MODULE 1

Numbers and its Properties

- Vedic Mathematics
- Number System

1 Vedic Mathematics

CHAPTER

LEARNING OBJECTIVES

After completion of this chapter, you should have a thorough understanding of the following:

- How to do Faster Calculations?
- Multiplications
- Squares
- Cubes
- Properties of Squares and Cubes

Introduction to the topic

Vedic Mathematics is the ancient system of Mathematics drawn from the Vedas. The Vedas are ancient texts that encompass a broad spectrum of knowledge, covering all aspects of life. These include the sutras (verses) pertaining to Mathematics. In the early 20th century, Swami Shri Bharati Krishna Tirthaji Maharaja claimed to have rediscovered a collection of 16 ancient mathematical sutras from the Vedas and published it in a book titled *Vedic Mathematics*. Historians, however, do not agree on whether or not these were truly a part of the Vedic tradition. If these sutras dated back to the Vedic era, they would be a part of an oral rather than a written tradition. Despite controversies, they are a novel and useful approach to computation; they are flexible in application and easy to remember. They can often be applied in the algebraic contexts and in simple arithmetic as well.

❍ TYPES OF CALCULATIONS

The different types of calculations that form the basis of mathematics are:

1. Addition
2. Subtraction
3. Multiplication
4. Division
5. Ratio comparison
6. Percentage calculations

When we talk about the techniques of calculations, addition and subtraction can simply not have any short-cuts. Since addition and subtraction are the basic units, we can at best only approximate the values.

In case of multiplication, the techniques of vedic Maths can be used.

Ratio comparison techniques are discussed in the chapter on ratio, proportion and variation and the percentage calculations in the chapter on percentage.

❍ VEDIC MATH TECHNIQUES IN MULTIPLICATION

There are several techniques of multiplication. We will discuss them one by one.

Method 1: Base Method

In this method, one number is used as a base; for example, 10, 50, 100, etc. The number that is closer to both the numbers should be taken as the base.

Example 1 105×107

Solution In this case, both the numbers are close to 100, so 100 is taken as the base. We will now find the deficit/surplus from the base.

Base = 100, Surplus = 5 and 7

105	+5
107	+7
112	35

The right part (after slash) ⇒ this is the product of the surplus. Since the base = 100 and the surpluses are 5 and 7, the product would be $5 \times 7 = 35$.

The left part (before slash) ⇒ It could be either of the numbers plus the surplus of the other multiplicand. Hence, the left part would be either $(105 + 7)$ or $(107 + 5) = 112$ (both will always be the same), i.e., 112.

The left part would be equivalent to the number × 100. In this case, $112 \times 100 = 11200$

Now, we add both the right part and the left part $= 11200 + 35 = 11235$

Hence, the result of the multiplication would be 11235.

> www To know more about Vedic Maths, go to *www.pims.math.ca/pi/issue6/page15–16.pdf*

Example 2 108×104

Solution

108	+8
104	+4
112	32

Example 3 111×112

Solution

111	+11
112	+12
123	132

Here, it is $11 \times 12 = 132$. But it can have only two digits. Thus, 1 will be carried over to the left part and the right part will be only 32. Left part will be either $111 + 12 + 1$ (1 for the carry over) or $(112 + 11 + 1)$, i.e., 124. So, the result will be 12432.

For 102×104, the answer will be 10608. Please note that the right part will be 08 and not simply 8.

Example 4 97×95

Solution

97	–3
95	–5
92	15

Base = 100, Deficit = $97 - 100 = -3$ and $95 - 100 = -5$

Example 5 97×102

Solution

97	–3
102	+2
99	–06

97×102

Base = 100, Deficit = $97 - 100 = -3$,

Surplus = $102 - 100 = 2$

The right part will now be $(-3) \times 2$, i.e., –06. To take care of the negative, we will borrow 1 from the left part, which is equivalent to borrowing 100 (because we are borrowing from the hundred digits of the left part). Thus, this part will be $100 - 06 = 94$.

So, the answer = 98 94

Example 6 62×63

Solution

62	+12
63	+13
75	156

We will assume here the base as 50 owing to the fact that the numbers are close to 50.

Base = 50, Surplus = 62 − 50 = 12,

Surplus = 63 − 50 = 13

The left hand side = 156 and the right hand side = 75. Since the base is assumed to be equal to 50, so the value of the right hand side = 75 × 50 = 3750. Besides, only two digits can be there on the right hand side, so 1(100) is transfered to the left hand side leaving 56 only on the left hand side.

So, the value on the right hand side = 3750 + 100 = 3850

Value on the left hand side = 56

Net value = 3850 + 56 = 3906

Let us do the same multiplication by assuming 60 as the base.

$$\begin{array}{c|c} 62 & +2 \\ 63 & +3 \\ \hline 65 & 06 \end{array}$$

Base = 60, Surplus = 62 − 60 = 2, Surplus = 63 − 60 = 3

Since the base is assumed to be equal to 60, the value of the right hand side = 65 × 60 = 130 × 30 = 3900

So, net value = 3906

Method 2: Place Value Method

In this method of multiplication, every digit is assigned a place value and the multiplication is done by equating the place values of multiplicands with the place value of the product.

Let us see this with some examples:

$$\begin{array}{ccccccc} & & & 1_3 & 2_2 & 5_1 & 4_0 \\ & \times & & 3_3 & 3_2 & 2_1 & 1_0 \\ \hline 6 & 5 & 4 & 3 & 2 & 1 & 0 \\ \hline \end{array}$$

Conventionally, the unit digit is assigned a place value 0, the tens place digit is assigned a place value 1, the hundreds place digit is assigned a place value 2, the thousands place digits is assigned a place value 3 and so on.

This multiplication is a two-step process.

Step 1 Add the place values of the digits of the numbers given (1254 × 3321) to obtain the place value of the digits of the product.

For example, using the place values of the multiplicands, i.e., using 0, 1, 2 and 3 of the number 1254 and the same place values 0, 1, 2 and 3 of the another multiplicand 3321, we can get 0 place value in the product in just one way, i.e., adding 0 and 0.

$$\begin{array}{ccccccc} & & & 1_3 & 2_2 & 5_1 & 4_0 \\ & \times & & 3_3 & 3_2 & 2_1 & 1_0 \\ \hline 6 & 5 & 4 & 3 & 2 & 1 & 0 \\ \hline \end{array}$$

Place value 1 in the product can be obtained in two ways.

$$\begin{array}{ccccccc} & & & 1_3 & 2_2 & 5_1 & 4_0 \\ & \times & & 3_3 & 3_2 & 2_1 & 1_0 \\ \hline 6 & 5 & 4 & 3 & 2 & 1 & 0 \\ \hline \end{array}$$

Place value 2 can be obtained in three ways.

$$\begin{array}{ccccccc} & & & 1_3 & 2_2 & 5_1 & 4_0 \\ & \times & & 3_3 & 3_2 & 2_1 & 1_0 \\ \hline 6 & 5 & 4 & 3 & 2 & 1 & 0 \\ \hline \end{array}$$

Place value 3 can be obtained in four ways.

$$\begin{array}{ccccccc} & & & 1_3 & 2_2 & 5_1 & 4_0 \\ & \times & & 3_3 & 3_2 & 2_1 & 1_0 \\ \hline 6 & 5 & 4 & 3 & 2 & 1 & 0 \\ \hline \end{array}$$

Place value 4 can be obtained in three ways.

$$\begin{array}{ccccccc} & & & 1_3 & 2_2 & 5_1 & 4_0 \\ & \times & & 3_3 & 3_2 & 2_1 & 1_0 \\ \hline 6 & 5 & 4 & 3 & 2 & 1 & 0 \\ \hline \end{array}$$

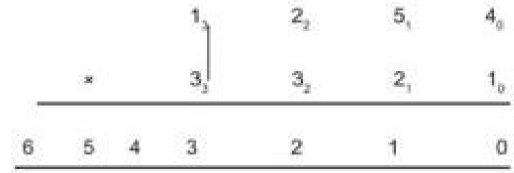
Place value 5 can be obtained in two ways.

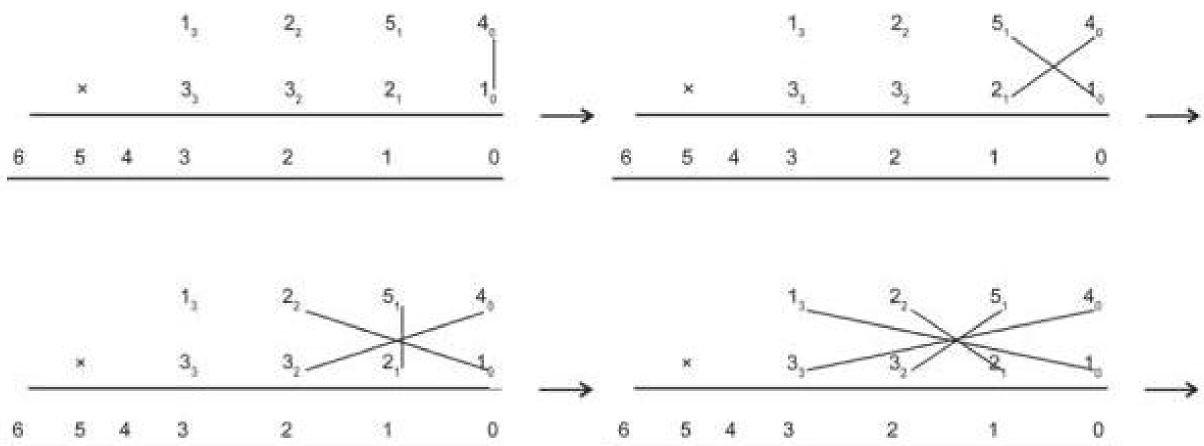
$$\begin{array}{ccccccc} & & & 1_3 & 2_2 & 5_1 & 4_0 \\ & \times & & 3_3 & 3_2 & 2_1 & 1_0 \\ \hline 6 & 5 & 4 & 3 & 2 & 1 & 0 \\ \hline \end{array}$$

Place value 6 can be obtained in one way.

And this is the maximum place value that can be obtained.

Step 2 Multiply the corresponding numbers one by one.

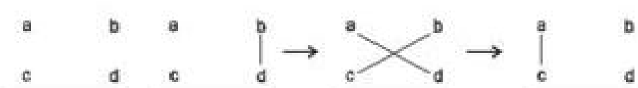




In this manner, we can find the product = 4164534

This method is most useful in case of the multiplications of 2 digits × 2 digits or 2 digits × 3 digits or 3 digits × 3 digits multiplication.

Example $ab \times cd$



Similarly, we can have a proper mechanism of multiplication of 2 digits × 3 digits or 3 digits × 3 digits also using the place value method.

Method 3: Units Digit Method

This method of multiplication uses the sum of the units digit, provided all the other digits on the left hand side of the unit digit are the same.

Example 7 75×75

Solution

1.0 + 7	5
7	5
56	25

The sum of the units digit = 10, so we add 1.0 in one of the digits on the left hand side.

Example 8 62×63

Solution

0.5 + 6	3
6	2
39	06

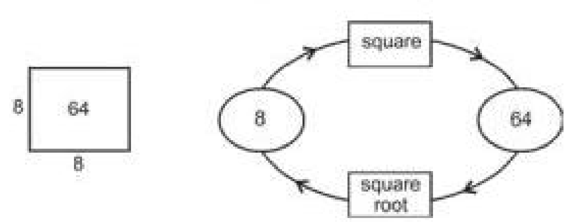
The sum of the units digit = 5, so we add 0.5 in one of the digits on the left hand side.

❍ SQUARING

A **square number**, also called a **perfect square**, is an integer that can be written as the square of some other integer. In other words, a number whose square root is an integer is known as the square number of a perfect square.

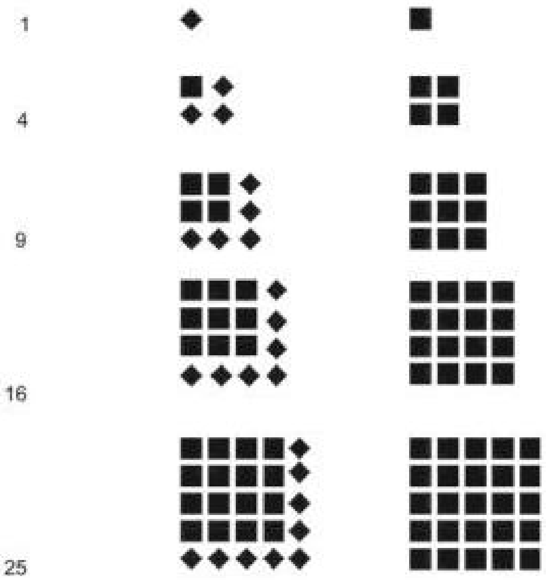
For example, 9 is a square number since it can be written as 3 × 3.

This can be seen through the following flow-chart also.



Properties of a Square Number

1. The number N is a square number if it can be arranged as N points in a square:



Thus, it can be deduced that the formula for the nth square number is n^2. This is also equal to the sum of the first n odd numbers $n^2 = \sum_{k=1}^{n}(2k-1)$, as can be seen in the above figure, where a square results from the previous one by adding an odd number of points (marked as '◆'). For example, $5^2 = 25 = 1 + 3 + 5 + 7 + 9$.

It should be noted that the square of any number can be represented as the sum $1 + 1 + 2 + 2 + ... + n - 1 + n - 1 + n$. For instance, the square of 4 or 4^2 is equal to $1 + 1 + 2 + 2 + 3 + 3 + 4 = 16$. This is the result of adding a column and row of thickness 1 to the square graph of three. This can also be useful for finding the square of a big number quickly. For instance, the square of $52 = 50^2 + 50 + 51 + 51 + 52 = 2500 + 204 = 2704$.

2. A square number can only end with digits 00, 1, 4, 6, 9, or 25 in base 10, as follows:
3. If the last digit of a number is 0, its square ends in 00 and the preceding digits must also form a square.
4. If the last digit of a number is 1 or 9, its square ends in 1 and the number formed by its preceding digits must be divisible by four.
5. If the last digit of a number is 2 or 8, its square ends in 4 and the preceding digit must be even.
6. If the last digit of a number is 3 or 7, its square ends in 9 and the number formed by its preceding digits must be divisible by four.
7. If the last digit of a number is 4 or 6, its square ends in 6 and the preceding digit must be odd.
8. If the last digit of a number is 5, its square ends in 25 and the preceding digits (other than 25) must be 0, 2, 06, or 56.
9. A square number cannot be a perfect number. (If the sum of all the factors of a number excluidng the number itself is equal to the number, then the number is known to be a perfect number.)
10. The digital sum of any perfect square can be only 0, 1, 4, 9, 7. (Digital sum of any number is obtained by adding the digits of the number until we get a single digit. Digital sum of $385 = 3 + 8 + 5 = 1 + 6 = 7$)

An easy way to find the squares is to find two numbers which have a mean of it. This can be seen through the following example:

To find the square of 21, take 20 and 22, then multiply the two numbers together and add the square of the distance from the mean: $22 \times 20 = 440 + 1^2 = 441$. Here, we have used the following formula $(x - y)(x + y) = x^2 - y^2$ known as the difference of two squares. Thus,

$(21 - 1)(21 + 1) = 21^2 - 1^2 = 440$.

Odd and Even Square Numbers

Squares of even numbers are even, since $(2n)^2 = 4n^2$.

Squares of odd numbers are odd, since $(2n + 1)^2 = 4(n^2 + n) + 1$.

Hence, we can infer that the square roots of even square numbers are even, and square roots of odd square numbers are odd.

Methods of Squaring

As we have seen in the case of multiplication, there are several methods for squaring also. Let us see the methods one by one.

Method 1: Base 10 Method

Understand it by taking few examples:

- Let us find out the square of 9. Since 9 is 1 less than 10, decrease it still further to 8. This is the left side of our answer.
- On the right hand side put the square of the deficiency that is 1^2. Hence, the answer is 81.
- Similarly, $8^2 = 64$, $7^2 = 49$.
- For numbers above 10, instead of looking at the deficit we look at the surplus. For example,

  $11^2 = (11 + 1); 10 + 1^2 = 121$
  $12^2 = (12 + 2); 10 + 2^2 = 144$
  $14^2 = (14 + 4); 10 + 4^2 = 18; 10 + 16 = 196$
  and so on.

This is based on the identities $(a + b)(a - b) = a^2 - b^2$ and $(a + b)^2 = a^2 + 2ab + b^2$.

We can be use this method to find the squares of any number, but after a certain stage, this method loses its efficiency.

Method 2: Base 50*n* Method here, (*n* is any natural number)

This method is nothing but the application of $(a + b)^2 = a^2 + 2ab + b^2$.

This can be seen in the following example:

Example 9 Find the square of 62.

Solution Because this number is close to 50, we will assume 50 as the base.

$(62)^2 = (50 + 12)^2 = (50)^2 + 2 \times 50 \times 12 + (12)^2 = 2500 + 1200 + 144$

To make it self explanatory a special method of writring is used.

$(62)^2 = [100\text{'s in (Base)}]^2 + \text{Surplus} \mid \text{Surplus}^2$

$= 25 + 12 \mid 144 = 38 \mid 44$ [Number before the bar on its left hand side is number of hundreds and on its right hand side are last two digits of the number.]

$(68)^2 = 25 + 18 \mid 324 = 46 \mid 24$

$(76)^2 = 25 + 26 \mid 676 = 57 \mid 76$

$(42)^2 = 25 - 8 \mid 64 = 17 \mid 24$ $[(a - b)^2 = a^2 - 2ab + b^2]$

Example 10 Find the square of 112.

Solution Since this number is closer to 100, we will take 100 as the base.

$(112)^2 = (100 + 12)^2 = (100)^2 + 2 \times 100 \times 12 + (12)^2 = 10000 + 2 \times 1200 + 144$

$(112)^2 = [100\text{'s in (Base)}]^2 + 2 \times \text{Surplus} \mid \text{Surplus}^2$

$= 100 + 2 \times 12 \mid 12^2 = 125 \mid 44$

Alternatively, we can multiply it directly using base value method.

Had this been 162, we would have multiplied 3 in surplus before adding it into $[100\text{'s in (Base)}]^2$ because assumed base here is 150.

$(162)^2 = [100\text{'s in (Base)}]^2 + 3 \times \text{Surplus} \mid \text{Surplus}^2$

$= 225 + 3 \times 12 \mid 12^2 = 262 \mid 44$

Method 3: 10^n Method

This method is applied when the number is close to 10^n.

With base as 10^n, find the surplus or deficit (×)

Again answer can be arrived at in two parts

$(B + 2x) \mid x^2$

The right-hand part will consist of *n* digits. Add leading zeros or carry forward the extra to satisfy this condition.

$108^2 = (100 + 2 \times 8) \mid 8^2 = 116 \mid 64 = 11664$

$102^2 = (100 + 2 \times 2) \mid 2^2 = 104 \mid 04 = 10404$

$93^2 = (100 - 2 \times 7) \mid (-7)^2 = 86 \mid 49 \Rightarrow 8649$

$1006^2 = (1000 + 2 \times 6) \mid 6^2 = 10|12 \mid 036 = 1012036$

The right-hand part will consist of 2 digits. Add leading zeros or carry forward the extra to satisfy this condition.

$63^2 = (25 + 13) \mid 13^2 = 38 \mid 169 = 3969$

$38^2 = (25 - 12) + (-12)^2 = 13 \mid 144 = 1444$

Square Mirrors

$14^2 + 87^2 = 41^2 + 78^2$
$15^2 + 75^2 = 51^2 + 57^2$
$17^2 + 84^2 = 71^2 + 48^2$
$26^2 + 97^2 = 62^2 + 79^2$
$27^2 + 96^2 = 72^2 + 69^2$

Some Special Cases

1. Numbers ending with 5

If *a* number is in the form of *n*5, the square of it is $n(n + 1) \mid 25$

Example $45^2 = 4 \times 5 \mid 25 = 2025$

$135^2 = 13 \times 14 \mid 25 = 18225$

This is nothing but the application of the multiplication method using the sum of units digits.

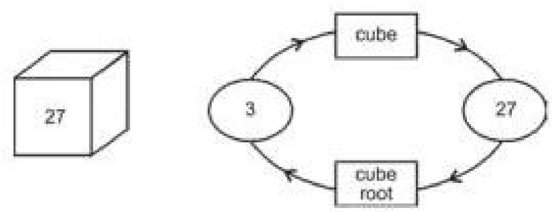
We can use this method to find out the squares fractions like $1\frac{1}{2}, 2\frac{1}{2}, 3\frac{1}{2} \ldots$ also.

Process: Multiply the integral portion by the next higher integer and add $\frac{1}{4}$.

For example, $\left(6\frac{1}{2}\right) = 6 \times 7 + \frac{1}{4} = 42\frac{1}{4}$

❍ CUBING

A number whose cube root is an integer is called a perfect cube.



Properties of a Cube

1. The sum of the cubes of any number of consecutive integers starting with 1 is the square of some integer. (For example, $1^3 + 2^3 = 9 = 3^2$, $1^3 + 2^3 + 3^3 = 36 = 6^2$, etc.)
2. Unit digit of any cube can be any digit from 0 – 9.

Methods of Cubing

We can find the cube of any number close to a power of 10 say 10^n with base = 10^n by finding the surplus or the defict (x). The answer will be obtained in three parts. $B + 3x \mid 3 . x^2 \mid x^3$

The left two parts will have n digits.

104^3

Base B = 100 and surplus = $x = 4$

$(100 + 3 \times 4) | 3 \times 4^2 | 4^3 = 112|48|64 = 1124864$

109^3

Base B = 100 and $x = 9$

$(100 + 3 \times 9) | 3 \times 9^2 | 9^3 = 127|243|729 = 1295029$

98^3

Base B = 100 and $x = -2$

$(100 - 3 \times 2) | 3 \times (-2)^2 | (-2)^3 = 94 | 12 | -8 = 94 | 11 | 100 - 8 = 941192$

❍ VEDIC MATHS TECHNIQUES IN ALGEBRA

1. If one is in ratio, the other one is zero

This formula is often used to solve simple simultaneous equations which may involve big numbers. But these equations in special cases can be visually solved because of a certain ratio between the co-efficients. Consider the following example:

$6x + 7y = 8$

$19x + 14y = 16$

Here, the ratio of co-efficients of y is the same as that of the constant terms. Therefore, the "other" is zero, i.e., $x = 0$. Hence, the solution of the equations is $x = 0$ and $y = 8/7$.

Alternatively,

$19x + 14y = 16$ is equivalent to. $(19/2)x + 7y = 8$.

Thus, x has to be zero and no ratio is needed, just divide by 2!

Note that it would not work if both had been "in ratio":

$6x + 7y = 8$

$12x + 14y = 16$

This formula is easily applicable to more general cases with any number of variables. For instance,

$ax + by + cz = a$

$bx + cy + az = b$

$cx + ay + bz = c$

which yields $x = 1, y = 0, z = 0$.

2. When samuccaya is the same, that samuccaya is zero

Consider the following symbols: N_1 – Numerator 1, N_2 – Numerator 2, D_1 – Denominator 1, D_2 – Denominator 2 and so on.

This formula is useful for solving equations that can be solved visually. The word "samuccaya" has various meanings in different applications. For instance, it may mean a term, which occurs as a common factor in all the terms concerned. For example, an equation "$12x + 3x = 4x + 5x$". Since "x" occurs as a common factor in all the terms, therefore, $x = 0$ is the solution. Alternatively, samuccaya is the product of independent terms. For instance, in $(x + 7)(x + 9) = (x + 3)(x + 21)$, the samuccaya is $7 \times 9 = 3 \times 21$, therefore, $x = 0$ is the solution. It is also the sum of the denominators of two fractions having the same numerical numerator, for example: $1/(2x - 1) + 1/(3x - 1) = 0$ means $5x - 2 = 0$

The more commonly used meaning is "combination" or total. For instance, if the sum of the numerators and the sum of denominators are the same then that sum is zero. Therefore,

$$\frac{2x+9}{2x+7} = \frac{2x+7}{2x+9}$$

Therefore, $4x + 16 = 0$ or $x = -4$

This meaning ("total") can also be applied in solving the quadratic equations. The total meaning not only imply sum but also subtraction. For instance, when given $N_1D_1 = N_2/D_2$, if $N_1 + N_2 = D_1 + D_2$ (as shown earlier) then this sum is zero. Mental cross multiplication reveals that the resulting equation is quadratic (the co-efficients of x^2 are different on the two sides). So, if $N_1 - D_1 = N_2 - D_2$ then that samuccaya is also zero. This yields the other root of a quadratic equation.

The interpretation of "total" is also applied in multi-term RHS and LHS. For instance, consider

$$\frac{1}{x-7} + \frac{1}{x-9} = \frac{1}{x-6} + \frac{1}{x-10}$$

Here, $D_1 + D_2 = D_3 + D_4 = 2x - 16$. Thus $x = 8$.

There are several other cases where samuccaya can be applied with great versatility. For instance, "apparently cubic" or "biquadratic" equations can be easily solved as shown below:

$$(x - 3)^3 + (x - 9)^3 = 2(x - 6)^3$$

Note that $x - 3 + x - 9 = 2(x - 6)$. Therefore, $(x - 6) = 0$ or $x = 6$.

Consider

$$\frac{(x+3)^3}{(x+5)^3} = \frac{x+1}{x+7}$$

Observe: $N_1 + D_1 = N_2 + D_2 = 2x + 8$

Therefore, $x = -4$

2 Number System

CHAPTER

LEARNING OBJECTIVES

After completion of this chapter, you should have a thorough understanding of the following:

- Numbers and their different types
- Definitions and properties of these numbers
- Concepts attached to these numbers
- Kind of questions which are asked in the CAT
- Methods of solving questions

Introduction to the Number System

Starting with the relative importance of Number System with respect to CAT preparation, it has been one of the important topics in QA historically. In the last 15 years CAT paper, it is observed that almost 20% of QA paper consisted of questions from Number System every year. **Numbers in this chapter do not have that important role to play, as has logic**. In other words, we can say that logical processes outweigh calculations in finding solution to phenomenally lengthy mathematical problems in Number System. Students are expected to get a clear understanding of the definitions as well as concepts and develop a keen insight about numbers and their properties. Apart from these, try to maximize learning with every question which you solve.

❍ QUESTIONS ARE ASKED FROM THIS TOPIC IN TWO WAYS

1. Based on definitions and properties of numbers In this section, questions will be based upon the definitions of different kinds of numbers. A part from this, questions can be asked from some of the very basic calculations, formula or properties of numbers.

2. Based on concepts Some of the concepts on which questions are being asked are:

i. LCM and HCF
ii. Divisibility rules (For base 10)
iii. Divisibility rules (For base other than 10)
iv. Number of divisors
v. Number of exponents
vi. Remainders
vii. Base system
viii. Units digit
ix. Tens' digit
x. Pigeon-Hole principle

❍ CLASSIFICATION OF NUMBERS/ INTEGERS

Natural Numbers

Natural numbers are counting numbers, i.e., the numbers which we use to count any number of things. E.g., 1, 2, 3, ...

Lowest natural number is 1.

Whole Numbers

When zero is included in the list of natural number, then they are known as whole numbers. E.g., 0, 1, 2, ...

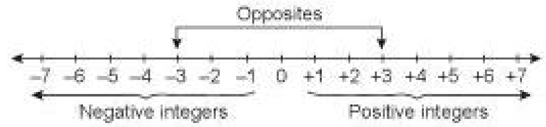
Lowest whole number is 0.

Integers

Integers are whole numbers, negative of whole numbers, and zero. For example, 43, 434235, 28, 2, 0, −28, and −3030 are integers, but numbers like 1/2, 4.00032, 2.5, Pi, and −9.90 are not whole numbers.

Number Line

The number line is used to represent the set of real numbers. Below is the brief representation of the number line:



Properties of Number Line

- The number line goes on till infinity in both directions. This is indicated by the arrows.
- Integers greater than zero are called positive integers. These numbers are to the right of zero on the number line.
- Integers less than zero are called negative integers. These numbers are to the left of zero on the number line.
- The integer zero is neutral. It is neither positive nor negative.
- The sign of an integer is either positive (+) or negative (−), except zero, which has no sign.
- Two integers are opposites if each of them is at the same distance from zero, but on opposite sides of the number line. One will have a positive sign, the other a negative sign. In the number line above, +3 and −3 are labelled as opposites. In other words, the whole negative number scale looks like a mirror image of the positive number scale, with a number like −15 being the same distance away from 0 as 15 is.
- The number half way between −1 and −2 is −1.5; just as the number half way between 1 and 2 is 1.5.
- We represent positive numbers without using a +ve sign. For example, we would write 29.1 instead of +29.1. But when we talk of negative numbers, the sign must be there.

Prime Numbers and Composite Numbers

Prime Numbers

Among natural numbers, we can distinguish prime numbers and composite numbers.

All the numbers which are divisible by 1 and itself only are known as prime numbers.

Again as said above, primes can be natural numbers only.

In other words, we can say that all the numbers which have only two factors are known as prime numbers.

Prime numbers can also be seen as the building blocks. And we combine two or more than two same or distinct prime numbers to create numbers bigger than these prime numbers, E.g., → $3 \times 2 = 6$

List of all prime numbers less than 1000

2	3	5	7	11	13	17	19	23	29	31	37	41	43	47	53	59	61
67	71	73	79	83	89	97	101	103	107	109	113	127	131	137	139	149	151
157	163	167	173	179	181	191	193	197	199	211	223	227	229	233	239	241	251
257	263	269	271	277	281	283	293	307	311	313	317	331	337	347	349	353	359
367	373	379	383	389	397	401	409	419	421	431	433	439	443	449	457	461	463
467	479	487	491	499	503	509	521	523	541	547	557	563	569	571	577	587	593
599	601	607	613	617	619	631	641	643	647	653	659	661	673	677	683	691	701
709	719	727	733	739	743	751	757	761	769	773	787	797	809	811	821	823	827
829	839	853	857	859	863	877	881	883	887	907	911	919	929	937	941	947	953
967	971	977	983	991	997												

With this, we can find out number of prime numbers between every 100 numbers.

Numbers from-to	1-100	101-200	201-300	301-400	401-500	501-600	601-700	701-800	801-900	901-1000
Number of primes	25	21	16	16	17	14	16	14	15	14

Largest prime number till date and history of prime number. The largest known prime today is the 7, 816, 230 digit prime number $2^{25964951} - 1$ found in early 2005 but how big have the "largest known primes" been historically?, and when might we see the first billion–digit prime number?

Records before Electronic Computers

Number	*Digits*	*Year*	*Prover*	*Method*
$2^{17} - 1$	6	1588	Cataldi	Trial division
$2^{19} - 1$	6	1588	Cataldi	Trial division
$2^{31} - 1$	10	1772	Euler	Trial division
$(2^{59} - 1)/179951$	13	1867	Landry	Trial division
$2^{127} - 1$	39	1876	Lucas	Lucas sequences
$(2^{148} + 1)/17$	44	1951	Ferrier	Proth's theorem

Prime number found by Lucas in 1876 was accepted as the largest prime number till 1951. In 1951, Ferrier used a mechanical desk calculator and techniques based on partial inverses of Fermat's little theorem (see the pages on remainder theorem) to slightly better this record by finding a 44 digits prime.

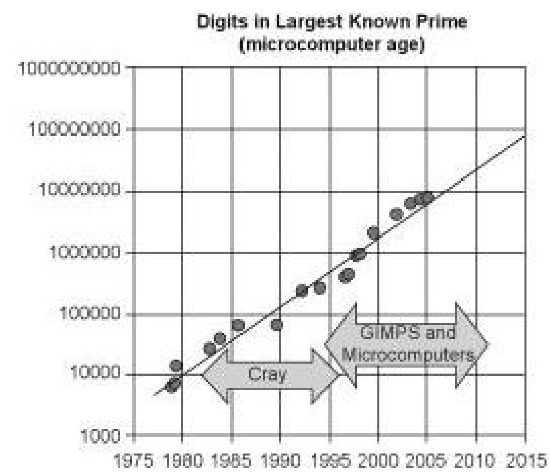
In 1951, Ferrier found the prime $(2^{148} + 1)/17 = 20988936657440586486151264256610222593863921$.

However, this record was very short-lived. In the same year 1951, advent of electronic computers helped human being in finding a bigger prime number.

In 1951, Miller and Wheeler began the electronic computing age by finding several primes as well as the new 79 digit record: $2^{127} - 1$

And we know, this was the computer age and everybody was working hard to find out the primes with the help of computers. Records were broken with a never-before pace.

When will we have a one billion digit prime?



Can we have a Single Formula Representing all the Prime Numbers?

Till now, all the attempts done in this regard have proved to be fruitless. It is all because there is no symmetricity between the differences among the prime numbers. Sometimes, two consecutive prime numbers differ by 2, sometimes by 4, and sometimes even it can be 10,000 or more. So, there is no standard formula that can represent the prime numbers.

However, there are some standard notations which give us limited number of prime numbers:

$N^2 + N + 41 \rightarrow$ For all the values of N from −39 to +39, this expression gives us a prime number.

$N^2 + N + 17$ another similar example.

Remember

All the prime numbers (>3) are of the form $6n \pm 1$ form, (where n is any natural number), i.e., all the prime numbers (>3) when divided by 6 give either 1 or 5 as the remainder.

NOTE: It is important to know here that if a number gives a remainder of 1 or 5 when divided by 6, it is not necessarily a prime number. For example, 25 when divided by 6 gives remainder = 1, but 25 is not a prime number.

Composite Numbers

A number is composite if it is the product of two or more than two distinct or same prime numbers. E.g., → 4, 6, 8,….

$4 = 2^2$

$6 = 2^1 \times 3^1$

Lowest composite number is 4.

All the composite numbers will have at least 3 factors.

Even and Odd Numbers

Suppose N is an integer. If there exists an integer P such that $N = 2P + 1$, then N is an **odd number**. If there exists an integer P such that $N = 2P$, then N is an **even number**.

Putting in a simple language, even numbers are those integers which are divisible by 2 and odd numbers are those integers which are not divisible by 2.

Even and odd numbers can be positive as well as negative also.

In other words, if x is an integer (even or not), then $2x$ will be an even integer, because it is a multiple of 2. Also x raised to any positive integer power will be an even number, so x^2, x^3, x^4, etc., will be even numbers.

Any integer that is not a multiple of 2 is called an *odd* number. For instance, –1, 3, 6883 and –8147 are all odd numbers. Any odd number raised to a positive integer power will also be an odd number, so if x is an odd number, so will x^2, x^3, x^4, etc., be odd numbers.

The concept of even and odd numbers are most easily understood in the binary base. Above definition simply states that even numbers end with a 0, and odd numbers end with a 1.

Comparing Integers

We can compare two different integers by looking at their positions on the number line. For any two different places on the number line, the integer on the right is greater than the integer on the left. Note that every positive integer is greater than any negative integer.

Examples: $9 > 4$, $6 > -9$, $-2 > -8$, and $0 > -5$ $-2 < 1$, $8 < 11$, $-7 < -5$, and $-10 < 0$

Points to remember:
i. 1 is neither prime nor composite.
ii. 0 is neither positive nor negative.

Example 1 Two of a, b, c and d are even and two are odd, not necessarily in order. Which of the following is definitely even? ***(CAT 1997)***

(a) $a + b + c - 2d$
(b) $a + 2b - c$
(c) $a + b - c + d$
(d) $2a + b + c - d$

Solution Since we do not know which two are even and which two are odd, we will have to do a bit of hit-and-trial to solve this problem with the help of options.

In option (a), if a and b are even, and c and d are odd, then this will lead us to odd number.

In option (b), if a and b are even, and c is odd, then this will lead us to odd number.

In option (d), if a and b are odd, and c and d are even, then this will lead us to odd number.

In option (c), whatever is the value of a, b, c and d, it is always going to be an even number.

Explanation Whatever kind of calculation we do with two even and two odd numbers, we will always get an even result.

So, answer is option (c).

Example 2 If N, $N + 2$ and $N + 4$ are prime numbers, then the number of possible solutions for N is/are ***(CAT 2003)***

(a) 1 (b) 2
(c) 3 (d) None of these

Solution There is only one triplet of prime numbers where difference between any two prime number is 2, that is 3, 5 and 7. So, $N = 3$ is the only solution.

Hence, answer is (a).

Proof of above example We know that prime numbers are of the form $6M \pm 1$ (except 2 and 3). Now if N is of the format $6M + 1$, then $N + 2$ will be of $6M + 3$ format and $N + 4$ will be of $6M + 5$ format. Out of these three numbers, since $N + 2$ is of $6M + 3$ format, it will be divisible by 3.

Similarly, if N is of the format $6M - 1$, then $N + 2$ will be of $6M + 1$ format and $N + 4$ will be of $6M + 3$ format. Out of these three numbers, since $N + 4$ is of $6M + 3$ format, it will be divisible by 3.

In both the cases, we find that one number out of given three number is divisible by 3. In the example given above (3, 5 and 7), one of the given three numbers is divisible by 3.

Example 3 Let x and y be positive integers such that x is prime and y is composite. Then which of the following is true? ***(CAT 2003)***

(a) $y - x$ cannot be an even integer.
(b) xy cannot be an even integer.

(c) $\frac{x+y}{x}$ cannot be an even integer.

(d) None of these

Solution Eliminating the options,

To eliminate option (a): If $y = 4$ and $x = 2$, then $y - x$ can be even.

To eliminate option (b): If $y = 4$ and $x = 2$, then yx can be even.

To eliminate option (c): If $y = 6$ and $x = 2$, then it is also be even.

So, answer is option (d).

❍ QUESTIONS BASED UPON CONCEPTS

1. LCM and HCF

Meaning of LCM

A common multiple is a number that is a multiple of two or more than two numbers. The common multiples of 3 and 4 are 12, 24, ...

The Least Common Multiple (LCM) of two numbers is the smallest positive number that is a multiple of both.

Multiples of 3 → 3, 6, 9, 12, 15, 18, 21, 24,...............

Multiples of 4 → 4, 8, 12, 16, 20, 24, 28,..................

So, LCM of 3 and 4 will be 12, which is the lowest common multiple of 3 and 4.

First of all, the basic question which lies is—What kind of numbers we can use LCM for?

Let us see this through an example: LCM of 10, 20 and 25 is 100. It means that 100 is the lowest number which is divisible by all these three numbers.

But cannot the LCM be (−100)? Since (−100) is lower than 100 and divisible by each of 10, 20 and 25. Or, it can be zero also.

Or, what will be the LCM of (−10) and 20?

Will it be (−20) or (−200) or (−2000) or smallest of all the numbers, i.e., $-\infty$?

Answer to all these questions is very simple: LCM is a concept defined only for positive numbers be it an integer or a fraction, i.e., **LCM is not defined for negative numbers or zero**.

Now we will define a bit different method for finding out LCM of two or more than two positive integers.

Process to find out LCM

Step 1 Factorize all the numbers into their prime factors.

Step 2 Collect all the distinct factors.

Step 3 Raise each factor to its maximum available power and multiply.

Example 4 LCM of 10, 20, 25.

Solution

Step 1 $10 = 2^1 \times 5^1$

$20 = 2^2 \times 5^1$

$25 = 5^2$

Step 2 2, 5

Step 3 $2^2 \times 5^2 = 100$

The biggest advantage of using this method lies in the fact that we can find out LCM of any number of numbers in a straight line without using the conventional method. It can be understood in the following way with the previous example:

First of all, find out the LCM of 10, 20 = 20, and now LCM of 20 and 25 = 100 (For this you will have to check that which factor of 25 is not present there in 20 and then multiplying by this factor. Since 25 is having 5^2 and 20 is having 5^1 only, so we will multiply 20 by 5.)

Example 5 LCM of 35, 45, 55.

Solution First of all, find out LCM of 35 and 45.

Now $35 = 5^1 \times 7^1$ and $45 = 3^2 \times 5^1$.

So, it can be observed here that 35 is not having 3^2 in it, so we will multiply 35 by 3^2.

So, LCM of 35 and 45 = 35×3^2. (You can start with 45 also to find out about the missing factors of 35 in 45.)

Now, we will find out LCM of 35×3^2 and $55 = 5^1 \times 11^1$

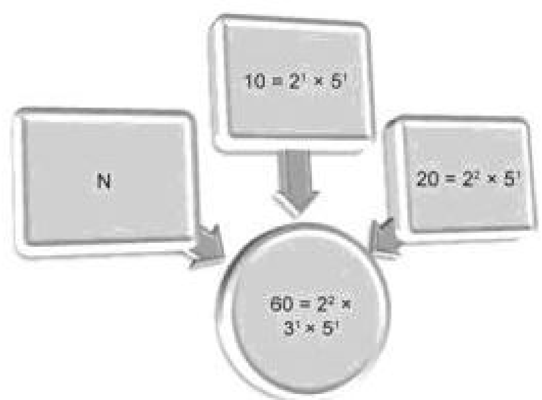
$55 = 5^1 \times 11^1$

Now, 11^1 is not there with 35×3^2. So, we will multiply 35×3^2 with 11^1.

So, finally LCM = $35 \times 3^2 \times 11^1 = 3465$.

Example 6 LCM of three natural numbers 10, 20 and $N = 60$. How many values of N are possible?

Solution We have already seen that to generate the LCM we multiply the prime numbers with the highest available power. So let us start with factorizing the numbers:



$2^2 \times 5^1$ is already present in 20, however, 3 is not present in either 10 or 20. So we can conclude that 3^1 has to come from N. This is the minimum value of $N = 3$. Secondly, we can also say that N may contain powers of 2 and 5 as long as maximum power of 2 = 2 and maximum power of 5 = 1 (as in $2^2 \times 5^1$).

So, total different values of $N = (3^1 \times 2^0 \times 5^0), (3^1 \times 2^1 \times 5^0), (3^1 \times 2^2 \times 5^0), (3^1 \times 2^0 \times 5^1), (3^1 \times 2^1 \times 5^1), (3^1 \times 2^2 \times 5^1)$ = 3, 6, 12, 15, 30, 60 = 6 values

Meaning of Highest Common Factor (HCF)

Factors are those positive integral values of a number, which can divide that number. HCF, which is known as Greatest Common Divisor (GCD) also, is the highest value which can divide the given numbers.

Factors of 20 – 1, 2, 4, 5, 10, 20.
Factors of 30 – 1, 2, 3, 5, 6, 10, 15, 30.
So, 10 will be the HCF of 20 and 30.

Process to find out HCF

Step 1 Factorize all the numbers into their prime factors.
Step 2 Collect all the common factors.
Step 3 Raise each factor to its minimum available power and multiply.

Example 7 HCF of 100, 200 and 250

Solution

Step 1 $100 = 2^2 \times 5^2$
$200 = 2^3 \times 5^2$
$250 = 5^3 \times 2^1$
Step 2 2, 5
Step 3 $2^1 \times 5^2 = 50$

Alternatively, to find out HCF of numbers like 100, 200 and 250, one is required to observe the quantity which one can take out common from these numbers. To do this, we can write these numbers as $(100x + 200y + 250z)$ and now it can be very easily observed that we can take 50 common out of these numbers.

Summarizing LCM and HCF

It is very essential to understand the mechanism of find out LCM and HCF. We can simply understand the mechanism to find out lowest common multiple and highest common factor through this example:

Example 8 Find out LCM and HCF of 16, 12, 24.

Solution

No.	Multiples	Factors
16	16, 32, 48, 64, 80, 96, 112, 128,.......	1, 2, 4, 8, 16
12	12, 24, 36, 48, 60, 72, 84, 96, 108,....	1, 2, 3, 4, 6, 12
24	24, 48, 72, 96, 120, 144, 168, 192,....	1, 2, 3, 4, 6, 8, 12, 24

Common Multiple	Common Factor
48	1, 2, 3, 4
Lowest common multiple	Highest common factor
48	4

Standard Formula

1. LCM × HCF = Product of two numbers.

   This formula can be applied only in case of two numbers. However, if the numbers are relatively prime to each other (i.e., HCF of numbers = 1), then this formula can be applied for any number of numbers.

2. LCM of fractions = LCM of numerator of all the fractions/HCF of denominator of fractions.
3. HCF of fractions = HCF of numerator of all the fractions/LCM of denominator of fractions.
4. HCF of (sum of two numbers and their LCM) = HCF of numbers.

Example 9 HCF of two natural numbers A and B is 120 and their product = 10,000. How many set of values of A and B is/are possible?

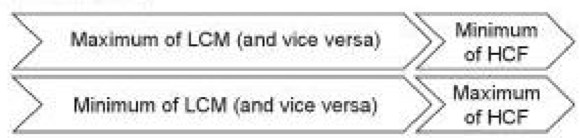
Solution HCF(A, B) = 120 ⇒ 120 is a common factor of both the numbers (120 being the HCF). Hence, 120 is present in both the numbers. So the minimum product of A and B = 120 × 120 = 14400. Hence, no set of A and B are possible satisfying the conditions.

Maxima and Minima in case of LCM/HCF

If product of two numbers is given, and none of LCM or HCF is given, then this gives rise to the case of maxima and minima.

Primarily, the formula that we are going to use is—LCM × HCF = Product of two numbers. Although this formula only provides the basic framework, and to solve these questions we would be required to visualize the situation.

Going by the formula, LCM × HCF = Product of two number, we can say that, since RHS is constant, LHS will be inversely proportional to HCF (subject to the values being natural numbers).



Example 10 Product of two natural numbers = 144. What is the (a) largest possible (b) smallest possible HCF of these two natural numbers?

Solution Let us first factorize $144 = 12 \times 12$
$= (2^2 \times 3) \times (2^2 \times 3)$

Largest possible HCF occurs when LCM = HCF ⇒ when LCM = HCF, numbers are equal.

We already know that product of two natural numbers = LCM × HCF

Since numbers have to be equal, each of the numbers = 12, and largest possible HCF = 12.

(b) Smallest possible HCF, obviously, has to be equal to 1. (Possible set of numbers = 144, 1)

Example 11 Product of two natural numbers = 144. How many different values of LCM are possible for these two natural numbers?

Solution We have already seen in the above question that largest possible value of HCF = 12. And consequently, smallest possible value of LCM = 12.

Let us see the different values of HCF and corresponding values of LCM:

HCF = 12	HCF = 6	HCF = 4	HCF = 3	HCF = 2	HCF = 1
LCM = 12	LCM = 24	LCM = 36	LCM = 48	LCM = 72	LCM = 144

So, total different values of LCM = 6.

❍ SOME QUESTIONS BASED UPON STANDARD APPLICATION OF LCM AND HCF

Case 1 Time and Work

Example 12 Tatto can do a work in 10 days and Tappo can do same work in 12 days. How many days will it take if both of them start working together?

Solution Let us assume total work = LCM of (10, 12) units = 60 units. Now 60 units of work is being done by Tatto in 10 days, so Tatto is doing 6 units of work per day and similarly, Tappo is doing 5 units of work per day. Hence, they are doing 11 units of work in one day together. So, finally they will take $\frac{60}{11} = 5\frac{5}{11}$ days to complete the work.

Case 2 Time, Speed and Distance: Circular Motion

Example 13 Speed of A is 15 m/s and speed of B is 20 m/s. They are running around a circular track of length 1000 m in the same direction. Find after how much time will they meet at the starting point if they start running at same time.

Solution Time taken by A in taking one circle = 66.66 sec

Time taken by B in taking one circle = 50 sec

LCM (66.66, 50) = 200 sec

Case 3 Number System: Tolling the bell

Example 14 There are two bells in a temple. Both the bells toll at a regular interval of 66.66 sec and 50 sec respectively. After how much time will they toll together for the first time?

Solution Time taken by 1st bell to toll = 66.66 sec

Time taken by 2nd bell to toll = 50 sec

LCM (66.66, 50) = 200 sec

It can be observed here that mathematical interpretation of both the questions are same, just the language has been changed.

Case 4 Number System: Number of Rows

Example 15 There are 24 peaches, 36 apricots and 60 bananas and they have to be arranged in several rows in such a way that every row contains same number of fruits of one type. What is the minimum number of rows required for this to happen?

Solution We can put one fruit in one row, and still in (24 + 36 + 60) 120 rows, we can arrange all the fruits. Or, even we can put two fruits in one row and can arrange all the fruits in 60 rows. But for the rows to be minimum, number of fruits should be maximum in one row.

HCF of 24, 36, 60 = 12, so 12 fruits should be there in one row.

Hence, number of rows = 10

Case 5 Number System: Finding Remainder

Example 16 There is a number which when divided by 4 and 5 gives 3 as the remainder. What is the lowest three digit number that satisfies this condition?

Solution Let us assume that there is no remainder. So, number has to be a multiple of LCM of 4 and 5. Now, LCM (4, 5) = 20

But there is a remainder of 3 when divided by 4 and 5. So, the number will be in the form of $(20N + 3)$.

Hence, numbers are 23, 43, 63, 83, 103 and so on....

So, 103 is the answer.

❍ DIVISIBILITY RULES (FOR DECIMAL SYSTEM)

Divisibility rules are quite imperative because with the help of this, we can infer if a particular number is divisible by other number or not, without actually dividing it.

Divisibility rules of numbers are specific to that particular number only. It simply means that divisibility rules of different numbers will be different. We shall now see a list of divisibility rules for some of the natural numbers:

Divisibility Rules

For 2 If unit digit of any number is 0, 2, 4, 6 or 8, then that number will be divisible by 2.

For 3 If sum total of all the digits of any number is divisible by 3, then the number will be divisible by 3. (E.g., 123, 456, etc.)

Example 17 How many values of A are possible if $3245684A$ is divisible by 3?

Solution Sum total of the number = $32 + A$

For this number to be divisible by 3, A can take three values namely 1 or 4 or 7. (No other values are possible since A is the unit digit of the number)

For 4 If last two digit of a number is divisible by 4, then that number will be divisible by 4. (E.g., 3796, 248, 1256, etc.)

For 5 If last digit of the number is 5 or 0, then that number will be divisible by 5.

For 6 If last digit of the number is divisible by two and sum total of all the digits of number is divisible by 3, then that number will be divisible by 6.

For 7 The integer is divisible by 7 if and only if the difference of the number of its thousands and the remainder of its divisible by 1000 is divisible by 7.

For 7 If the difference between the numbers of tens in the number and twice the unit digits divisible by 7 then the given number is divisible by 7.

Example: Let us take the number 795. The units digit is 5 and when it is doubled it, we get 10. The remaining part of the number (i.e., the tens) is 79. If 10 is subtracted from 79 we get 69. Since this result is not divisible by 7, the original number 695 is not divisible by 7.

For 8 If last 3 digits of number is divisible by 8, then the number itself will be divisible by 8. E.g., 128, 34568, 76232, etc.

For 9 If sum of digits of the number is divisible by 9, then the number will be divisible by 9. E.g., 129835782.

$1 + 2 + 9 + 8 + 3 + 5 + 7 + 8 + 2 = 45$. Since 45 is divisible by 9, number will be divisible by 9.

Example 18 How many pairs of A and B are possible in number $89765A4B$ if it is divisible by 9, given that last digit of number is even?

Solution Sum of digits of number is $8 + 9 + 7 + 6 + 5 + A + 4 + B = 39 + A + B$.

So, $A + B$ should be 6 or 15. Next value should be 24 but since A and B are digits so it cannot be more than 18. Possible pairs of A and B are:

A	B
0	6
1	5
2	4
3	3
4	2
5	1
6	0
7	8
8	7
9	6
6	9

Since B is even, six possible set of values of A and B are there.

For 11 A number is divisible by 11, if the difference between the sum of the digits at even places and the sum of the digits at odd places is divisible by 11 (zero is divisible by 11).

Example: 6595149 is divisible by 11 as the difference of $6 + 9 + 1 + 9 = 25$ and $5 + 5 + 4 = 14$ is 11.

For 12 If the number is divisible by 3 and 4, then the number will be divisible by 12. E.g., 144, 348.

For 13 $(A + 4B)$, where B is the unit's place digit and A is all the remaining digits.

Example: Checking the divisibility of 1404 by 13: Here $A = 140$ and $B = 4$, then $A + 4B = 140 + 4 \times 4 = 156$. This 156 is divisible by 13, so 1404 will be divisible by 13.

For 14 If the number is divisible by 2 and 7 both, then the number will be divisible by 14.

For 15 A number is divisible by 15, if the sum of the digits is divisible by 3 and unit digit of the number is 0 or 5.

Example: 225, 450, 375, etc.

For 16 A number is divisible by 16, if the number formed by the last 4 digits of the given number is divisible by 16.

Example: 12578320 is divisible by 16, since last 4 digits of the number, 8320 is divisible by 16.

For 17 $(A - 5B)$ – Where B is the unit's place digit and A is all the remaining digits.

For 18 Number should be divisible by 9 and 2 both.

For 19 $(A + 2B)$ Where B is the unit's place digit and A is all the remaining digits.

If the sum of the number of tens in the number and twice the unit digit is divisible by 19, then the number is divisible by 19.

For example, let us take the number 665. The units digit is 5 and when it is doubled, we get 10. The remaining part of the number is 66. If 10 (which is the unit digit doubled) is added to 66 we get 76. Since this result 76 is divisible by 19, it means the original number 665 is also divisible by 19.

For 20 Number should be divisible by 4 and 5.

Process to find out the divisibility rule for Prime numbers

Process is simple, but difficult to express in words. Let us see. We are creating the divisibility rule for P, a prime number.

Step 1 Find the multiple of P, closest to any multiple of 10. (This will be essentially of the form $10K + 1$ or $10K - 1$.)

Step 2 If it is $10K - 1$, then the divisibility rule will be $A + KB$, and if it is $10K + 1$, then the divisibility rule will be $A - KB$, where B is the unit's place digit and A is all the remaining digits.

Example Finding out the divisibility rule of 23: Lowest Multiple of 23, which is closest to any multiple of 10 = 69 = 7 × 10 − 1

So, rule is $A + 7B$.

Number of Divisors

If one integer can be divided by another integer an exact number of times then the first number is said to be a **multiple** of the second, and the second number is said to be a **factor** of the first.

For example, 48 is a **multiple** of 6 because 6 goes into 48 an exact number of times (8 times in this case). In other words, if I have 48 apples, I can distribute this among 6 persons equally without cutting any apple.

Similarly, 6 is a **factor** of 48. On the other hand, 48 is not a multiple of 5, because 5 does not go into 48 an exact number of times. So, 5 is not a factor of 48.

When we talk about number of divisors of any number, we are talking about positive integral divisor of that number.

For example, it can be observed that 20 has six divisors namely, 1, 2, 4, 5, 10 and 20.

Mechanism of Formation of Divisors

$20 = 2^2 \times 5^1$

Now start thinking that 20 will be divisible by which numbers:

$\frac{2^2 \times 5^1}{7}$ Yes/No

$\frac{2^2 \times 5^1}{2^1}$ Yes/No

$\frac{2^2 \times 5^1}{2^3}$ Yes/No

$\frac{2^2 \times 5^1}{2^1 \times 5^1}$ Yes/No

Answer to the above posers can be given in the following order—No, Yes, No, Yes.

We can observe that denominator should have powers of only 2 and 5—powers of 2 should be from 0–2 and powers of 5 should be 0–1.

$$\frac{2^2 \times 5^1}{2^{0-2} \times 5^{0-1}}$$

Hence, we will take three powers of 2 viz. 2^0, 2^1 and 2^2 and two powers of 5 viz., 5^0 and 5^1.

Divisors will come from all the possible arrangements of powers of 2 and powers of 5.

$2^0 \times 5^0 = 1$
$2^0 \times 5^1 = 5$
$2^1 \times 5^0 = 2$
$2^1 \times 5^1 = 10$
$2^2 \times 5^0 = 4$
$2^2 \times 5^1 = 20$

Summarizing the above, following formula can be derived:

If N is any number which can be factorized like $N = a^p \times b^q \times c^r \times$ ......, where a, b and c are prime numbers.

Number of divisors = $(p + 1)(q + 1)(r + 1)$.................

Example 19 Find the number of divisors of $N = 420$.

Solution $N = 420 = 2^2 \times 3^1 \times 7^1 \times 5^1$

So, number of divisors = $(2 + 1)(1 + 1)(1 + 1)(1 + 1) = 24$

Example 20 Find the total number of even and prime divisors of $N = 420$.

Solution $N = 420 = 2^2 \times 3^1 \times 7^1 \times 5^1$

Odd divisors will come only if we take zero power of 2 (since any number multiplied by any power (≥ 1) of 2 will give us an even number)

So, odd divisors will come if we take $N_1 = 2^0 \times 3^1 \times 7^1 \times 5^1$

So, number of odd divisors = $(0 + 1)(1 + 1)(1 + 1)(1 + 1) = 8$

So, total number even divisors = Total number of divisors − Number of odd divisors = 24 − 8 = 16

Alternatively, we can also find out the number of even divisors of $N = 420$ directly (Or, in general for any number).

$$420 = 2^2 \times 3^1 \times 7^1 \times 5^1$$

To obtain the factors of 420 which are even, we will not consider 2^0, since $2^0 = 1$

So, number of even divisors of 420 = $(2)(1 + 1)(1 + 1)(1 + 1) = 16$

(We are not adding 1 in the power of 2, since we are not taking 2^0 here, i.e., we are not taking one power of 2.)

Prime divisor = 4 (namely 2, 3, 5 and 7 only)

Example 21 $N = 2^7 \times 3^5 \times 5^6 \times 7^8$. How many factors of N are divisible by 50 but not by 100?

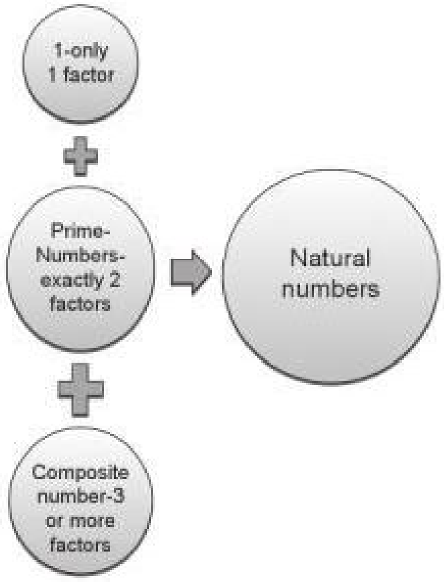
Solution All the factors which are divisible by 50 but not divisible by 100 will have at least two powers of 5, and one power of 2.

And its format will be $2^1 \times 5^{2+y}$

So, number of divisors = 1 × 6 × 5 × 9 = 270

Finding Prime Factors and Composite Factors

We know that natural number line (starting from 1, 2, 3, ...) can be classified on the basis of number of factors to the natural number.



Above graphics also shows that:

i. On the basis of number of factors, natural number line can be categorized in three parts: (a) 1, (b) Prime Number, (c) Composite Factors
ii. Lowest composite number = 4.

Essence of the whole discussion lies in the fact that total number of factors of any natural number = 1 (number 1 is a factor of all the natural numbers) + prime factors + composite factors.

So once we have done the prime factorization, to find out the number of prime factors, we just need to count the number of prime factors are there. To calculate the number of composite factors, we will subtract the number of prime factors and 1 from the total number of factors.

Example 22 Find the number of prime factors and composite factors of $N = 420$?

Solution $420 = 2^2 \times 3^1 \times 5^1 \times 7^1$

Number of prime factors = 4 (namely 2, 3, 5, 7).

Total number of factors = $(2 + 1)(1 + 1)(1 + 1)(1 + 1) = 3 \times 2 \times 2 \times 2 = 24$

So, total number of composite factors = Total number of factors − Prime factors − 1 = 24 − 4 − 1 = 19

Finding Factors which are Perfect Squares or Cubes or Higher Power

A number will be perfect square only if all the prime factors of this number will have even powers. So a number of the format 2^x will be a perfect square only if $x = 0, 2, 4, 6, 8$, etc.

And similarly, a number will be perfect cube only if all the prime factors of this number will have powers divisible by 3. So a number of the format 2^x will be a cube only if $x = 0, 3, 6, 9$, etc.

Example 23 How many factors of the number $N = 720$ will be (a) perfect square, (b) cube, (c) a perfect square and cube both?

Solution $N = 720 = 2^4 \times 3^2 \times 5^1$

(a) For a factor of $N = 720$ to be a perfect square, it should have only the following powers of its prime factors:

Powers of 2	Powers of 3	Powers of 5
2^0	3^0	5^0
2^2	3^2	
2^4		

Number of powers of 2 used = 3
Number of powers of 3 used = 2
Number of powers of 5 used = 1
Hence, total number of factors of $N = 720$ that are perfect square = $3 \times 2 \times 1 = 6$

(b) For a factor of $N = 720$ to be a cube, it should have only the following powers of its prime factors:

Powers of 2	Powers of 3	Powers of 5
2^0	3^0	5^0
2^3		

Number of powers of 2 used = 2
Number of powers of 3 used = 1
Number of powers of 5 used = 1
Hence, total number of factors of $N = 720$ that are cubes $= 2 \times 1 \times 1 = 2$

(c) For a factor of $N = 720$ to be a cube and a square both, it should have only the following powers of its prime factors:

Powers of 2	Powers of 3	Powers of 5
2^0	3^0	5^0

Number of powers of 2 used = 1
Number of powers of 3 used = 1
Number of powers of 5 used = 1
Hence, total number of factors of $N = 720$ that are cubes $= 1 \times 1 \times 1 = 1$

Condition for two Divisors of any Number *N* to be co-prime to Each other

Two numbers are said to be co-prime to each other if their HCF = 1. This can happen only if none of the factors of first number (other than 1) is present in the 2nd number and vice versa.

Let us see it for $N = 12$

Total number of factors of 12 = 6 (namely 1, 2, 3, 4, 6, 12)

Now if we have to find out set of factors of this number which are co-prime to each other, we can start with 1.

Number of factors which are co-prime to 1 = 5 (namely, 2, 3, 4, 6, 12)

Next in the line is the number of factors which are co-prime to 2 = 1 (namely 3)

So total number of set of factors of 12 which are co-prime to each other = 6

So, we can induce that if we have to find out the set of factors which are co-prime to each other for $N = a^p \times b^q$, it will be equal to $[(p + 1)(q + 1) - 1 + pq]$.

If there are three prime factors of the number viz., $N = a^p \times b^q \times c^r$, then set of co-prime factors can be given by $[(p + 1)(q + 1)(r + 1) - 1 + pq + qr + pr + 3pqr]$

Alternatively, we can find out set of co-prime factors of this number by pairing up it first and then finding it out with the third factor.

Example 24 Find the set of co-prime factors of the number $N = 720$.

Solution $720 = 2^4 \times 3^2 \times 5^1$

Using the formula for three prime factors $[(p + 1)(q + 1)(r + 1) - 1 + pq + qr + pr + 3pqr]$

We get, $[(4 + 1)(2 + 1)(1 + 1) - 1 + 4.2 + 2.1 + 4.1 + 3.4.2.1] = 67$

Alternatively, let us find it out first for $2^4 \times 3^2 = [(4 + 1)(2 + 1) - 1 + 4.2] = 22$

Now $p^{22} \times 5^1$ will give us $[(22 + 1)(1 + 1) - 1 + 22.1] = 67$

Sum of Divisors

Like number of divisors of any number, we can find out the sum of divisors also.

If N is any number which can be factorized like $N = a^p \times b^q \times c^r \times ......$, where a, b and c are prime numbers.

Then sum of the divisors = $\dfrac{(a^{p+1} - 1)(b^{q+1} - 1)(c^{r+1} - 1)}{(a - 1)(b - 1)(c - 1)}$

Remainders

Dividend = Quotient × Divisor + Remainder

Basic framework of remainder

i. If N is a number divisible by 7, it can be written as: $7K = N$, where K is the quotient.

ii. When N is divided by 7, remainder obtained is 3 $\Rightarrow$ it can be written as: $7K + 3 = N$, where K is the quotient.

iii. When N is divided by 7, remainder obtained is 3 is equivalent of saying remainder obtained is (–4) when divided by 7. It can be understood that When N is divided by 7, remainder obtained is 3 $\Rightarrow N$ is 3 more than a multiple of 7 $\Rightarrow$ So N is 4 short of another multiple of 7. So remainder obtained = –4.

iv. When divided by 8, different remainders obtained can be = 0, 1, 2, 3, 4, 5, 6, 7 (8 different remainders)
Similarly, when divided by 5, different remainders obtained can be = 0, 1, 2, 3, 4 (5 different remainders)

❍ BASICS OF REMAINDER

1. When any positive number A is divided by any other positive number B, and if $B > A$, then the remainder will be A itself. In other words, if numerator is smaller than denominator, then numerator is the remainder.

   E.g., Remainder of $\dfrac{5}{12} = 5$

   Remainder of $\dfrac{21}{45} = 21$

2. Remainder should always be calculated in its actual form, i.e., you can not reduce the fraction to its lower ratio.

   E.g., Remainder of 1/2 = 1
   Remainder of 2/4 = 2
   Remainder of 3/6 = 3

   It can be observed that despite all the fractions being equal, remainders are different in each case.

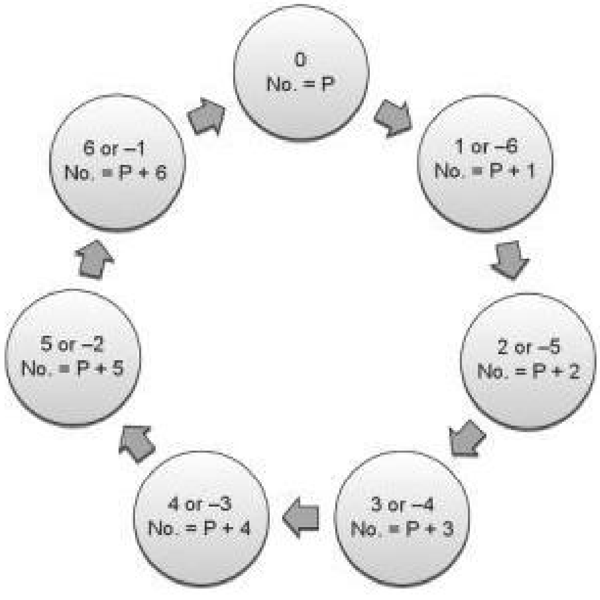
Example 25 What is the remainder when 5×10^5 is divided by 6×10^6?

Solution As we know that we cannot reduce the fractions to its lower terms and numerator is less than denominator, remainder obtained will be equal to 5×10^5.

3. Concept of negative remainder—As obvious from the name, remainder implies that something has been left out or something remains there. So, remainder simply can never be negative. Its minimum value can be zero only and not negative.

   E.g., What is the remainder when –50 is divided by 7?

   Solution → $\dfrac{-50}{7} = \dfrac{-56+6}{7}$; which gives a remainder of 6.
   Or, when we divide –50 by 7, we get –1 as the remainder. Now, since remainder has to be non-negative, so we add 7(quotient) to it which makes final remainder as → –1 + 7 = 6.
   It can be seen below also:
   Assume that when P is divided by 7, remainder obtained = 0.



So when $P + 1$ will be divided by 7, remainder obtained will be either 1 or –6. Similarly, when $P + 2$ is divided by 7, remainder obtained will be 2 or –5, and so on.

Now, there are two methods to find out the remainder of any expression:

1. Cyclicity Method
2. Remainder Theorem Method

1. Cyclicity Method

For every expression of remainder, there comes attached a specific cyclicity of remainders.

Example 26 What is the remainder when 4^{1000} is divided by 7?

Solution To find the cyclicity, we keep finding the remainders until some remainder repeats itself. It can be understood with the following example:

Number/7	→	4^1	4^2	4^3	4^4	4^5	4^6	4^7	4^8
Remainder	→	4	2	1	4	2	1	4	2

Now, 4^4 gives us the same remainder as 4^1, so the cyclicity is of 3 (Because remainders start repeating themselves after 4^3.)

So, any power of 3 or a multiple of 3 will give remainder of 1. So, 4^{999} will give 1 as the remainder.

Final remainder = 4

Example 27 What is the remainder when 4^{96} is divided by 6? ***(CAT 2003)***

Solution: Finding out the cyclicity,

Number/6	→	4^1	4^2	4^3	4^4	4^5	4^6	4^7	4^8
Remainder	→	4	4	4	4	4	4	4	4

Remainder in all the cases is 4, so final remainder will be 4. Actually, we are not needed to find remainders till 4^8 or even 4^3. 4^2 itself gives us a remainder of 4 when divided by 6, which is same as the remainder obtained when 4^1 is divided by 6. So, length of cycle = 1.

Hence, final remainder = 4

It also can be observed here that if we write $4^{100}/6 = 2^{200}/6 = 2^{199}/3$, then remainder obtained will be 2, which is not the right answer (as given in the CAT brochure of next year, i.e., CAT 2004.)

2. Remainder Theorem Method

Product of any two or more than two natural numbers has the same remainder when divided by any natural number, as the product of their remainders.

Lets understand this through an example:

Example 28 Remainder $\frac{12 \times 13}{7}$ = Remainder $\frac{156}{7}$ = 2

Solution

Normal way of doing this is—Product →→→ Remainder

Theorem method: Remainder →→→
Product →→→ Remainder

So, first of all we will find out the remainders of each individual number and then we will multiply these individual remainders to find out final remainder.

Remainder 12/7 = 5

Remainder 13/7 = 6

Remainder $\frac{12 \times 13}{7}$ = Remainder $(5 \times 6)/7$ = Remainder 30/7 = 2

Example 29 What is the remainder obtained when $(1421 \times 1423 \times 1425)$ is divided 12? ***(CAT 2000)***

Solution Remainder of 1421/12 = 5
Remainder of 1423/12 = 7
Remainder of 1425/12 = 9

Remainder $(1421 \times 1423 \times 1425)/12$ = Remainder $(5 \times 7 \times 9)/12$ = Remainder $(5 \times 63)/12$ = Remainder $(5 \times 3)/12 = 3$

❍ CONCEPT OF SUCCESSIVE DIVISION

Suppose we say that N is any number which is divided successively by 3 and 5, then what we mean to say is—At first, we divide N by 3 and then the quotient obtained is divided by 5.

Example: Let us see the case when 50 is divided by 5 and 3 successively.

50 divided by 5 gives 10 as the quotient. Now, we will divide 10 by 3. It gives finally a quotient of 3 and remainder of 1.

Example 30 When a number N is divided successively by 3 and 5, remainder obtained are 1 and 2 respectively. What is the remainder when N is divided by 15?

Solution It can be seen that we are required to calculate it from back-end.

Family of numbers which when divided by 5 gives remainder $2 = 5S + 2$

So, $N = 3(5S + 2) + 1 = 15S + 7$

Now, when N is divided by 15, remainder = 7

❍ FERMAT'S REMAINDER THEROREM

Let P be a prime number and N be a number not divisible by P. Then remainder obtained when A^{P-1} is divided by P is 1.

(Remainder obtained when $\frac{A^{P-1}}{P} = 1$, if HCF $(A, P) = 1$)

Example 31 What is the remainder when 2^{100} is divided by 101?

Solution Since it satisfies the Fermat's theorem format, remainder = 1

Derivations

1. $(A + 1)^N$ will always give 1 as the remainder.(For all natural values of A and N) A

Example 32 What is the remainder when 9^{100} is divided by 8?

Solution For $A = 8$, it satisfies the above condition. So, remainder = 1

Alternatively, we can apply either of cyclicity or theorem method to find the remainder. (Do this yourself).

2. A^N When N is even, remainder is 1 and when N is odd, remainder is A itself.
   $A + 1$

Example 33 What is the remainder when 2^{10} is divided by 3?

Solution Since here N is even, so remainder = 1

3. i. $(a^n + b^n)$ is divisible by $(a + b)$, if n is odd.
   Extension of the above formula $(a^n + b^n + c^n)$ is divisible by $(a + b + c)$, if n is odd and a, b and c are in Arithmetic Progression.

Example 34 What is the remainder obtained when $\frac{7^7 + 10^7 + 13^7 + 16^7}{46}$?

Solution It can be seen that 7, 10, 13 and 16 are in Arithmetic Progression and power n = Odd. Further, denominator = 7 + 10 + 13 + 16 = 46. Hence, it will be divisible. Hence, remainder obtained = 0.

Similarly, the above situation can be extended for any number of terms.

ii. $(a^n - b^n)$ is divisible by $(a + b)$, if n is even.

iii. $(a^n - b^n)$ is divisible by $(a - b)$, if n is even.

Example 35 What is the remainder when $(15^{23} + 23^{23})$ is divided by 19? ***(CAT 2004, 2 marks)***

Solution It can be observed that $(15^{23} + 23^{23})$ is divisible by 38, so it will be divisible by 19 also. Hence, remainder = 0.

Alternatively, this problem can be done either by cyclicity method or theorem method.

Example 36 What is the remainder when $(16^3 + 17^3 + 18^3 + 19^3)$ is divided by 70? ***(CAT 2005, 1 mark)***

Solution We know, this is a basic multiplication and division question. But using the above approach makes it a lot simple.

We know that $(a^n + b^n)$ is divisible by $(a + b)$, if n is odd. Taking cue from this we can say that $(a^n + b^n + c^n)$ is divisible by $(a + b + c)$, if n is odd and similarly $(a^n + b^n + c^n + d^n)$ is divisible by $(a + b + c + d)$. Now 16 + 17 + 18 + 19 = 70, so remainder is zero.

Some More Types of Problems

	Problem states that.....	*Solution*
1	Find the greatest number that will exactly divide a, b and c	Required number = HCF of a, b and c
2	Find the greatest number that will divide x, y and z leaving remainders a, b and c respectively.	Required number (greatest divisor) = HCF of $(x - a)$, $(y - b)$ and $(z - c)$
3	Find the least number which is exactly divisible by a, b and c	Required number = LCM of a, b and c
4	Find the least number which when divided by x, y and z leaves the remainders a, b and c respectively, and $(x - a) = (y - b) = (z - c) = N$	Required number = LCM of (x, y and z) $- N$
5	Find the least number which when divided by x, y and z leaves the same remainder 'r' each case.	Required number = (LCM of x, y and z) + r

❍ UNIT DIGIT

As we have seen the cyclicity of remainders above, cyclicity exists for Unit digit of the numbers also.(But always keep in your mind that there is no relation between the cyclicity of remainders and unit digit.) Taking a very simple example $-2^5 = 32$, and so we know that unit digit of 2^5 is 2. But problem occurs when we start getting big numbers like 25678^{2345}, etc. To find out the unit digit of these kinds of numbers, we have some standard results, which we use as formula.

(Any even number)^{4n} = ...6

It means that any even number raised to any power, which is a multiple of 4, will give us 6 as the unit digit.

(Any odd number)^{4n} = ...1

It means that any odd number raised to any power, which is a multiple of 4, will give us 1 as the unit digit.

Exception: 0, 1, 5, 6 [these are independent of power, and unit digit will be the same respectively]

Example 37 Find the unit digit of $25678^{2345} \times 3485^{4857}$.

Solution Unit digit of 25678^{2345} = Unit digit of 8^{45}

(To find out unit digit, we need to have unit digits only. And similarly, to find out tens digit we need to have the tens and units digit only. In the present case, we are considering only last two digits of the power because divisibility rule of 4 needs only the last two digits of the number)

$$8^{45} = 8^{44+1} = 8^{44} \times 8^1 = (\ldots\ldots\ldots 6) \times 8 = \ldots\ldots 8$$

Example 38 What is the unit digit of $_{32}32^{32}$?

Solution 32 is an even number which is having a power of the form $4n$. So, it will give 6 as the unit digit.

Example 39 When 3^{32} is divided by 50, it gives a number of the format ($asdf$........•xy) (xy being the last two digits after decimal). Find y.

Solution It can be observed that unit digit of 3^{32} = 1. Now any number having 1 as the unit digit will always give 2 at the unit place when divided by 50.

So, answer is 2.

Example 40 What is the last non-zero digit of the number 30^{2720}? ***(CAT 2005, 2 marks)***

Solution $30^{2720} = [30^4]^{680} = \ldots 1$

Unit digit can also be found out by cyclicity method as well.

It can be seen that

Unit digit of $2^1 = 2$

Unit digit of $2^2 = 4$

Unit digit of $2^3 = 8$

Unit digit of $2^4 = 6$

Unit digit of $2^5 = 2$

So, it can be inferred that Unit digit of 2^1 = Unit digit of 2^5 = Unit digit of 2^9

Hence, cyclicity of 2 = 4, i.e., every fourth power of 2 will give same unit digit.

Similarly, cyclicity of 3 = 4

Cyclicity of 4 = 2

Cyclicity of 7 = 4

Cyclicity of 8 = 4

Cyclicity of 9 = 2

Cyclicity of 0 or Cyclicity of 1 or Cyclicity of 5 or Cyclicity of 6 = 1

❍ TENS' DIGIT

Method 1: Cyclicity Method

Digits		Cyclicity
2, 3, 8	–	20
4, 9	–	10
5	–	1
6	–	5
7	–	4

Example 41 What is the tens' place digit of 12^{42}?

Solution For this, we need to break 12^{42} first by using binomial theorem as $(10 + 2)^{42}$. Obviously, this expression will have 43 terms, and out of these 43 terms first 41 terms will have both of their tens and units place digit as 0.

Last two terms will be $\rightarrow {}^{42}C_{41} \times 10^1 \times 2^{41} + {}^{42}C_{42} \times 10^0 \times 2^{42}$

Now we will find the tens place digit of all these terms individually.

Tens digit of ${}^{42}C_{41} \times 10^1 \times 2^{41} = 42 \times 10 \times (02)$ [Cyclicity of 2 is 20, so 2^{41} will have same tens digits as 2^1] = 840, so 40 are the last two digits.

Similarly, ${}^{42}C_{42} \times 10^0 \times 2^{42} = 1 \times 1 \times 04 = 04$

So, finally last two digits are $\rightarrow 40 + 04 = 44$, so 4 is the tens place digit.

> **Note:** $(25)^n$ and $(76)^n$ will always give 25 and 76 as the last two digits for any natural number value of n.

Method 2: Generalization Method

(i) (Any even number)^{20N} will give 76 as its last two digits. [Where N is any natural number]

Exception If unit digit = 0, then it will give '00' as the last two digits.

(ii) (Any Odd number)^{20N} will give 01 as its last two digits. [Where N is any natural number]

Exception If unit digit = 5, then it will give '25' as the last two digits.

Let us solve the previous worked-out example once again using this method.

Example 42 What is the tens' place digit of 12^{42}?

Solution Using Generalization (i) as given above, we get 12^{20} = ...76 (76 as last two digits)

$$12^{20} \times 12^{20} = 12^{40}$$
$$= (\ldots\ldots\ldots\ldots\ldots\ldots 76)$$
$$\times (\ldots\ldots\ldots\ldots\ldots\ldots 76)$$
$$= (\ldots\ldots\ldots\ldots\ldots 76)$$

$$12^{42} = 12^{40} \times 12^2 = (\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 76) \times (144)$$

Since we are required to calculate last two digits, we will focus only upon last two digits of both the numbers.

(………………………76) × (44) = 3344. Hence, 44 is the last two digits of 12^{42}.

Note: we are not sure if 3 is at 100s place of this number.

Example 43 Find the tens place digit of 784^{1000}.

Solution Tens place digit of 784^{1000} = Tens place digit of 84^{1000}

As we have seen above, (any even number)^{20N} will give 76 as the last two digits.

$84^{1000} = (84)^{20\times 50} = (84)^{20N}$. This will have 76 as last two digits.

❍ NUMBER OF EXPONENTS

Let us take a simple number—10^5

This is read as—10 to the power 5, or we say that exponent of 10 is 5 here.

In simple terms, exponents are also known as Power.

Example 44 What is the maximum value of s if $N = (35 \times 45 \times 55 \times 60 \times 124 \times 75)$ is divisible by 5^x?

Solution If we factorize $N = (35 \times 45 \times 55 \times 60 \times 124 \times 75)$, then we can see that 5 appears 6 times, it means N is divisible by 5^6.

So, maximum value of $x = 6$

Exponent of any prime number P in n!

$$= \left[\frac{n}{p}\right] + \left[\frac{n}{p^2}\right] + \left[\frac{n}{p^3}\right] + \ldots\ldots\ldots\ldots \left[\frac{n}{p^x}\right], \text{ where } n \geq p^x \text{ and}$$

[.] denotes the greatest integer value, i.e., we have to consider only the integral value.

Let us find out exponent of 5 in 1000! $= \frac{1000}{5} + \frac{1000}{5^2} + \frac{1000}{5^3} + \frac{1000}{5^4} = 200 + 40 + 8 + 1 = 249$

Example 45 What is the highest power of 5 which can divide $N = (22! + 17894!)$?

Solution Number of times this number is divisible by 5 is same as number of zeroes at the end of this number. Since 22! have 4 zeroes at its end, so N will also be having only four zeroes at its end. Hence, highest power of 5 which can divide N is 4.

Process to find out the exponent of any composite number in n!

We have got three different kinds of composite numbers:

1. Product of two or more than two prime numbers with unit power of all the prime numbers
   E.g., $15(5 \times 3)$, $30(2 \times 3 \times 5)$ etc.
2. (Any prime number)^n, where $n > 1$
   E.g., $4(2^2)$, $27(3^3)$
3. Product of two or more than two prime numbers with power of any one prime number more than 1.
   E.g., $12(2^2 \times 3)$, $72(2^3 \times 3^2)$ etc.

Let us find out the exponents of the above written composite numbers one by one:

1. Let us find out the exponent of 15 in 100!
   15 is the product of two distinct prime numbers 5 and 3. So, to find out the exponents of 15, we need to find out the exponents of 5 and 3 individually.
   So, we will apply the same formula of finding out the exponents for any prime number in both of these cases individually, and minimum of those two will be the answer.
   $100/5^x = [100/5] + [100/5^2] = 20 + 4 = 24$
   $100/3^x = [100/3] + [100/3^2] + [100/3^3] + [100/3^4] = 33 + 11 + 3 + 1 = 48$
   Obviously, 24 is going to be the answer.
2. Let us find out the exponent of 25 in 100!
   $25 = 5^2$
   In this case, we will first find out the exponents of 5 and then divide it by 2 (actually the power) to find out the exponents of 25.
   $100/5^x = [100/5] + [100/5^2] = 20 + 4 = 24$
   So, $100/25^x = 24/2 = 12$
3. Similarly, we can find out for third category numbers also.

❍ BASE SYSTEM

In our decimal system of writing the numbers, we use 10 digits (0 – 9). In this system, largest number of single digit = 9, and the moment we have to form a number bigger than this no, we are needed to take resort to two-digit numbers starting from 10. Similarly, largest number of two digits = 99 and after this we have 100, a number of three digits. And it is very plain and simple.

Now let's assume a system of writing where we use only 6 digits (0 – 5). Largest single digit number in this system will be 5 and next to this will be 10. Similarly, largest two digit number will be 55 and next to this is 100.

This whole procedure can be summed up in the following table:

$(0-9)_{10}$	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
$(0-8)_9$	0	1	2	3	4	5	6	7	8	10	11	12	13	14	15	16	17	18
$(0-7)_8$	0	1	2	3	4	5	6	7	10	11	12	13	14	15	16	17	20	21
$(0-6)_7$	0	1	2	3	4	5	6	10	11	12	13	14	15	16	20	21	22	23
$(0-5)_6$	0	1	2	3	4	5	10	11	12	13	14	15	20	21	22	23	24	25
$(0-3)_4$	0	1	2	3	10	11	12	13	20	21	22	23	30	31	32	33	100	101
$(0-2)_3$	0	1	2	10	11	12	20	21	22	100	101	102	110	111	112	120	121	122

Questions from this concept are asked in three different ways:

1. $(\text{Base})_{10}$ to any other base and vice versa
2. $(\text{Base})_x$ to $(\text{Base})_y$ and vice versa; none of x and y being equal to 10 but x and y will be given.
3. $(\text{Base})_x$ to $(\text{Base})_y$, value of x and y will not be given.

1. $(\text{Base})_{10}$ to any other base and vice versa

Method 1:

Let us see in case of $(74)_{10}$.

$(74)_{10} = 7 \times 10^1 + 4 \times 10^0$, since the base is 10.

Now if we have to convert this number in 9 base, then we will try to write it in terms of powers of 9.

$$(74)_{10} = 8 \times 9^1 + 2 \times 9^0 = (82)_9$$
$$(74)_{10} = 1 \times 8^2 + 1 \times 8^1 + 2 \times 8^0 = (112)_8$$
$$(74)_{10} = 1 \times 7^2 + 3 \times 7^1 + 4 \times 7^0 = (134)_7$$
$$(74)_{10} = 2 \times 6^2 + 0 \times 6^1 + 2 \times 6^0 = (202)_6$$

While converting the numbers from decimal system to any other system of writing the numbers, we should be concerned with following two rules:

i. Take maximum possible power of the base and then keep writing rest of the number with the help of lesser power of base (as illustrated in above example).
ii. Once we have used $(\text{base})^n$, where n is the maximum power, that we will be required to

Write the co-efficients of all the powers of base from 0 to $(n-1)$ as in the case of $(74)_{10} = (202)_6$.

Now, suppose we have to convert $(356)_7$ in the base of 10.

$(356)_7 = 3 \times 7^2 + 5 \times 7^1 + 6 \times 7^0 = (188)_{10}$

Method 2:

Converting $(74)_{10}$ to the base of $(\ )_9$:

Base	74	Remainder
9	8	2

So, $(74)_{10} = (82)_9$

Converting $(74)_{10}$ to the base of $(\ )_8$:

Base	74	Remainder
9	9	2
8	1	1

So, $(74)_{10} = (112)_8$

Converting $(74)_{10}$ to the base of $(\ )_7$:

Base	74	Remainder
7	10	4
7	1	3
	1	

So, $(74)_{10} = (134)_7$

Converting $(74)_{10}$ to the base of $(\ )_6$:

Base	74	Remainder
6	12	2
6	2	0
	Quotient	

So, $(74)_{10} = (202)_6$

Task for students

Convert $(123)_{10}$ into base 9, base 8, base 7, base 15, base 20.

Answer at the end of topic.

2. $(\text{Base})_x$ to $(\text{Base})_y$ and vice versa; none of x and y being equal to 10 but x and y will be given

Converting $(345)_8$ to the base of $(\ )_7$:

We will do this problem with the help of creating a bridge of base 10 between base 8 and base 7.

Step 1 Convert $(345)_8$ into base 10.
$345 = 3 \times 8^2 + 4 \times 8^1 + 5 \times 8^0 = (229)_{10}$

Step 2 Now convert this number in base 10 into base 9.
$(229)_{10} = 2 \times 9^2 + 7 \times 9^1 + 4 \times 9^0 = (274)_9$

However, if new base is a power of old base and vice versa, then it can be converted directly also in the new base, i.e., we are not needed to go to base 10 for these kinds of conversions.

E.g., for, $(\text{Base})_2$ to $(\text{Base})_4$ or $(\text{Base})_2$ to $(\text{Base})_8$—conversion does not require a bridge of base 10.

Converting $(101110010)_2$ to Octal $(\ )_8$ system:

At first we will club three digits of binary number into a single block and then will write the decimal equivalent of each group (left to right).

So, $(101110010)_2$ is now $(101)_2(110)_2(010)_2$
Now, $(101)_2 = 1 \times 2^2 + 0 + 1 \times 2^0 = 5$
$(110)_2 = 1 \times 2^2 + 1 \times 2^1 + 0 \times 2^0 = 6$
$(010)_2 = 0 \times 2^2 + 1 \times 2^1 + 0 \times 2^0 = 2$
So, $(101110010)_2 = (562)_8$

Converting $(101110010)_2$ to Hexa-decimal $(\ )_{16}$ system:

At first, we will club four digits of binary number into a single block and then will write the decimal equivalent of each group (left to right).

So, $(101110010)_2$ is now $(0001)_2(0111)_2(0010)_2$
Now, decimal equivalent of $(0001)_2 = 1$
Decimal equivalent of $(0111)_2 = 7$
Decimal equivalent of $(0010)_2 = 2$
$(101110010)_2 = (172)_{16}$

3. $(\text{Base})_x$ to $(\text{Base})_y$, value of x and y will not be given

In these types of questions, normally some calculation is given in some unknown system of writing numbers and on the basis of that we will be required to solve questions based upon that.

Example 46 In a system of writing of N digits, $4 \times 6 = 30$ and $5 \times 6 = 36$. What will be the value of $N = 3 \times 4 \times 5$ in the same system of writing?

Solution Let us assume that there are N digits in this system of writing.

So, $(30)_N = 3 \times N^1 + 0 \times N^0 = 24$
$\Rightarrow$ $3N = 24$
$\Rightarrow$ $N = 8$

So, this system of writing has 8 digits.

In this system $3 \times 4 \times 5 = 60$ will be written as 74. $(60 = 7 \times 8^1 + 4 \times 8^0)$

Alternatively, since this system is having 6 as one of its digits, so minimum value of N will be 7. Again, 24 is written as 30 in this system, so N is less than 10. Now use hit and trial for $N = 7$ or 8 or 9 to find out N in $24 = (30)_N$

❍ DECIMAL CALCULATION

So far we have seen the calculations involving natural numbers only. Let us work now with decimals.

Converting decimal system numbers to any other system:

Suppose (12.725) is a number in decimal system which is required to be converted into octal system (8 digits).

We will first convert 12 into octal system.

$(12)_{10} = (14)_8$

Now to convert $(0.725)_{10}$ into $(\ )_8$, we will apply following method:

$0.725 \times 8 = \mathbf{5}.8$	Take out integral part from here.
$0.8 \times 8 = \mathbf{6}.4$	Take out integral part from here.
$0.4 \times 8 = \mathbf{3}.2$	Take out integral part from here.
$0.2 \times 8 = \mathbf{1}.6$	Take out integral part from here.

And keep doing this till the moment we get decimal part as zero, i.e., the product should be an integer.

$(0.725)_{10} = (0.5632\ldots\ldots)_8$
So, $(12.725) = (14.5632\ldots\ldots)_8$

Converting any other system numbers to decimal system:

Now suppose if $(15.453)_7$ is to be converted into decimal system, then the process is as follows:

We will first convert $(15)_7$ into decimal system.

$(15)_7 = 1 \times 7^1 + 5 \times 7^0 = (12)_{10}$
Now $(0.453)_7 = 4 \times 7^{-1} + 5 \times 7^{-2} + 3 \times 7^{-3}$
So, $(15.453)_7 = (12.)_{10}$

Basic Algebraic Calculations Involving Base Systems

Addition

$$\begin{array}{r} 325_7 \\ +\,456_7 \\ \hline \end{array}$$

Start with the units place digit, $5 + 6 = 11$ which is 14_7. So, unit digit is 4 and carry over is 1.

Next is tens place digit, $2 + 5 + 1$ (carry over) $= 8$ which is 11_7. So, tens digit is 1 and carry over is again 1.

Next is $3 + 4 + 1$ (carry over) $= 8$ which is 11_7.

$$\begin{array}{r} 325_7 \\ +\,456_7 \\ \hline 1114_7 \\ \hline \end{array}$$

Subtraction

$456_8 - 367_8$

Starting with the units digit, since 6 is smaller than 7, we will borrow 1 from the tens place digit. So, now it is 14 (when the base is 10, we get 10 but here base is 8, so will get 8.) and 7 subtracted from it = 14 − 7 = 7, which is the units digit.

Next, tens digit is now 4 and we have to subtract 5 from it. We will again borrow 1 from hundred's place digit. So, now it is 12, and 12 − 6 = 6, which is the tens place digit.

Now, hundred's place digit is 3(4 − 1), so 3 − 3 = 0

$$\begin{array}{r} 456_8 \\ -\,367_8 \\ \hline 67_8 \end{array}$$

Note: Another method of doing these kinds of calculations is to convert these values (in whatever base) into decimal system, then do the actual calculation in decimal system itself and finally converting the numbers into the required or given system.

Some standard system of writing:

Decimal system
Digits used—0, 1, 2, 3, 4, 5, 6, 7, 8, 9.
Total digits used = 10 digits

Hexa-decimal system
Digits used—0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F.
Total digits used = 16

Octal system
Digits used—0, 1, 2, 3, 4, 5, 6, 7.
Total digits used = 8

Binary system
Digits used—0, 1
Total Digits used—2

Divisibility Rules for Systems Other than Decimal System

I would like to emphasize that different number systems are just different ways to write numbers. Thus the divisibility of one number by another does not depend on the particular system in which they are written.

At the same time, in each system there are some tricks to determine divisibility by certain specific numbers. These are the divisibility tests.

Let us investigate now other, less trivial, divisibility tests. Perhaps the most well-known of these are the tests for divisibility by 3 and 9. We will try to generalize these tests for any number base system.

Is 123456564231_7 divisible by 6?

We know the divisibility rule for 9—Sum of digits of the number should be divisible by 9.

Sum of digits of this number is 42.

Now we can answer this question easily: since the sum of the digits (which is 42_{10}) is divisible by 6, so the number itself is also divisible by 6.

In general,

Thus the sum of the digits of a number written in the base *n* system is divisible by (*n* − 1) if and only if the number itself is divisible by (*n* − 1).

So, divisibility rule for 4 in a base system of 5—Sum of digits of the number should be divisible by 4. For example, 31_5 is divisible by 4.

Similarly, if we have to find out the divisibility rule of 12 in the base of 11, it will be nothing but same as the divisibility rule of 11 in the base of 10. Generalizing this whole concept, divisibility rule of any natural number N in the base of $(N - 1)$ will be same as divisibility rule of 11 on base 10.

Pigeon-Hole Principle

Despite not being very much in vogue with respect to the CAT preparation (only a few questions have been asked from this concept so far in CAT) importance of this topic lies in the fact that this concept is purely logical.

General Statement of Pigeon Hole Principle

If we put $(N + 1)$ or more pigeons in N holes (nests), then at least one hole will be there which will have 2 or more pigeons.

Example 47 What is the minimum number of people in any group of five people who have an identical number of friends within the group, provided if A is friend of B, then B is also friend of A?

Solution Since there are five persons in the group, so possible number of friends is 0, 1, 2, 3, 4. It seems here that everybody is having different number of friends, so answer is zero. But anybody having four friends ensures that nobody is having 0 friends. So, at least two persons must have same number of friends.

PRACTICE EXERCISES

WARM UP

Q.1. Which of the following is the smallest?
(a) $5^{1/2}$ (b) $6^{1/3}$ (c) $8^{1/4}$ (d) $12^{1/6}$

Q.2. A number N is divisible by 6 but not divisible by 4. Which of the following will not be an integer?
(a) N/3 (b) N/2 (c) N/6 (d) N/12

Q.3. If a, b and c are consecutive positive integers, then the largest number which always divides $(a^2 + b^2 + c^2)$
(a) 14 (b) 55
(c) 3 (d) None of these

Q.4. $\dfrac{(3.134)^3 + (1.866)^3}{(3.134)^2 - 3.134 \times 1.866 + (1.866)^2} = ?$
(a) 25 (b) 2.68 (c) 1.038 (d) 5

Q.5. If n^2 is a perfect cube, then which of the following statements is always true?
(a) n is odd
(b) n is even
(c) n^3 is a perfect square
(d) n is a perfect cube

Q.6. If $(5x + 11y)$ is a prime number for natural number values of x and y, then what is the minimum value of $(x + y)$?
(a) 2 (b) 3 (c) 4 (d) 5

Q.7. For what values of x is $25^x + 1$ divisible by 13?
(a) All real values of x
(b) Odd natural values of x
(c) Even values of x
(d) All the integral values of x

Q.8. Which of the following numbers lies between 5/6 and 6/7?
(a) 71/84 (b) 31/42 (c) 129/168 (d) 157/339

Q.9. By multiplying with which of the following numbers does the product of $8 \times 9 \times 10 \times 11 \times 12$ become a perfect square?
(a) 55 (b) 11 (c) 165 (d) 310

Q.10. What is the difference between the sum of the cubes and that of squares of the first ten natural numbers?
(a) 5,280 (b) 2,640 (c) 3,820 (d) 4,130

Q.11. If $3 - 9 + 15 - 21 + \dots$ up to 19 terms $= x$, then x is a/an
(a) odd number (b) even number
(c) prime number (d) irrational number

Q.12. What is the unit's digit of $21^3 \times 21^2 \times 34^7 \times 46^8 \times 77^8$?
(a) 4 (b) 8 (c) 6 (d) 2

Q.13. If the unit's digit in the product $(47n \times 729 \times 345 \times 343)$ is 5, what is the maximum number of values that n may take?
(a) 9 (b) 3 (c) 7 (d) 5

Q.14. In how many ways can 846 be resolved into two factors?
(a) 9 (b) 11
(c) 6 (d) None of these

Q.15. If a number is divided by 15, it leaves a remainder of 7. If thrice the number is divided by 5, then what is the remainder?
(a) 5 (b) 6 (c) 7 (d) 1

Q.16. A number when divided by 391 gives a remainder of 49. Find the remainder when it is divided by 39.
(a) 10 (b) 9
(c) 11 (d) Cannot be determined

Q.17. p and q are two prime numbers such that $p < q < 50$. In how many cases would $(q + p)$ be also a prime number?
(a) 5 (b) 6
(c) 7 (d) None of these

Q.18. How many distinct factors of 1,600 are perfect cubes?
(a) 3 (b) 4 (c) 6 (d) 2

Q.19. The LCM of 96,144 and N is 576. If their HCF is 48, then which of the following can be one of the values of N?
(a) 168 (b) 192 (c) 144 (d) 244

Q.20. If p and q are consecutive natural numbers (in increasing order), then which of the following is true?
(a) $q^2 < p$ (b) $2p > p^2$
(c) $(q + 1)^2 > p^2$ (d) $(p + 2)^3 < q^3$

Q.21. $(17^{21} + 19^{21})$ is not divisible by
(a) 36 (b) 8 (c) 9 (d) 18

Q.22. Which of the following will divide $11^{12296} - 1$?
(a) 11 and 12 (b) 11 and 10
(c) 10 and 12 (d) 11 only

Q.23. If a, b, c and d are consecutive odd numbers, then $(a^2 + b^2 + c^2 + d^2)$ is always divisible by
(a) 5 (b) 7 (c) 3 (d) 4

Q.24. Four bells toll at intervals of 14, 21 and 42 minutes respectively. If they toll together at 11:22 am, when will they toll together for the first time after that?
(a) 11:56 am (b) 12:04 pm
(c) 12:06 pm (d) 11:48 am

Q.25. When x is divided by 6, remainder obtained is 3. Find the remainder when $x^4 + x^3 + x^2 + x + 1$ is divided by 6.
(a) 3 (b) 4 (c) 1 (d) 5

Q.26. I have 7^7 sweets and I want to distribute them equally among 2^4 students. After each of the student got maximum integral sweets, how many sweets are left with me?
(a) 8 (b) 5
(c) 1 (d) None of these

Q.27. When I distribute some chocolates to my 40 students, three chocolates will be left. If I distribute the same number of chocolates to my students and my colleague Manoj Dawrani, seven chocolates are left. Find the minimum number of chocolates I have.
(a) 1,443 (b) 1,476
(c) 1,480 (d) None of these

Q.28. The LCM of two numbers is 40 times their HCF. The sum of the LCM and HCF is 1,476. If one of the numbers is 288, find the other numbers?
(a) 169 (b) 180 (c) 240 (d) 260

Q.29. 1010101 ... 94 digits is a 94 digits number. What will be the remainder obtained when this number is divided by 375?
(a) 10 (b) 320
(c) 260 (d) None of these

Q.30. Chandrabhal adds first N natural numbers and finds the sum to be 1,850. But actually one numbers was added twice by mistake. Find the difference between N and that number.
(a) 40 (b) 33 (c) 60 (d) 17

Q.31. When I distribute a packet of chocolate to 7 students, I am left with 4 chocolates. When I distribute the same packet of chocolate to 11 students, I am left with 6 chocolates. How many chocolates will be left with me if I distribute the same packet of chocolate among 13 students (a packet of chocolate contains total number of chocolates N, $1000 < N < 1050$)?
(a) 2 (b) 0 (c) 6 (d) 7

Q.32. How many prime numbers are there between 80 and 105?
(a) 3 (b) 4 (c) 5 (d) 8

Q.33. If x and y are consecutive natural numbers in an increasing order, then which of the following is always true?
(a) $x^y > y^x$
(b) $y^x > x^y$
(c) $x^x > y^y$
(d) $y^y > x^x$

Q.34. What is the remainder when 5^{79} is divided by 7?
(a) 1 (b) 0 (c) 5 (d) 4

FOUNDATION

Q.1. LCM of two natural numbers is 590 and their HCF is 59. How many sets of values are possible?
(a) 1 (b) 2 (c) 5 (d) 10

Q.2. MUL has a waiting list of 5005 applicants. The list shows that there are at least 5 males between any two females. The largest number of females in the list could be
(a) 920 (b) 835 (c) 721 (d) 1005

Q.3. HCF of two numbers A and B is 24. HCF of two other numbers C and D is 36. What will be the HCF of A, B, C and D?
(a) 12 (b) 24
(c) 36 (d) 6

Q.4. How many zeroes will be there at the end of $25 \times 35 \times 40 \times 50 \times 60 \times 65$?
(a) 6 (b) 8 (c) 5 (d) 7

Q.5. What is the unit digit of $576847 \times 564068 \times 96467 \times 458576$?
(a) 2 (b) 4 (c) 6 (d) 8

Q.6. What is the unit digit of $1! + 2! + 3! + \ldots + 99! + 100!$?
(a) 3 (b) 1 (c) 5 (d) 6

Q.7. How many divisors will be there of the number 1020?
(a) 12 (b) 20
(c) 24 (d) 36

Q.8. In question 7, what is the difference between the number of even divisors and number of prime divisors?
(a) 13 (b) 12
(c) 11 (d) None of these

Q.9. $N = 7!^3$. How many factors of N are multiples of 10?
(a) 736 (b) 1008 (c) 1352 (d) 894

Q.10. A number N has odd number of divisors. Which of the following is true about N?
(a) All the divisors of this number will be odd.
(b) There will be at least $(N - 11)$ prime divisors.
(c) N will be a perfect square.
(d) At least one divisor of the number should be odd.

Q.11. How many zeroes will be there at the end of the expression $N = 2 \times 4 \times 6 \times 8 \times \ldots \times 100$?
(a) 10 (b) 12
(c) 14 (d) None of these

Q.12. How many zeroes will be there at the end of the expression $N = 10 \times 20 \times 30 \ldots \times 1000$?
(a) 1280 (b) 1300
(c) 1320 (d) None of these

Q.13. How many zeroes will be there at the end of the expression $N = 7 \times 14 \times 21 \times \ldots \times 777$?
(a) 24 (b) 25
(c) 26 (d) None of these

Q.14. The number from 1 to 33 are written side by side as follows: 123456… 33. What is the remainder when this number is divided by 9?
(a) 0 (b) 1 (c) 3 (d) 6

Q.15. The number 444444 … (999 times) is definitely divisible by
(a) 22 (b) 44
(c) 222 (d) All of these

Q.16. Find the unit digit of $N = {}_{17}27!^{37!}$
(a) 1 (b) 3 (c) 7 (d) 9

Q.17. How many divisors of $N = 420$ will be of the form $4n + 1$, where n is a whole number?
(a) 3 (b) 4 (c) 5 (d) 8

Q.18. $N = 2^3 \times 5^3 \times 7^2$. How many sets of two factors of N are co-prime?
(a) 72 (b) 64
(c) 36 (d) None of these

Q.19. What is the unit digit of $2^{3^{4^5}}$?
(a) 2 (b) 4 (c) 8 (d) 6

Q.20. How many zeroes will be there at the end of $1003 \times 1001 \times 999 \times \ldots \times 123$?
(a) 224 (b) 217
(c) 0 (d) None of these

Q.21. How many zeroes will be there at the end of $36!^{36!}$?
(a) $7 \times 6!$ (b) $8 \times 6!$ (c) $7 \times 36!$ (d) $8 \times 36!$

Q.22. The number formed by writing any digit 6 times (e.g., 111111, 444444, etc.) is always divisible by
(i) 7 (ii) 11 (iii) 13
(a) i and ii (b) ii and iii
(c) i and iii (d) i, ii and iii

Q.23. What is the maximum value of HCF of $[n^2 + 17]$ and $(n + 1)^2 + 17]$?
(a) 69 (b) 85
(c) 170 (d) None of these

Q.24. What is the number of pairs of values of (x, y), which will satisfy $2x - 5y = 1$, where $x \leq 200$, and x and y are positive integers?
(a) 38 (b) 39 (c) 40 (d) 41

Q.25. $N = 2^3 \times 5^3$. How many sets of two distinct factors of N are co-prime to each other?
(a) 12 (b) 24 (c) 23 (d) 11

Q.26. What is the sum of digits of the least multiple of 13, which when divided by 6, 8 and 12 leave 5, 7 and 11 as the remainder?
(a) 5 (b) 6 (c) 7 (d) 8

Q.27. What is the unit digit of $7^{11^{22^{33}}}$?
(a) 1 (b) 3 (c) 7 (d) 9

Q.28. What is the remainder when $(1! + 2! + 3! + \ldots 1000!)$ is divided by 5?
(a) 1 (b) 2 (c) 3 (d) 4

Q.29. $A = 3^{150} \times 5^{76} \times 7^{140}$, $B = 3^{148} \times 5^{76} \times 7^{141}$, $C = 3^{148} \times 5^{80} \times 7^{139}$, $D = 3^{151} \times 5^{80} \times 7^{142}$, then the order of A, B, C and D from largest to smallest is
(a) DACB (b) CDBA
(c) CDAB (d) DCAB

Q.30. The HCF of 0.3, 0.15, 0.225, 0.0003 is
(a) 0.0003 (b) 0.3 (c) 0.15 (d) 0.0015

Q.31. How many numbers between 1 and 250 are divisible by 5 but not by 9?
(a) 98 (b) 97
(c) 101 (d) None of these

Q.32. A and B are two distinct digits. If the sum of the two-digit numbers formed by using both the digits is a perfect square, what is the value of (A + B)?
(a) 9 (b) 11 (c) 13 (d) 17

Q.33. A number N = 897324P64Q is divisible by both 8 and 9. Which of the following is the value of P + Q?
i. 2 ii. 11 iii. 9
(a) either i or ii (b) either ii or iii
(c) either i or ii or iii (d) None of these

Direction for questions 34 and 35: Read the passage below and solve the questions based on it.

A = Set of first N positive numbers. There are 16 numbers in A which are divisible by both X and Y. There are 50 numbers in A which are divisible by X but not by Y and 34 numbers in A divisible Y but not by X.

Q.34. How many numbers in A are divisible by any of the two numbers?
(a) 100 (b) 50
(c) 200 (d) None of these

Q.35. How many numbers in N are divisible by X?
(a) 42 (b) 56
(c) 66 (d) None of these

Q.36. Nitin had forgotten his 6 digit bank account number but only remembered that it was of the form X515X0 and was divisible by 36. What was the value of X?
(a) 4 (b) 7 (c) 8 (d) 9

Q.37. Students from the Delhi Public School are writing their exams in Kendriya Vidyalaya. There are 60 students writing their Hindi exams, 72 students writing there French exam and 96 students writing their English exam. The authorities of the Kendriya Vidyalaya have to make arrangements such that each classroom contains equal number of students. What is the minimum number of classrooms required to accommodate all students of Delhi Pubic School?
(a) 19 (b) 38 (c) 13 (d) 6

Q.38. In the Jyotirmayi school, all classes started at 9:00 am. The school has three sections: primary, middle and secondary. Each class for the primary section lasts for half an hour, for the middle section for forty five minutes and for the secondary section for half an hour. A lunch break has to be given for the entire school when each of three sections have just finished a respective class and are free. What is the earliest time for the lunch break?
(a) 11:00 am (b) 10:30 am
(c) 12:00 pm (d) 12:30 pm

Q.39. In the firing range, 4 shooters are firing at their respective targets. The first, the second, the third and the fourth schooter hits the target once every 5s, 6s, 7s, 8s respectively. If all of them hit their target at 10:00 am, when will they hit their target together again?
(a) 10:14 am (b) 10:28 am
(c) 10:30 am (d) 10:31 am

Q.40. Two friends Harry and Jayesh were discussing about 2 numbers. They found the two numbers to be such that one was twice the other. However, both had the same number of prime factors while the larger one had 4 more factors than the smaller one. What are the numbers?
(a) 40, 80 (b) 20, 40
(c) 30, 60 (d) 50, 100

Q.41. To celebrate their victory in the World Cup, the Sri Lankans distributed sweets. If the sweets were distributed among 11 players, 2 sweets were left. When the sweets were distributed among 11 players, three extra's and 1 coach, even then 2 sweets were left. What is the minimum number of sweets in the box?
(a) 167 (b) 334 (c) 332 (d) 165

Q.42. The first 20 natural numbers from 1 to 20 are written next to each other to form a 31 digit number N = 12345 67891011121314151617181920. What is the remainder when this number is divided by16?
(a) 0 (b) 4 (c) 7 (d) 9

Q.43. Two friends Kanti and Sridhar were trying to find the HCF of fifty distinct numbers. If they were finding the HCF of two numbers at a time, how many times this operation should be repeated to find the HCF of 50 numbers?
(a) 20 (b) 25 (c) 49 (d) 50

Q.44. How many zeroes will be there at the end of $N = 18! + 19!$?
(a) 3 (b) 4
(c) 5 (d) Cannot be determined

Q.45. Manish was dividing 2 numbers by a certain divisor and obtained remainders of 437 and 298 respectively. When he divides the sum of the two numbers by the same divisor, the remainder is 236. What is the divisor?
(a) 499 (b) 735
(c) 971 (d) None of these

Q.46. I purchased a ticket for the football match between France and Italy in the World Cup. The number on the ticket was a 5 digit perfect square such that the first and the last digit were the same and the 2nd and 4th digit were the same. If the 3rd digit was 3, then what was the ticket number?
(a) 24,342 (b) 12,321
(c) 21,312 (d) None of these

Q.47. How many integers N in the set of integers {1, 2, 3, …, 100} are there such that $N^2 + N^3$ is a perfect square?
(a) 5 (b) 7 (c) 9 (d) 11

Q.48. In a birthday party, all the children were given candy bars. All the children got three candy bars each except the child sitting at the end who got only 2 candy bars. If each child had been given only 2 candy bars there would have been 8 candy bars remaining. How many children were there and how many candy bars were distributed?
(a) 9, 26 (b) 6, 26 (c) 9, 18 (d) 6, 15

Q.49. A natural number N satisfies following conditions.
(A) Number is having all the 9s.
(B) It is divisible by 13.
How many digits are there in N?
(a) 5 (b) 6 (c) 7 (d) 8

Q.50. What is the minimum number of identical square tiles required to cover a floor of dimension 3.78 m × 4.8 m?
(a) 2,520 (b) 3,780
(c) 5,040 (d) 6,480

Q.51. What is the smallest five-digit number which when divided by 7, 11 and 21 leaves a remainder of 3 in each case?
(a) 10,019 (b) 10,001
(c) 10,111 (d) 10,167

Q.52. A milkman has 3 jars containing 57 litres, 129 litres and 177 litres of pure milk respectively. A measuring can leaves the same amount of milk unmeasured in each jar after a different number of exact measurements of milk in each jar. What is the volume of largest such can?
(a) 12 litres (b) 16 litres
(c) 24 litres (d) 48 litres

Q.53. A boy was carrying a basket of eggs. He fell down and some of the eggs were broken. The boy has 10 eggs left with him. When asked by his mother how many eggs were broken, the boy could not recall. However he recalled that when he counted the total number of eggs 3 at a time 1 egg was left. When counted 4 at a time, 1 egg was left and when counted 5 at time a no egg was left. How many eggs were broken?
(a) 15 (b) 25 (c) 30 (d) 35

Q.54. How many prime numbers are there which, when divided by another prime number, gives a quotient which is same as the remainder?
(a) 0 (b) 1
(c) 2 (d) More than 2

Q.55. Let A, B, and C be digits such that $(100A + 10B + C)(A + B + C) = 2005$. What is the value of A?
(a) 4 (b) 2 (c) 3 (d) 1

Q.56. Find the product of all the factors of 3^{16}.
(a) 3^{33} (b) 3^{68}
(c) 3^{136} (d) 3^{128}

Q.57. What is the remainder when 90^{91} is divided by 13?
(a) 0 (b) 7 (c) 12 (d) 1

Q.58. Find the remainder when the product of 10 consecutive natural numbers starting from 8641 is divided by 8640.
(a) 1 (b) 55 (c) 10 (d) 0

Q.59. A faulty car odometer proceeds from digit 3 to digit 5, always skipping the digit 4, regardless of position. If the odometer now reads 002005, how many miles has the car actually traveled?
(a) 1404 (b) 1462
(c) 1604 (d) 1605

Q.60. How many numbers are there between 400 and 600 in which 8 occurs only once?
(a) 36 (b) 18
(c) 19 (d) 38

Q.61. If $n^2 = 123.45654321$, which of the following is the exact value of n.
(a) 11.1001 (b) 11.1101
(c) 11.1111 (d) 11.1011

Q.62. A mule said to a horse, "If I take one sack off your back, my load will be double of yours and if you take one off my back our loads will be the same." How many sacks in all were they carrying?
(a) 5 (b) 7 (c) 12 (d) 14

Q.63. Divide 45 into 4 parts such that if the first is increased by 2, the second is decreased by 2, the third multiplied by 2 and the fourth divided by 2, the result is the same.
(a) 20, 8, 5, 12
(b) 12, 5, 20, 8
(c) 5, 8, 12, 20
(d) 8, 12, 5, 20

Q.64. Find the remainder when $3x^2 - x^6 + 31x^4 + 21x + 5$ is divided by $x + 2$.
(a) 10 (b) 12
(c) 11 (d) None of these

Q.65. Four prime numbers are in ascending order of their magnitudes. The product of the first three is 385 and that of last three is 1001. The largest given prime number is
(a) 11 (b) 13 (c) 17 (d) 19

Q.66. What is the remainder when 4^{44} is divided by 15?
(a) 1 (b) 2 (c) 3 (d) 4

Q.67. LCM of two integers P and Q is 211. What is the HCF of P and Q?
(a) 37 (b) 1
(c) 3 (d) Cannot be determined

Q.68. How many times does the digit 6 appear when we count from 11 to 400?
(a) 34 (b) 74 (c) 39 (d) 79

Q.69. In Q.68, how many numbers will be having 8 as its digit?
(a) 74 (b) 75 (c) 76 (d) 77

Q.70. S is a number formed by writing 8 for 88 times. What will be the remainder of this number when divided by 7?
(a) 4 (b) 5 (c) 8 (d) 1

Q.71. We are writing all the multiples of 3 from 111 to 324. How many times will we write digit 3?
(a) 18 (b) 19 (c) 21 (d) 22

Q.72. What is the remainder when 7 + 77 + 777 + 7777 + … (till 100 terms) is divided by 8?
(a) 0 (b) 2 (c) 4 (d) 6

Q.73. A number has exactly 15 composite factors. What can be the maximum number of prime factors of this number?
(a) 2 (b) 3 (c) 4 (d) 5

Q.74. $N = 204 \times 221 \times 238 \times 255 \times \ldots \times 850$. How many consecutive zeroes will be there at the end of this number N?
(a) 8 (b) 10 (c) 11 (d) 12

Q.75. 1st 126 natural numbers are put side by side in the ascending order to create a large number N = 123456… 125126. What will be the remainder when N is divided by 5625?
(a) 5126 (b) 26 (c) 126 (d) 156

Q.76. When a number S is divided by 3, 4 and 7 successively, remainders obtained are 2, 1 and 4 respectively. What will be the remainder when the same number is divided by 84?
(a) 43 (b) 53 (c) 63 (d) 73

Q.77. What is the remainder when $1714 \times 1715 \times 1717$ is divided by 12?
(a) 3 (b) 8 (c) 2 (d) 9

Q.78. $N^2 = 12345678987654321$. Find N.
(a) 101010101 (b) 11111
(c) 111111111 (d) 1000000001

Q.79. If a, b, c and d are distinct integers in the range 10 to 15 (both inclusive), the greatest value of $(a + b)(c + d)$ is:
(a) 750 (b) 731 (c) 700 (d) 729

Q.80. The smallest natural number which is a perfect square and is of the form abbb lies in between
(a) 1,000 to 2,000 (b) 2,000 to 3,000
(c) 3,000 to 4,000 (d) 4,000 to 5,000

MODERATE

Q.1. How many number of zeroes will be there at the end of 12! expressed in base 6?
(a) 4 (b) 5 (c) 6 (d) 7

Q.2. Find the remainder when $2222^{5555} + 5555^{2222}$ is divided by 7.
(a) 1 (b) 3 (c) 0 (d) 5

Q.3 LCM of first 100 natural numbers is N. What is the LCM of first 105 natural numbers?
(a) 5! × N (b) 10403 N
(c) 105N/103 (d) 4N

Q.4. How many divisors of 10^5 end with a zero?
(a) 1 (b) 3 (c) 9 (d) 16

Q.5. Following expression holds true if we replace some of '+' signs by '×' signs.
1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10 = 100
How many '+' signs are needed to be replaced by '×'?
(a) 2 (b) 3 (c) 4 (d) 1

Q.6. In a particular country, all the numbers are expressed with the help of three alphabets a, b and c.
15 is written as abc.
6 is written as bc.
60 is written as bcbc.
How would one write 17 in that country?
(a) abb (b) bab (c) baa (d) aba

Q.7. When a certain two-digit number is added to another two digit number having the same digits in reverse order, the sum is a perfect square. How many such two-digit numbers are there?
(a) 4 (b) 6 (c) 8 (d) 10

Q.8. What is the remainder when ${}_{32}32^{32}$ is divided by 7?
(a) 2 (b) 3 (c) 4 (d) 6

Q.9. N is a 1001 digit number consisting of 1001 sevens. What is the remainder when N is divided by 1001?
(a) 7 (b) 700
(c) 777 (d) None of these

Q.10. Find four positive numbers such that the sum of the first, third and fourth exceeds the second by 8; the sum of the squares of the first and second exceeds the sum of the squares of the third and fourth by 36; the sum of the products of the first and second, and of the third and fourth is 42; the cube of the first is equal to the sum of the cubes of the second, third, and fourth.
(a) 2, 1, 9,3 (b) 2, 4, 6, 8
(c) 6, 5, 4, 3 (d) None of these

Q.11. Digital sum of a number is obtained by adding all the digits of a number until a single digit is obtained. Find the digital sum of 19^{100}.
(a) 1 (b) 4 (c) 7 (d) 9

Q.12. Find the HCF of $(2^{100} - 1)$ and $(2^{120} - 1)$.
(a) $2^{10} - 1$ (b) $2^{20} - 1$
(c) 1 (d) None of these

Q13. Let S be the set of positive integers n for which $\frac{1}{n}$ has the repeating decimal representation $0.\overline{ab} = 0.ababab\ldots\ldots$, with a and b different digits. What is the sum of the elements of S?
(a) 11 (b) 44 (c) 110 (d) 143

Q.14. An intelligence agency forms a code of two distinct digits selected from 0, 1,…, 9 such that the first digit of the code is non-zero. The code, handwritten on a slip, however, can potentially create confusion when read upside down, e.g., the code 91 may appear as 16. How many codes are there for which no such confusion can arise?
(a) 80 (b) 63
(c) 71 (d) None of these

Q.15. If p, $p + 2$, $p + 4$ are prime numbers, then the number of possible solutions for p is
(a) 0 (b) 1
(c) 2 (d) None of these

Q.16. Suppose N is an integer such that the sum of the digits of N is 2, and $10^9 < N < 10^{10}$. How many values of N are possible?
(a) 11 (b) 10 (c) 9 (d) 8

Q.17. Ten students solved a total of 35 questions in a Maths Olympiad. Each question was solved by exactly one student. There is at least one student who solved exactly one problem, at least one student who solved exactly two problems and at least one student who solved exactly three problems. What is the minimum number of students who has/have solved at least five problems?
(a) 1 (b) 2
(c) 3 (d) None of these

Q.18. N has 37 zeroes at its end. How many values of N is/are possible?
(a) 0 (b) 1 (c) 5 (d) Infinite

Q.19. In the above question, how many values of N will be even?
(a) 0 (b) 2 (c) 3 (d) Infinite

Q.20. N! is having 30 zeroes at its end. How many values of N is/are possible?
(a) 0 (b) 1 (c) 5 (d) Infinite

Q.21. What is the remainder when $(1^1 + 2^2 + 3^3 + \ldots + 100^{100})$ is divided by 4?
(a) 0 (b) 1 (c) 2 (d) 3

Q.22. A 3-digit number in which all the 3-digits are odd is such that if the cubes of the digit are added, the sum would be equal to the number itself. If one of the digit is 7, find the number.
(a) 171 (b) 371 (c) 575 (d) 775

Q.23. A teacher said that there were 100 students in his class, 24 of whom were boys and 32 were girls. Which base system did the teacher use in this statement?
(a) 9 (b) 5 (c) 6 (d) 8

Q.24. What is the remainder when 3^{450} is divided by 108?
(a) 3 (b) 1 (c) 27 (d) 81

Q.25. P is a natural number. 2P has 28 divisors and 3P has 30 divisors. How many divisors of 6P will be there?
(a) 35 (b) 40 (c) 45 (d) 48

Q.26. pqr is a three digit natural number such that $pqr = p^3 + q^3 + r^3$. What is the value of r?
(a) 0 (b) 1
(c) 3 (d) Cannot be determined

Q.27. There are two three-digit numbers. When one number is divided by another number, quotient obtained is 6 and remainder is 0. Sum total of both the numbers is a multiple of 504. What is the difference between the numbers?
(a) 720 (b) 360 (c) 120 (d) 420

Q.28. LCM of 12^{24}, 16^{18} and N is 24^{24}. Number of all the possible values of N = S. What is the value of S?
(a) 25 (b) 1,800
(c) 1,825 (d) None of these

Q.29. Each of P, Q, R and S equals either 0 or 1. It is given that
If Q = 0, then R = 1
If R = 0, then P = S
If S = 0, then P = 1
Assume R = 0, find the value of (P + Q + R + S)?
(a) 0 (b) 1 (c) 2 (d) 3

Q.30. When asked about his date of birth in 1996, Mayank replied that "last two digits of my birth year stands for my age." When Siddharth was asked about his age, he also replied the same. But Siddharth is older to Mayank. What is the difference in their age?
(a) 46 (b) 50
(c) 0 (d) Cannot be determined

Direction for questions 31 to 35: Read the passage below and solve the questions based on it.

There is a prison with 100 cells inside it. Cells are numbered from 1 to 100 and every cell is occupied by one prisoner only. One day jailer decides to release some of the prisoners and for this he defines an algorithm of 100 steps which follows:

Q.6. A three-digit number ABC is a perfect square and the number of factors of this number is also a perfect square. If (A + B + C) is also a perfect square, then what is the number of factors of the 6-digit number ABCABC?

(a) 32 (b) 52
(c) 72 (d) Cannot be determined

Q.7. How many divisors of 10^5 will have at least one zero at its end?

(a) 9 (b) 12 (c) 15 (d) 25

Q.8. Let $V_1, V_2, V_3, \ldots, V_{100}$ be hundred positive integers such that $V_i + V_{i+1} + V_{i+2} + V_{i+3} = K$, where K is a constant and i = 1, 2, 3, … , 97. If $V_3 = 9$, then what is the value of V_{99}?

(a) 9 (b) K − 9
(c) (K/2 − 9) (d) Cannot be determined

Q.9. In the above question, if $V_5 = 7$, then what is the value of V_{90}?

(a) 7 (b) K − 7
(c) (K − 7)/2 (d) Cannot be determined

Q.10. What is the largest integer that is a divisor of $(n + 1)(n + 3)(n + 5)(n + 7)(n + 9)$ for all positive even integers n?

(a) 3 (b) 5 (c) 11 (d) 15

Q.11. If K is any natural number, such that $100 \le K \le 200$, how many values of K exist such that K! has 'z' zeroes at its end and (K + 2)! has 'z + 2' zeroes at its end?

(a) 2 (b) 4
(c) 6 (d) None of these

Q.12. Tatto bought a notebook containing 96 pages leaves and numbered them which came to 192 pages. Tappo tore out the latter 25 leaves of the notebook and added the 50 numbers she found on those pages. Which of the following is not true?

(a) She could have found the sum of pages as 1990
(b) She could have found sum of pages as 1275
(c) She could have got sum of pages as 1375
(d) None of these

Direction for questions 13 to 15: Read the passage below and solve the questions based on it.

There are 50 integers a1, a2, a3, … ,a50; not all of them necessarily different. Let the greatest integer of these 50 integers be referred to as G and smallest integer be referred to as L. The integers a1 to a24 form a sequence S1 and the rest form a sequence S2. Each member of S1 is less than or equal to each member of S2.

Q.13. All values in S1 are changed in sign, while those in S2 remain unchanged. Which of the following statements is true?

(a) Every member of S1 is greater than or equal to every member of S2
(b) G is in S1
(c) If all the numbers originally in S1 and S2 had the same sign, then after the change of sign, the largest number of S1 and S2 is in S1
(d) None of these

Q.14. Elements of S1 are in ascending order and those of S2 are in descending order. a24 and a25 are interchanged then which of the following is true?

(a) S1 continues to be in ascending order
(b) S2 continues to be in descending order
(c) Both (a) and (b)
(d) Cannot be determined

Q.15. Every element of S1 is made greater than or equal to every element of S2 by adding to each element of S1 an integer x. Then, x cannot be less than

(a) 2^{10}
(b) The smallest value of S2
(c) The largest value of S2
(d) (G-L)

Q.16. Twenty-five boxes of sweets are delivered to Mr Roy's home. Mr Roy had ordered sweets of three different types. What is the minimum number of boxes of sweets which are having sweets of same type?

(a) 1 (b) 8
(c) 9 (d) Cannot be determined

Q.17. A warehouse contains 200 shoes of size 8, 200 shoes of size 9 and 200 shoes of size 10. Of these 600 shoes, there are 300 left shoes and 300 right shoes. What is the minimum number of usable shoes?

(a) 50 (b) 100
(c) 200 (d) None of these

Q.18. A teacher was doing some calculation exercise on the blackboard. When the teacher went out, a naughty student Chunmun erased some of the numbers written on the blackboard. Now it appeared like this

```
  2 3 _ 5 _
+ 1 _ 6 4 2
-----------
  4 2 4 2 3
```

When teacher entered the room, he realized that still this calculation was right, but in some other system of writing (i.e., not 10). How many digits are there in that system?

(a) 11 (b) 9 (c) 7 (d) 8

Q.19. Tatto, Tappo and Bubbly were solving problems from a problem book. Each solved exactly 60 problems, but they solved only 100 problems altogether. Any problem is known as "easy" if it was solved by all of them, and "difficult" if it was solved by only one of them. What is the difference between the number of "difficult" problems and number of "easy" problems?

(a) 10 (b) 20
(c) 30 (d) 40

Q.20. LCM of two numbers A and B = $P^x \times Q^y$, where P and Q are prime numbers and x and y are positive whole numbers. How many set of values are possible for A and B?
(a) $xy(x + y)$ (b) $xy(x - y)$
(c) $x^2y^2(x + y)$ (d) None of these

Q.21. When 7179 and 9699 are divided by another natural number N, remainder obtained is same. How many values of N will be ending with one or more than one zeroes?
(a) 24 (b) 124
(c) 46 (d) None of these

Q.22. There exists a 5 digit number N with distinct and non-zero digits such that it equals the sum of all distinct three digit numbers whose digits are all different and are all digits of N. Then the sum of the digits of N is a necessarily.
(a) Perfect square (b) Cube
(c) Even (d) None of these

Q.23. Starting with 1, positive integers are written one after the other. What is the 40,0000th digit that will be written?
(a) 3 (b) 6
(c) 8 (d) None of these

Q.24. Which of the following would always divide a six-digit number of the form ababab?
(a) 10,101 (b) 11,111
(c) 10,001 (d) None of these

Q.25. If in the number system of a particular country, 25 means 5 tens and 2 units, 467 means 7 hundreds, 6 tens and 4 units. Then find the value of 173 × 425?
(a) 4,04,491 (b) 7,35,255
(c) 6,22,744 (d) 5,25,376

Q.26. Let A be the set of integers N such that
i. $100 \le N \le 500$
ii. N is even
iii. N is divisible by either 2 or 3 or 4 but not by 7.
How many elements are there in set A?
(a) 171 (b) 172 (c) 170 (d) 173

Q.27. Three distinct prime numbers, less than 10 are taken and all the numbers that can be formed by arranging all the digits are taken. Now, difference between the largest and the smallest number formed is equal to 495. It is also given that sum of the digits is more than 13. What is the product of the numbers?
(a) 30 (b) 70 (c) 105 (d) 315

Q.28. What is the remainder when ${}_{55}56^{57}$ is divided by 17?
(a) 1 (b) 4 (c) 13 (d) 17

Q.29. What is the remainder when ${}_{20}51^{97}$ is divided by 17?
(a) 10 (b) 9 (c) 15 (d) 7

Q.30. $[111 \ldots 111(200 \text{ digits}) - 222 \ldots 22(100 \text{ digits})]^{1/2}$ is equal to
(a) 1313 … 1313(100 digits)
(b) 2121 … 2121(100 digits)
(c) 1111 … 1111(100 digits)
(d) 3333 … 333(100 digits)

Q.31. N is a number which when divided by 10 gives 9 as the remainder,
when divided by 9 gives 8 as the remainder,
when divided by 8 gives 7 as the remainder,
when divided by 7 gives 6 as the remainder,
when divided by 6 gives 5 as the remainder,
when divided by 5 gives 4 as the remainder,
when divided by 4 gives 3 as the remainder,
when divided by 3 gives 2 as the remainder,
when divided by 2 gives 1 as the remainder.
What is N?
(a) 2519 (b) 841 (c) 839 (d) 2521

Q.32. What is the remainder when $(10^3 + 9^3)^{752}$ is divided by 12^3?
(a) 1 (b) 729 (c) 752 (d) 1000

Q.33. $f(x, x) = x^x$ and $f(f(x, x) = {}_x x^x$ and so on its keeps on going. What is the value of $f(f(f(f(7,7))))$ is divided by 5?
(a) 1 (b) 2 (c) 3 (d) 4

Q.34. The students of class 10th of Morgan High School took a test which had a maximum of 50 marks. The teacher misplaced the text notebooks of two of the students—Robin and Garry, but remembered that Garry had scored something between 10 and 15 and Robin something between 32 to 40. She also remembered that the product of the marks obtained by the two students was also equal to 10 times the marks obtained by two of them. How many marks did Garry scored?
(a) 11 (b) 12 (c) 13 (d) 14

Q.35. The History teacher was referring to a year in the 19th century. Rohan found an easy way to remember the year. He found that the number, when viewed in a mirror, increased 4.5 times. Which year was the teacher referring to?
(a) 1,801 (b) 1,810
(c) 1,818 (d) More than one value

Q.36. Srini wrote his class 10th board examination this year. When the result came out he searched for his hall ticket to see his roll number but could not trace it. He could remember only the first three digits of the 6 digit number as 267. His father, however, remembered that the number was divisible by 11. His mother gave the information that the number was also divisible by 13. They tried to recollect the number when all of a sudden Srini told that the number was a multiple of 7. What was the unit digits of the number?
(a) 5 (b) 7
(c) 2 (d) Cannot be determined

Q.37. Prof. Mathur and Prof. Singh attended the All India Historian's meet last week. Prof. Mathur told Prof. Singh, "I found out that your teaching experience is twice that of mine". Prof. Singh replied in the affirmative. Prof. Mathur continued, "But last time when both of us came for the same meet, I remember that your teaching experience was thrice that of mine". "That was 2 years ago." Prof. Singh said. How many years has Prof. Singh been working?
(a) 8 yrs (b) 10 yrs (c) 12 yrs (d) 16 yrs

Direction for questions 38 and 39: Read the passage below and solve the questions based on it.

ABCDEF is a 6-digit number with distinct digits. Further, the number is divisible by 11 and the sum of its digits is 24. Further, A > C > E and B > D > F.

Q.38. The sum A + C + E is equal to
(a) 12 (b) 6
(c) 8 (d) Cannot be determined

Q.39. A + B is always
(a) 10 (b) 9
(c) 6 (d) Cannot be determined

Q.40. Raju had to divide 1080 by N, a two-digit number. Instead, he performed the division using M which is obtained by reversing the digits of N and ended up with a quotient which was 25 less than what he should have obtained otherwise. If 1080 is exactly divisible both by N and M, find the sum of the digits of N.
(a) 6 (b) 8
(c) 9 (d) None of these

Q.41. Let S = {1, 2, 3,... n} be a set of N natural numbers. Let T be a subset of S such that the sum of any three elements of T is not less than N. Find the maximum number of elements in any such subset T for N = 40?
(a) 26 (b) 27
(c) 28 (d) None of these

Q.42. The last digit of the LCM of $(3^{2003} - 1)$ and $(3^{2003} + 1)$ is
(a) 8 (b) 2 (c) 4 (d) 6

Q.43. a, b and c are positive integers such that, $a + b + c = 2003$. Let $E = (-1)^a + (-1)^b + (-1)^c$. Find the number of possible values of E.
(a) 2004 (b) 3 (c) 1003 (d) 2

Q.44. Ajay took a 4-digit number in base 5 notation. He subtracted the sum of the digits of the numbers from the number. From the result, he struck off one of the digits. The remaining 3 digits were 1, 0 and 2. Then the digit struck off by Ajay was:
(a) 2 (b) 1
(c) 4 (d) Cannot be determined

Direction for questions 45 and 46: Read the passage below and solve the questions based on it.

N is a single digit integer satisfying the following two conditions.
i. N is non-zero.
ii. N is the right most digit of the number $(n!)^4$, where n is a natural number greater than 1.

Q.45. What is the number of possible values of N?
(a) 1 (b) 2
(c) 0 (d) None of these

Q.46. If condition (a) is relaxed, the number of possible values of N is
(a) 1 (b) 2
(c) 0 (d) More than 2

Q.47. A teacher wrote a number on the blackboard and the following observations were made by the students
The number is a four-digit number.
The sum of the digits equals the product of the digits.
The number is divisible by the sum of the digits.
The sum of the digits of the number is
(a) 8 (b) 10 (c) 12 (d) 14

Q.48. The N of odd numbers are taken. Product of these odd numbers is of the form $(4n + 1)$, where n is any natural number Which of the following is true regarding the number of numbers?
(a) There must have been an odd number of numbers of the form $4n + 1$
(b) There must have been an even number of numbers of the form $4n + 1$
(c) There must have been an even number of numbers of the form $4n + 3$
(d) None of these

Q.49. 16 students were writing a test in a class. Rahul made 14 mistakes in the paper, which was the highest number of mistakes made by any student. Which of the following statements is definitely true?
(a) At least two students made the same number of mistakes
(b) Exactly two students made the same number of mistakes
(c) At most two students made the same number of mistakes
(d) All students made different number of mistakes.

Q.50. The sum of the factorials of the three-digits of a 3-digit number is equal to the three-digit number formed by these three digits, taken in the same order. Which of the following is true of the number of such three-digit numbers, if no digit occurs more than once?
(a) No such number exists
(b) Exactly one such number exists
(c) There is more than one such number, but they are finite in number
(d) There are infinite such numbers

5. A, B, C, D are four natural numbers. If we know the LCM of A, B and LCM of C, D separately, then it is always possible to find out the LCM of A, B, C, D.

   State whether True or False.

6. A, B, C, D are four natural numbers. If we know the HCF of A, B and HCF of C, D separately, then it is always possible to find out the HCF of A, B, C, D.

   State whether True or False.

7. If we know the total number of odd factors of a number, then we can always find out the total number of factors of that number.

   State whether True or False.

8. If we know the total number of even factors of a number, then we can always find out the total number of factors of that number.

   State whether True or False.

9. If a number is odd, then it cannot have total number of factors as an even number.

   State whether True or False.

ANSWERS

Warm Up

Q. No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Answer	(d)	(d)	(d)	(d)	(d)	(d)	(b)	(a)	(c)	(b)	(a)	(a)	(d)	(c)	(d)	(d)	(b)	(a)	(b)	(c)
Q. No.	21	22	23	24	25	26	27	28	29	30	31	32	33	34						
Answer	(b)	(c)	(d)	(b)	(c)	(d)	(d)	(b)	(d)	(a)	(b)	(c)	(d)	(c)						

Foundation

Q. No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Answer	(b)	(b)	(a)	(a)	(a)	(a)	(c)	(b)	(b)	(c)	(b)	(d)	(c)	(c)	(c)	(a)	(b)	(d)	(a)	(c)
Q. No.	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
Answer	(d)	(d)	(a)	(c)	(b)	(d)	(c)	(c)	(d)	(a)	(b)	(b)	(a)	(a)	(c)	(c)	(a)	(b)	(a)	(c)
Q. No.	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60
Answer	(a)	(a)	(c)	(b)	(a)	(b)	(c)	(a)	(b)	(c)	(d)	(c)	(a)	(b)	(a)	(c)	(c)	(d)	(b)	(a)
Q. No.	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80
Answer	(c)	(c)	(d)	(d)	(b)	(a)	(b)	(d)	(b)	(b)	(d)	(d)	(a)	(b)	(c)	(b)	(c)	(c)	(d)	(a)

Moderate

Q. No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Answer	(b)	(c)	(b)	(c)	(b)	(a)	(c)	(c)	(b)	(c)	(a)	(b)	(d)	(c)	(b)	(b)	(a)	(c)	(c)	(a)
Q. No.	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
Answer	(a)	(b)	(c)	(d)	(a)	(d)	(a)	(c)	(d)	(b)	(b)	(b)	(c)	(b)	(b)	(a)	(a)	(c)	(b)	(c)
Q. No.	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60
Answer	(d)	(b)	(c)	(b)	(d)	(c)	(a)	(b)	(a)	(c)	(d)	(d)	(c)	(b)	(b)	(c)	(a)	(b)	(a)	(a)
Q. No.	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75					
Answer	(a)	(d)	(a)	(a)	(c)	(c)	(d)	(b)	(a)	(d)	(b)	(b)	(b)	(c)	(d)					

Advanced

Q. No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Answer	(d)	(a)	(a)	(a)	(c)	(d)	(d)	(a)	(d)	(d)	(c)	(a)	(d)	(a)	(d)	(c)	(b)	(c)	(b)	(d)
Q. No.	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
Answer	(c)	(a)	(d)	(a)	(a)	(b)	(b)	(a)	(a)	(d)	(a)	(a)	(c)	(d)	(c)	(b)	(a)	(a)	(c)	(c)
Q. No.	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60
Answer	(d)	(c)	(d)	(b)	(a)	(b)	(a)	(c)	(a)	(b)	(c)	(c)	(d)	(a)	(d)	(b)	(c)	(d)	(b)	(d)

True/False

1. True

   To find out the remainder when M divided by P, we simply need to divide R by P.

2. False

   If we divide M by P, we would get a range of remainders in terms of R and not the exact value of remainder in all the cases.

3. True

   LCM, by its meaning, is the lowest number divisible by all the numbers constituting it. Final LCM will be the LCM of the pairs of numbers.

4. True

   HCF, by its meaning, is the highest number that can divide the numbers constituting it. Final HCF will be the HCF of the pairs of numbers.

5. True

   LCM, by its meaning, is the lowest number divisible by all the numbers constituting it. Final LCM will be the LCM of the pairs of numbers.

6. True

   HCF, by its meaning, is the highest number that can divide the numbers constituting it. Final HCF will be the HCF of the pairs of numbers.

7. False
8. False
9. False

   Total number of factors do not have any relationship with the number being odd or even. For example, all the perfect squares (irrespective of being odd or even) have total number of factors = Odd number.

HINTS AND SOLUTIONS

Warm Up

1. Numbers are $5^{1/2}$, $6^{1/3}$, $8^{1/4}$ and $12^{1/6}$
To solve such questions, we raise each number to a common power so that the powers of the numbers are natural numbers.
In this case, raise each number to the power 12 (LCM of 2, 3, 4, and 6)
So numbers obtained = $(5^{1/2})^{12}$, $(6^{1/3})^{12}$, $(8^{1/4})^{12}$ and $(12^{1/6})^{12} = 5^6, 6^4, 8^3$ and 12^2
Now the smallest number in these numbers is 12^2
Therefore smallest number = $12^{1/6}$

2. $\frac{N}{12} = \frac{N}{3} \times \frac{N}{4}$
But it is given that $\frac{N}{4}$ is not an integer, So $\frac{N}{12}$ will not be a integer also.

3. Let the numbers are $(x - 1)$, x and $(x + 1)$
Then $(x - 1)^2 + x^2 + (x + 1)^2 = 3x^2 + 2$
When $x = 2$, then $3x^2 + 2 = 14$
And when $x = 3$, then $3x^2 + 2 = 29$
So the largest number which will always divide $(a^2 + b^2 + c^2) = 1$

4. Let $3.134 = a$ and $1.866 = b$
Then $\frac{a^3 + b^3}{a^2 - ab + b^2} = \frac{(a+b)^3 - 3ab(a+b)}{a^2 - ab + b^2}$
$= \frac{(a+b)[a^2 - ab + b^2]}{a^2 - ab + b^2} = (a + b)$
So $\frac{(3.134)^3 + (1.866)^3}{(3.134)^2 - 3.134 \times 1.866 + (1.866)2}$
$= 3.134 + 1.866 = 5$
If n is a perfect cube, then n^2 will also be a perfect cube. So answer is option (d).

6. For $5x + 11y = 31$
The value of x and will be 4 and 1
Which are the minimum value of x and y. Then $x + y = 5$

7. $\frac{25^x + 1}{13} = \frac{(-1)^x}{13} + \frac{1}{13}$
Hence, for odd natural values of x, $25^x + 1$ will be divisible.

8. Solve it through actual calculation. Number is 71/84.

9. Let the number is x
Hence, $x \times 8 \times 9 \times 10 \times 11 \times 12 = x \times 2^6 \times 3^2 \times 3 \times 5 \times 11$
Here we can say that for being a perfect square, x should be $3 \times 5 \times 11 = 165$.

10. General term would be $n^3 - n^2 = n^2(n - 1)$
So, summation would be $= 0 + 4 + 18 + 48 + 100 + 180 + 294 + 448 + 648 + 900 = 2640$

11. $3 - 9 + 15 - 21 +$ _______ 19 terms
$(3 \times 1) - (3 \times 3) + (3 \times 5) - (3 \times 7) + (3 \times 9)$ _______ 19 terms
From here we can say that every term of this series will be an odd number.
Hence, odd − odd = even number.
So we can say that till 18th term, they all will become even numbers and 19th term is an odd number.

12. Unit digit of $21^3 \times 21^2 \times 34^7 \times 46^8 \times 77^8$
$= 1 \times 1 \times 4 \times 6 \times 1 = 24$
So unit digit = 4

13. $47n \times 729 \times 345 \times 343 = 47n \times 86266215$
It is given that unit digit of $47n \times 86266215$
Or $n \times 5$ is 5.
So the values of n are all odd digits.
Hence, option (d) is the answer.

14. Total number of factors of $846 = 2 \times 3^2 \times 47$ are $(1 + 1)(2 + 1)(1 + 1) = 2 \times 3 \times 2 = 12$
So total sets are $= \frac{12}{2} = 6$

15. Let the original number is $x + 7$. Hence, thrice the number $= 3(x + 7) = 3x + 21$
It is given that x is divisible by 15, then $3x$ will also be divisible by 15 or by 5
So remainder obtain when $3x + 21$ divide by 5 = remainder obtain,
When 21 divide by 5 = 1

16. The first number is 49, Next number is $(391 + 49)$, next number $= (2 \times 391 + 49)$
And so on……
Since there are many numbers, therefore the answer is cannot be determined.

17. Do it from actual calculation.
The values of $P + Q = 5, 7, 13, 19, 31, 43$
Hence, answer is 6.

18. Prime factors of $1600 = 2^6 \times 5^2$
Hence, for a perfect cube, we can take the values of $2 = 2^0$, 2^3 and 2^6 and the value of 5 is 5^0.
So number of perfect cube factors $= 3 \times 1 = 3$

19. LCM of 96, 144 and $N = 576$
Or, LCM of $(2^5 \times 3, 2^4 \times 3^2$ and $N) = 2^6 \times 3^2$
From here N should be $2^6 \times 3^2$ or $2^6 \times 3$ or 2^6
But it is given that HCF is $48 = 2^4 \times 3$
Hence, $N = 2^6 \times 3$

20. It is given that p and q are consecutive natural numbers, such that $p < q$
Hence, option (a) is incorrect for every possible value of p and q

Option (b) is incorrect for p equals to 1 and q equals to 2

Option (d) is incorrect for every possible value of p and q

So, answer is option (c).

21. We know that $a^n + b^n$ is divisible by $a + b$ if n is a odd number

It means $(17^{21} + 19^{21})$ is divisible by 36 and all the factors of 36.

So answer is 8 because 8 is not a factor of 36.

22. $11^{12296} - 1$ is divisible by 10 and 12.

Because $\frac{11^{12296}-1}{10} = \frac{(1)^{12296}-1}{10} = \frac{1-1}{10} = 0$

And $\frac{11^{12296}-1}{12} = \frac{(-1)^{12296}-1}{12} = \frac{1-1}{12} = 0$

23. **Method 1:** Assume that the numbers are $(2a - 3)$, $(2a - 1)$, $(2a + 1)$ and $(2a + 3)$.

Given that: $(2a-3)^2 + (2a-1)^2 + (2a+1)^2 + (2a+3)^2$

$4a^2 - 12a + 9 + 4a^2 - 4a + 1 + 4a^2 + 1 + 4a + 4a^2 + 9 + 12a$

$= 16a^2 + 20 = 4\,(4a^2 + 5)$

Method 2: Assume numbers to be 3, 5, 7 and 9.

So, $a^2 + b^2 + c^2 + d^2 = 3^2 + 5^2 + 7^2 + 9^2$

$= 9 + 25 + 49 + 81 = 164$

This is divisible by 4(maximum value).

Hence, option (d) is the answer.

24. LCM of 14, 21 and 42 is 42.

It means that after every 42 minutes all bells will tall together.

Then after 11:22 am they will toll at 11:22 + 42 = 11:64 – 12:04 pm

25. $\frac{x^4+x^3+x^2+x+1}{6} = \frac{(3)^4+(3)^3+(2)^2+(2)+1}{6}$

$\left[\text{since } \frac{x}{6} = \frac{3}{6}\right] = \frac{121}{6} = 1$

So remainder is 1.

26. Question is asking about the remainder when we divide 7^7 by 2^4.

Remainder is 7.

27. Let the number is 41K + 7. Now divide 41K + 7 by 40

$\frac{41K+7}{40} = \frac{40K+K+7}{40} = \frac{K+7}{40}$

Now put the value of K for which $x + 7$ will give a remainder of 3.

Which is $K = 36$

So the original number $= 41K + 7 = 41 \times 36 + 7 = 1479$

28. Let the HCF is x

Then LCM + HCF = 1476

$40x + x = 1476$, or, $x = 36$

So HCF = 36 and LCM = $40x$ = 1440

We know that Product of numbers = LCM × HCF

Now, you can solve the equation. Answer is 180.

29. 101010........94 digits can be written as:

$\frac{101010\ldots 100000\ (94 \text{ digits}) + 1010}{125 \times 3}$

$\frac{101010\ldots100000}{125 \times 3} + \frac{1010}{375}$

Remainder obtained when $\frac{101010\ldots100000}{125 \times 3} = 0$

Remainder obtained when $\frac{1010}{375} = 260.$

Hence, net remainder = 260

Hence, option (d) is the answer.

30. Sum of 1st 60 numbers $= \frac{60 \times 61}{2} = 1830$

So the number which has been added twice

= 1850 – 1830 = 20

Hence, $N - 20 = 60 - 20 = 40.$

31. Let the number is $11x + 6$

Divide $11x + 6$ by 7

$\frac{11x+6}{7} = \frac{7x+4x+6}{7} = \frac{4x+6}{7}$

Now put the value of x in $4x + 6$, so that the remainder will be 4, which is $x = 3$.

So the value of $11x + 6 = 39$

Now the remainder 39 when divided by 13 is zero.

It remains same for every number which satisfy the given condition

32. Count the number by actual counting method.

The numbers are—83, 89, 87, 101, 103.

33. Solve the question by taking different value of x and y.

For option (b) $x = 2$ and $y = 3$

$y^x = 3^2 = 9$ and $x^y = 2^3 = 8$

34. Using Fermat's theorem

$\frac{5^{79}}{7} = \frac{5^{78} \times 5}{7} = \frac{5^{6(13)} \times 5}{7} = \frac{15}{7} = 5$

Foundation

1. It is given that LCM = 590 = 59 × 2^2 × 5 and HCF = 59

So numbers can be assumed as $59a$ and $59b$

We know that Product of two numbers = LCM × HCF

So, $59a \times 59b = 590 \times 59$

Hence, $ab = 10 \Rightarrow$ Sets possible for a and b = (10,1) and (5, 2).

From here the sets of value of a and b are

(i) 59 × 2 and 59 × 5

(ii) 59 × 2 × 5 and 59

2. Let the first applicant is female. The remaining applicants = 5005 − 1 = 5004
For maximum female applicants, for every six applicants, there should be a female.
Therefore number of females = $1 + \frac{5004}{6} = 1 + 834 = 835$

3. HCF of A and $B = 24 = 2^3 \times 3$
And HCF of C and $D = 36 = 2^2 \times 3^2$
Then HCF of A, B, C and D = HCF of 24 and $36 = 2^2 \times 3 = 12$

4. $25 \times 35 \times 40 \times 50 \times 60 \times 65 = (5)^2 \times (5 \times 7) \times (5 \times 8) \times (5^2 \times 2) \times (5 \times 12) \times (5 \times 13)$
$= 5^8 \times 2^6 \times 3 \times 7 \times 13$
There are eight 5s and six 2s.
Number of zeroes = Number of sets of 2 and 5 = Minimum of (Number of 2s and number of 5s) = 6.

5. Unit digit of $576847 \times 564068 \times 96467 \times 458576$ = unit digit of $7 \times 8 \times 7 \times 6 = 56 \times 42 = 6 \times 2 = 12 = 2$

6. Unit digit of 1! + 2! + 3! + 4! + 5! + 6! ....
= 1 + 2 + 6 + 24 + 120 + 0 .... = 3
Note: We know that unit digit of 5! or for all the numbers greater than 5! is zero.

7. Factors of 1020 will divide 1020 properly.
So factors of $1020 = 2^2 \times 3 \times 5 \times 7$
$= (2 + 1)(1 + 1)(1 + 1)(1 + 1) = 24$

8. Number of prime divisors or factors = 4
(namely: 2, 3, 5 and 7)
Number of even factors = $2 \times 2 \times 2 \times 2 = 16$
So required factors = 16 − 4 = 12

9. Prime factorization of $(7!)^3 = (2^4 \times 3^2 \times 5 \times 7)^3$
$= 2^{12} \times 3^6 \times 5^3 \times 7^3$
Now for a multiple of 10, there should be at least one 5 and at least one 2 present in the number.
So the number can be like $= 2^{1-12} \times 3^{0-6} \times 5^{1-3} \times 7^{1-3}$
Hence, number of factors = $12 \times 7 \times 3 \times 4 = 1008$

10. If a number has odd number of divisors then it means, it is a perfect square.

11. $N = 2 \times 4 \times 6 \times 8 \times$ 100
Count the number of five's in N which is 12. So number of zeroes are 12.

12. $N = 10 \times 20 \times 30$ .... $\times 1000$
There is one 5 in the multiple of 10
There are two 5s in the multiple of 25
There are three 5s in the multiple of 125
Now count the multiple of 5s in the expression, which are 100 + 20 + 4 = 124

13. $N = 7 \times 14 \times 21 \times$ ..... $\times 777$
Method 1:
In this expression every fifth term is a multiple of 5.
Now there are 111 terms in the expression.
So number of 5s = $\frac{111}{5} + \frac{111}{25} = 22 + 4 = 26$
Method 2:
$N = 7 \times 14 \times 21 \times$ ..... $\times 777 = (7 \times 1) \times (7 \times 2) \times (7 \times 3)$.......................... $\times (7 \times 111) = 7^{111} \times (1 \times 2 \times 3 \times$ .... $\times 111) = 7^{111} \times 111!$
Number of zeroes in 111! = $\frac{111}{5} + \frac{111}{5^2} = 22 + 4 = 26.$

14. If the sum of digits is divisible by 9 then the number will also be divisible by 9.
So sum of 1 to 33 = $\frac{33 \times 34}{2} = 561$
Now the remainder, when 561 is divided by 9 = 3

15. Since there are 999 terms in the number, then it is divisible by 222.
Because every term will be divisible by 222 so all 999 terms will also be divisible by 222.

16. $7^1 = 7$, $7^2 = 9$, $7^3 = 3$ and $7^4 = 1$
So the cycle of 7 is 4 and $27!^{37!}$ is divisible by 4. So unit digit is 1.

17. $N = 420 = 2^2 \times 3 \times 5 \times 7$
Odd factors in N = 1, 3, 5, 7, 15, 21, 35, 105
Now $4n + 1$ format → Remainder obtain when divided by 4 is 1
So $4n + 1$ format number = 1, 5, 21, 105

18. $N = 2^3 + 5^3 \times 7^2$
First number of sets of co-prime factors in $2^3 \times 5^3 = (x + 1)(y + 1) + xy = 16 + 9 = 25$
Now number of sets of co-prime factors in $2^3 \times 5^3 \times 7^2 = A^{25} \times 7^2 = (x + 1)(y + 1) + xy$
$= 26 \times 3 + 50 = 128$

19. We know that $2^1 = 2$, $2^2 = 4$, $2^3 = 8$, $2^4 = 16$ and $2^5 = 32$
So cycle of 2 is 4
Now $\frac{3^{4^5}}{4} = \frac{(-1)^{4^5}}{4} = \frac{1}{4}$
So remainder is 1 so unit digit = $2^1 = 2$

20. Since all the numbers in the expression are odd. So product of all odd numbers would also be odd.
Hence, number of zeros is zero.

21. Number of 5s in 36! = $\left[\frac{36}{5}\right] + \left[\frac{36}{25}\right] = 7 + 1 = 8$
So zeros in $(36!)^{36!} = 8 \times 36!$

22. See the divisibility rule of 7, 11 and 13. These types of number will always divisible by 3, 7, 11, 13 and 37.

24. It is given that $2x - 5y = 1$
Smallest positive value of x is 3 when y is 1
And next sets are: (8, 3), (13, 5) and so on ....

Now it is clear that in every five consecutive numbers, there is a value of x, which satisfy $2x - 5y = 1$

Then number of values of $x = 1 + \frac{200-2}{5} = 1 + 39 = 40$

25. For $N = 2^3 \times 5^3$
The number of sets of factors co-prime to each other $= (x+1)(y+1) + xy = (3+1)(3+1) + 3 \times 3 = 25$
But for co-prime set (1, 1), factors are not distinct.
Therefore number of sets $= 25 - 1 = 24$

26. Do this question by actual calculation and the number is 143.
So sum of digits $= 1 + 4 + 3 = 8$

27. We know that $7^1 = 7$, $7^2 = 9$, $7^3 = 3$ and $7^4 = 1$
So cycle of 7 is four.
Now divide $11^{22^{33}}$ by 4
$\frac{11^{22^{33}}}{4} = \frac{(-1)^{22^{33}}}{4} = \frac{1}{4}$
Remainder obtain = 1
So unit digit $= 7^1 = 7$

28. We know that 5! Or greater than 5! will be divisible by 5
So, Remainder when (1! + 2! + 3!…… 1000!) is divisible by 5 equals to when
(1! + 2! + 3! + 4!) is divided by 5
So $\frac{1!+2!+3!+4!}{5} = \frac{33}{5} = \frac{3}{5}$
Hence, remainder obtained = 3

29. Let $x = 3^{148}$, $y = 5^{76}$ and $z = 7^{139}$
Then $A = x \times 3^2 \times y \times z \times 7 = 63xyz$
$B = x \times y \times z \times 7^2 = 49xyz$
$C = x \times y \times 5^4 \times z = 625xyz$
$D = x \times 3^3 \times y \times 5^4 \times z \times 7^3 = 5788125xyz$
So the order of A, B, C and D = DCAB

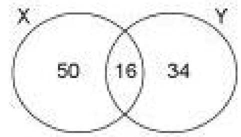
30. Smallest number is 0.0003 and it will also divide all the others number properly.
So HCF = 0.0003

31. Numbers divided by 5 but not by $9 = \left[\frac{550}{5} - \frac{550}{5 \times 9}\right] =$
$110 - 12 = 98$

32. It is given that $AB + BA$ = perfect square
$(10A + B) + (10B + A)$ = perfect square
$11(A + B)$ = perfect square.
For being a perfect square, $(A + B)$ should be 11.

33. N = 897324P64Q
For N divisible by 8, last three digits should be divisible by 8.
But 64Q is divisible by 8 when Q equals 0 and 8
And for N divisible by 9, sum of digits should be divisible by 9.
Now if $Q = 0$, then P should be 2,
And if $Q = 8$, then P should be 3.
Then $P + Q = 2$ and 11

Answers to Q.34 to 35:



34. Number in A, divisible by any of the two numbers $= 50 + 16 + 34 = 100$

35. Numbers, divisible by $X = 50 + 16 = 66$

36. Divisibility rule of 9 is that sum of all digits should be divisible by 9
So $\frac{x+5+1+5+x+0}{9} = \frac{11+2x}{9}$
From here x should be 8
So the number is 851580, which is also divisible by 4.

37. For minimum number of classrooms maximum number of students should be in a classroom.
This can be obtained by calculating the HCF of 60, 72 and 96 = 12
It means, every classroom should contain 12 student
Hence, number of classroom $= \frac{60}{12} + \frac{72}{12} + \frac{96}{12} = 19$

38. It is only asking about the LCM of 30 minutes, 45 minutes and 30 minutes.
So LCM = 90 minutes = 1 hour 30 minutes
Hence, earliest time for the lunch break = 9 am + 1 hour 30 minutes = 10:30 am

39. This question is asking about the LCM of 5s, 6s, 7s and 8s
Then LCM of 5s, 6s, 7s and 8s = 840 sec = 14 minutes
Hence, the time, when they hit target together = 10:14 am

40. Go through the options.
And the answer is option (c).
Because, number of factors of 30 $(2 \times 3 \times 5) = 8$
And number of factors of 60 $(2^2 \times 3 \times 5) = 12$

41. This question is asking about a number which when divided by 11, gives remainder 2 and when divided by 15, gives remainder 2 again.
Now find the number from actual calculation and the number is 167.

42. If last four digits of a number is divisible by 16, then the number will also be divisible by 16.

43. For n numbers, the operation should be repeated for $(x - 1)$ times, therefore for 50 numbers, the operation should be repeated for $50 - 1 = 49$ times

44. The number of 5s in $18! = \frac{18}{5} = 3$
And in $19! = \frac{19}{5} = 3$
So number zeroes in 18! is 3 and in 19! is 3.
Hence, number of zeroes in 18! + 19! = 3 zeroes.

45. Let the number is x
It is given that if we divide the sum of two numbers, then the remainder is 236.
Hence, it means when we divide $(437 + 298)$ by x, then the remainder is 236.
From here, the number x should be 499.

46. For being a perfect square, the last digit of the number should be 1, 4, 5, 6 and 9.
And the digital sum of the number should be: 1, 4, 9 and 7.

47. $N^2 + N^3 = N^2 (N + 1)$
For $N^2 + N^3$ be a perfect square, $(N + 1)$ should be a perfect square.
And we know that there are 10 perfect square till 100.
But we cannot take $N + 1 = 1 \rightarrow N = 0$
So there are 9 numbers for which $N^2 (N + 1)$ will be a perfect square.

48. Go through the options.

49. Any number of format *abcabc* or *aaaaaa* will be divisible by 7, 11 and 13.

50. For minimum tiles, the sides of tiles should be the HCF of 3.78 m and 4.8 m
HCF of 3.78 and 4.8 = 0.06 m

$$\text{Hence, number of tiles} = \frac{\textit{Area of floor}}{\textit{Area of tile}} = \frac{3.78 \times 4.8}{0.06 \times 0.06} = 5040$$

51. Number should be like: (multiple of LCM of 7, 11 and 21) + 3
Then find the smallest five digit multiple of LCM of 7, 11 and 21 and add 3 to that number.

52. Answer should be HCF of $(57 - x)$, $(129 - x)$ and $(177 - x)$.
In other words, the largest number which gives the same remainder when dividing 57, 129 and 177 is the answer.
Now go through the options. Answer is 24 litres.

53. Let us first find the number which is divided by 3, 4 and 5 gives remainder 1, 1 and 0 respectively. It is equal to 25.
It is given the 10 eggs are left now. It means 25 – 10 = 15 eggs has been broken.

54. There is only one set of prime number which satisfy the given condition.
And the set of prime number is (2, 3)

55. Clearly the two quantities are both integers, so we check the prime factorization of $2005 = 5 \times 401$. It can be seen that $(A, B, C) = (4, 0, 1)$ satisfies the relation. Hence, option (a) is the answer.

56. There are 17 factors of 3^{16} which are $3^0, 3^1, 3^2, 3^3, 3^4, \ldots\ldots, 3^{16}$
Product of factors $= 3^0 \times 3^1 \times 3^2 \times 3^3 \times 3^4 \ldots \times \ldots\ldots \times 3^{16} = 3^{(0+1+2+3+\ldots\ldots+16)} = 3^{136}$

57. Remainder obtained when $\frac{90^{91}}{13} = \frac{(-1)^{91}}{13} = \frac{-1}{13}$
Hence, remainder is –1 or 12.

58. $$\frac{8641}{8640} \times \frac{8642}{8640} \times \ldots \times \frac{8650}{8640} = \frac{1 \times 2 \times 3 \times \ldots\ldots \times 10}{8640} = \frac{10!}{8640} = \frac{3628800}{8640}$$
This is divisible by 8640 as can be seen through the actual calculation. Hence, remainder = 0.

59. **Method 1:**
We find the number of numbers with a 4 and subtract from 2005. Quick counting tells us that there are 200 numbers with a 4 in the hundreds place, 200 numbers with a 4 in the tens place, and 201 numbers with a 4 in the units place (counting 2004). There are 20 numbers with a 4 in the hundreds and in the tens, and 20 for both the other two intersections. The intersection of all three sets is just 2. So we get:
$2005 - (200 + 200 + 201 - 20 - 20 - 20 + 2) = 1462$.
Hence, option (b) is the answer.

Method 2:
Alternatively, consider that counting without the number 4 is almost equivalent to counting in base 9; only, in base 9, the number 9 is not counted. Since 4 is skipped, the symbol 5 represents 4 miles of travel, and we have traveled 2004_g miles. By basic conversion, $2005_g = 9^3(2) + 9^0(5) = 729(2) + 1(5) = 1458 + 5 = 1463$. $1463 - 1 = 1462$. Hence, option (b) is the answer.

60. These are exactly 18 numbers between 400 and 500 and 18 numbers between 500 and 600 where 8 occurs only once. So total number = 18 + 18 = 36

61. $n^2 = 123.45654321 = 12345654321 \times 10^{-8}$
So $n^2 = (111111 \times 10^{-4})^2$. So $n = 11.1111$

62. Let the load of mule is x and load of horse is y.
Now from the question
$2(x - 1) = y + 1$
$2x - 2 = y + 1 \rightarrow 2x - y = 3$ .... (i)
And $x + 1 = y - 1$
$x - y = -2$ ...... (ii)
Now from equation (i) and (ii) $x = 5$ and $y = 7$ then $x + y = 12$

64. Remainder obtained when $3x^2 - x^6 + 31x^4 + 21x + 5$ is divided by $x + 2$ can be obtained by putting $x + 2 = 0$ in the original expression.
Putting $x = -2$ in the given expression:
$3x^2 - x^6 + 31x^4 + 21x + 5 = 3(-2)^2 - (-2)^6 + 31\ (-2)^4 + 21\ (-2) + 5 = 406$
So option (d) is the answer.

65. Let the numbers are: a, b, c and d
It is given that $a \times b \times c = 385$ .... (i)
And $b \times c \times d = 1001$ .... (ii)
Now divide equation (ii) by equation (i)
$\frac{b \times c \times d}{a \times b \times c} = \frac{1001}{385} \to \frac{d}{a} = \frac{13}{5}$
Hence, largest number (d) = 13

66. Remainder, when $\frac{4^{44}}{15} = \frac{16^{22}}{15} = \frac{(1)^{22}}{15} = \frac{1}{15}$
Hence, remainder = 1

67. Since 211 is a prime number, So P and Q = (1 and 211) or (211 and 1)
Hence, HCF = 1

68. We know that in every consecutive 100 numbers, every digit comes 10 times at unit's place and 10 times at ten's place. Then from 11 – 100, 6 will appear for 19 times from 100 – 400, 6 will appear for 3 × 20 = 60 times
Hence, answer = 19 + 60 = 79 times

69. In every 100 consecutive natural numbers, every digit will appear in 19 numbers (a total of 20 times). Now solve the question.

70. A number like *aaaaaa* is divisible by 7. It means 8 written 84(6 × 14) times is divisible by 7.
Now divide the last four digits of the number by 7 and find the remainder.
Hence, remainder obtained = $\frac{8888}{7} = 5$

71. Do it from actual counting.

72. $\frac{7 + 77 + 777........(till\ 100\ terms)}{8} = \frac{7}{8} + \frac{77}{8} + \frac{777}{8}$ .... (till 100 terms)
$\frac{7 + 5 + 1 + 1 + 1........(till\ 100\ terms)}{8}$
$= \frac{7 + 5 + 98}{8} = \frac{6}{8}$. Hence, remainder = 6.

73. Total number of factors of any number = 1 + Prime factors of that number + Composite factors of that numbers
Now we will verify the number of prime factors one by one.
If number of prime factor = 1, then total number of factors = 1 + 1 + 15 = 17.
If the number is like 2^{16}, it will have 17 factors. Hence, using one prime factor, it is possible to make total 17 factors (or total number of composite factors = 15).
Let prime factors are two,
Then number of total factor = 15 + 2 + 1 = 18
This is possible for $a^2 \times b^5$
When prime factors are three,
Then number of total factors = 19, which is not possible because 19 cannot be broken down in three parts.
When prime factors are four,
Total number of factors = 20, which is also not possible because 20 cannot be broken down in four parts.
Similarly, when prime factors are five, minimum number of factors of $a \times b \times c \times d \times e = 32$
Hence, maximum value of prime factors = 2.

74. $N = 204 \times 221 \times 238 \times 255 \times .... \times 850 = (17 \times 12) \times (17 \times 13) \times (17 \times 14) \times (17 \times 15) \times .... (17 \times 50)$
We are required to count the number of 5s in N = Number of zeroes in N.
To count the number of 5s, we can count it from $[(17 \times 1) \times (17 \times 2)............. (17 \times 12) \times (17 \times 13) \times (17 \times 14) \times (17 \times 15) \times ....(17 \times 50)]$ and then subtract the number of 5s in $[(17 \times 1) \times (17 \times 2)............. (17 \times 10) \times (17 \times 11)]$
Number of 5s in $[(17 \times 1) \times (17 \times 2)............. (17 \times 12) \times (17 \times 13) \times ....(17 \times 50)] = 12$
Number of 5s in $[(17 \times 1) \times (17 \times 2)............. (17 \times 11)] = 2$
Hence, number of 5s in $(17 \times 12) \times (17 \times 13) \times (17 \times 14) \times (17 \times 15) \times ....(17 \times 50) = 12 - 2 = 10$.

75. $\frac{123456.....125126}{5625} = \frac{123456......125000 + 126}{5^4 \times 3^2}$
$= \frac{123456.....124125 \times 10^3}{5^4 \times 3^2} + \frac{126}{5^4 \times 3^2}$
Now number 12345.....124125 is divisible by 9 because sum of digits is divisible by 9 and it is also divisible by 5^4 because 10^3 is divisible by 5^3 and number 123456.....125 is divisible by 5.
Hence, the remainder = Remainder obtained when $\frac{126}{5625} = 126$

76. Number will be like: $3(4(7x + 4) + 1) + 2 = 84x + 53$
When this number is divided by 84, remainder obtained = 53
Alternatively, go through the options.

77. Remainder, when $\frac{1714 \times 1715 \times 1717}{12} = \frac{10 \times 11?13}{12}$
$= \frac{2}{12}$
Hence, the remainder is 2.

78. Following pattern can be observed:
$(11)^2 = 121$
$(111)^2 = 12321$
$(1111)^2 = 1234321$
....
$(111111111)^2 = 12345678987654321$

79. For largest value of the product, difference between $(a + b)$ and $(c + d)$ should be as less as possible.
Then for this condition, Let $a = 12$, $b = 15$, $c = 13$, $d = 14$
So $(a + b)(c + d) = (12 + 15)(13 + 14) = 27 \times 27 = 729$

80. There is only one number of form *abbb*, and which is $38^2 = 1444$

Moderate

1. In case of decimal system, we obtain 10 by multiplying 5 and 2, and then to find the number of zeroes, we search the exponents of 5. In case of base 6, 10 will be obtained by multiplying 3 and 2. So, here we will check for the exponents of 3 to know about the number of zeroes. And obviously it is 5[12/3 + 12/9].

2. The remainder obtained when $2222^{5555} + 5555^{2222}$ is divided by 7 will be the same as the remainder when $3^{5555} + 4^{2222}$ is divided by 7. Now find the individual remainder and solve it.

3. If we look at the numbers $100 < N \leq 105$, we see only 101 and 103 do not have their factors in N (because these are primes). So, obviously the new LCM will be $101 \times 103 \times N$.

4. $10^5 = 2^5 \times 5^5$
Now all the factors of 10^5 which will end in one zero will be zero power of 2 and 1 – 5 powers of 5 and vice versa. This will be equal to 9.

5. $1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10 = 55$
So, by replacing the signs we need to make 45 extra. This is possible only if we write in this way:
$1 \times 2 + 3 \times 4 + 5 + 6 + 7 \times 8 + 9 + 10 = 55 + 45 = 100$

6. The key is the fact that in this country only three symbols are used to write numbers.
So, $6 = (20)_3 = (bc)_3$
So, $b = 2$, $c = 0$ and $a = 1$
$17 = (122)_3 = abb$

7. Let the number is AB
For Perfect square $= AB + BA = (10A + B) + (10B + A) = 11(A + B)$
For being a perfect square $A + B$ should be equal to 11. Then $A + B = 11$. Now find the sets of values of A and B.

8. Remainder of (${}_{32}32^{32}$ divided by 7) = Remainder of (${}_{4}32^{32}$ divided by 7)
Now find cyclicity of remainder of (${}_{4}32^{n}$ divided by 7).
Remainder when ${}_{4}32^{1}$ divided by 7 = 2
Remainder when ${}_{4}32^{2}$ divided by 7 = 4
Remainder when ${}_{4}32^{3}$ divided by 7 = 2
So, the cyclicity is 2, 4, 2, 4 and so on.
For every even value of n, remainder = 4
So, answer is option (d).

9. $1001 = 7 \times 11 \times 13$
We know that any digit written 6 times consecutively (like 111111, or 222222, etc.) will be divisible by 3, 7, 11, 13, 37. So, this question is—what is the remainder when 11111 is divided by 1001. Find it out by actual division method.

10. **Method 1:**
It is given that
$a + b + c = d + 8$ .... (i)
$a^2 + b^2 = c^2 + d^2 + 36$ .... (ii)
$ab + cd = 42$ .... (iii)
$a^3 = b^3 + c^3 + d$ .... (iv)
Now go through the options.

11. Digital sum of $(19)^{100}$ = Digital sum of $(1 + 9)^{100}$ = Digital sum of $(10)^{100}$ = Digital sum of $(1 + 0)^{100}$ = Digital sum of $(1)^{100} = 1$

12. Use the formula given in the concepts.

13. **Method 1:**
Note that $\frac{1}{11} = 0.\overline{09}$.
Dividing by 3 gives $\frac{1}{33} = 0.\overline{03}$, and dividing by 9 gives $\frac{1}{99} = 0.\overline{01}$.
$S = \{11, 33, 99\}$
$11 + 33 + 99 = 143$
The answer must be at least 143, but cannot be 155. Hence, option (d) is the answer.

Method 2:
Let us begin by working with the condition $0.\overline{ab} = 0.ababab....$
Let $x = 0.ababab....$ So, $100x - x = ab$. Or, $x = \frac{ab}{99}$.
In order for this fraction x to be in the form $\frac{1}{n}$, 99 must be a multiple of ab. Hence, the possibilities of ab are 1, 3, 9, 11, 33, 99. Checking each of these,
$\frac{1}{99} = 0.\overline{01}$, $\frac{3}{99} = \frac{1}{33} = 0.\overline{03}$, $\frac{9}{99} = \frac{1}{11} = 0.\overline{09}$, $\frac{11}{99} = \frac{1}{9} = 0.\overline{1}$, $\frac{33}{99} = \frac{1}{3} = 0.\overline{3}$, and $\frac{99}{99} = 1$. So the only values of n that have distinct a and b are 11, 33, and 99. So, $11 + 33 + 99 =$ (d) 143.

14. Digits which can create confusion = 1, 6, 8, 9 (0 cannot create confusion because passwords has to be two-digit numbers).
Total two digit numbers with distinct digit = 81
Two digit numbers created by 1, 6, 8, 9 = 12
So, total numbers left = 69
But 69 and 96 wont create confusion (it looks same upside down), so total numbers = 71

15. There is only one set possible.
Where $p = 3$, $p + 2 = 5$ and $p + 4 = 7$
In every other set, one number will be divisible by 3, and hence, that number will not be a prime number.

16. See the solution of CAT '04 given at the end of this book.

17. For minimum number of students, who has/have solved at least five questions, the case is:
Exactly one student has solved one question,
Exactly one student has solved two questions,
Exactly one student has solved three questions,
Exactly six students have solved four questions,
And exactly one student has solved five questions.

18. $N!$ is having 37 zeroes at its end, so $N = 150$ (can be arrived at by a guess).
Obviously, $150 \le N < 155$ is the answer.

19. From the previous question, we have found that the range of $N = 150 \le N \le 155$
Then odd values of $N = 151$, 153 and 155

20. There is no number having 30 zeroes at its end. Because 124! has 28 zeroes at its end and 125! has 31 zeroes at its end.

21. There are 50 odd numbers and 50 even numbers. Every even number will be divisible by 4. And in odd numbers half of them having 1 as the remainder and half of them having −1 as the remainder. Then overall remainder is zero.

22. We cannot take 9 and 7 together because $9^3 + 7^3 = 1072$ (four digit number)
We cannot 9 and 5 together, because $9^3 + 5^3 = 854$ (8 is a even number)
We cannot take 9, 3 and 1 together because $9^3 + 3^3 + 1^3 = 757$
We cannot take 7 and 5 together, because $7^3 + 5^3 = 468$ (4 is a even number)
We can take 7, 3, 1 as the digits, because $3^3 + 7^3 + 1^3 = 371$

23. The question is: In which system of writing, 24 + 32 = 100. Go through options.

24. $108 = 3^3 \times 4 = 27 \times 4$
Remainder obtained when 3^{450} is divided by 108 is same as the number obtained when 3^{450} is divided by 27 and 4.
Remainder obtained when 3^{450} is divided by $3^3 = 0$
Remainder obtained when 3^{450} is divided by $4 = 1$
Now we are required to find a number which when divided by 27 gives 0 as the remainder and when divided by 4 gives 1 as the remainder = 81.
Hence, option (d) is the answer.

25. 2P is having 28 (7 × 4) divisors but 3P is not having a total divisor which is divisible by 7. So, the first part of the number P will be 2^5.
Similarly, 3P is having 30 (3 × 10) divisors but 2P does not have a total divisor which is divisible by 3. So, 2nd part of the number P will be 3^3.
So, $P = 2^5 \times 3^3$.

26. pqr can be 370 or 371. So, it is not possible to arrive at a unique answer.

27. Let the smaller number = x. Then the larger number = $6x$
So from the question, $6x + x = 504\,K$
Here the only value of K should be 2.
Then $6x + x = 504 \times 2$. Hence, $x = 144$
Then $6x - x = 864 - 144 = 720$

28. It is given that LCM of 12^{24}, 16^{18} and $N = 24^{24}$
Or $3^{24} \times 2^{48}$, 2^{72} and $N = 2^{72} \times 3^{24}$
From here, the value of N can be: $2^{0-72} \times 3^{0-24}$
Then total number of value = (72 + 1) (24 + 1) = 1825

29. It is given that if $Q = 0$, then $R = 1$
But $R = 0$, so $Q = 1$ ...... (i)
It is also given that if $S = 0$, then $P = 1$
But for $R = 0$, $P = S$,
So $P = S = 1$ ...... (ii)
Then $(P + Q + R + S) = 1 + 1 + 0 + 1 = 3$

30. Mayank DOB = 1948 and Siddharth DOB = 1898

31. Let us discuss the fate of any particular cell number as per the algorithm given:
Cell Number 45
Initially - Closed

After Step 1	After Step 2	After Step 3	After Step 4	After Step 5	After Step 6
Open	Open	Close	Close	Open	Open

After Step 7	After Step 8	After Step 9	After Step 15	After Step 16	After Step 45
Open	Open	Close	Open	Open	Close