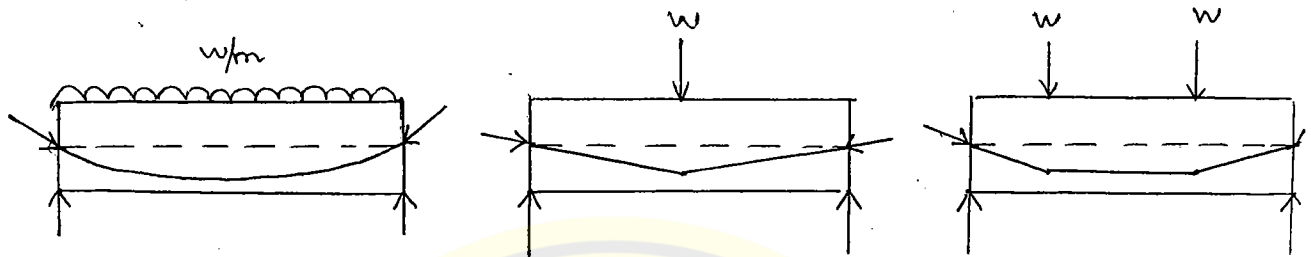
In prestressed concrete beam design (post tensioning) if the cable profile is similar to that of BMD of external load, as a SSB, then the beams are subj. to axial compression only. This is called 'Load Balancing Concept'.

Similar approach is followed in arch analysis also.

◉ 'Linear arch or Theoretical Arch or Pressure line' :-

If the shape of the arch similar to that of BMD, neither BM nor SF at any section. Such an arch is called linear arch.

◉ Eddy's Theorem

If the BMD of given load system is not similar to that of the arch provided, then the BM at any section is proportional to the difference of ordinates of BMD and the arch provided — Eddy's Theorem

$$M \propto (y_2 - y_1)$$

(3)

(72)

Q.

$$2H_1 = \frac{wl^2}{8h}$$

$$\therefore \quad H_1 = \frac{wl^2}{16h}$$

Q.

$$H = 2H_1$$

$$2H_1 = \frac{wl^2}{8H} \implies H_1 = \frac{wl^2}{16h}$$

Q.

$$\Sigma M_c = 0 \implies \frac{w}{2} \times \frac{l}{2} = H \times h$$
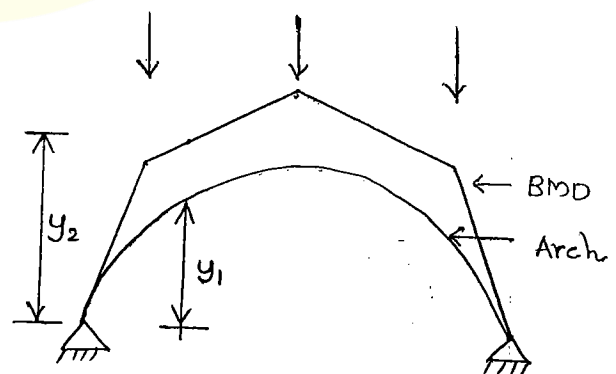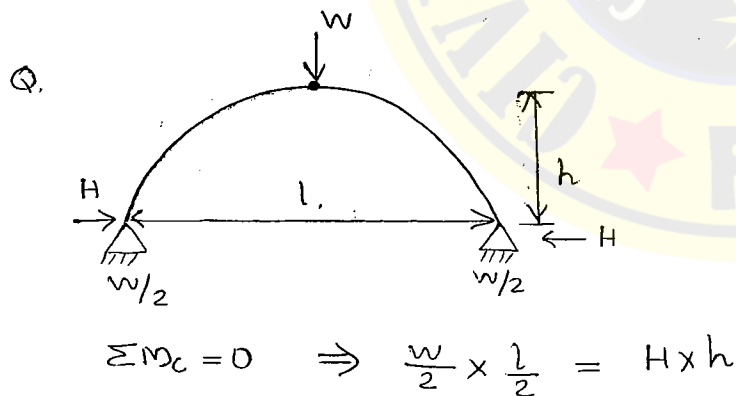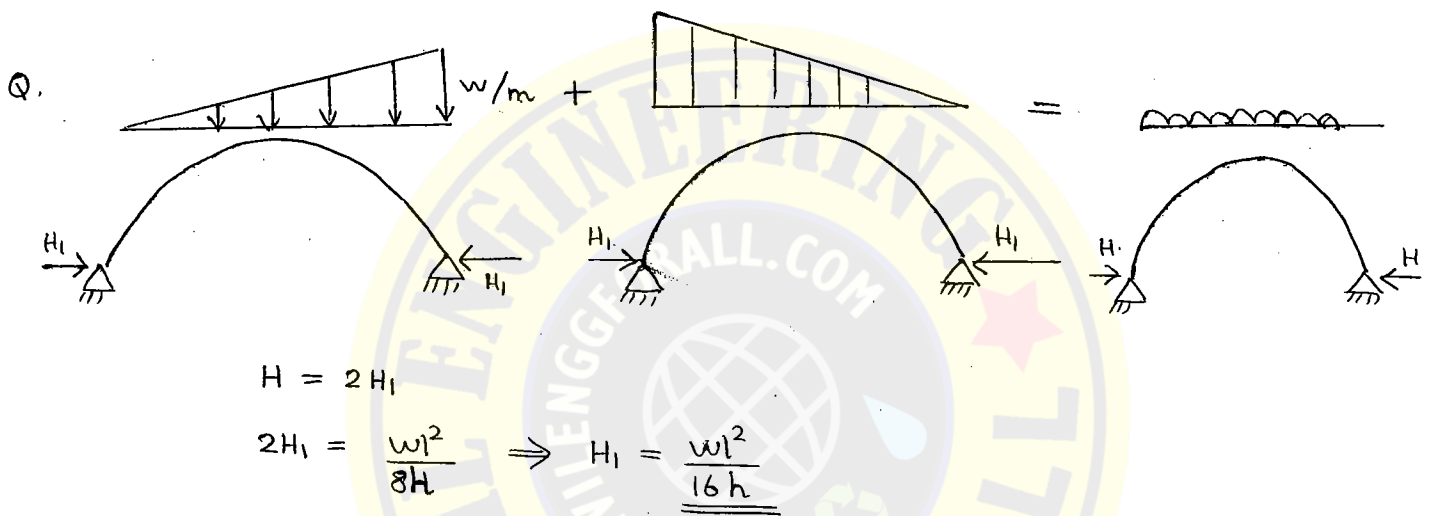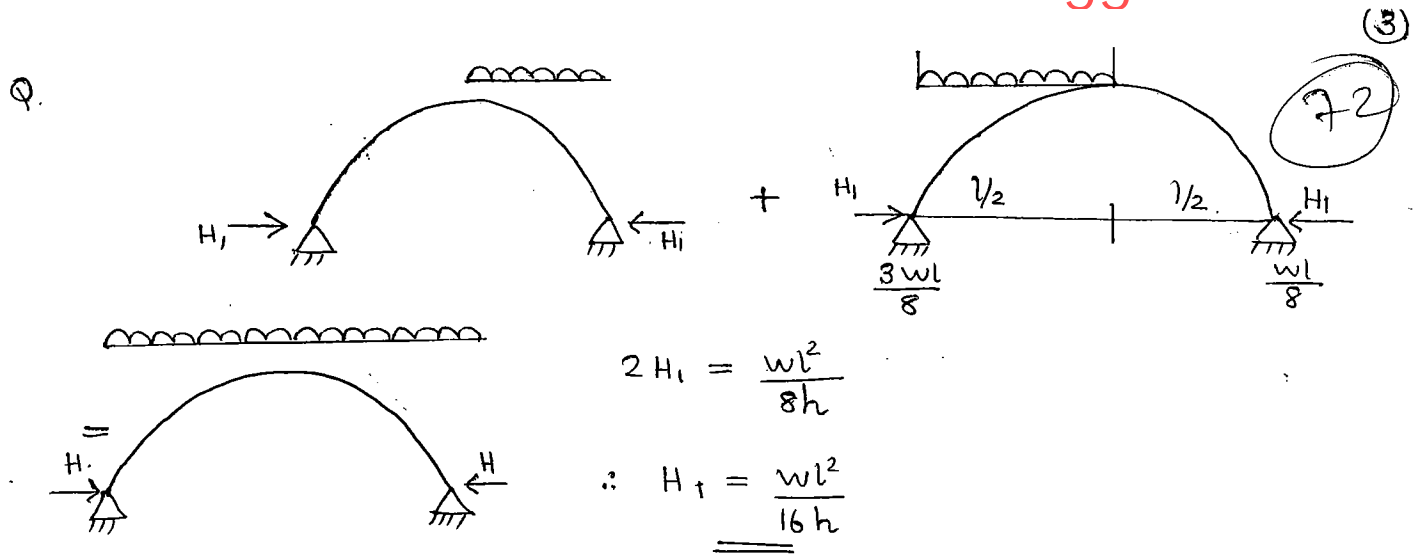
$$\boxed{H = \frac{wl}{4h}}$$

＊ ILD for 3-hinged Arches.

(i) ILD for Horizontal Thrust.
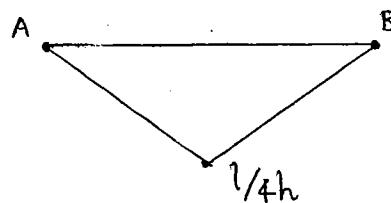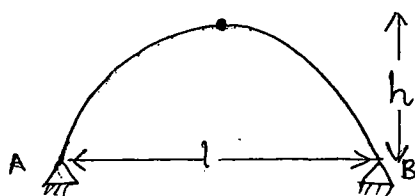
when unit lo
At supports
horizontal
thrust ≈ 0.

✳ (ii) ILD for M_x.

$$M_{arch} = M_{beam} - Hy$$

x & y are the coordinates of the chosen section where ILD is to be drawn for BM.

$$\frac{x(l-x)}{l} \qquad Hy = \frac{ly}{4h}$$

ILD for y
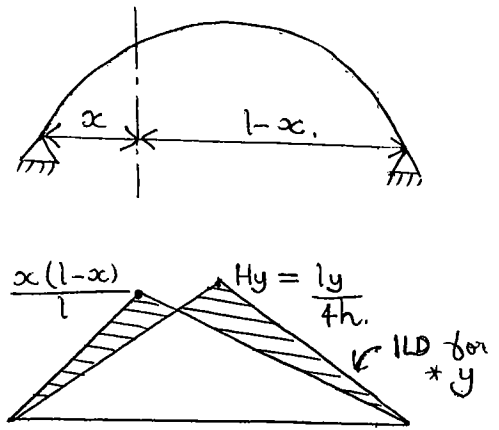
Q. Calculate horizontal and vertical reaction reaction.
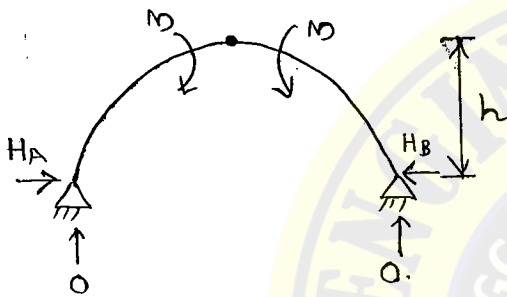
$$V_A = V_B = \frac{Net\ moment}{l} = \frac{M-M}{l} = 0$$

$$\Sigma M_c = 0 \ (from\ left)$$

$$\Rightarrow H_A \times h = M$$

$$\boxed{H = \frac{M}{h}}$$

Q.
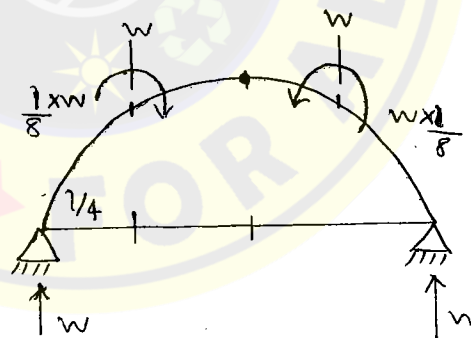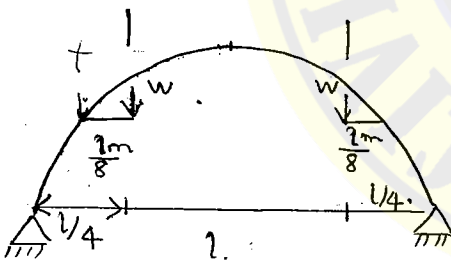
$$\Sigma M_c = 0$$

$$\Rightarrow H_A \times h + w \times \frac{l}{4} = \frac{wl}{2} + \frac{wl}{8}$$

$$H_A = \left(\frac{wl}{4} + \frac{wl}{8}\right)\frac{1}{h} \qquad \Rightarrow \boxed{H = \frac{3wl}{8h}}$$

✳ Reaction at A:

$$R_A = \sqrt{H_A^2 + V_A^2}$$

$$= \sqrt{\left(\frac{3wl}{8h}\right)^2 + w^2} = w\sqrt{\frac{9l^2}{64h^2} + 1}$$

✳ Inclination with horizontal.

$$\tan\theta = \frac{V_A}{H_A} = \frac{w \times 8h}{3wl} = \frac{8h}{3l} \Rightarrow \theta = \tan^{-1}\left(\frac{8h}{3l}\right)$$

b) Supports at Different level.   ④

                                                      ⑦₃

Q.

$$\downarrow 20 \text{ kN/m}$$

4

c

A

6

30 m.

l_1 l_2 B
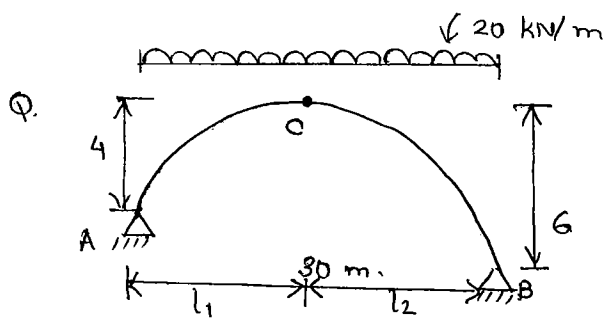
Calculate reactions at
the supports.

(i) Three hinged Parabolic Unsymmetric Arches

Step 1: Calculate horizontal distances of AC & BC

We know $\dfrac{x^2}{y} = $ const. (for parabolic arch wrt crown
as origin)

$$\frac{x}{\sqrt{y}} = \text{const.}$$

$$\frac{l_1}{\sqrt{h_1}} = \frac{l_2}{\sqrt{h_2}} = \text{const.} = \frac{l_1 + l_2}{\sqrt{h_1} + \sqrt{h_2}} = \frac{l}{\sqrt{h_1} + \sqrt{h_2}}$$

$$\frac{l_1}{\sqrt{4}} = \frac{30}{\sqrt{4} + \sqrt{6}}$$

$$\Rightarrow \quad l_1 = \frac{60}{2 + \sqrt{6}} = 13.48 \text{ m}$$

$$l_2 = 16.51 \text{ m}$$

As supports are not at same level, we cannot calculate
vertical reactions by treating like a SSB initially.

☞ Apply $\Sigma M_C = 0$ (from left)

$$R_A \times 13.48 = 20 \times \frac{13.48^2}{2} + 4H$$

$$R_A = 0.3H + 134.9. \longrightarrow ①$$

Apply $\Sigma M_C = 0$ (from right)

$$R_B \times 16.51 = 20 \times \frac{16.51^2}{2} + 6H.$$

$$R_B = 165.1 + 0.363H. \longrightarrow ②$$

Apply $\Sigma V = 0$. for the entire arch,

$$R_A + R_B = 20 \times 30$$

$$H = \cancel{500} \; 455.2 \text{ kN} \qquad (\text{from } \textcircled{1} \; \& \; \textcircled{2}).$$

$$N_A = 269.64 \text{ kN}$$

$$N_B = \underline{\underline{330.36}} \text{ kN}$$

⊙ For an unsymmetrical 3 hinged arch subj. to udl througho max. reaction occurs at the deepest support. Similarly, in case of unsymmetrical cable subj. to udl throughout, max. reaction occurs at highest support
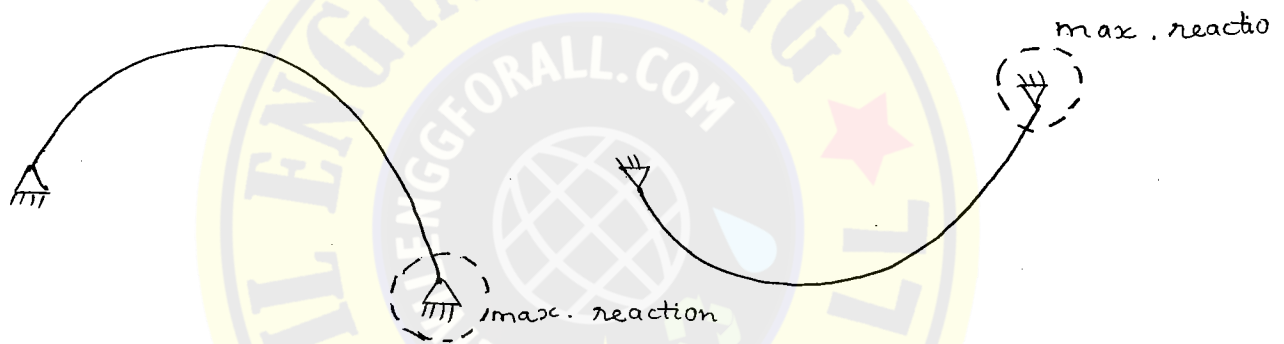
<u>4th Sept,</u>
THURSDAY

→ Radial Shear & Normal Thrust

Consider free body diagram of part AD
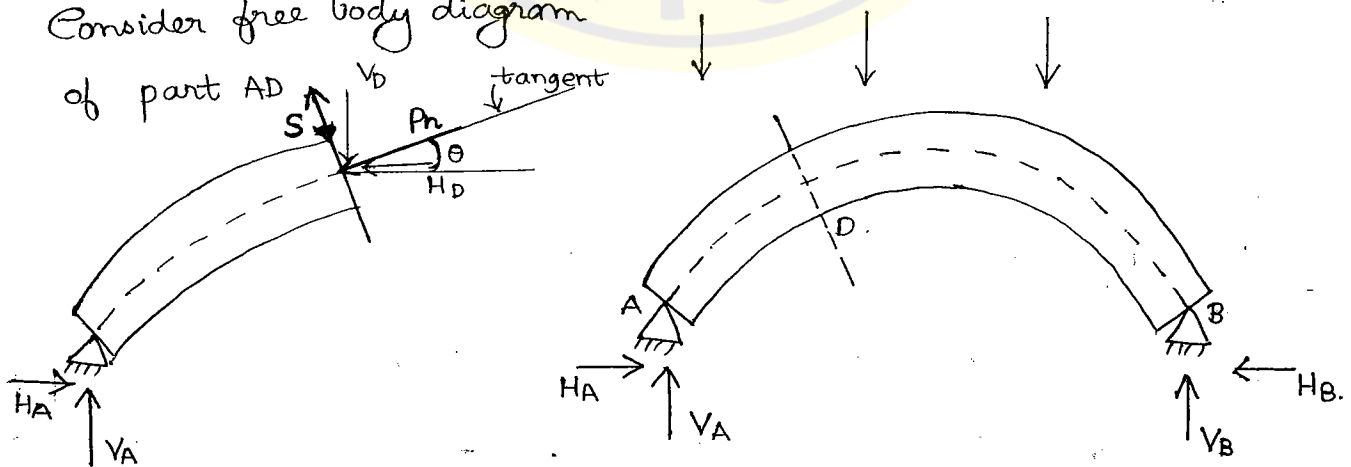
V_D → net vertical reaction at D

H_D → net horizontal reaction at D

θ → angle b/w the tangent at D and horizontal

$P_n \rightarrow$ normal thrust or axial compression

$S \rightarrow$ radial SF.

⦿ P_n is the resultant of H_D & V_D. resolved in the direction of P_n

$$\boxed{P_n = H_D \cos\theta + V_D \sin\theta.}$$

⦿ Radial shear, S is the resultant of H_D & V_D in the direction of S.

$$\boxed{S = H_D \sin\theta - V_D \cos\theta}$$

⦿ Prove that the shear force at any section of a 3-hinged parabolic arch subjected to udl throughout is zero.

For parabolic arch, at any section,

$$\frac{dy}{dx} = \frac{4h}{l^2}(l-2x) = \tan\theta.$$

$$\sin\theta = \frac{4h(l-2x)}{\sqrt{l^4 + 16h^2(l-2x)^2}}$$

$$\cos\theta = \frac{l^2}{\sqrt{l^4 + 16h^2(l-2x)^2}}$$
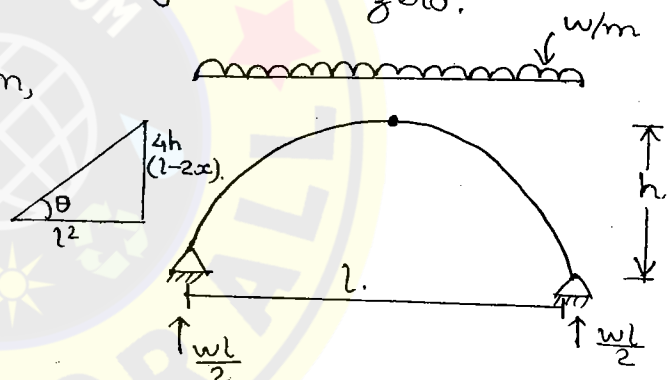
$$V_D = \frac{wl}{2} - wx$$

$$H_D = \frac{wl^2}{8h}$$

$$S = H_D \sin\theta - V_D \cos\theta$$

$$= \frac{wl^2}{8h} \times \frac{4h(l-2x)}{\sqrt{l^4 + 16h^2(l-2x)^2}} - \frac{wl}{2} \cdot \frac{l^2}{\sqrt{l^4 + 16h^2(l-2x)^2}} + wx \cdot \frac{l^2}{\sqrt{l^4 + 16h^2(l-2x)^2}}$$

$$= \underline{\underline{0}}$$

→ Effect of Temperature on 3 hinged Arches

As 3 hinged arch is statically determinate, no thermal stresses are developed. We know stresses depend upon B.M. at a section. No stresses means no change in the moment of 3 hinged arch due to temperature change.
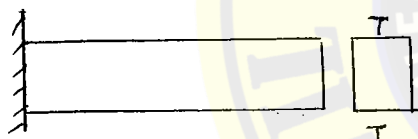
$$M = fz \implies f = \frac{M}{Z}$$
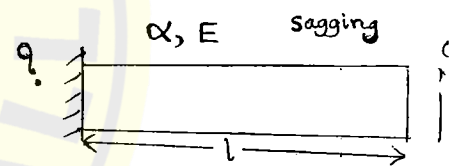
$$\boxed{d = \left(\frac{l^2 + 4h^2}{4h}\right) \alpha T}$$

As $T \uparrow$, $y \uparrow$, $H \downarrow$

$$M_{Arch} = M_{beam} - Hy.$$

$$\frac{dH}{H} = -\frac{dh}{h} \; ; \quad \text{-ve indicates that } H \text{ and } h \text{ vary in opposite directions.}$$

$$\frac{dT}{dy} = 0$$

A cantilever subj. to temp change uniformly or temperature gradient ($\frac{dT}{dy} = 0$) zero, then the cantilever is free to elongate. Hence no resistance against deformation or no resistance against strain. No resistance means no stress.
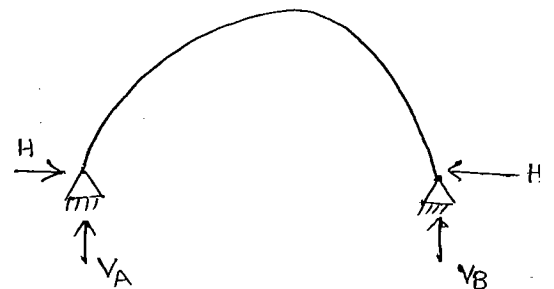
→ Two Hinged Arches:

— Assume supports of two hinged arch will not yield laterally.

According to Castigliano's theorem, if no deformation, assuming hori· reaction as redundant,

$$\frac{\partial U}{\partial H} = 0 \qquad \left(\frac{\partial U}{\partial R} = \delta\right)$$

$$\implies \boxed{H = \frac{\int My\, ds}{\int y^2\, ds}} \; ; \quad m = \text{beam moment}$$
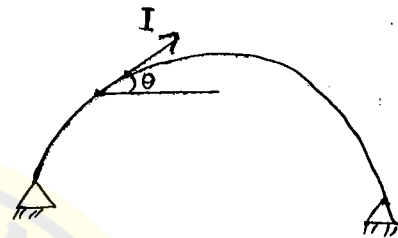
⑥

⊙ $H = \dfrac{\int M_0 y \, ds}{\int y^2 \, ds}$ is useful for arches like $\frac{2}{3}$-hinged

arch with udl throughout. For unsymmetrical loads, numerator
and denominator of above equation are not integrable.
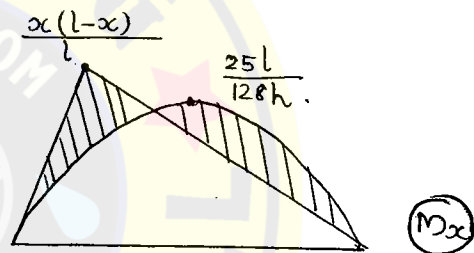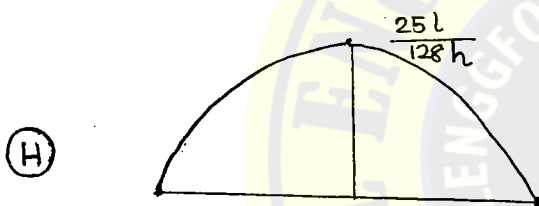
In order to analyse it is assumed that, $I = I_0 \sec\theta$ at
any section; where I_0 is moment of inertia at the crown.
With this assumption,
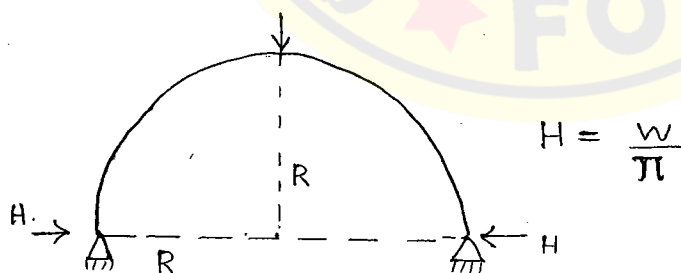
$$H = \dfrac{\int M_0 y \, dx}{\int y^2 \, dx}$$

→ ILD for 2 hinged Arch:

$\dfrac{25l}{128h}$

Ⓗ

$\dfrac{x(l-x)}{l}$ $\dfrac{25l}{128h}$

Ⓜx

→ Two hinged Semi circular Arches
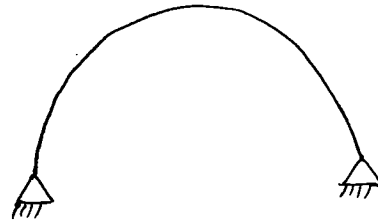 (i) Point load at Crown.

$H = \dfrac{W}{\pi}$

(ii)

→ Temperature effect on 2 hinged Arches

$$M_{arch} = M_{beam} - Hy$$
(change)   (const)   (changes)

As there is no hinge at the crown, y won't change.

But H changes.

As T↑, H↑

If temperature increases, H increases. If temp. increases, no change in the value of rise. Hence temp will try to push the supports out. But they will not. In this process, H will increase. As H↑, Hy increases. ⟹ M_{arch} decreases

→ Effect of Rib Shortening in 2 hinged Arches

The effect of normal thrust in the arch is to shorten the rib of the arch and thus release part of horizontal thrust.
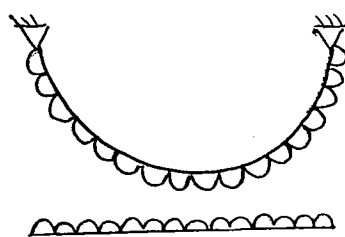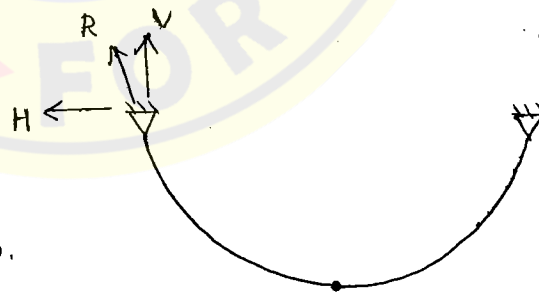
→ **Cables**

✱ Assumptions:

(i) Cable is flexible.
    BM @ every point is zero.

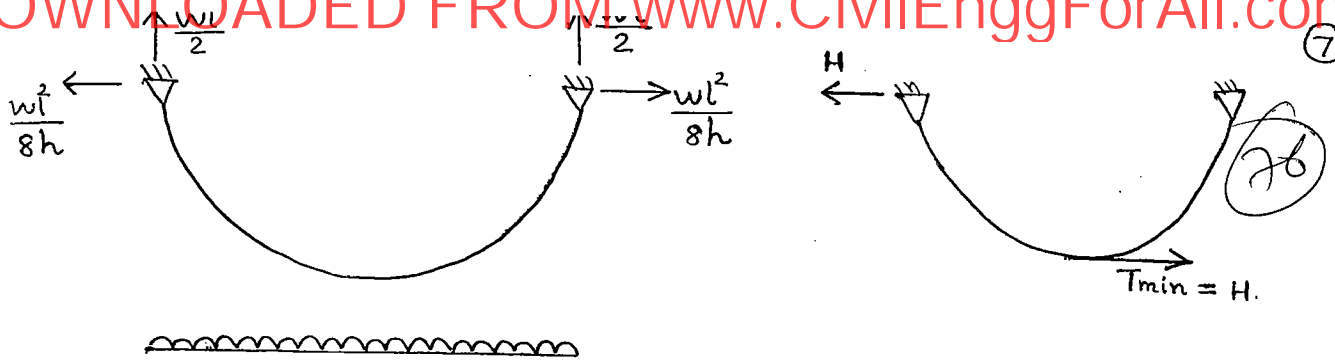(ii) Self weight is neglected.

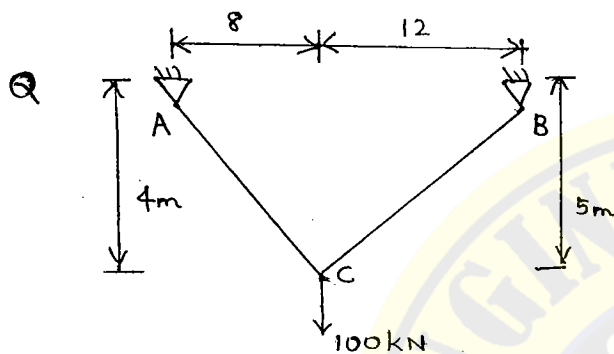⊙ Load along the horizontal span
  – Shape of cable is <u>parabola</u>.

⊙ udl is along the curve.
  – shape of cable is <u>catenary</u>

– In chain surveying also, correction to sag is catenary

$$\frac{wl}{2} \qquad \frac{wl}{2}$$

$$\frac{wl^2}{8h} \leftarrow \qquad \rightarrow \frac{wl^2}{8h}$$

$$H \leftarrow$$

$$T_{min} = H.$$

Shape of the cable due to point loads is similar to that of BMD. Hence BM at every point is zero

Q

8   12

A   B

4m   5m

C

100kN

Calculate reactions at supports.

$$\Sigma M_C = 0 \ (\text{from left})$$

$$12 \, V_B = 5H$$

$$\Sigma M_C = 0 \ (\text{from right}).$$

$$8 \, V_A = 4H.$$

$$V_A + V_B = 100$$

$$\frac{5}{12} H + 0.5H = 100 \Rightarrow H = 109.1 \ kN$$

$$V_A = 0.5 H = 54.54 \ kN.$$

$$V_B = 45.46.$$

Max tension at support A, $R_A = \sqrt{V_A^2 + H_A^2}$

$$= \sqrt{54.54^2 + 109.1^2}$$

$$= 121.97 \ kN$$

Q

V_A V_B

$H \leftarrow$ $\rightarrow H$

3   5

l_1 l_2

20
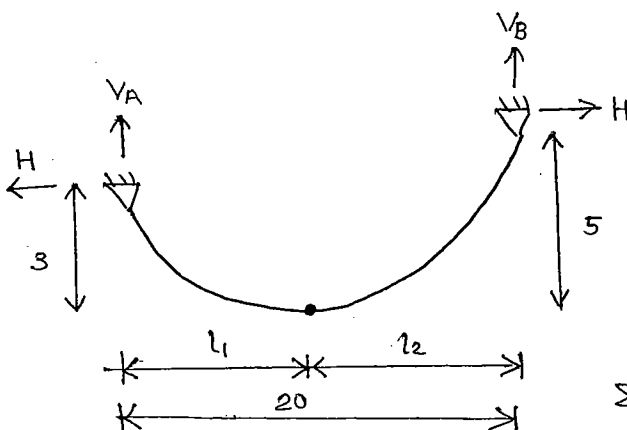
$$l_1 = \frac{l \cdot \sqrt{h_1}}{\sqrt{h_1} + \sqrt{h_2}}$$

$$= \frac{20\sqrt{3}}{\sqrt{3} + \sqrt{5}} = 8.73 \ m.$$

$$l_2 = 11.27 \ m.$$

$$\Sigma M_C = 0 \ (\text{from left})$$

$$8.73 \, V_A = 3H + 20 \times \frac{8.73^2}{2}$$

$$V_A = 0.344 \, H + 87.3$$

$$11.27 \, V_B = 5H + 20 \times \frac{11.27^2}{2}$$

$$V_B = 0.44365 \, H + 112.7$$

$$V_A + V_B = 0.78765 \, H + 200$$

$$20 \times 20 = 0.7876 \, H + 200$$

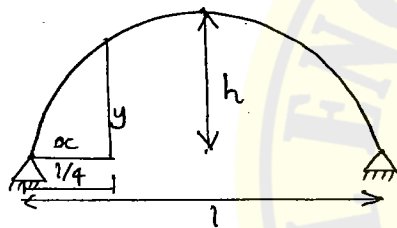$$H = 253.92 \text{ kN}$$

$$V_A = 174.648 \text{ kN}$$

$$V_B = 225.351 \text{ kN}$$

Max tension, $R_B = \sqrt{253.92^2 + 225.351^2} = 339.74$ kN
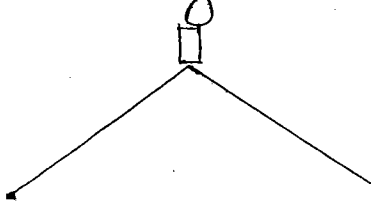
$$T_{min} = H = 253.92 \text{ kN}$$

P-111.

10.

$$y = \frac{4h}{l^2}(x)(1-x)$$

$$= \frac{4h}{l^2}\left(\frac{l}{4}\right)\left(1 - \frac{1}{4}\right) = \frac{3h}{4}$$
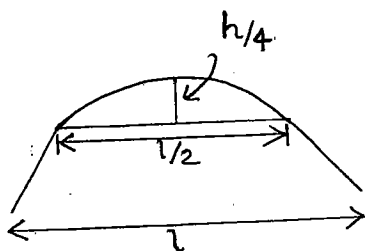
Rise at quarter point = 75% rise at mid span.

$$= \frac{3 \times 10}{4} = 7.5 \text{ m}$$

Initially cambers were provided as shown below:

But this causes stress concentration when vehicle wheel moves over the centre of camber causing failure of the camber.

This problem was solved by providing camber as shown:

Rise at quarter point is only 75% h. Smooth and comfort overtaking conditions provided by parabolic portion in 1/2 distance for fast moving vehicles.

Similarly for slow moving vehicles, comfort conditions are provided by straight end portions
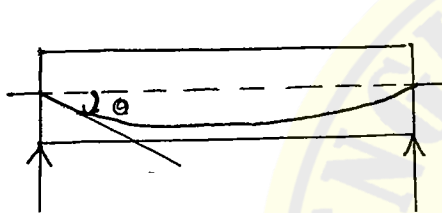
5th Sept,
FRIDAY

11. MATRIX METHODS

→ Stiffness Matrix (Displacement Method):

- Force per unit displacement.

⊙ Axial force → Axial deformation → Axial Stiffness.

Moment → Rotation or Slope → Flexural Stiffness
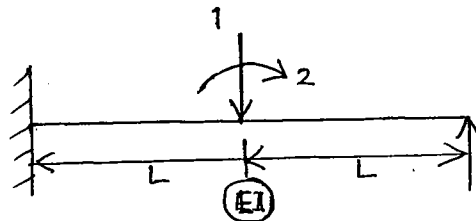
Torsion → Twisting angle → Torsional Stiffness.

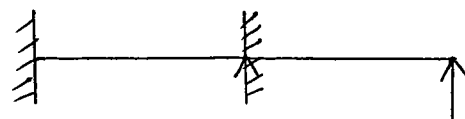Slope or rotation is the change in angle wrt the longitudinal axis
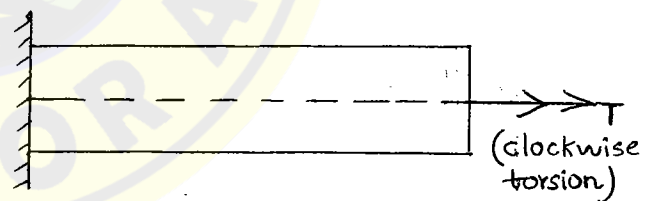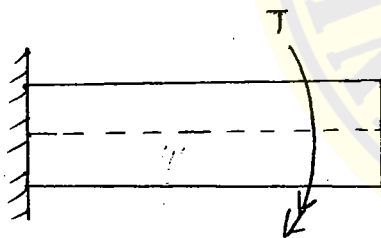
Twisting angle is the change in angle wrt c/s axis.

→ Generation of Stiffness Matrix.

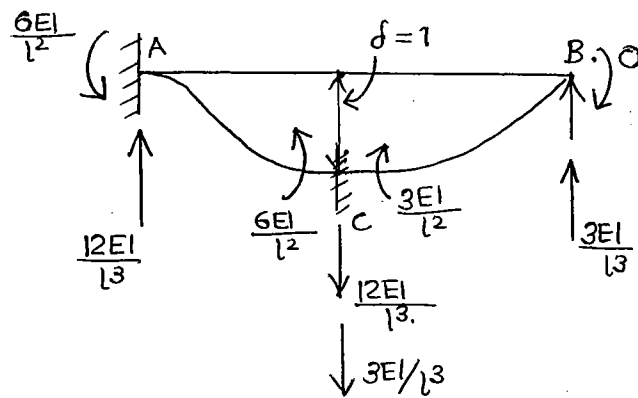Q. Generate stiffness matrix for co-ordinates shown.

Step 1: Restrain the structure in the co-ordinates shown.

Restrained Structure.

Step 2: Allow unit displacements in the direction 1. by releasing 1. But structure is restrained in the direction 2.

K_{11} = Force developed in the direction ① due to unit displacement in its own direction.

$$= \frac{12EI}{l^3} + \frac{3EI}{l^3} = \frac{15EI}{l^3}$$

$$K_{11} = \frac{15EI}{l^3}$$

NOTE:

If the co-ordinate given and the force developed arc in the same direction, then 'tve value'.

$$K_{ij}$$
at ← → due to

Stiffness element, K_{ij} = Force developed in the direction 'i' due to unit displacement in the direction 'j'

K_{21} = Force developed in the direction ② due to unit displacement in the direction ①.

$$K_{21} = \frac{3EI}{l^2} - \frac{6EI}{l^2} \Rightarrow K_{21} = \frac{-3EI}{l^2}$$

Step 3: Allow or release unit displacement (notation) in the direction ②. One is restrained again.
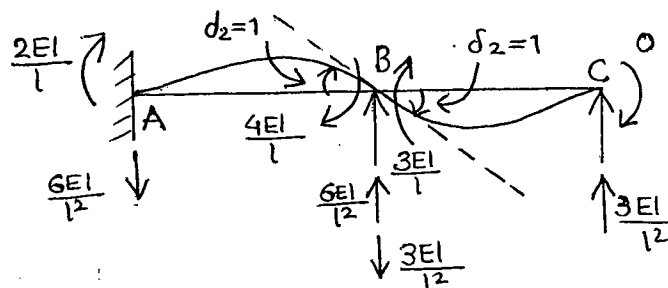
K_{22} = Force developed in direction ② due to unit rotation in its own direction

$$= \frac{4EI}{l} + \frac{3EI}{l} = \frac{7EI}{l}$$

② ②⑥

K_{12} = Force developed in the direction ① due to unit rotation in direction ②,

$$= -\frac{6EI}{l^2} + \frac{3EI}{l^2} = -\frac{3EI}{l^2}$$

$$\boxed{K_{ij} = K_{ji}}$$ Maxwell's Law is the basis for Matrix Approach.

Stiffness matrix, $[K] = \begin{bmatrix} K_{11} & K_{12} & K_{13} \\ K_{21} & K_{22} & K_{23} \\ K_{31} & K_{32} & K_{33} \end{bmatrix}$

$$= \begin{bmatrix} \frac{15EI}{l^3} & -\frac{3EI}{l^2} \\ -\frac{3EI}{l^2} & \frac{7EI}{l} \end{bmatrix} = \frac{EI}{L} \begin{bmatrix} \frac{15}{l^2} & -\frac{3}{L} \\ -\frac{3}{L} & 7 \end{bmatrix}$$

Q In this problem, generate flexibility matrix

* Steps in Flexibility Matrix:

This method is also called 'Force method' as we apply unit forces one after the other and we calculate corresponding displacements.

Step 1: Apply unit force in the direction ①. Calculate the displacements in the co-ordinate directions ① & ②.
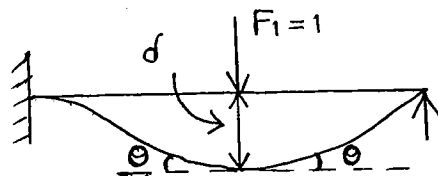
In this problem, calculation of displacements is very difficult and hence flexibility matrix method is complicated for this type of problem. However flexibility matrix can be generated as follows.

Flexibility = Displacement per unit force.

$\therefore$ Flexibility $\times$ Stiffness $= 1$

$$[\delta][k] = [I]$$
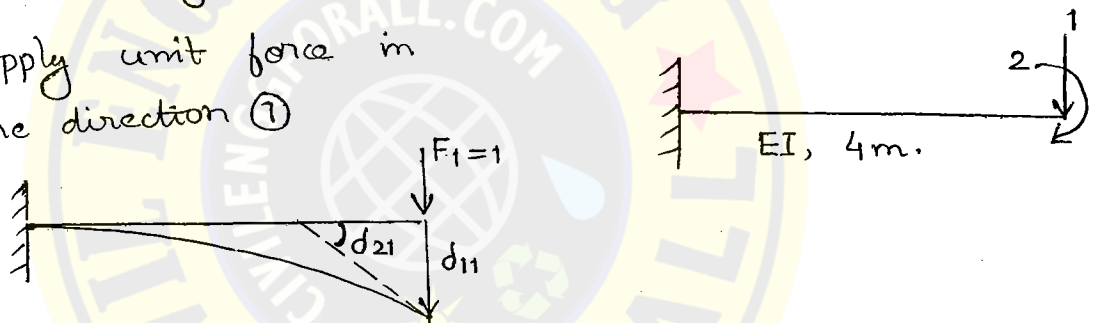
$$\therefore \quad [\delta] = [k]^{-1}$$

$$= \frac{L \times L^2}{EI \times 96} \begin{bmatrix} 7 & 3/L \\ 3/L & 15/L^2 \end{bmatrix}$$

$$= \frac{L^3}{96 EI} \begin{bmatrix} 7 & 3/L \\ 3/L & 15/L^2 \end{bmatrix} \qquad \frac{105}{L^2} - \frac{9}{L^2} = 9$$

Q. Generate flexibility matrix for the cantilever shown.

Step 1: Apply unit force in the direction ①

$\delta_{ij} \to$ flexibility element for i,j

$\delta_{11} =$ displacement in direction ① due to unit force at in the same direction.

$$= \frac{F_1 L^3}{3EI} = \frac{L^3}{3EI}$$

$\delta_{21} =$ displacement in direction ② (rotation) due to unit force in the direction ①.

$$= \frac{F_1 L^2}{2EI} = \frac{L^2}{2EI}$$

Step 2: Apply unit force (moment) in the direction ②

$$d_{12} = \frac{ML^2}{2EI} = \frac{L^2}{2EI}$$

79 ③

$$d_{12} = d_{21} \quad \text{(Maxwell's Law of reciprocal deflection}$$

$$d_{22} = \frac{ML}{EI} = \frac{L}{EI}.$$
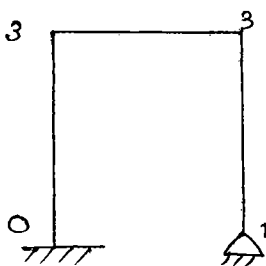
Flexibility matrix, $\delta = \begin{matrix} d_{11} & d_{12} \\ d_{21} & d_{22} \end{matrix}$

$$= \begin{bmatrix} \dfrac{L^3}{3EI} & \dfrac{L^2}{2EI} \\[3mm] \dfrac{L^2}{2EI} & \dfrac{L}{EI} \end{bmatrix} = \frac{L}{EI} \begin{bmatrix} \dfrac{L^2}{3} & \dfrac{L}{2} \\[3mm] \dfrac{L}{2} & 1 \end{bmatrix}$$

⊙ Generate stiffness matrix for above problem.

→ Order of Stiffness Matrix.

Stiffness matrix deals with unknown joint displacements or degrees of freedom neglecting axial deformations
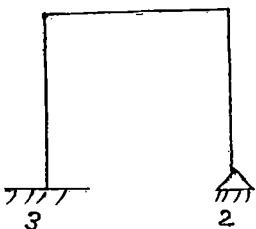

$$7 - 3 = 4.$$

$$[K]_{4 \times 4}$$

→ Order of Flexibility Matrix.

Flexibility matrix deals with redundant forces, ie, static indeterminacy.

$$D_{se} = 3 + 2 = 5 - 3 = \underline{\underline{2}}$$
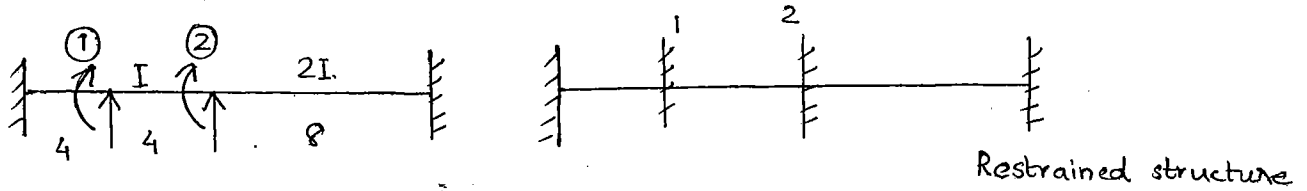
$$D_{si} = 0$$

$$[\delta]_{2 \times 2}$$

As $D_s < D_K$, flexibility matrix method may be used for this problem
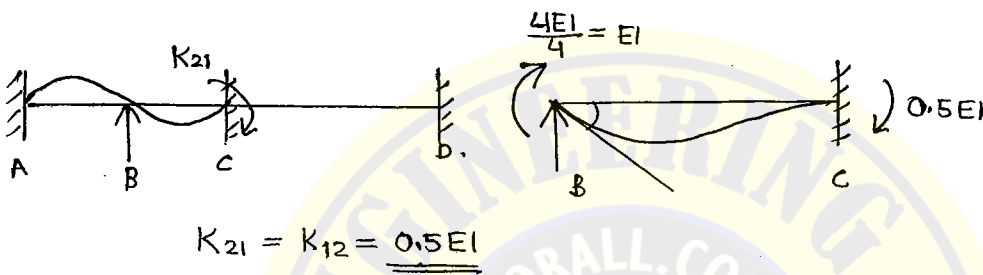
P-119

1. $\delta = \dfrac{WL^3}{3EI} = \dfrac{W}{K}$
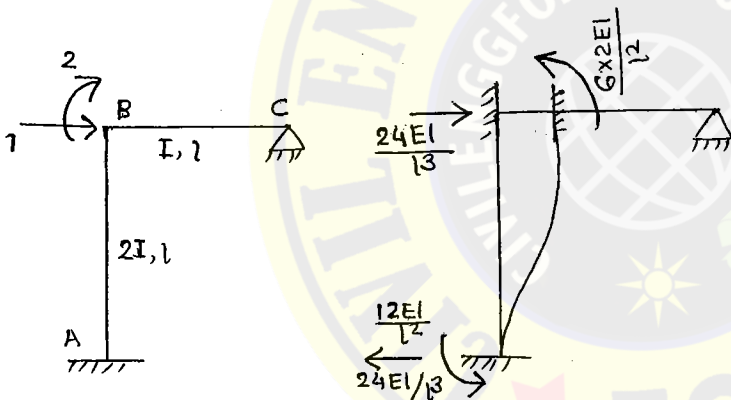
   K doubled, δ halved.

02.

Restrained structure

Restrain the structure at co-ordinates ① & ②

Release the structure at ①

$\dfrac{4EI}{4} = EI$

$K_{21} = K_{12} = \underline{0.5EI}$

$K_{21} = K_{12} = \underline{\underline{0.5EI}}$

04.

$\dfrac{24EI}{l^3}$

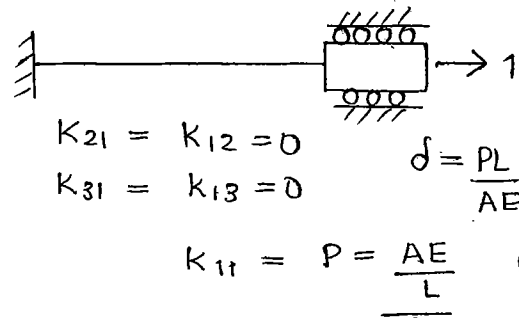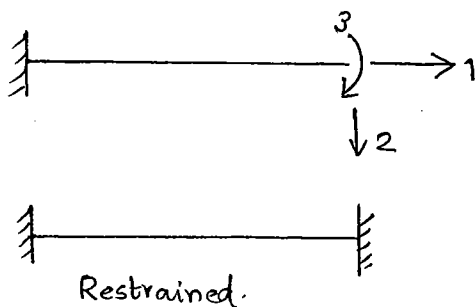$\dfrac{6 \times 2EI}{l^2}$

$\dfrac{12EI}{l^2}$

$\dfrac{24EI/l^3}$

Restrain the structure at co-ordinates shown

Allow unit displacement in direction ① but without releasing ②.

$$K_{11} = \dfrac{24EI}{l^3}$$

05. $[\delta] = [k]^{-1} = \dfrac{L}{2EI \times 3}\begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$

P.117
E.1.

Restrained.

$K_{21} = K_{12} = 0$
$K_{31} = K_{13} = 0$

$\delta = \dfrac{PL}{AE}$

$K_{11} = P = \dfrac{AE}{L} \quad (\delta = 1)$