1.1. Define following terms

(a) Current

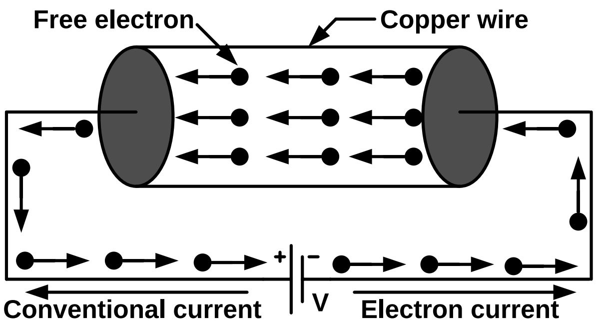


Figure 1.1Concept of electric current

- Flow of electron in closed circuit is called current.
- Amount of charge passing through the conductor in unit time also called current.
- Unit of current is charge/second or Ampere (A).

$$I = \frac{Q}{t}$$

$Where,\ I =$ Current

$Q =$ Charge

$t =$ Time

(b) Potential or Voltage

- The capacity of a charged body to do work is called potential.
- Unit of potential is joule/coulomb or Volt (V).

$$V = \frac{W}{Q}$$

$Where,\ V =$ Potential or Voltage

$W =$ Workdone

(c) Potential difference

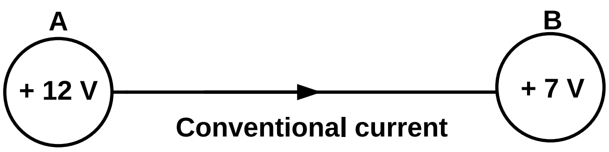


Figure 1. 1Potential differences

- The difference of electrical potential between two charged bodies is called potential difference.
- Unit of Potential Difference is Volt (V).
- If potential of body A is +12V and potential of body B is +7V then potential difference is +5V.
  i.e. (+12V) - (+7V) = +5V

(d) Electro Motive Force (emf)

- The force is required to move electron from negative terminal to positive terminal of electrical source in electrical circuit is called emf.
- Unit of emf is volt (V).
- Emf is denoted as ε.

(e) Energy

- Ability to do work is called energy.
- Unit of energy is Joule or Watt-sec or Kilowatt-hour (KWh).
- 1KWh is equal to 1 Unit.

$$W = P \times t = VIt = I^2Rt = \frac{V^2t}{R}$$

Where, W=Energy

P =Power

t =Time

(f) Power

- Energy per unit in time is called power.
- Unit of Power is Joule/Second or Watt (W).

$$P = \frac{W}{t}$$

(g) Resistance

- Property of a material that opposes the flow of electron is called resistance.
- Unit of resistance is Ohm (Ω).

$$R = \frac{V}{I}$$

Where, $R =$ Resistance

(h) Conductance

- Property of a material that allows flow of electron.
- It is reciprocal of resistance.
- Unit of conductance is (Ω^{-1}) or mho or Siemens(S).

$$G = \frac{1}{R}$$

Where, $G =$ Conductance

(i) Resistivity or Specific Resistance

- Amount of resistance offered by 1m length of wire of $1m^2$ cross-sectional area.
- Resistivity is denoted as a ρ.
- Unit of Resistivity is Ohm-meter (Ω-m).

$$R \propto \frac{l}{a}$$

$$R = \rho \frac{l}{a}$$

$$\rho = \frac{Ra}{l}$$

$Where, R =$ Resistance

$\rho =$ Resistivity

$l =$ Length of wire

$a =$ Cross section area of wire

(j) Conductivity

- Ability of a material to allow flow of electron of a given material for 1 m length & 1 m^2cross-sectional area is called conductivity. Unit of conductivity is$\Omega^{-1}m^{-1}$ or Siemens m^{-1}.

$$\sigma = \frac{1}{\rho}$$

$Where, \sigma =$ Conductivity

1.2. Explain types of electrical energysource

- Electrical source is an element which supplies energy to networks. There are two types of electrical sources.

(a) Independent sources

Independent voltage source

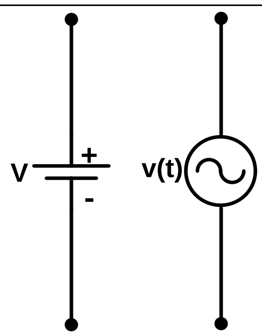


Figure 1. 2Independent voltage source

- It is a two terminal element that provide a specific voltage across its terminal.
- The value of this voltage at any instant is independent of value or direction of the current that flow through it.

Independent current source

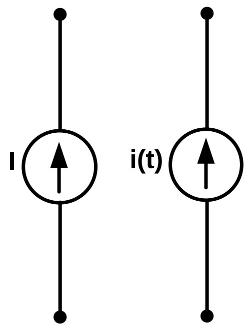


Figure 1. 3Independent current source

- It is two-terminal elements that provide a specific current across its terminal.
- The value and direction of this current at any instant is independent of value or direction of the voltage that appears across the terminal of source

(b) Dependent sources

Voltage controlled voltage source (VCVS)

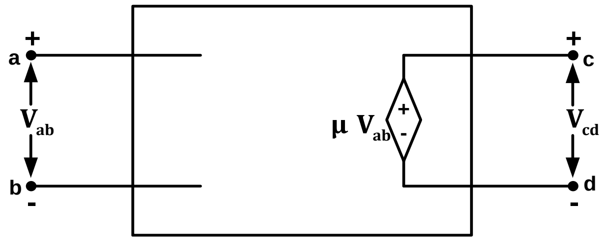


Figure 1.5VCVS

- Voltage controlled voltage source is four terminal network components that established a voltage V_{cd} between two-point c and d.

  $V_{cd} = \mu V_{ab}$
- The voltage V_{cd} depends upon the control voltage V_{ab} and μ is constant so it is dimensionless.
- μ is known as a voltage gain.

Voltage controlled current source (VCCS)

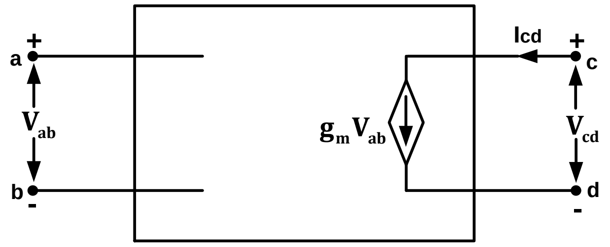


Figure 1.6VCCS

- Voltage controlled current source is four terminal network components that established a current i_{cd} in the branch of circuit.

  $i_{cd} = g_m V_{ab}$
- i_{cd} depends only on the control voltage V_{ab} and constant g_m ,is called trans conductance or mutual conductance.
- Unit of transconductance is Ampere/Volt or Siemens(S).

Current controlled voltage source (CCVS)

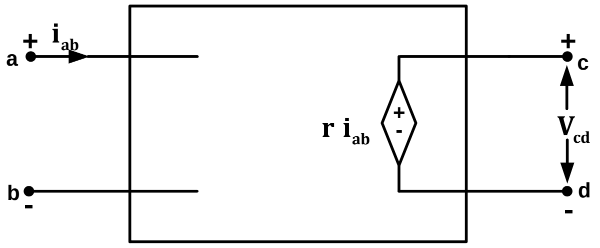


Figure 1.7CCVS

- Current controlled voltage source is four terminal network components that established a voltage V_{cd} between two-point c and d.

  $V_{cd} = r i_{ab}$
- V_{cd} depends on only on the control current i_{ab} and constant r and r is called trans resistance or mutual resistance.
- Unit of transresistance is Volt/Ampere or Ohm (Ω).

Current controlled current source (CCCS)

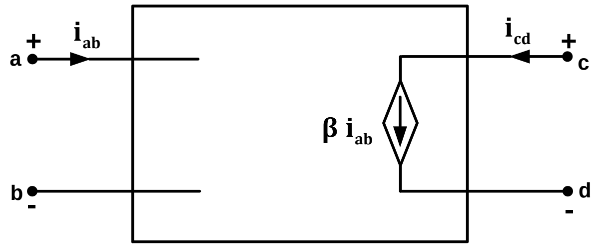


Figure 1.8CCCS

- Current controlled current source is four terminal network components that established a current I_{cd} in the branch of circuit.

  $i_{cd} = \beta i_{ab}$
- i_{cd} depends on only on the control current i_{ab} and constant β and β is called current gain. Current gain is constant.
- Current gain is dimensionless.

1.3. Explain source conversion

- A voltage source with a series resistor can be converted into an equivalent current source with a parallel resistor. Conversely, a current source with a parallel resistor can be converted into a voltage source with a series resistor.
- Open circuit voltages in both the circuits are equal and short circuit currents in both the circuit are equal.Source transformation can be applied to dependent source as well.

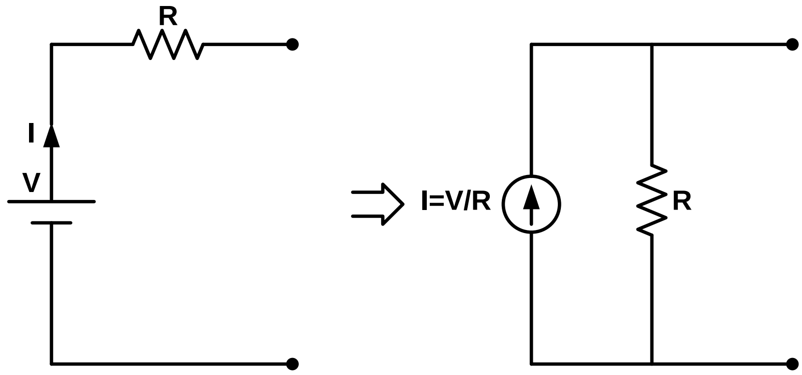
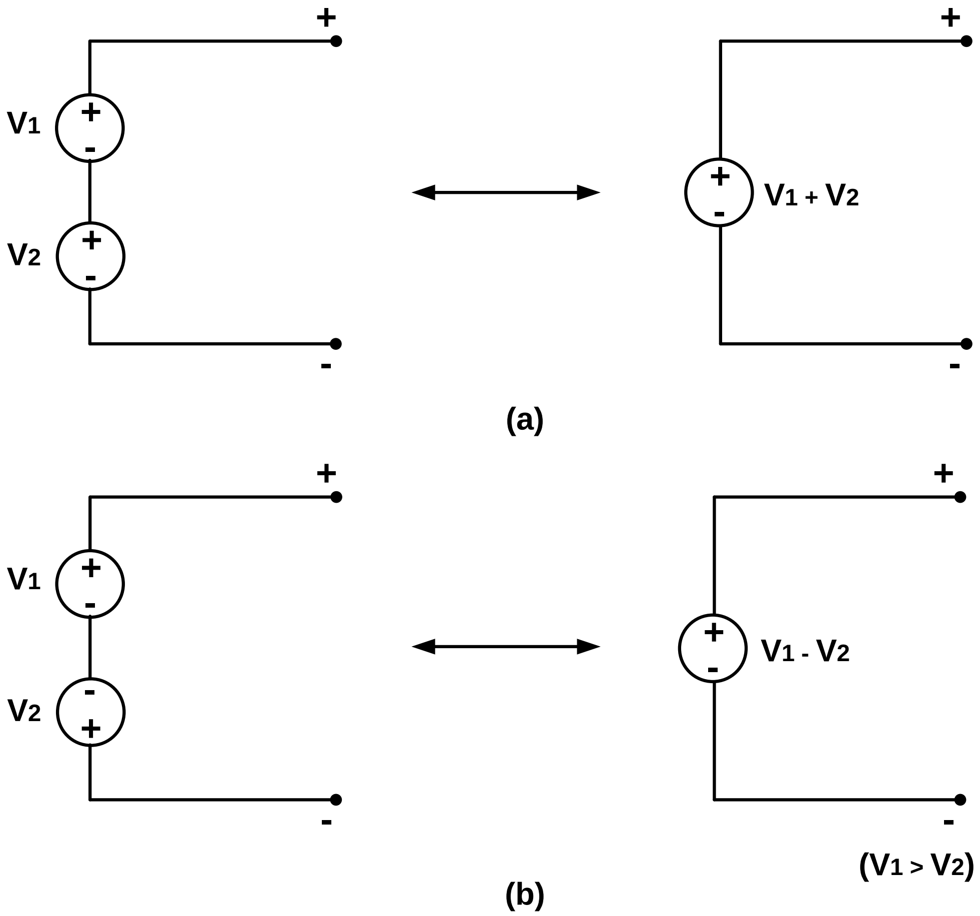
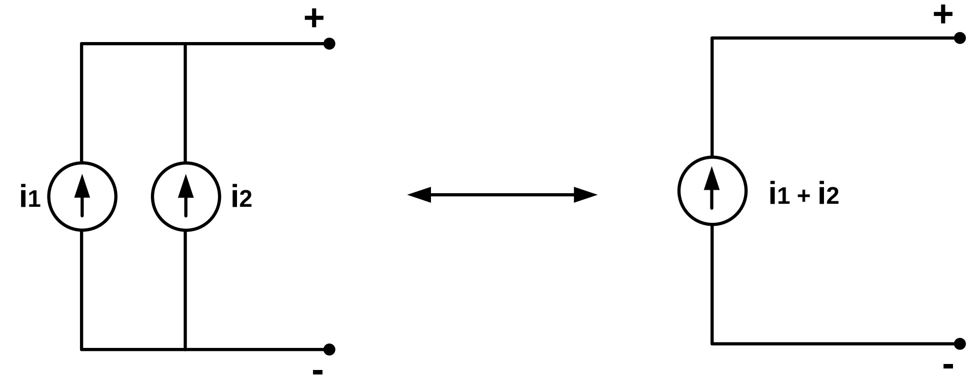


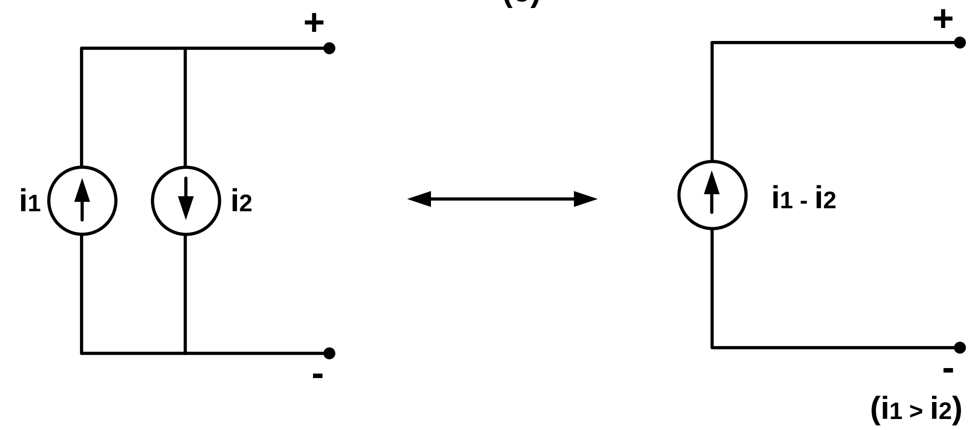
Figure 1. 9Source conversion

Network simplification techniques

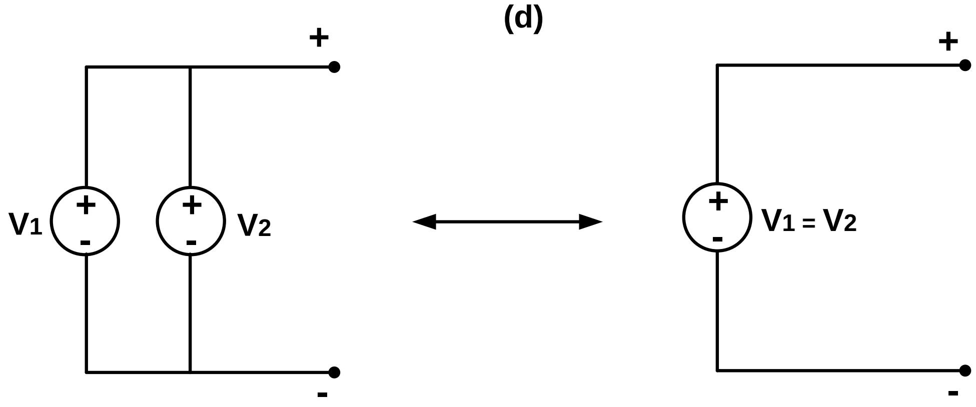




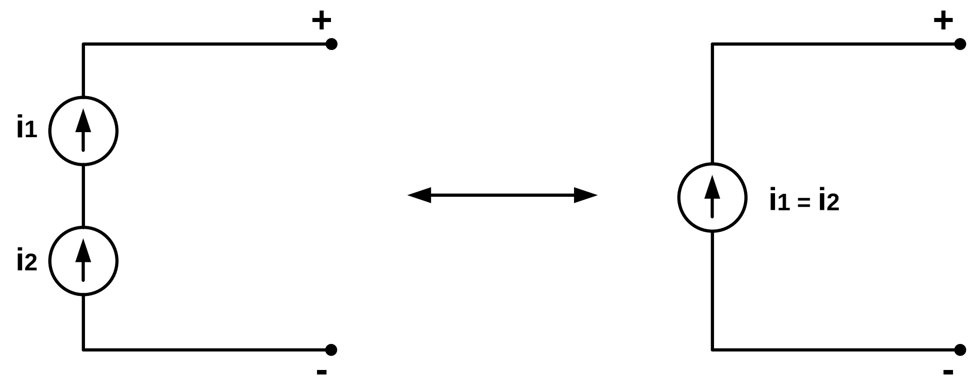
(c)



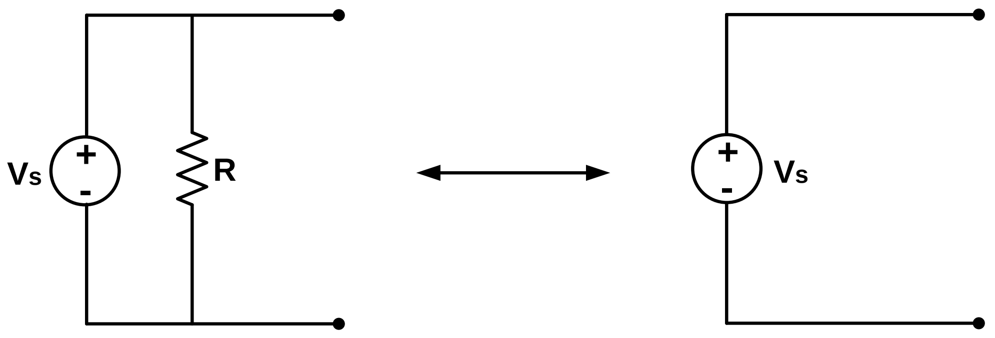
(d)



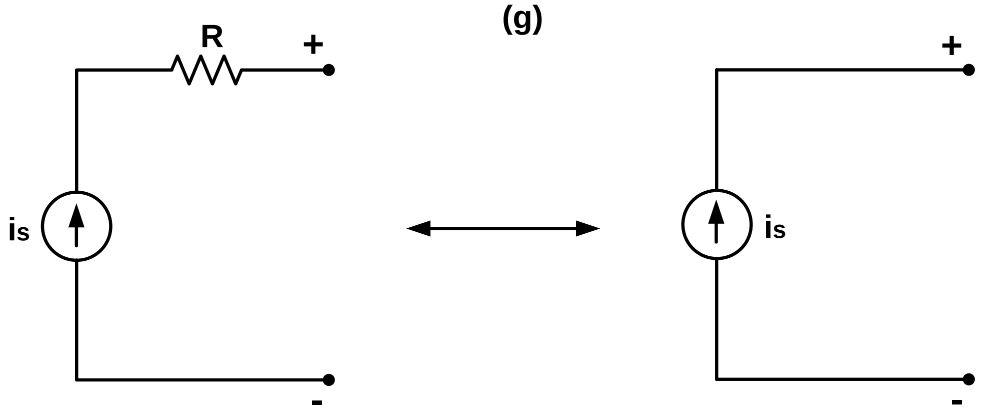
(e)



(f)



(g)



(h)

Figure 1.10Rules under which source may be combined and separated

1.4. Explain ideal electrical circuit element.

- There are major three electrical circuit elements which are discussed below.

(a) Resistor

- Resistor is element which opposes the flow of current.

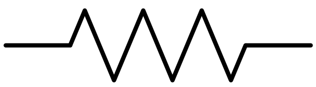


Figure 1.11Resistor

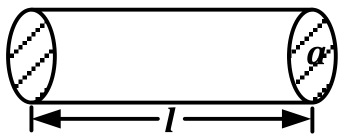


Figure 1.12Conductor

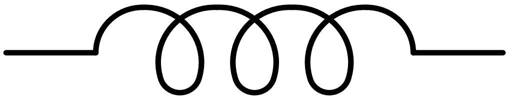
- Resistance is property of material which opposes the flow current. It is measured in Ohms (Ω).
- Value of resistance of conductor is
  - ✓ Proportional to its length.
  - ✓ Inversely proportional to the area of cross section.
  - ✓ Depends on nature of material.
  - ✓ Depends on temperature of conductor.

$$R \propto \frac{l}{a}$$

$$R = \frac{\rho l}{a}$$

(b) Inductor

- An inductor is element which store energy in form of magnetic field.
- The property of the coil of inducing emf due to the changing flux linked with it is known as inductance of the coil.
- Inductance is denoted by L and it is measured in Henry (H).



1.13Inductor

- Value of inductance of coil is
  - ✓ Directly proportional to the square of number of turns.
  - ✓ Directly proportional to the area of cross section.
  - ✓ Inversely proportional to the length.
  - ✓ Depends on absolute permeability of magnetic material.

$$\Phi = \frac{F}{S} = \frac{NI}{S} = \frac{NI}{\dfrac{l}{\mu_0 \mu_r A}} = \frac{NI\mu_0 \mu_r A}{l}$$

$$Now, L = \frac{N\Phi}{I} = \frac{N\left(\dfrac{NI\mu_0 \mu_r A}{l}\right)}{I} = \frac{N^2 \mu_0 \mu_r A}{l}$$

$Where, L$ = Inductance of coil

N = Number of turns of coil

Φ = Flux link in coil

F = Magneto motive force(MMF)

I = Current in the coil

l = Mean length of coil

μ_0 = Permiability of free space

μ_r = Relative permiability of magnetic material

A = Cross sectional area of magnetic material

(c) Capacitor

- Capacitor is an element which stored energy in form of charge.
- Capacitance is the capacity of capacitor to store electric charge.
- It is denoted by C and measured in Farad (F).

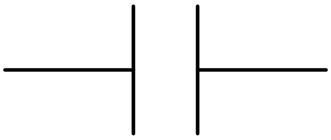


Figure 1.14Capacitor

- Value of capacitance is
  - ✓ Directly proportional to the area of plate.
  - ✓ Inversely proportional to distance between two plates.
  - ✓ Depends on absolute permittivity of medium between the plates.

$C \propto \dfrac{A}{d}$

$C = \dfrac{\varepsilon A}{d}$

$C = \dfrac{\varepsilon_0 \varepsilon_r A}{d}$

$Where, C$=Capacitance of capacitor

A =Cross sectional area of plates

d =Distance between two plates

ε = Abolute Permittivity

ε_0= Permittivity of free space

ε_r= Relative permittivity of dielectric material

1.5. Explain effect of temperature on resistance.

- The resistance of material changes with temperature due to change in resistivity of material caused by the changing activity of the atoms. For different type of materials, amount of change in resistance due to change in temperature is different.
- The resistance of all pure metals increases linearly with increase in temperature over a limited temperature range. As temperature increases, the ion inside the metal gets energy and start to oscillating about their mean positions. This vibrating ion is collides with electrons. So, flow of electron become decrease and resistance of metal increase. Hence resistance increases with increase in temperature.
- The resistance of almost all alloys increases with increase in temperature but the rate of change of resistance is less than the metals. Some alloys like a manganin, Eureka do not change resistance by change in temperature.

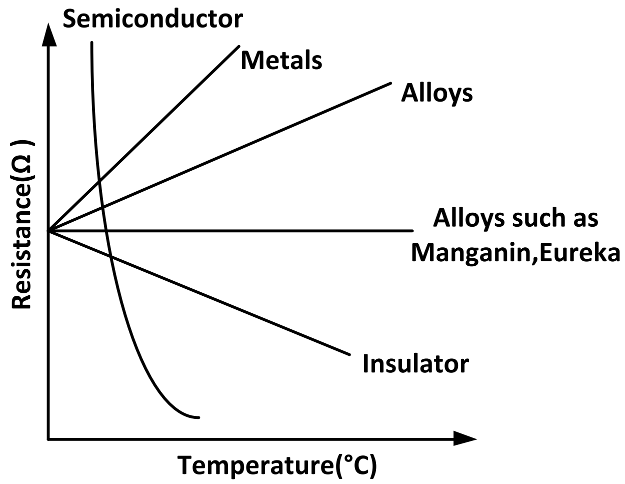


Figure 1.15Effect of temperature on resistance

- In case of insulator and semiconductor resistance of material decreases with increase in temperature. As a temperature increases, some electrons get energy and become free for conduction. Collision between ion and electron will decrease in semiconductor as compare to metal and flow of electron become increases. Hence, conductivity increases and resistance decreases with increase in temperature.

- The change in resistance of material with rise in temperature can be expressed by mean of temperature coefficient of resistance.

Let,

R_{t_1} = Resistance at $t_1\,^0C$

R_{t_2} = Resistance at $t_2\,^0C$

$R_{t_2} - R_{t_1}$ = Change in resistance for the change in temperature $(t_2 - t_1)^0C$

α_{t_1} = Temperature coefficient of resistance at $t_1\,^0C$

- Change in resistance i.e. $(R_{t_2} - R_{t_1})$
  - ✓ Directly proportional to initial resistance.
  - ✓ Directly proportional to the rise in temperature.
  - ✓ Depending on the nature of the material.

$$R_{t_2} - R_{t_1} \propto R_{t_1}$$
$$\propto (t_2 - t_1)$$
$$\therefore R_{t_2} - R_{t_1} = \alpha_{t_1} R_{t_1} (t_2 - t_1)$$
$$R_{t_2} = R_{t_1} + \alpha_{t_1} R_{t_1} (t_2 - t_1)$$
$$R_{t_2} = R_{t_1}\left(1 + \alpha_{t_1}(t_2 - t_1)\right)$$

Now, suppose $t_1 = 0$ and $t_2 = t$

$$\therefore R_t = R_0\left(1 + \alpha_0 t\right)$$

- When resistance of material increases with increase in temperature, it has positive temperature coefficient. For Example Metal, Alloy.
- When resistance of material decreases with increase in temperature, it has negative temperature coefficient. For Example Insulator, Semiconductor.

1.6. Derive an expression for temperature coefficient at temperature t, $\alpha_{t_2} = \alpha_{t_1} / \left(1 + \alpha_{t_1}(t_2 - t_1)\right)$ and $\alpha_t = \alpha_0 / \left(1 + \alpha_0 t\right)$

- As the temperature of conductor increase from $t_1\,^0C$ to $t_2\,^0C$, Change in resistance

$$R_{t_2} = R_{t_1}\left(1 + \alpha_{t_1}(t_2 - t_1)\right) \qquad (i)$$

Same material is cooled from $t_2\,^0c$ to $t_1\,^0c$, Change in resistance

$$R_{t_1} = R_{t_2}\left(1 + \alpha_{t_2}(t_1 - t_2)\right)$$

Solving equation (i) *and* (ii) $\qquad (ii)$

$$R_{t_2} = R_{t_2}\left(1 + \alpha_{t_2}(t_1 - t_2)\right)\left(1 + \alpha_{t_1}(t_2 - t_1)\right)$$
$$1 = \left(1 - \alpha_{t_2}(t_2 - t_1)\right)\left(1 + \alpha_{t_1}(t_2 - t_1)\right)$$
$$\left(1 - \alpha_{t_2}(t_2 - t_1)\right) = \frac{1}{\left(1 + \alpha_{t_1}(t_2 - t_1)\right)}$$

$$\alpha_{t_2}(t_2-t_1)=1-\frac{1}{\left(1+\alpha_{t_1}(t_2-t_1)\right)}$$

$$\alpha_{t_2}(t_2-t_1)=\frac{1+\alpha_{t_1}(t_2-t_1)-1}{\left(1+\alpha_{t_1}(t_2-t_1)\right)}$$

$$\alpha_{t_2}(t_2-t_1)=\frac{\alpha_{t_1}(t_2-t_1)}{\left(1+\alpha_{t_1}(t_2-t_1)\right)}$$

$$\alpha_{t_2}=\frac{\alpha_{t_1}}{\left(1+\alpha_{t_1}(t_2-t_1)\right)}$$

$$\alpha_{t_2}=\frac{1}{\left(\frac{1}{\alpha_{t_1}}+(t_2-t_1)\right)}$$

- Now $t_1=0^0c$, $t_2=t^0c$ in below equation

$$\alpha_{t_2}=\frac{\alpha_{t_1}}{\left(1+\alpha_{t_1}(t_2-t_1)\right)}$$

$$\alpha_t=\frac{\alpha_0}{1+\alpha_0 t}$$

1.7. Explain Ohm's law and its limitations.

- Current flowing through the conductor is directly proportional to the potential difference applied to the conductor, provided that no change in temperature.

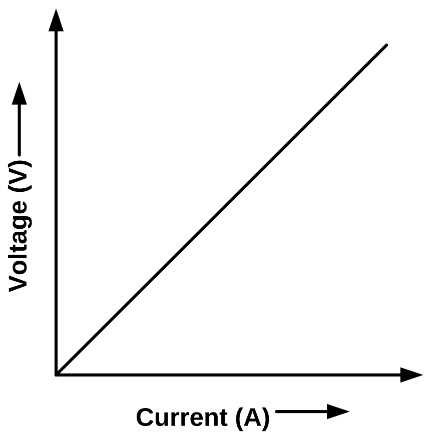


Figure 1.16Change in current w.r.t change in voltage for conducting material

$$V \propto I$$

$$\therefore V = IR$$

- Where R is constant which is called resistance of the conductor.

$$\therefore R=\frac{V}{I}$$

- Limitations of Ohm's Law:
  - ✓ It cannot be applied to non-linear device e.g. Diode, Zener diode etc.

- ✓ It cannot be applied to non-metallic conductor e.g. Graphite, Conducting polymers
- ✓ It can only be applied in the constant temperature condition.

1.8. State and explain the Kirchhoff's current and voltage laws

(a) Kirchhoff's current law (KCL)

- **Statement**:
  "Algebraic sum of all current meeting at a junction is zero"
- Let, Suppose
  - ✓ Branches are meeting at a junction 'J'
  - ✓ Incoming current are denoted with (+ve) sign
  - ✓ Outgoing currents are denoted with (-ve) sign

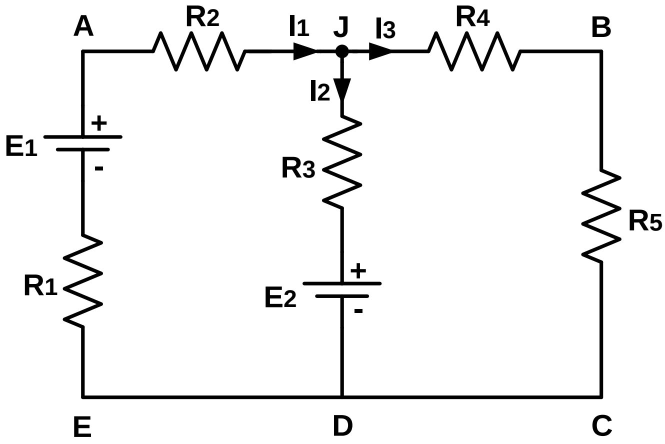


Figure 1.17Kirchhoff's law diagram

- Then,

$$\sum I = 0$$

$$(+I_1)+(-I_2)+(-I_3)=0$$

$$I_1 - I_2 - I_3 = 0$$

$$I_1 = I_2 + I_3$$

$\therefore$ *Incoming current* $=$ *Outgoing current*

(b) Kirchhoff's voltage law (KVL)

- **Statement**:
  "Algebraic sum of all voltage drops and all emf sources in any closed path is zero"
- Let, Suppose
  - ✓ Loop current in clockwise or anticlockwise direction
  - ✓ Circuit current and loop current are in same direction than voltage drop is denoted by (-ve) sign.
  - ✓ Circuit current and loop current are in opposite direction than voltage drop is denoted by (+ve) sign.
  - ✓ Loop current move through (+ve) to (-ve) terminal of source than direction of emf is (-ve).
  - ✓ If Loop current move through (-ve) to (+ve) terminal of source than direction of emf is (+ve).

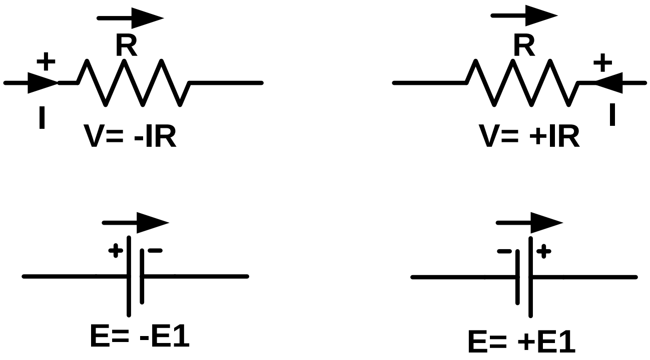


Figure 1.18Sign convention for Kirchhoff's voltage law

$$\therefore \sum IR + \sum E = 0$$

KVL to loop AJDEA

$$-I_1R_2 - I_2R_3 - E_2 - I_1R_1 + E_1 = 0$$

KVL to loop JBCDJ

$$-I_3R_4 - I_3R_5 + E_2 + I_2R_3 = 0$$

1.9. Explain series and parallel combination of resistor

Series combination of resistor

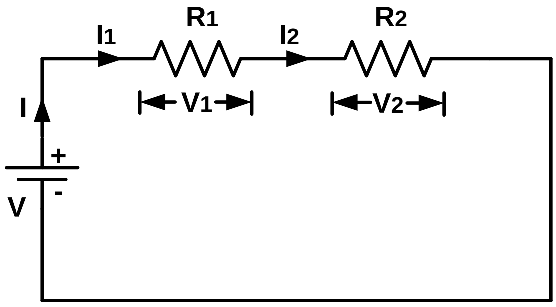


Figure 1.19Series combination of resistors

$$Here, I_1 = I_2 = I$$

As per KVL,

$$V = V_1 + V_2$$

$$V = IR_1 + IR_2$$

$$V = I(R_1 + R_2)$$

$$\frac{V}{I} = (R_1 + R_2)$$

$$R_{eq} = R_1 + R_2$$

For n resistor are connected in series

$$R_{eq} = R_1 + R_2 + R_3 + \dots\dots\dots + R_n$$

Parallel combination of resistor

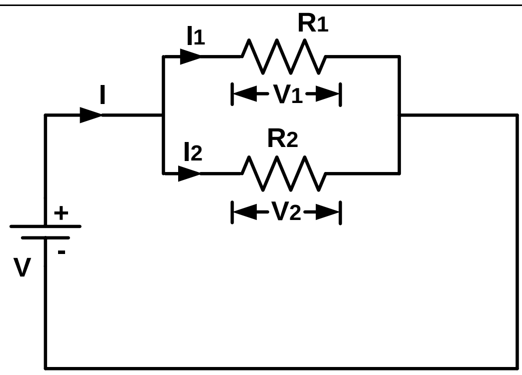


Figure 1.20Parallel combinations of resistors

$$Here, V_1 = V_2 = V$$

As per KCL,

$$I = I_1 + I_2$$

$$I = \frac{V}{R_1} + \frac{V}{R_2}$$

$$I = V\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

$$\frac{I}{V} = \left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

$$\frac{1}{R_{eq}} = \left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

- Value of equivalent resistance of series circuit is bigger than the biggest value of individual resistance of circuit.

For n resistor are connected in Parallel

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \dots\dots + \frac{1}{R_n}$$

- Value of equivalent resistance of parallel circuit is smaller than the smallest value of individual resistance of circuit.

1.10. Explain Voltage divider law and current divider Law.

Voltage Divider Law

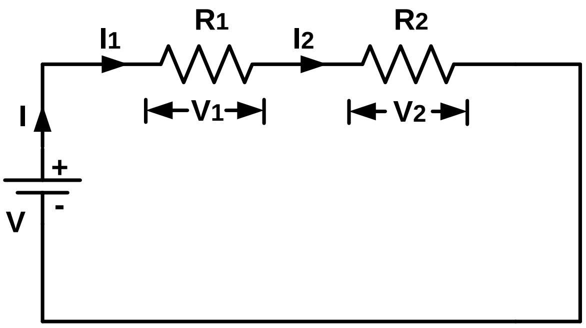


Figure 1.21Voltage divider circuit

$$Here, I_1 = I_2 = I$$
$$As\ per\ KVL,$$
$$V = V_1 + V_2$$
$$V = I_1R_1 + I_2R_2$$
$$V = IR_1 + IR_2$$
$$V = I(R_1 + R_2)$$
$$I = I_1 = I_2 = \frac{V}{(R_1 + R_2)}$$
$$Now, V_1 = I_1R_1$$
$$V_1 = \frac{V}{R_1 + R_2}R_1$$
$$V_1 = V\left(\frac{R_1}{R_1 + R_2}\right)$$
$$Now, V_2 = I_2R_2$$
$$V_2 = \frac{V}{R_1 + R_2}R_2$$
$$V_2 = V\left(\frac{R_2}{R_1 + R_2}\right)$$

Current Divider Law

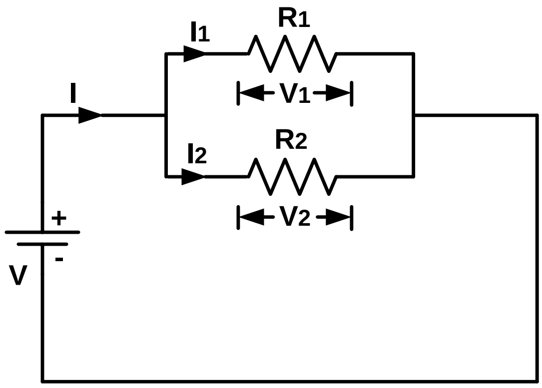


Figure 1.22Current divider circuit

$$Here, V_1 = V_2 = V$$
$$As\ per\ KCL,$$
$$I = I_1 + I_2$$
$$I = \frac{V_1}{R_1} + \frac{V_2}{R_2}$$
$$I = \frac{V}{R_1} + \frac{V}{R_2}$$
$$I = V\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$
$$\frac{I}{V} = \left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$
$$V = V_1 = V_2 = I\left(\frac{R_1R_2}{R_1 + R_2}\right)$$
$$Now, I_1 = \frac{V_1}{R_1}$$
$$I_1 = \frac{I\left(\frac{R_1R_2}{R_1 + R_2}\right)}{R_1}$$
$$I_1 = I\left(\frac{R_2}{R_1 + R_2}\right)$$

$$Now, I_2 = \frac{V_2}{R_2}$$

$$I_2 = \frac{I\left(\frac{R_1 R_2}{R_1 + R_2}\right)}{R_2}$$

$$I_2 = I\left(\frac{R_1}{R_1 + R_2}\right)$$

1.11. Derive the equation of delta to star and star to delta transformation

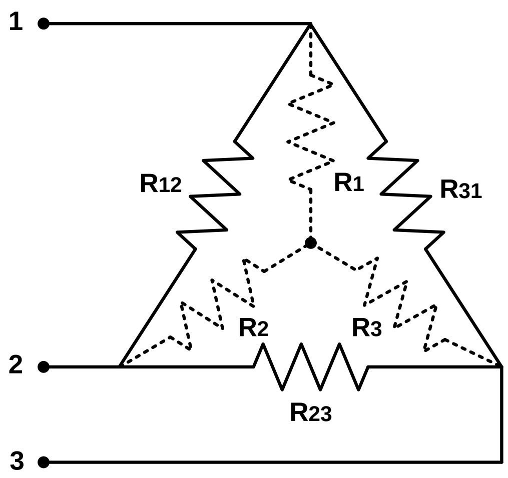


Figure 1.23Delta connected network

Resistance between terminal (1) & (2)

$$= R_{12} \parallel (R_{23} + R_{31})$$

$$= \frac{R_{12}(R_{23} + R_{31})}{R_{12} + R_{23} + R_{31}}$$

Resistance between terminal (2) & (3)

$$= R_{23} \parallel (R_{12} + R_{31})$$

$$= \frac{R_{23}(R_{12} + R_{31})}{R_{12} + R_{23} + R_{31}}$$

Resistance between terminal (3) & (1)

$$= R_{31} \parallel (R_{12} + R_{23})$$

$$= \frac{R_{31}(R_{12} + R_{23})}{R_{12} + R_{23} + R_{31}}$$

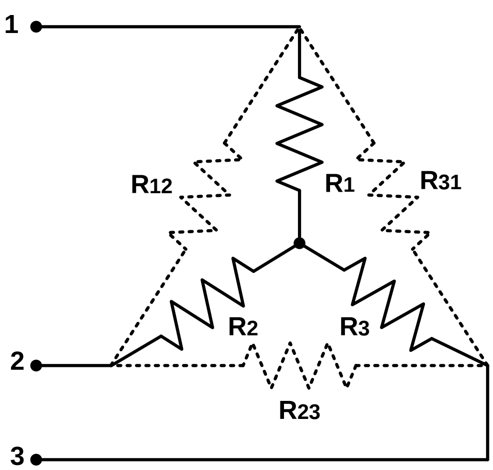


Figure 1.24Star connected network

Resistance between terminal (1) & (2)

$$= R_1 + R_2$$

Resistance between terminal (2) & (3)

$$= R_2 + R_3$$

Resistance between terminal (3) & (1)

$$= R_3 + R_1$$

Resistance between terminals (1) & (2) *in delta equal to resistance between terminals* (1) & (2) *in star*

$$R_1 + R_2 = \frac{R_{12}(R_{23} + R_{31})}{R_{12} + R_{23} + R_{31}} \qquad (i)$$

Similarly,

$$R_2 + R_3 = \frac{R_{23}(R_{12} + R_{31})}{R_{12} + R_{23} + R_{31}} \qquad (ii)$$

$$R_3 + R_1 = \frac{R_{31}(R_{12}+R_{23})}{R_{12}+R_{23}+R_{31}} \qquad (iii)$$

(a) Delta to star conversion

Simplify $(i)+(ii)-(iii)$ *on both the side of equations*

$$R_1+R_2+R_2+R_3-R_3-R_1 = \frac{R_{12}(R_{23}+R_{31})}{R_{12}+R_{23}+R_{31}} + \frac{R_{23}(R_{12}+R_{31})}{R_{12}+R_{23}+R_{31}} - \frac{R_{31}(R_{12}+R_{23})}{R_{12}+R_{23}+R_{31}}$$

$$= \frac{(R_{12}R_{23}+R_{12}R_{31})}{R_{12}+R_{23}+R_{31}} + \frac{(R_{23}R_{12}+R_{23}R_{31})}{R_{12}+R_{23}+R_{31}} - \frac{(R_{31}R_{12}+R_{31}R_{23})}{R_{12}+R_{23}+R_{31}}$$

$$= \frac{(R_{12}R_{23}+R_{12}R_{31}+R_{23}R_{12}+R_{23}R_{31}-R_{31}R_{12}-R_{31}R_{23})}{(R_{12}+R_{23}+R_{31})}$$

$$2R_2 = \frac{2R_{12}R_{23}}{R_{12}+R_{23}+R_{31}}$$

$$R_2 = \frac{R_{12}R_{23}}{R_{12}+R_{23}+R_{31}}$$

Similarly , $R_1 = \dfrac{R_{12}R_{31}}{R_{12}+R_{23}+R_{31}}$

$$R_3 = \frac{R_{23}R_{31}}{R_{12}+R_{23}+R_{31}}$$

(b) Star to delta conversion

Simplify $(i)(ii)+(ii)(iii)+(iii)(i)$ *on both the side of equation*

$$(R_1+R_2)(R_2+R_3)+(R_2+R_3)(R_3+R_1)+(R_3+R_1)(R_1+R_2)$$

$$= \left(\frac{R_{12}(R_{23}+R_{31})}{R_{12}+R_{23}+R_{31}}\right)\left(\frac{R_{23}(R_{12}+R_{31})}{R_{12}+R_{23}+R_{31}}\right) + \left(\frac{R_{23}(R_{12}+R_{31})}{R_{12}+R_{23}+R_{31}}\right)\left(\frac{R_{31}(R_{12}+R_{23})}{R_{12}+R_{23}+R_{31}}\right) + \left(\frac{R_{31}(R_{12}+R_{23})}{R_{12}+R_{23}+R_{31}}\right)\left(\frac{R_{12}(R_{23}+R_{31})}{R_{12}+R_{23}+R_{31}}\right)$$

$$R_1R_2+R_1R_3+R_2^2+R_2R_3+R_2R_3+R_2R_1+R_3^2+R_3R_1+R_3R_1+R_3R_2+R_1^2+R_1R_2$$

$$= \left(\frac{R_{12}R_{23}+R_{12}R_{31}}{R_{12}+R_{23}+R_{31}}\right)\left(\frac{R_{23}R_{12}+R_{23}R_{31}}{R_{12}+R_{23}+R_{31}}\right) + \left(\frac{R_{23}R_{12}+R_{23}R_{31}}{R_{12}+R_{23}+R_{31}}\right)\left(\frac{R_{31}R_{12}+R_{31}R_{23}}{R_{12}+R_{23}+R_{31}}\right) + \left(\frac{R_{31}R_{12}+R_{31}R_{23}}{R_{12}+R_{23}+R_{31}}\right)\left(\frac{R_{12}R_{23}+R_{12}R_{31}}{R_{12}+R_{23}+R_{31}}\right)$$

$$3R_1R_2+3R_2R_3+3R_3R_1+R_1^2+R_2^2+R_3^2$$

$$= \left(\frac{R_{23}^2R_{12}^2+R_{12}R_{23}^2R_{31}+R_{12}^2R_{23}R_{31}+R_{12}R_{23}R_{31}^2}{(R_{12}+R_{23}+R_{31})^2}\right) + \left(\frac{R_{12}^2R_{23}R_{31}+R_{12}R_{23}^2R_{31}+R_{12}R_{23}R_{31}^2+R_{23}^2R_{31}^2}{(R_{12}+R_{23}+R_{31})^2}\right) + \left(\frac{R_{12}^2R_{23}R_{31}+R_{12}^2R_{31}^2+R_{12}R_{23}^2R_{31}+R_{12}R_{23}R_{31}^2}{(R_{12}+R_{23}+R_{31})^2}\right)$$

$$= \frac{R_{23}^2R_{12}^2+R_{12}R_{23}^2R_{31}+R_{12}^2R_{23}R_{31}+R_{12}R_{23}R_{31}^2+R_{12}^2R_{23}R_{31}+R_{12}R_{23}^2R_{31}+R_{12}R_{23}R_{31}^2+R_{23}^2R_{31}^2+R_{12}^2R_{23}R_{31}+R_{12}^2R_{31}^2+R_{12}R_{23}^2R_{31}+R_{12}R_{23}R_{31}^2}{(R_{12}+R_{23}+R_{31})^2}$$

$$= \frac{(R_{12}R_{23}^2R_{31}+R_{12}^2R_{23}R_{31}+R_{12}R_{23}R_{31}^2+R_{12}^2R_{23}R_{31}+R_{12}R_{23}^2R_{31}+R_{12}R_{23}R_{31}^2+R_{12}^2R_{23}R_{31}+R_{12}R_{23}^2R_{31}+R_{12}R_{23}R_{31}^2)+(R_{23}^2R_{12}^2+R_{23}^2R_{31}^2+R_{12}^2R_{31}^2)}{(R_{12}+R_{23}+R_{31})^2}$$

$$= \frac{R_{12}R_{23}R_{31}(R_{23}+R_{12}+R_{31}+R_{12}+R_{23}+R_{31}+R_{12}+R_{23}+R_{31})}{(R_{12}+R_{23}+R_{31})^2} + \frac{(R_{23}^2R_{12}^2+R_{23}^2R_{31}^2+R_{12}^2R_{31}^2)}{(R_{12}+R_{23}+R_{31})^2}$$

$$= \frac{R_{12}R_{23}R_{31}(3R_{12}+3R_{23}+3R_{31})}{(R_{12}+R_{23}+R_{31})^2} + \left(\frac{R_{23}^2R_{12}^2}{(R_{12}+R_{23}+R_{31})^2} + \frac{R_{23}^2R_{31}^2}{(R_{12}+R_{23}+R_{31})^2} + \frac{R_{12}^2R_{31}^2}{(R_{12}+R_{23}+R_{31})^2}\right)$$

$$= \frac{3R_{12}R_{23}R_{31}(R_{12}+R_{23}+R_{31})}{(R_{12}+R_{23}+R_{31})^2} + \left(\frac{R_{23}^2R_{12}^2}{(R_{12}+R_{23}+R_{31})^2} + \frac{R_{23}^2R_{31}^2}{(R_{12}+R_{23}+R_{31})^2} + \frac{R_{12}^2R_{31}^2}{(R_{12}+R_{23}+R_{31})^2}\right)$$

$$= 3R_3R_{12}+R_2^2+R_3^2+R_1^2$$

Now equation become

$$3R_1R_2+3R_2R_3+3R_3R_1+R_1^2+R_2^2+R_3^2 = 3R_3R_{12}+R_2^2+R_3^2+R_1^2$$

$$3R_1R_2+3R_2R_3+3R_3R_1 = 3R_3R_{12}$$

$R_{12} = R_1 + R_2 + \frac{R_1 R_2}{R_3}$

Similarly

$R_{23} = R_2 + R_3 + \frac{R_2 R_3}{R_1}$

$R_{31} = R_3 + R_1 + \frac{R_3 R_1}{R_2}$

1.12. Explain Node analysis

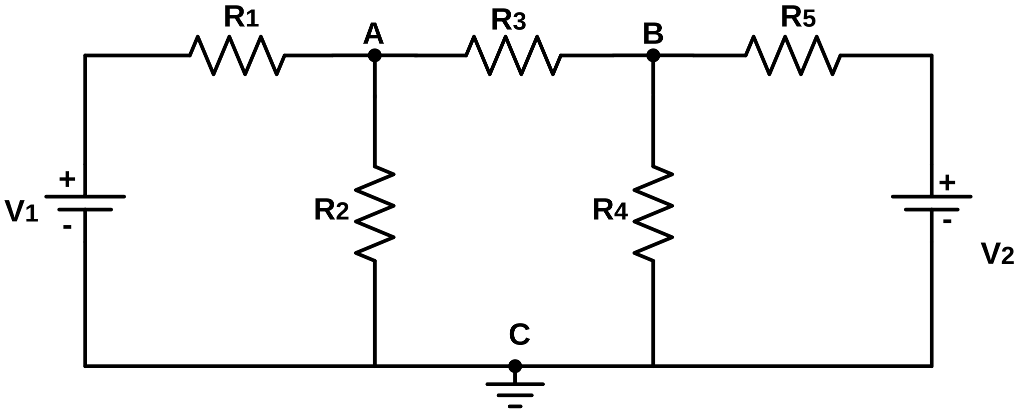


Figure 1.25Node analysis network

- **Node**: Node refers to any point on circuit where two or more circuit elements meet.
- Node analysis based on Kirchhoff's current law states that algebraic summation of currents meeting at junction is zero.
- Node C is taken as reference node in this network. If there are n nodes in any network, the number of equation to be solved will be (n-1).
- Node A,B and C are shown in given network and their voltages areV_A,V_B andV_C . Value of node V_C is zero because V_C is reference node.
- Steps to follow in node analysis:
  - ✓ Consider node in the network, assign current and voltage for each branch and node respectively.
  - ✓ Apply the KCL for each node and apply ohm's law to branch current.
  - ✓ Solve the equation for find the unknown node voltage.
  - ✓ Using these voltages, find the required branch currents.
- **Node A**

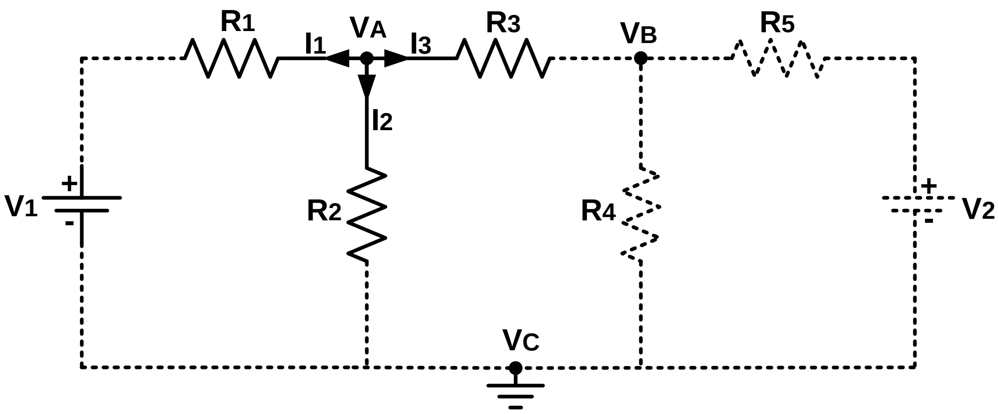


Figure 1.26Node analysis network for node A

Apply KCL at node A,

$$(-I_1)+(-I_2)+(-I_3)=0$$

$$I_1+I_2+I_3=0$$

$$\frac{V_A - V_1}{R_1}+\frac{V_A - V_C}{R_2}+\frac{V_A - V_B}{R_3}=0$$

$$V_A\left[\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}\right]+V_B\left[-\frac{1}{R_3}\right]=\frac{V_1}{R_1} \qquad (i)$$

- **Node B**

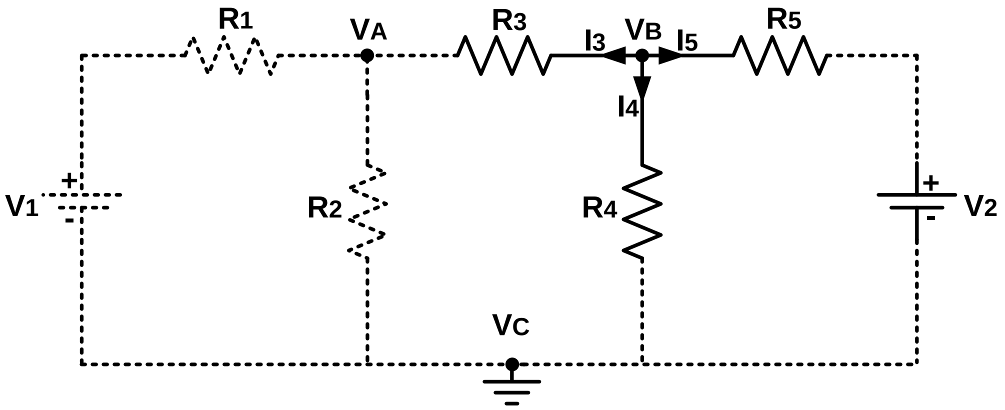


Figure 1.27Node analysis network for node B

Apply the KCL at node B,

$$(-I_3)+(-I_4)+(-I_5)=0$$

$$I_3+I_4+I_5=0$$

$$\frac{V_B - V_A}{R_3}+\frac{V_B - V_C}{R_4}+\frac{V_B - V_2}{R_5}=0$$

$$V_A\left[-\frac{1}{R_3}\right]+V_B\left[\frac{1}{R_3}+\frac{1}{R_4}+\frac{1}{R_5}\right]=\frac{V_2}{R_5} \qquad (ii)$$

From, equation (i) & (ii)

$$\begin{pmatrix} \frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3} & -\frac{1}{R_3} \\ -\frac{1}{R_3} & \frac{1}{R_3}+\frac{1}{R_4}+\frac{1}{R_5} \end{pmatrix} \begin{pmatrix} V_A \\ V_B \end{pmatrix} = \begin{pmatrix} \frac{V_1}{R_1} \\ \frac{V_2}{R_5} \end{pmatrix}$$

- One can easily find branch current of this network by solving equation (i) and (ii),if V_1 , V_2 and all resistance value are given.

1.13. Explain Mesh analysis

- **Mesh:** It is defined as a loop which does not contain any other loops within it.
- The current in different meshes are assigned continues path that they do not split at a junction into a branch currents.
- Basically, this analysis consists of writing mesh equation by Kirchhoff's voltage law in terms of unknown mesh current.

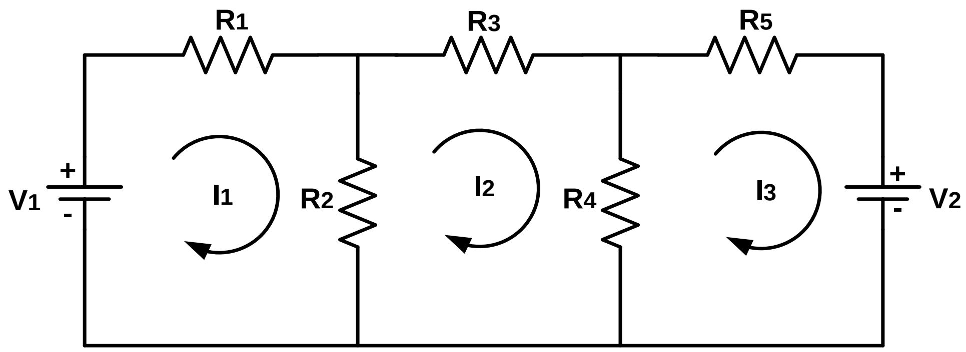


Figure 1.28Mesh analysis network

- Steps to be followed in mesh analysis:
  - ✓ Identify the mesh, assign a direction to it and assign an unknown current in it.
  - ✓ Assigned polarity for voltage across the branches.
  - ✓ Apply the KVL around the mesh and use ohm's law to express the branch voltage in term of unknown mesh current and resistance.
  - ✓ Solve the equations for unknown mesh current.
- **Loop 1**

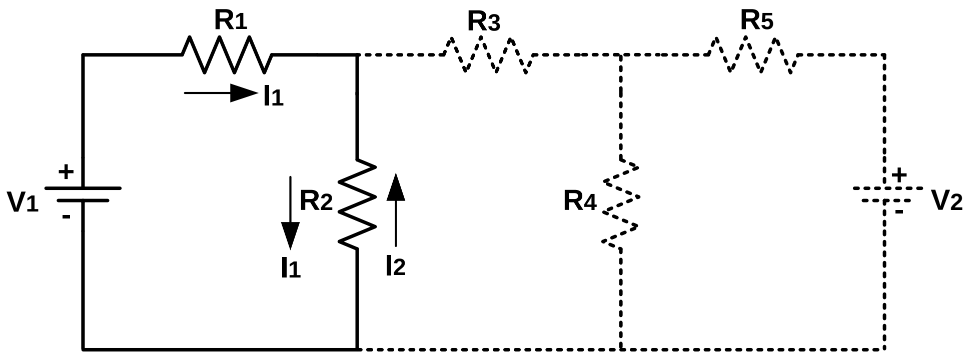


Figure 1.29Mesh analysis network for loop-1

Now apply the KVL in loop -1,

$$-I_1R_1 - (I_1 - I_2)R_2 + V_1 = 0$$

$$-I_1R_1 - I_1R_2 + I_2R_2 + V_1 = 0$$

$$-(R_1 + R_2)I_1 + R_2I_2 = -V_1 \qquad \text{(i)}$$

- **Loop 2**

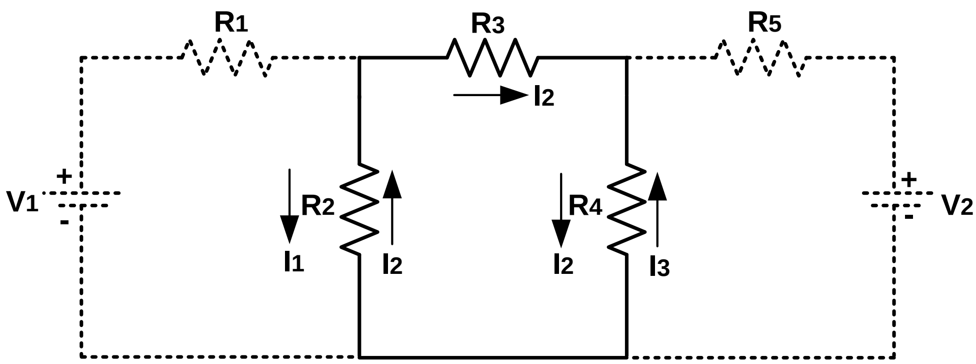


Figure 1.30Mesh analysis network for loop-2

$$\textit{Now Apply the KVL loop}-2,$$
$$-I_2R_3-(I_2-I_3)R_4-(I_2-I_1)R_2=0$$
$$-I_2R_3-I_2R_4+I_3R_4-I_2R_2+I_1R_2=0$$
$$I_1R_2-I_2(R_3+R_4+R_2)+I_3R_4=0$$
$$R_2I_1-(R_3+R_4+R_2)I_2+R_4I_3=0 \qquad \text{(ii)}$$

- **Loop 3**

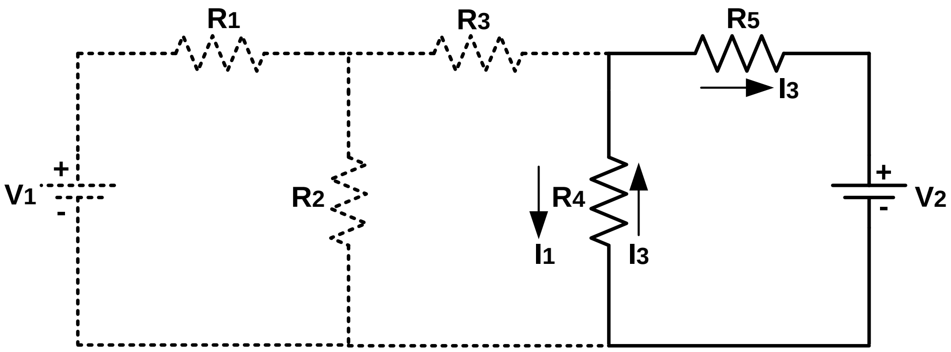


Figure 1.31Mesh analysis network for loop-3

$$\textit{Now Apply the KVL loop}-3,$$
$$-I_3R_5-V_2-(I_3-I_2)R_4=0$$
$$-I_3R_5-V_2-I_3R_4+I_2R_4=0$$
$$I_2R_4-I_3(R_5+R_4)=V_2$$
$$R_4I_2-(R_5+R_4)I_3=V_2 \qquad \text{(iii)}$$

$$\textit{From equation } (i),(ii)\ \&(iii)$$

$$\begin{pmatrix} -(R_1+R_2) & R_2 & 0 \\ R_2 & (R_3+R_4+R_2) & R_4 \\ 0 & R_4 & -(R_5+R_4) \end{pmatrix} \begin{pmatrix} I_1 \\ I_2 \\ I_3 \end{pmatrix} = \begin{pmatrix} -V_1 \\ 0 \\ V_2 \end{pmatrix}$$

$$\Delta = \begin{pmatrix} -(R_1+R_2) & R_2 & 0 \\ R_2 & (R_3+R_4+R_2) & R_4 \\ 0 & R_4 & -(R_5+R_4) \end{pmatrix}$$

$$\Delta_1 = \begin{pmatrix} -V_1 & R_2 & 0 \\ 0 & (R_3+R_4+R_2) & R_4 \\ V_2 & R_4 & -(R_5+R_4) \end{pmatrix}$$

$$\Delta_2 = \begin{pmatrix} -(R_1+R_2) & -V_1 & 0 \\ R_2 & 0 & R_4 \\ 0 & V_2 & -(R_5+R_4) \end{pmatrix}$$

$$\Delta_3 = \begin{pmatrix} -(R_1+R_2) & R_2 & -V_1 \\ R_2 & (R_3+R_4+R_2) & 0 \\ 0 & R_4 & V_2 \end{pmatrix}$$

$$\textit{Now},$$

$$I_1=\frac{\Delta_1}{\Delta},\ I_2=\frac{\Delta_2}{\Delta},\ I_3=\frac{\Delta_3}{\Delta}$$