8th Semester, Mechanical Engg.

Course Name: Production & Operations Mgmt.

Unit : II → Operation Research-II

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Transportation Model Problems:→

Introduction: In the previous semester, the general nature of LPP & its solution (graphical, simplex, etc.) was discussed. However, as the number of variables & constraints increase, the computation becomes more difficult. One such model requiring simplified calculations is the distribution model or the transportation model. Transportation problem is one of the sub-classes of LPP in which the main objective is to transport various quantities of a single homogenous commodity that are initially stored at various origins, to different destinations in such a way ~~tada~~ that total transportation cost is minimized. To achieve this objective, we must know the amount & location of available supplies, the quantity demanded, in addition to the involvement of cost associated with each sending.

Definition of T. Model:

→ Feasible Solution: A feasible soln. to a transportation problem is a set of non-negative allocations (units transported per route from origin to destination), x_{ij} that satisfies the rim (row & column) restrictions.

→ Basic Feasible Solution. A soln that contains not more than $m+n-1$ non-negative allocations, where m is the no. of rows & n is the no. of columns of T. problem.

→ Optimal Solution: A feasible soln (not necessarily basic) that minimizes (maximises) the transportation cost (profit) is called an Optimal Solution.

Tabular Representation of T. Model :-

ORIGIN/SOURCE	DESTINATIONS					
$i \downarrow$ $j \rightarrow$	d_1	d_2	d_3	$d_4 \ldots$	d_n	Supply / Availability
O_1	C_{11} $\boxed{x_{11}}$	C_{12} $\boxed{x_{12}}$	C_{13} $\boxed{x_{13}}$	C_{14} $\boxed{x_{14}}$	C_{1n} $\boxed{x_{1n}}$	S_1
O_2	C_{21} $\boxed{x_{21}}$	C_{22} $\boxed{x_{22}}$	C_{23} $\boxed{x_{23}}$	C_{24} $\boxed{x_{24}}$	C_{2n} $\boxed{x_{2n}}$	S_2
$\vdots$ O_m	C_{m1} $\boxed{x_{m1}}$	C_{m2} $\boxed{x_{m2}}$	C_{m3} $\boxed{x_{m3}}$	C_{m4} $\boxed{x_{m4}}$	C_{mn} $\boxed{x_{mn}}$	S_m
Requirement / Demand	b_1	b_2	b_3	b_4	b_n	$\sum_{i=1}^{m} S_i = \sum_{j=1}^{n} b_j$

C_{11} is the transportation cost from origin 1 to destination 1.

x_{11} is the allocation (no. of units transported per route from origin 1 to dest. 1)

$b_1, b_2 \ldots b_n$ is the requirement at respective destinations.

Methods used to Solve Transportation Problems:

1) North-West Corner Method
2) Row Minima Method
3) Column Minima Method
4) Lowest Cost or Matrix Minima Method.
5) Vogel's Approximation Method (VAM) or Penalty Method
6) Modified Distribution Method or MODI Method or u-v Method.

Solution rules to above methods on book (OR by AP Verma).

N_{11} Solve the following problem using NWCM, RMM, CMM & MMM. (above 1st 4 methods).

Origin	d_1	d_2	d_3	d_4	Supply
O_1	21	16	25	13	11
O_2	17	18	14	23	13
O_3	32	27	18	41	19
Demand	6	10	12	15	Σ Supply = Σ Demand 43 = 43

NORTH - WEST CORNER METHOD:

Origin	d_1	d_2	d_3	d_4	Supply
O_1	21 $\boxed{6}$	16	25	13	~~11~~ 5
O_2	17	18	14	23	13
O_3	32	27	18	41	19
Demand	~~6~~	10	12	15	

→

Origin	d_2	d_3	d_4	Supply
~~O_1~~	~~16~~ $\boxed{5}$	~~25~~	~~13~~	~~5~~
O_2	18	14	23	13
O_3	27	18	41	19
Demand	~~10~~ 5	12	15	

↓

Origin	d_2	d_3	d_4	Supply
O_2	18 $\boxed{5}$	14 $\boxed{8}$	23	~~13~~ 8
O_3	27	18 $\boxed{4}$	41 $\boxed{15}$	~~19~~ 15
Demand	5	~~12~~ 4	15	

m = no. of rows = 4

n = no. of columns = 3

m+n-1 = 6 reqd. no. of allocations.

& Actual no. of allocations after solving = 6.

∴ we have got basic feasible soln, & is optimal.

Total cost = Σ (allocation x total cost)

$= C_{11} \times x_{11} + C_{12} \times x_{12} + C_{22} \times x_{22} + C_{23} \times x_{23} + C_{33} \times x_{33} + C_{34} \times x_{34}$.

$= 21 \times 6 + 16 \times 5 + 18 \times 5 + 14 \times 8 + 18 \times 4 + 41 \times 15$

= Rs 1095

→ ROW - MINIMA METHOD:

Origin	d_1	d_2	d_3	d_4	Supply
O_1	~~21~~	~~16~~	~~25~~	~~13~~ $\boxed{11}$	~~11~~
O_2	17 $\boxed{1}$	18	14 $\boxed{12}$	23	~~13~~ 1
O_3	32 $\boxed{5}$	27 $\boxed{10}$	18	41 $\boxed{4}$	~~19~~ ~~9~~ 4
Demand	~~6~~ 5	10	12	~~15~~ 4	

No. of allocations = 6.

∴ Total Cost = 13x11 + 17x1 + 14x12 x 32x5 + 27x10 + 41x4

= Rs 922

3) Column Minima Method :

Origin	d_1	d_2	d_3	d_4	Supply
O_1	21	16 $\boxed{10}$	25	13 $\boxed{1}$	~~11~~ 1
O_2	17 $\boxed{6}$	18	~~14~~ $\boxed{7}$	23	~~13~~ 7
O_3	32	27	18 $\boxed{5}$	41 $\boxed{14}$	~~19~~ 14
Demand	6	10	~~12~~ 5	~~15~~ 14	

Total Cost : $17 \times 6 + 16 \times 10 + 14 \times 7 + 18 \times 5 + 13 \times 1 + 41 \times 14$
= Rs 1037.

4) Matrix Minima Method

~~Origin~~	d_1	d_2	d_3	d_4	Supply
O_1	~~21~~	~~16~~	~~25~~	~~13~~ $\boxed{11}$	~~11~~
O_2	~~17~~ $\boxed{1}$	~~18~~	~~14~~ $\boxed{12}$	~~23~~	~~13~~ 1
O_3	32 $\boxed{5}$	27 $\boxed{10}$	18	41 $\boxed{4}$	~~19~~ ~~9~~ 4
Demand	~~6~~ 5	10	12	~~15~~ 4	

Total Cost : $13 \times 11 + 17 \times 1 + 14 \times 12 + 32 \times 5 + 27 \times 10 + 41 \times 4$
= Rs 922

5. VAM (Impt.)

Here, we have to find penalties. Penalty means difference between lowest cost & next lowest cost. After determining penalty, we select the max. penalty in row & column. Then the lowest cost cell is taken in subsequent row or column & allocated.

Case: When there is tie in penalty between available & read, then penalty is shown having max. allocation. It is preferred to choose the cost difference corresponding to the largest no. of units can be assigned or corresponding to the cell chosen has minimum cost.

	d_1	d_2	d_3	d_4	Available	Row Penalities
O_1	1	2	1	4	30	1-1=0,
O_2	3	3	2	1 $\boxed{10}$	~~50~~ 40	2-1=0,
O_3	4	2	5	9	20	4-2=0,
Reqd.	20	40	30	~~10~~		
Column Penalities	3-1=2	2-2=0	2-1=ϕ	4-1=3		

	d_1	d_2	d_3	Available	R.P
O_1	1 $\boxed{20}$	2 $\boxed{10}$		~~30~~ ~~10~~	1-1=0, 0, 1
O_2	3	3 $\boxed{10}$	2 $\boxed{30}$	~~40~~ ~~10~~	3-2=1, 1, 1
O_3	4	2 $\boxed{20}$	5	~~20~~	4-2=2
Reqd.	20	~~40~~ ~~20~~ 10	30		
C.P	3-1=2, 2	2-2=0, 1, 1	2-1=1, 1, 1		

Total Allocations = 6.

$T.C = 1\times10 + 1\times20 + 2\times10 + 3\times10 + 2\times30 + 2\times20$
$= Rs\ 180.$

Unsolved Questions on VAM:

Q1)

	d_1	d_2	d_3	d_4	Supply
O_1	2	3	11	7	6
O_2	1	0	6	1	1
O_3	5	8	15	9	10
Reqd.	7	5	3	2.	

Q2)

		Warehouses				
		W_1	W_2	W_3	W_4	Capacity
factory	F_1	19	30	50	10	7
	F_2	70	30	40	60	9
	F_3	40	8	70	20	18
	Reqd.	5	8	7	14	

6) MODI METHOD → to find optimal solution. (6)

Q)

	W_1	W_2	W_3	Supply
F_1	16	20	12	200
F_2	14	8	18	160
F_3	26	24	16	90
Demand	180	120	150	

Step 1) Apply VAM to find most feasible soln.

	W_1	W_2	W_3	Supply	Row Penalties
F_1	~~16~~ [140]	~~20~~	~~12~~ [60]	~~200~~ 140	4, 4, 4
F_2	14 [40]	8 [120]	18	160	6, 4, 4
F_3	~~26~~	~~24~~	~~16~~ [90]	~~90~~	8, 10 ←
Demand	~~180~~ 40	~~120~~	~~150~~ 60		
Column Penalties	2, 2, 2, 2	12 ↑	4, 4, 6 ↑		

Check whether it is a basic feasible soln. or not.

Reqd. no. of allocations $= m+n-1 = 5$.

& actual no. of allocations $= 5$.

Step 2:) Assign dual variables:

Check which row or column has maximum no. of allocations. Assign that row or column, a dual variable $= 0$.

Let u_i = no. of rows

v_j = no. of columns.

Using formula, $u_i + v_j = c_{ij}$

	W_1	W_2	W_3	S	Dual variables u_i
F_1	16 [140]	20 / -10	12 [60]	200	$u_1 = 0$
F_2	14 [40]	8 [120]	18 / -8	160	$u_2 = -2$
F_3	26 / -6	24 / -10	16 [90]	90	$u_3 = 4$
Reqd.	180	120	150		
D.V, v_j	$v_1 = 16$	$v_2 = 10$	$v_3 = 12$		

Assign $u_1 = 0$ as Row 1 is having 2 allocations.

$\therefore u_1 + v_1 = 16$; Similarly find other dual variables.

$\Rightarrow v_1 = 16$

Step 3) Calculate Opportunity cost to other non-basic cells (non-allocated cells), using the formula; $u_i + v_j - c_{ij} = $ O.C

(5)

$u_1 + v_2 - 20 = -10$ $\{20/-10\}$.

$u_2 + v_3 - 18 = -8$

$u_3 + v_1 - 26 = -6$

$u_3 + v_2 - 24 = -10$

Since all the values of O.C. are negative, we need not to reduce further cost. We have reached the optimal solution.

∴ Total Cost = $16 \times 140 + 12 \times 60 + 14 \times 40 + 8 \times 120 + 16 \times 90$

= Rs 5920/=

Unsolved:

	d_1	d_2	d_3	d_4	Supply
S_1	23	27	16	18	30
S_2	12	17	20	51	40
S_3	22	28	12	32	53
Reqd.	22	35	25	41.	

Cases of MODI METHOD:

- Degeneracy Case
- Unbalanced Matrix → Dummy Line
- 0- allocations.