1. AC Fundamentals

Caution: In these experiments high voltages are involved, which could be harmful or fatal if one is exposed to them. Adhere to electrical safety rule at all times. Make sure that all of your connections are correct before turning on the power.

In 1831 Michael Faraday discovered the fundamental relationship between the voltage and magnetic flux in a circuit. This relationship is now known as Faraday's law of electromagnetic induction, which states:

If the flux linking a conductor forming a loop changes with time, a voltage is induced at its terminal. The magnitude of the induced voltage is proportional to the rate of change in the magnetic flux.

Mathematically the law is stated as follows:

$$E = N\frac{d\emptyset}{dt} \qquad (1)$$

where,

E = induced voltage (in volts)

N = number of turns in the conductor

Φ = Flux linking the conductor

Note that the induced voltage E depends on the rate of change of flux linking the conductor. The flux linkage can vary either by a magnetic field that is changing with time (due to AC current) or by moving the conductor in a stationary magnetic field. Fig. 1 shows a conductor being moved in a magnetic field. When the conductor is moved horizontally (right to left) a voltage is induced in the conductor due to the changing flux linkage. If the conductor forms a closed loop, the induced voltage would cause a current to flow though the conductor.
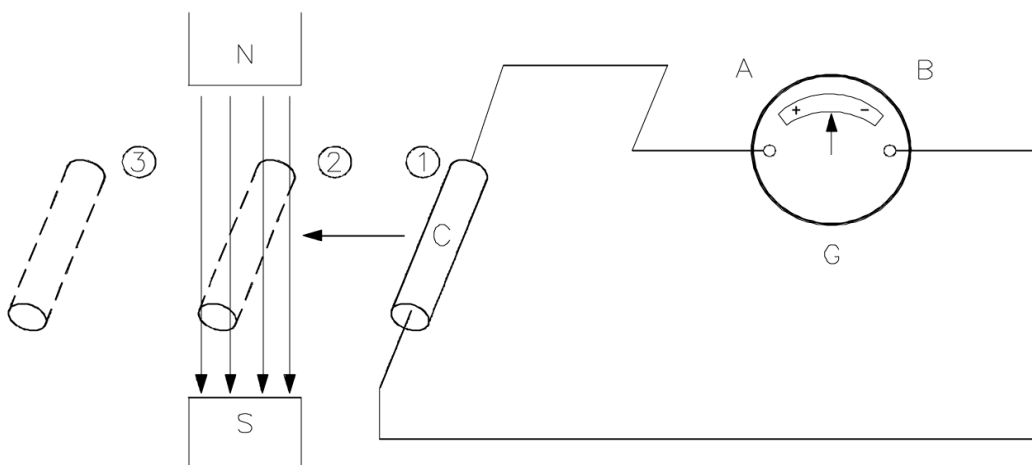

Figure 1: A conductor moving in a magnetic field*.

Note that the polarity of the induced voltage and the direction of the current through the conductor would reverse when the conductor is moved back from left to right. Furthermore, no voltage would be induced if the conductor in Fig. 1 is moved vertically because the flux through the conductor would not change in the vertical movement. Alternatively, the induced voltage can be expressed as:

$$E = Blv \qquad (2)$$

where,

$B = \frac{\emptyset}{A}$ is the flux density (in Weber/meter^2: Wb/m^2 or Telsa: T)

l = active length of the conductor in the magnetic field (m)

v = relative speed of the conductor (m/s)

A current carrying conductor, when placed in a magnetic field, experiences a force known as electromagnetic force (also called Lorentz force). This force is the basis of the operation of the electrical motors and generators. The maximum force is exerted on the current carrying conductor when it is perpendicular to the magnetic field and it is zero when the conductor is parallel to the field. Mathematically, electromagnetic force can be expressed as:

$$F = BlI \qquad (3)$$

where,

F = force exerted on the conductor (in Newton: N)

B = flux density (T)

l = active length of the conductor (m)

I = current carried by the conductor (A)

Single Phase AC Generator

There are two kinds of sources of electrical power:

(1) Direct current or voltage source (DC source) in which the current and voltage remains constant over time.
(2) Alternating current or voltage source (AC source) in which the current or voltage constantly changes with time. The voltage of the electrical power source that we use in our homes or offices (line voltage) is a sinusoidal signal that goes through a complete cycle 60 times in one second. In this section we will discuss how single phase AC voltage is generated.

Figure 2 shows a conductor placed in a magnetic field. A voltage is induced between its terminals (x, y) due to the change in flux linkage when the conductor is rotated in the magnetic field. The change in flux linkage is at its minimum when the conductor is moving parallel to the field and it is at its maximum when the conductor is moving perpendicular to the magnetic field. In a half

rotation, the conductor moves from being parallel to the field to being perpendicular to the field and eventually moving back to being parallel to the field. Accordingly, the induced voltage increases from zero to its maximum value and then back to zero at the end of the half rotation.
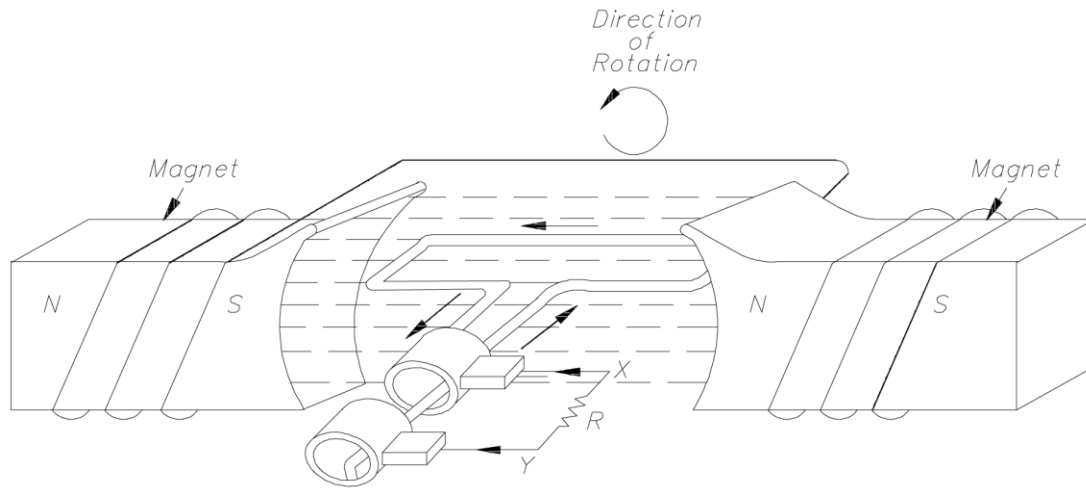

Figure 2: A rotating conductor in a magnetic field*.

Fig. 3 shows the changes in the induced voltage as the conductor rotates in the magnetic field. During the second half of the rotation the flux linkage changes through the conductor the same way as before; however, it induces the voltage with the opposite polarity because the position of the conductors is now reversed. Due to the shape of the poles and the rotary motion of the conductor the induced voltage turns out to be a sine wave.
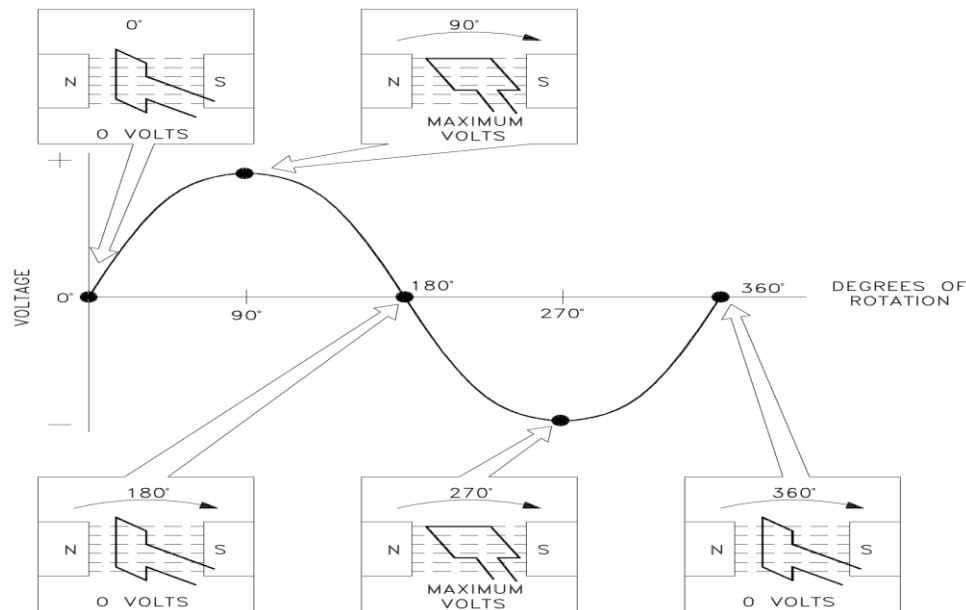

Figure 3: Induced voltage vs. rotation of the conductor*.

The frequency of the induced voltage depends on the number of rotations completed in one second as well as the number of poles in the generator. For the two-pole generator shown in Fig 2, a rotational speed of 60 revolutions per second (3600 rpm) would result in an induced voltage with a frequency of 60 Hz. The induced voltage goes through one peak to another peak in the opposite direction as the conductor is swept through the North (N) and the South (S) poles, respectively. For the two-pole generator in Fig 2, the frequency of the induced voltage is the same as the rotational speed of the generator. In general, the frequency of the induced voltage is given by

$$f = \frac{n \times p}{120} \qquad (4)$$

where,

f = frequency of the induced voltage (Hz)

n = rotational speed of the conductor (rpm)

p = number of poles in the generator.

Example: *A single phase generator has eight poles and it generates voltage at 60 Hz. What is the speed of the generator (in rpm)?*

Solution: Use the Eq. (4) to calculate the speed of the generator.

Frequency of the induced voltage = f = 60 Hz

Number of poles = p = 8

Generator speed = n = $\dfrac{f \times 120}{p} = \dfrac{60 \times 120}{8}$ = 900 rpm

Instantaneous and Effective Values of AC Voltage and Current

Figure41 show a graphical representation of AC voltage. The time taken to complete one cycle is called "time period," or simply "period" of the signal ("T' in Fig. 4). The number of cycles completed in one second is called "frequency." The unit of frequency is cycles/second or Hertz (Hz). The line voltage has a frequency of 60 Hz. The frequency "f" and the period "T" of a signal are related by the following equation:

$$f = \frac{1}{T} \qquad (5)$$

The frequency of a signal is also sometimes described in terms of the angular frequency "ω" (in radians/second). The frequency "f" of a signal and its angular frequency "ω" are related by the following equation:

$$\omega = 2\pi f \qquad (6)$$

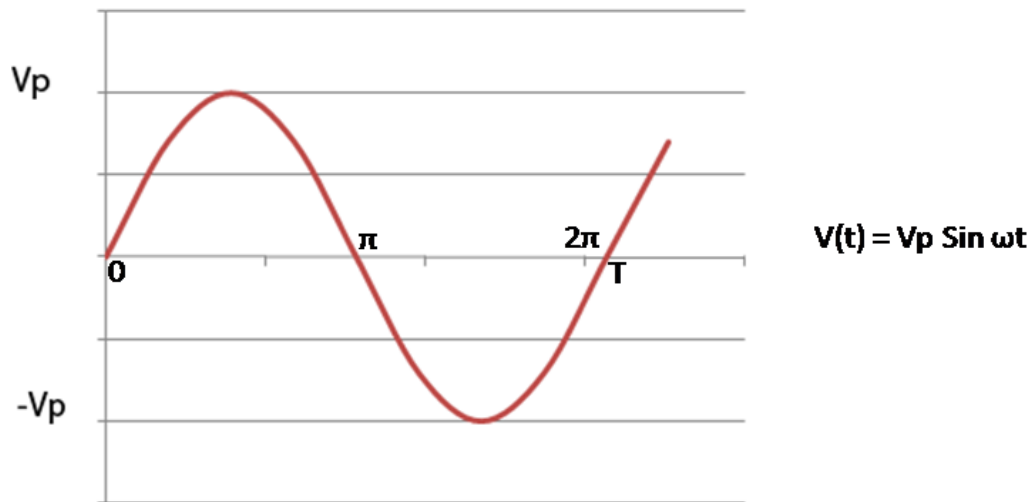Figure 4: AC voltage.

The AC voltage, V(t), is mathematically described as:

$$V(t) = Vp\, Sin\, \omega t$$

However, a more meaningful measure of AC quantities is their "root-mean-square" (RMS) value.

Assignment 1.1: Show that:

$$AVE[V(t)] = \frac{1}{T}\int_0^T Vp\, Sin\, \omega t = 0$$

And

$$RMS[V(t)] = \sqrt{\frac{1}{T}\int_0^T V_p^2 sin^2\omega t} = \frac{Vp}{\sqrt{2}}$$

Output Power of a Single Phase Generator

Figure 5 shows the instantaneous voltage, current, and power (which is a product of instantaneous voltage and current) of a single phase generator. In Fig. 5 the voltage and current are in phase and, therefore, the instantaneous power is positive throughout the whole cycle. Note that the output power of a single phase generator is not constant. Instead, it is pulsating at a frequency that is twice the frequency of the induced voltage. However, most of the mechanical energy sources that are used to create the rotary motion of the conductors (or the magnetic field) in an AC generator provide a constant mechanical power. This imbalance in the constant mechanical power at the input and the pulsating electrical power at the output of the generator results in machine vibrations and makes single phase generator undesirable.
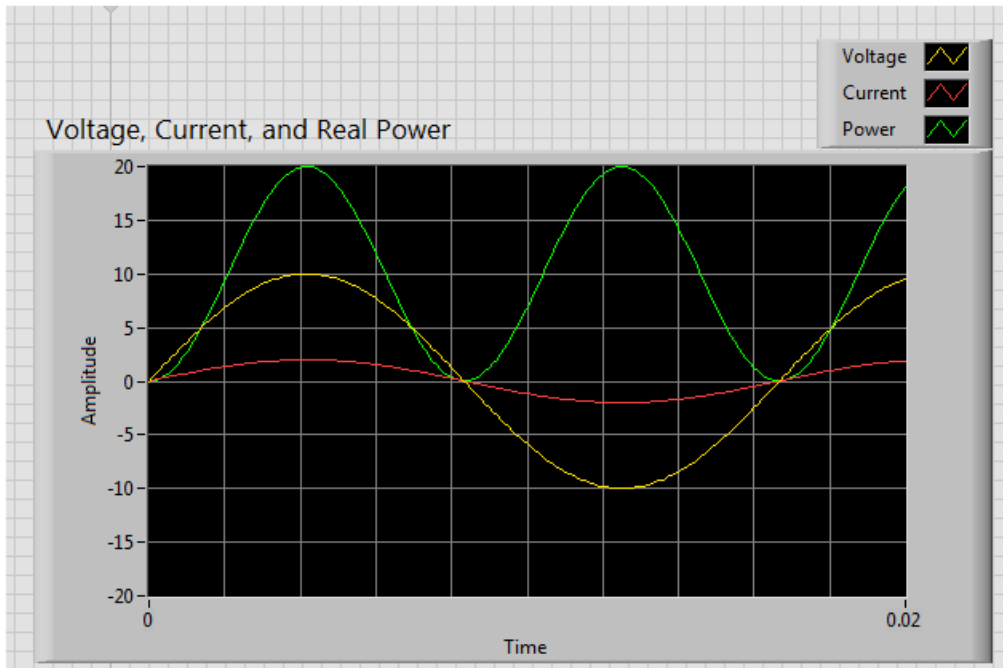
Figure 5: Instantaneous voltage, current, and power of a single phase AC generator.

Assignment 1.2: Study of RLC circuits with AC voltage source.

Components needed:

Resistor 10 Ω (2)

Capacitor 35 µF (1), 20 µF (1)

Inductor 39 mH (2), 10 mH (5), 100 mH (3)

Note: Measure the voltage by connecting the Voltmeter in parallel and measure the current by connecting the Ammeter in the series.

A. Parallel RLC Circuit

1. Build the circuit in Fig. 6 with $Z_L = 78$ mH (2x39), $Z_R = 20$ Ω, and $Z_C = 55$ µF.

2. Use a 30V AC source and measure the currents I, I_L, I_R, and I_C. Apply KCL to the circuit ($I = I_L + I_R + I_C$). Explain your results.

3. Increase Z_L to128mHin 10mH increments and record all the currents.

4. Increase Z_L to428 mH in 100 mH increments and record all the currents.

5. Plot I, I_L, I_R, and I_C vs Z_L. Note down the value of Z_L where I and I_R are closest to each other. What do you observe about I_L and I_C at that value of Z_L? Explain your results.
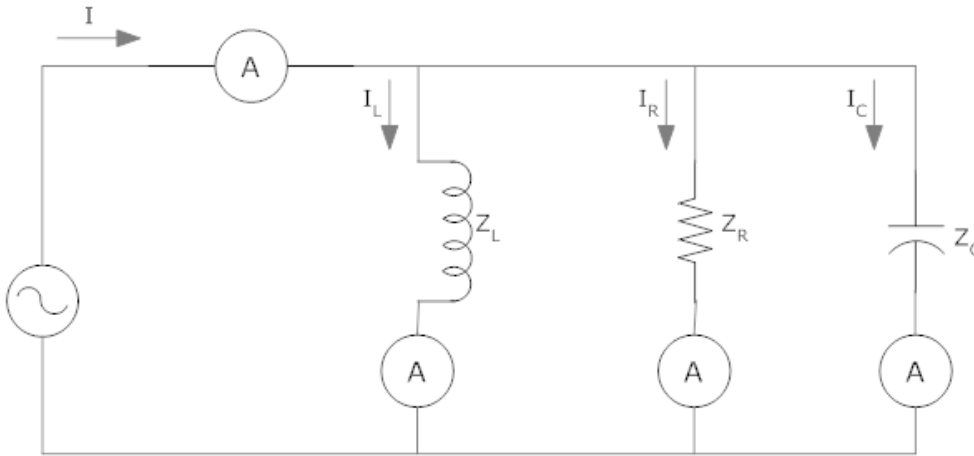

Figure 6: Parallel RLC circuit.

B. Series RLC Circuit

6. Build the circuit in Fig. 7 with Z_L = 78 mH (2x39), Z_{R1} = Z_{R2} = 75 Ω, and Z_C = 55 μF

7. Use a 30 V AC source and measure the voltages V_{AB}, V_{BC}, and V_{AC}. Apply KVL to the circuit ($V_{AC} = V_{AB} + V_{BC}$). Explain your results.

8. Increase Z_L to 128 mH in 10 mH increments and record all the voltages V_{AB}, V_{BC}, and V_{AC}.

9. Increase Z_L to 428 mH in 100mH increments and record all the voltages V_{AB}, V_{BC}, and V_{AC}.

10. Plot V_{AB}, V_{BC}, and V_{AC} vs. Z_L. Note down the value of Z_L where V_{AB} and V_{BC} are equal (or very close). Explain your results.
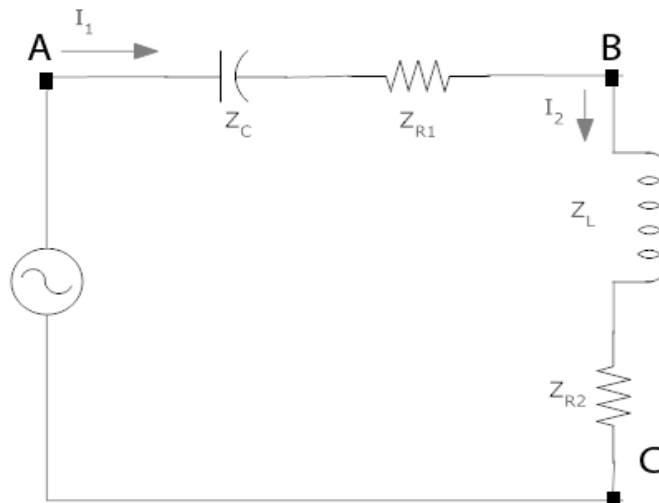

Figure 7: Series RLC Circuit.

2. Impedance and Phase

AC quantities are identified with two parameters: magnitude and phase (angle). Accordingly, AC quantities such voltage, current, power, etc. are expressed as complex numbers such as

$$A + jB \tag{7}$$

Where, A and B are called the real and reactive components of that quantity. This form of a complex number is called "rectangular" form. Alternatively, each quantity can also be expressed with the magnitude and associated phase (angle in electrical degrees). That is, for $A + jB$

$$Magnitude = |M| = \sqrt{A^2 + B^2} \tag{8}$$

$$Phase = \emptyset = \tan^{-1}\left(\frac{B}{A}\right) \tag{9}$$

When the magnitude and the phase of an electrical quantity are known, it is expressed in the "polar" form of a complex number as follows:

$$|M|\langle\phi = \sqrt{A^2 + B^2} \ \langle\tan^{-1}\left(\frac{B}{A}\right) \tag{10}$$

If the magnitude |M| and the phase angle Φ of an AC quantity are known, the real and reactive components of the AC quantity (A and B in Eq. 7) can be computed as follows:

$$A = |M| \times \cos\phi \tag{11}$$
$$B = |M| \times \sin\phi \tag{12}$$

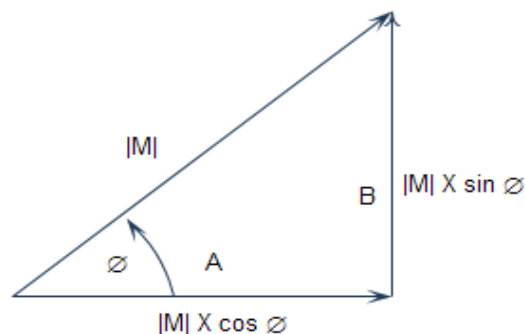Figure 6 shows the relationship between the polar and rectangular form of AC quantities.

Figure 6: Relationship between the polar and rectangular AC quantities.

Rules for manipulating complex numbers:

1. Addition or subtraction
    a. Use rectangular form
    b. The real part of one complex number can only be added to (or subtracted from) the real part of another complex number.
    c. The imaginary (reactive) part of a complex number can only be added to (or subtracted from) the imaginary (reactive) part of another complex number.

2. Multiplication or division
    a. Use polar form
    b. Multiplication: The magnitudes of the complex numbers are multiplied and their corresponding angles are added together.
    c. Division: The magnitude of the complex number in the numerator is divided by the magnitude of the complex number in the denominator. The angle of the complex number in the denominator is subtracted from the angle of the complex number in the numerator.

AC Voltage-Current Relationships for Resistance, Inductance, and Capacitance

If an AC voltage $V(t) = Vp\ Sin\ \omega t$ is applied across a resistor R, the current I_R though the resistor is given by

$$I_R = \frac{V_P}{R} Sin\ \omega t. \tag{13}$$

Note the current though a resistor is *in-phase* with the applied voltage.

If an AC voltage $V(t) = Vp\ Sin\ \omega t$ is applied across an inductor of inductance L, the current I_L though the inductor is given by

$$I_L = \frac{1}{L}\int V(t)dt \tag{14}$$

or

$$I_L = \frac{V_p}{L}\int Sin\ \omega t\ dt. \tag{15}$$

After integrating the R.H.S. we get

$$I_L = \frac{V_p}{\omega L}\cos \omega t = \frac{V_p}{\omega L}\sin\left(\omega t - 90^0\right). \tag{16}$$

Note the current through an inductor is *out-of-phase* with the applied voltage and it lags the applied voltage by 90^0. The figure shows the voltage-current relationship of an inductor. The quantity "ωL" is called the impedance of the inductor. We will represent the impedance of an inductor by X_l. Accordingly,

$$X_l = \omega L \tag{17}$$

and, therefore,

$$I_L = \frac{V_p}{X_l} \sin\left(\omega t - 90^0\right). \tag{18}$$
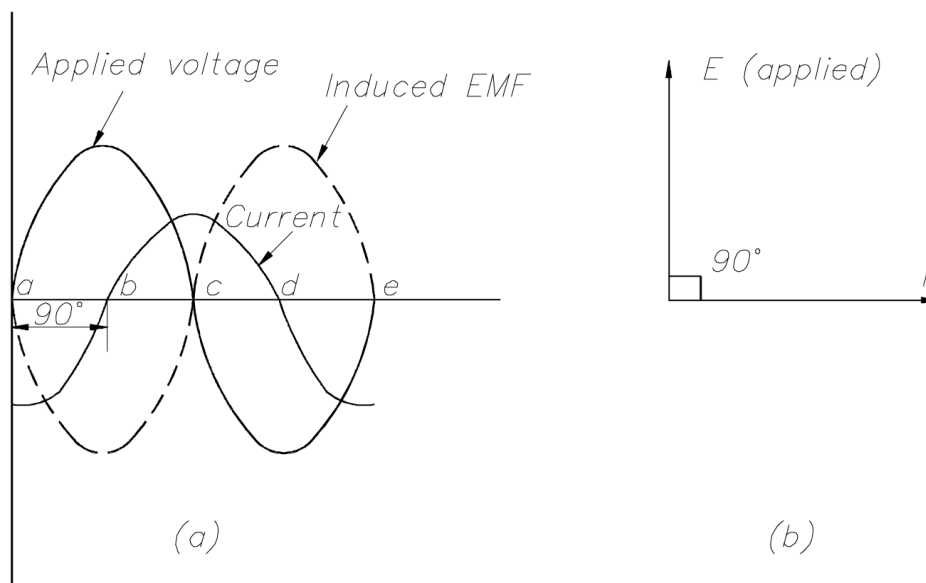

Figure 7: (a) Instantaneous voltage and current for an inductor. (b) Phasor diagram*.

If an AC voltage $V(t) = Vp\, Sin\, \omega t$ is applied across a capacitor of capacitance C farads, the current I_C though the capacitor is given by

$$I_C = C \frac{dV(t)}{dt} \tag{19}$$

or

$$I_C = C \, Vp \frac{d}{dt} Sin \, \omega t. \tag{20}$$

After differentiating the R.H.S. we get

$$I_C = -\omega C \times Vp \times \cos \omega t = \omega C \times Vp \, \sin\left(\omega t + 90^0\right). \tag{21}$$

Note the current though a capacitor is *out-of-phase* with the applied voltage and it leads the applied voltage by 90^0. Figure 8 shows the voltage-current relationship of a capacitor. The quantity "$\frac{1}{\omega C}$" is called the impedance of the capacitor. We will represent the impedance of a capacitor by X_C. Accordingly,

$$X_C = \frac{1}{\omega C} \tag{22}$$

and, therefore,

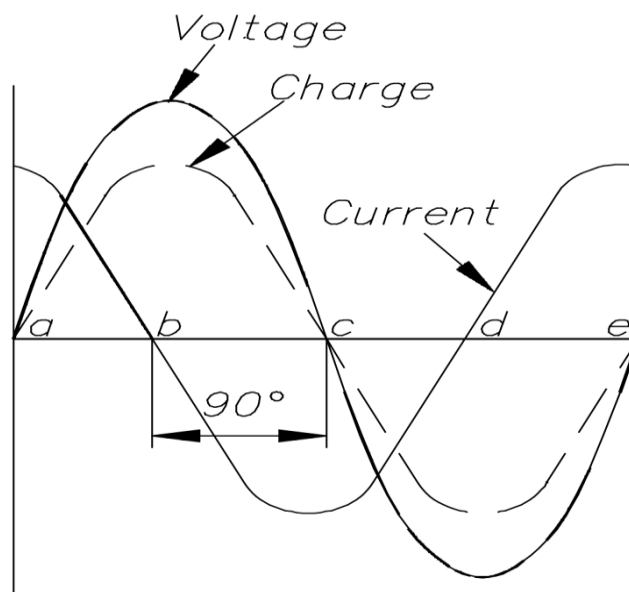$$I_C = \frac{V_p}{X_C} \sin\left(\omega t + 90^0\right). \tag{23}$$

Figure 8: Instantaneous voltage and current for a capacitor*.

Impedance:

In AC analysis, resistance, inductance, and capacitance are represented as part of the "generalized impedance" (or simply impedance) in the circuit and are expressed as complex numbers. The resistance constitutes the real part of the impedance and the inductance or capacitance constitutes the reactive (imaginary) part of the impedance. The unit for impedance is the ohm (Ω). The impedance Z can be expressed either in the rectangular or polar form just like any other AC quantity (voltage, current, power etc.). The impedance in rectangular form can be expressed as:

$$Z = R \pm jX$$

The reactive part is positive (+j) if the resultant reactance is inductive in nature and it is negative (-j) if the resultant reactance is capacitive in nature. Alternatively,

$$Z = R + jX_l$$

or

$$Z = R - jX_C$$

The magnitude of the inductive and capacitive reactance depends on the frequency of the signal they are processing. The angle of inductive reactance is +90 degrees and that of the capacitive reactance is -90 degrees. The inductive reactance can be calculated using the following formula:

$$Inductive\ Reactance = j\omega L$$

Where "ω" is the angular frequency of the AC signal ($\omega = 2\pi f$) and "L" is the inductance of the inductor expressed in Henry. The capacitive reactance can be calculated using the following formula:

$$Capacitive\ Reactance = \frac{1}{j\omega C} = -j\frac{1}{\omega C}$$

Where "C" is the capacitance of the capacitor expressed in Farad. Note that all the circuit analysis laws and techniques used for resistive networks with DC sources are applicable to AC analysis with the exception that the resistance is replaced by impedance and the analysis involves complex numbers.

Assignment 2.1: For the circuits in Figs.9 and 10, compute the impedance Z_L for L = 128 mH and Z_C for C= 55 µF for a line frequency of 60 Hz.

1. Compute the currents I, I_L, I_R, and I_C in Fig. 1 (Use Z_R = 20 Ω and a source voltage of 30V with angle = 0°). Apply KCL to the circuit (I = I_L + I_R + I_C) and draw the phasor diagram for the currents using I as the reference). Explain your results.

2. Compute the voltages V_{AB}, V_{BC}, and V_{AC} (Use Z_{R1} =Z_{R2} = 10 Ω and a source voltage of 30 V with angle = 0°). Apply KVL to the circuit (V_{AC} = V_{AB} + V_{BC}) and draw the phasor diagram for voltages using V_{AC} as the reference. Explain your results.
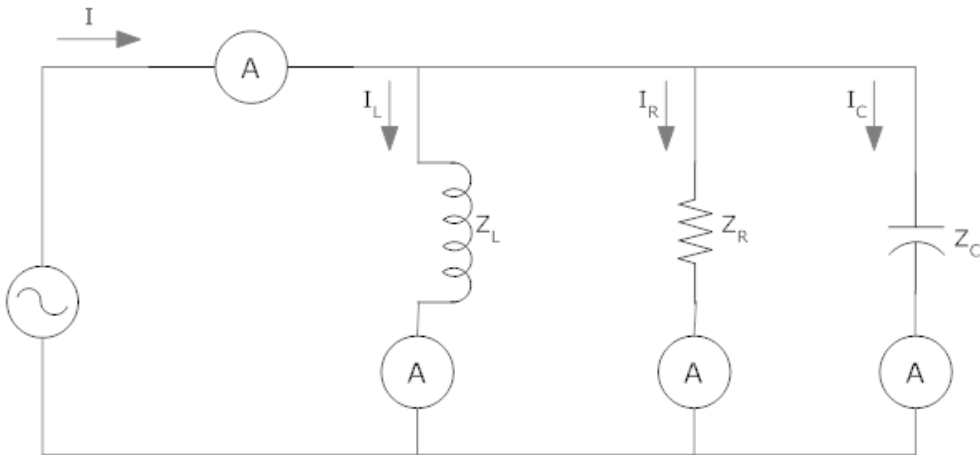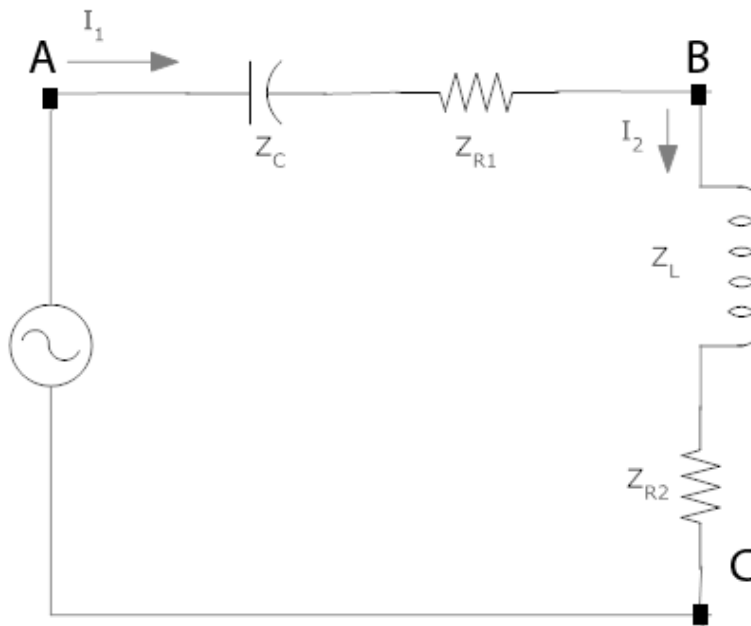
Figure 9: Parallel RLC circuit.

Figure 10: Series RLC circuit.

Assignment 2.2:

1. For the circuit in Fig. 11 compute I, V_L, V_C, and V_2. Assume the source voltage has an angle = 0°. Draw the phasor diagram and Apply KVL to the circuit. Explain your results.
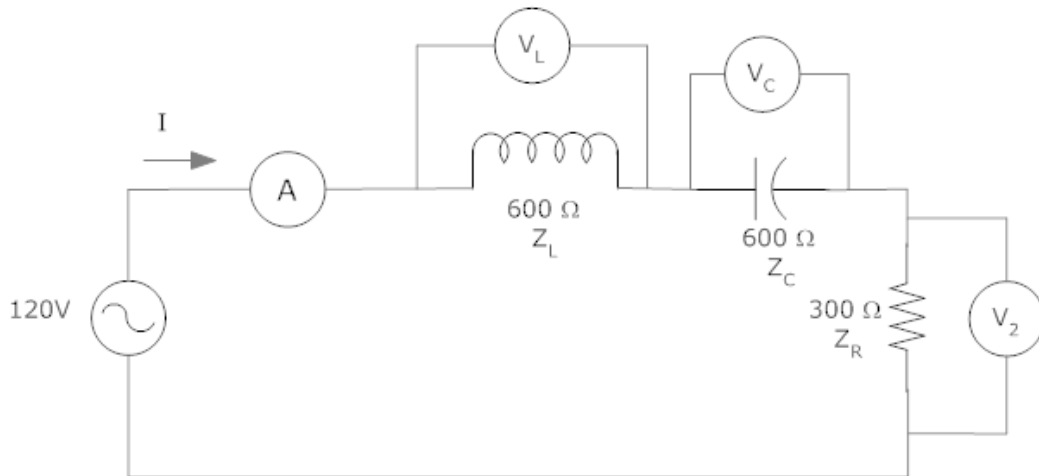

Figure 11: Series RLC circuit.

2. Build the circuit in Fig. 11 and measure I, V_L, V_C, and V_2. Assume the source voltage has an angle = 0°. Also measure the phase angles of the voltages V_L, V_C, and V_2 with respect to the source voltage. Compare your experimental results with the computed results in part 1. Explain discrepancies, if any.

3. Build the circuit in Fig. 12 and measure the power (both the real and reactive powers). If the reactive power is flowing towards the load, mark it as positive. If the reactive power is flowing towards the source mark it negative. Use X_l = 171 Ω, 200 Ω, 240 Ω, 300 Ω, 400 Ω, 600 Ω, and 1200 Ω. Plot reactive power vs. X_l. Explain your results.
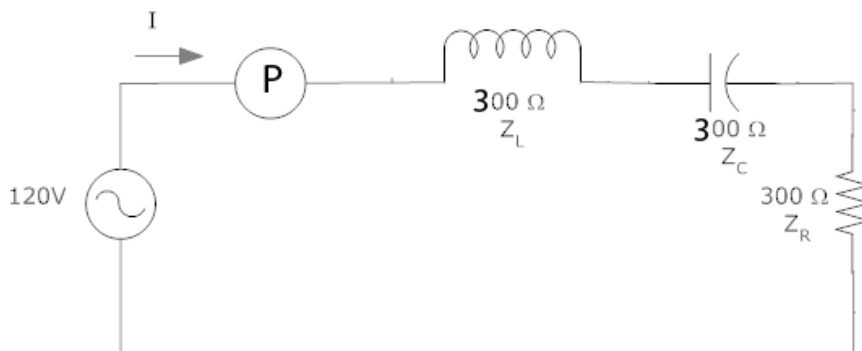

Figure 12: Series RLC circuit.

Per-Unit (pu) System of Measurements:

In per-unit system of measurements the quantities are expressed relative to a pre-defined reference quantity. The reference quantity is called *per-unit base of the system*. A system can have a single per-unit base quantity (such as voltage, current, power, impedance etc.). However, sometimes it is more useful to have more than one base quantity (especially in electrical systems) such as voltage and power as two per-unit base quantities. If we use power and voltage as the per-unit base quantities, we can derive several other useful per-unit base quantities from them. Per-unit quantities are represented with a subscript "B," that is,

P_B = per-unit base power

V_B = per-unit base voltage.

If we know the per-unit base power (P_B) and per-unit base voltage (V_B) of the system we can find per-unit current (I_B) and per-unit impedance (Z_B) of the system as follows

$$I_B = \frac{P_B}{V_B} \qquad (24)$$

and

$$Z_B = \frac{V_B}{I_B}. \qquad (25)$$

Example: A 10 Ω load (R_{load}) carries 25 A of current (I_{load}). If P_B = 20 KW and V_B = 400 V, calculate per-unit voltage, current, power, and load resistance.

Solution: We will, first, calculate per-unit base current (I_B) and impedance (Z_B) as follows

$$I_B = \frac{P_B}{V_B} = \frac{20000}{400} = 25 \; A$$

and

$$Z_B = \frac{V_B}{I_B} = \frac{400}{25} = 8 \; \Omega$$

We will now calculate load voltage (V_{load}) and power (P_{load}) as follows

$$V_{load} = I_{load} \times R_{load} = 25 \times 10 = 250 \; V$$

and

$$P_{load} = V_{load} \times I_{load} = 250 \times 25 = 6250 \; W$$

Finally, we can calculate p.u. quantities as follows

$$p.u.\,Power = \frac{P_{load}}{P_B} = \frac{6250}{20000} = 0.3125$$

$$p.u.\,Voltage = \frac{V_{load}}{V_B} = \frac{250}{400} = 0.625$$

$$p.u.\,Current = \frac{I_{load}}{I_B} = \frac{25}{50} = 0.50$$

$$p.u.\,Impedance = \frac{R_{load}}{Z_B} = \frac{10}{8} = 1.2$$

*These figures were adapted from *DOE Fundamentals Handbook: Electrical Science,* by US Department of Energy.

Table 1: Summary of Common Quantities, Units, and Symbols Used in Electrical Systems.

Quantity	Symbol (quantity)	Unit	Symbol (unit)
Time	t	Second	s
Length	l	Meter	m
Speed (linear)	v	Meters per second	m/s
Speed (angular)	ω	Radians per second	rad/s ($\omega = 2\pi f$)
Speed (rotational)	v	Revolutions per minute	rpm
Frequency	f	Cycles per second	Hz
Time period	T	Seconds / cycle	s ($T = 1/f$)
Force	F	Newton	N
Charge	q	Coulomb	C
Magnetic flux	ϕ	Weber	Web
Magnetic flux density	B	Webers per square meter (also Tesla)	Web/m^2 (T)
Magnetic field strength	H	Ampere turn	Amp-turn
Voltage	V	Volt	V
Current	I	Ampere	A
Power (apparent)	S	Volt-Ampere	VA
Power (real)	P	Watt	W
Power (reactive)	Q	var	var
Power (real in horse power)	P	Horse power	hp (1hp = 746 W)
Resistance	R	Ohm	Ω
Capacitance	C	Farad	f (F)
Inductance	L	Henry	H
Capacitive reactance	X_C ($X_C = 1/(\omega C)$)	Ohm	Ω
Inductive reactance	X_L ($X_L = \omega L$)	Ohm	Ω
Phase angle	ϕ, θ	Degree (radian)	^{o} (rad)
Conductance		Siemens (mho)	S
Resistivity		Ohm-meter	Ω-m

Kilo = 10^3 (K)
Mega = 10^6 (M)
Giga = 10^9 (G)
Terra = 10^{12} (T)
Milli = 10^{-3} (m)
Micro = 10^{-6} (μ)
Nano = 10^{-9} (n)
Pico = 10^{-12} (p)