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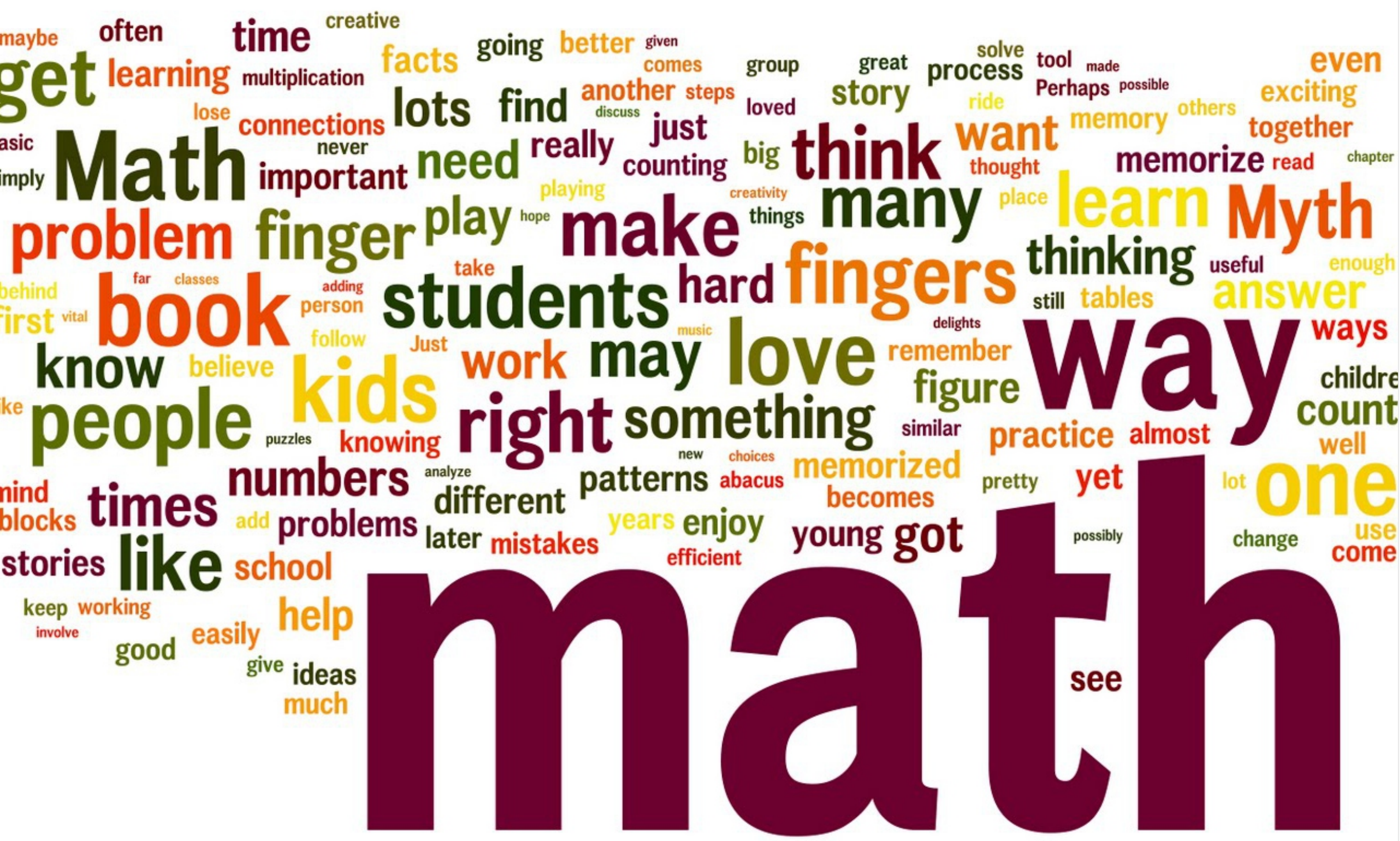

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Multiplying And Adding Numbers In The Form: ab + bc

A. From algebra we can factor:

$$ab + bc = b(a + c)$$

B. Using numbers instead of variables we get the following:

1. Take out the number that both sides have in common.
2. Add the remaining numbers.
3. Multiply the number in step 1 with the result in step 2 for the answer.

Ex [1] 15 x 12 + 15 x 8 =_________.

a) Rewrite in the form 15 x (12 + 8).
b) 15 x 20 = 300.
c) The answer is 300.

Ex [2] 16 x 16 + 16 x 17 =_________.

a) Rewrite in the form 16 x (16 + 17).
b) 16 x 33 = 48 x 11.
c) 48 x 11 = 528.

C. Ex [2] uses a variety of different methods. This is just how I would do the problem, but there are many different ways of going about solving this problem. This is up to you.

Adding a sequence in the form: 1 + 2 + ... + n:

A. This sequence is sometimes referred to as *Triangular Numbers*, and can be solved by the equation:

$$\sum_{i=1}^{n} i = 1 + 2 + \ldots + n = \frac{n \cdot (n+1)}{2}$$

B. Using numbers instead of variables we get the following:

1. Multiply the last number by that number plus 1, then divide by 2.
2. Notice one of these numbers is divisible by 2, so you can divide the even number by 2 and then multiply by the other number.

Ex [1] 1 + 2 + ... + 10 =_________.

a) From the equation we know this is equal to: $^{10 \text{ x } 11}/_2$ or 5 x 11 = 55.

b) The answer is 55.

Ex [2] 1 + 2 + ... + 50 =_________.

a) From the equation we know this is equal to: $^{50 \text{ x } 51}/_2$ or 25 x 51 = 1275. See *Multiplying by 25*.

b) The answer is 1275.

C. Sometimes there might be a number missing to throw you off, so you need to be careful.

Ex [3] 2 + 3 + 4 + ... + 25 =_________.

a) Notice that the number 1 is missing from the equation. Treat it as though it were there.

b) From the equation we know this is equal to: $^{25 \text{ x } 26}/_2$ or 25 x 13 = 325.

c) Since the number 1 is missing, you should subtract 1 from 325. The answer is 324.

Approximating - Adding A Series Of Numbers:

A. These types of problems will almost always be found on problem #10 and nowhere else.

B. Since these problems are approximations you can make it easier and faster on yourself by rounding some numbers with discretion. A basic rule of thumb is "the larger the numbers, the more you can round."

Ex [1] 558 + 243 - 132 + 69 = __________.

a. In this problem, the numbers are not large but small, so I would round with extreme discretion.

b. It is safe to use: 600 + 200 - 130 + 70, because the first number is less than 600 about the same distance as the second number is greater than 100. You would get 740.

c. However, you could use: 560 + 200 - 100 + 70, because 243 is almost the same distance from 200 as 132 is from 100. You would get 730.

d. The answers can be between 702 and 774.

Ex [2] 4589 + 6743 - 1237 + 555 = _________.

a. In this problem, the numbers are larger, so we have a greater leniency.

b. It is safe to use: 5000 + 6000 - 1000 + 600, because 4589 is close to the same distance from 5000 as 6743 is from 6000 and also 1237 is close to 1000 but to make sure round 555 up. You would get 10600.

c. The answer can be between 10118 and 11182.

C. Many times there will be numbers on the question that are insignificant and can be ignored.

Ex [3] 14141 - 1414 - 141 - 14 - 1 = __________.

a. In this problem we are dealing with big numbers and the small ones should be ignored. The first 2 numbers are the only important ones.

b. It is safe to use: 14400 - 1400, since the numbers are relatively big; we have more leniency. You would get 13000.

c. The answer can be between 11943 and 13199.

D. In short, there are numerous ways to going about solving these types of approximations. It takes practice to learn how much you can round.

E. If you do not feel comfortable rounding so much (as in Ex [3]), then you can round first and subtract a little off in the end just to be sure. This practice saved me a few times.

Adding And Subtracting Fractions:

A. The conventional way of adding and subtracting fractions is by changing the fractions to have the same denominators, otherwise known as the LCM or Least Common Multiple (see LCM in *Miscellaneous*).

Ex [1] $\frac{1}{3}+\frac{1}{2}=$ __________ (fraction)

Since $^1/_3$ and $^1/_2$ can not be added directly, each fraction needs to be changed to have a common denominator of 6.

a) $^1/_3 = {}^2/_6$ and $^1/_2 = {}^3/_6$.

b) $^2/_6 + {}^3/_6 = {}^5/_6$.

c) The answer is $^5/_6$.

B. Another way of adding and subtracting fractions is using the following rule from algebra (sometimes called cross-multiplication):

$$\frac{a}{b}+\frac{c}{d}=\frac{ad+bc}{b\cdot d}$$

Ex [2] $\frac{2}{7}-\frac{1}{6}=$ __________ (fraction)

a) $\frac{2}{7}-\frac{1}{6}=\frac{2\cdot 6-1\cdot 7}{6\cdot 7}$

b) $\frac{2\cdot 6-1\cdot 7}{6\cdot 7}=\frac{12-7}{42}=\frac{5}{42}$

c) The answer is $^5/_{42}$.

C. There are several precautions and suggestions that should be considered when adding and subtracting fractions.

1. Before using the method in part B, first look to see if one denominator is a multiple of the other. If it is, the method in part A is faster.
2. Before writing down the answer, make sure the fraction is reduced to its simplest form.
3. Always know what the question is asking for. Sometimes the answer can be given in improper fractions, while other times mixed numbers should be given.

Comparing Fractions:

A. When comparing fractions, we often need to know which fraction is smaller or larger.

$$\frac{a}{b} \; ? \; \frac{c}{d}$$

1. To solve this quickly you can use cross-multiplication to determine which fraction is smaller or larger:

$$ad \quad ? \quad bc$$

$$\frac{a}{b} \times \frac{c}{d}$$

2. In other words, if $ad > bc$ then the fraction on the left is larger. If $ad < bc$, then the fraction on the right is larger. If $ad = bc$, then the 2 fractions are equivalent.

Ex [1] Which is greater: $\frac{5}{6}$ or $\frac{7}{9}$?

a) Using the rule of cross-multiplication we can compare 9 x 5 and 6 x 7.

b) 9 x 5 = 45.

c) 6 x 7 = 42.

d) Since $45 > 42$, the fraction on the left is greater.

e) So the answer is $^5/_6$.

B. Sometimes instead of giving two fractions, the problem will give one fraction and one decimal. In problems like these, simply change the decimal to a fraction (it does not have to be in simplest terms) and compare using this method.

Ex [1] Which is smaller: .54 or $\frac{6}{11}$?

a) You can change .54 to $^{54}/_{100}$ (there is no need to simplify).

b) Using cross-multiplication we can compare 54 x 11 and 6 x 100.

c) 54 x 11 = 594

d) 6 x 100 = 600.

e) Since $594 < 600$, the fraction (or in this case the decimal) on the left is smaller.

f) The answer is .54.

C. In problems like Ex [1] Part B, it would be faster if you knew that $^{6}/_{11}$= .5454... Therefore, memorizing the fractions will be useful in situation like these.

Basic Conversions:

A. Below is a list of basic conversions that you should know:

Distance:

1000 mm = 1 m	12 in = 1 ft
100 cm = 1 m	3 ft = 1 yd
1000 m = 1 km	5280 ft = 1 mile
1 in = 2.54 cm	1760 yd = 1 mile

Volume:

1000 mL = 1 L	1 pint = 2 cups
1 Tbl = 3 Tsp	2 quarts = 1 pint
1 cup = 16 Tbl	4 quarts = 1 gal
1 cup = 8 oz	231 in^3 = 1 gal

Weight:

1000 mg = 1 g	1 lb = 16 oz
1000 g = 1 kg	1 ton = 2000 lbs

B. Examples

Ex [1] 36 yd = ______ in

a. First, convert yd to feet. 36 yd = 108 ft. (Multiply 36 x 3).

b. 108 ft = 12 x 108 in = 1296 in.

c. The answer is 1296.

Ex [2] How many pints are in 3 gallons? ________

a. First, convert gallons to quarts. 3 gallons = 3 x 4 quarts or 12.

b. Convert quarts to pints: 12 quarts = 12 x 2 pints = 24 pints.

c. The answer is 24.

Ex [3] 2.5 pounds = ______ oz.

a. There are 16 oz in a pound so multiply 2.5 by 16 or 40.

b. The answer is 40.

C. If you need more conversions, most can be found on the *basic conversion* page.

The following cubes should be memorized (at least through 20^3):

$1^3 = 1$	$2^3 = 8$	$3^3 = 27$	$4^3 = 64$	$5^3 = 125$
$6^3 = 216$	$7^3 = 343$	$8^3 = 512$	$9^3 = 729$	$10^3 = 1000$
$11^3 = 1331$	$12^3 = 1728$	$13^3 = 2197$	$14^3 = 2744$	$15^3 = 3375$
$16^3 = 4096$	$17^3 = 4913$	$18^3 = 5832$	$19^3 = 6859$	$20^3 = 8000$
$21^3 = 9261$	$22^3 = 10648$	$23^3 = 12167$	$24^3 = 13824$	$25^3 = 15625$

There are some numbers in which you should memorize even higher powers:

$2^4 = 16$	$2^8 = 256$	$3^4 = 81$	$4^5 = 1024$
$2^5 = 32$	$2^9 = 512$	$3^5 = 243$	$5^4 = 625$
$2^6 = 64$	$2^{10} = 1024$	$3^6 = 729$	$6^4 = 1296$
$2^7 = 128$	$2^{11} = 2048$	$4^4 = 256$	$7^4 = 2401$

Definitions:

A. There are many problems on the number sense tests that can be solved by simply knowing its definition.

Mean - The mean of a list of numbers is simply the average of all the numbers.

Ex [1] Find the mean of 12, 13, 14, 15, and 16.

a. On problems like this, you can add up all the numbers and divide by the number of terms like:

$(12 + 13 + 14 + 15 + 16) \div 5$ or

b. If each number is being added by a fixed number, you can simply write the middle number (if the list has an odd number of terms) or you can take the average of the middle two numbers (if the list has an even number of terms).

c. For this example, there are 5 terms, so the mean is 14.

d. The answer is 14.

Ex [2] Find the mean of 3, 7, 11, 15, 19, and 23.

a. In this example we can take the average of the middle two numbers: 11 and 15.

b. $(11 + 15) \div 2$ is $26 \div 2$ or 13.

c. The answer is 13.

*NOTE: Many times there is no shortcut and you must add all the terms and divide by the number of terms.

Median - The median of a set of numbers is simply the middle number. If a set has an even number of terms, then the median is the average of those two terms.

*NOTE: The numbers in the set MUST be in sequential or der.

Ex [3] Find the median of 2, 5, 6, 9, 12.

a. Since there are an odd number of terms, the median is the middle number or 6.

b. The answer is 6.

Ex [4] Find the median of 2, 2, 4, 4, 1, 2.

a. First, change this set to be {1, 2, 2, 2, 4, 4} mentally.

b. Since there is an even number of terms we take the average to the middle terms: 2 and 2.

c. The answer is 2.

Mode - The mode of a set of numbers is simply the number that repeats itself the most.

Ex [5] Find the mode of 2, 3, 7, 3, 7, 1, 3.

a. In this example there are more 3's than there are of other numbers so the mode is 3.

b. The answer is 3.

Range - The range of a set of numbers is simply the largest value minus the smallest value.

Ex [6] The range of 1, 7, -1, 8, 4, and 3 is _______.

a. The largest value is 8 and the smallest value is -1.

b. $8 - (^{-}1) = 9$.

c. The answer is 9.

Additive Inverse - The additive inverse of a number is the number that you would add to make the original number equal to 0.

Ex [7] The additive inverse of $-1\frac{4}{5}$ is __________.

a. The number that you would add to make this 0 is $1\ ^{4}/_{5}$.

b. The answer is $1\ ^{4}/_{5}$.

Multiplicative Inverse - The multiplicative inverse of a number is the number that you would multiply to make the original number equal 1.

Ex [8] The multiplicative inverse of $-^{4}/_{5}$ is _______.

a. The number that you would multiply to make this 1 is $-^{5}/_{4}$.

b. The answer is $-^{5}/_{4}$.

Negative Reciprocal - The negative reciprocal of a number is -1/n. If the number is a fraction simply flip the fraction and change the sign.

Ex [9] The negative reciprocal of $\frac{4}{5}$ is _______.

a. Flipping the fraction and changing the sign we get $-^5/_4$.

b. The answer is $-^5/_4$.

B. The following definitions are more advanced and therefore, probably will not be seen on an elementary test.

Geometric Mean - The geometric mean is equated by the following:

$$(a_i \times a_j \times \cdots \times a_n)^{1/n}$$

1. In other words it is the n^{th} root of the product of all the numbers in the set.
2. On a number sense test, most of the time you will be dealing with 2 numbers. If you are dealing with 3 or more, each value will have an even n^{th} root.

Ex [10] Find the geometric mean of 6 and 8.

a. According to the definition, the geometric mean is the square root of 6 x 8.

b. 6 x 8 = 48.

c. $\sqrt{48} = \sqrt{16} \times \sqrt{3} = 4\sqrt{3}$

d. The answer is $4\sqrt{3}$

Ex [11] Find the geometric mean of 64, 27, and 8.

a. In this example, we are going to have to find the cube root of the product of these numbers.

b. However, each number is a perfect cube. So take the cube root of each individual number and multiply them together.

c. $\sqrt[3]{64} = 4$; $\sqrt[3]{27} = 3$; $\sqrt[3]{8} = 2$. So 4 x 3 x 2 = 24.

d. The answer is 24.

Palindrome - A palindrome is a number (or a word) that is the same frontward and backward.

Ex [12] The smallest palindrome greater than 768 is _______.

a. The answer is 777. Since this is the next number that is a palindrome.

Ex [13] The largest palindrome less than 1024 is _______.

a. The answer is 1001.

DIVISION -

A number is divisible by another number if after dividing, the remainder is zero. For example, 18 is divisible by 3 because 18 ÷ 3 = 6 with 0 remainder. However, 25 is not divisible by 4 because 25 ÷ 4 = 6 with a remainder of 1. There are several mental math tricks that can be used to find the remainder after division without actually having to do the division.

Dividing By 2: A number is divisible by 2 if the last digit is even.

Dividing By 3: A number is divisible by 3 if the sum of all the digits is divisible by 3.

Ex [1] 34,164 is divisible by 3 because 3+4+1+6+4 = 18 which is divisible by 3.

*To find the remainder of a number divided by 3, add the digits and find that remainder. So if the digits added together equal 13 then the number has a remainder of 1 since 13 divided by 3 has a remainder of 1.

Dividing By 4: A number is divisible by 4 if the last 2-digits are divisible by 4.

Ex [1] 34,164 is divisible by 4 because 64 is divisible by 4.

*To find the remainder of a number divided by 4 take the remainder of the last 2 digits. So if the last 2-digits are 13 then the number has a remainder of 1 since 13 divided by 4 has a remainder of 1.

Dividing By 5: A number is divisible by 5 if the last digit is a 5 or a 0.

*To find the remainder of a number divided by 5 simply use the last digit. If it is greater than 5, subtract 5 for the remainder.

Dividing By 6: A number is divisible by 6 if it is divisible by 2 and by 3.

Ex [1] 34,164 is divisible by 6 because it is divisible by 2 and 3.

Dividing By 7: A number is divisible by 7 if the following is true:

1. Multiply the ones digit by 2.
2. Subtract this value from the rest of the number.
3. Continue this pattern until you find a number you know is or is not divisible by 7.

Ex [1] 7203 is divisible by 7 because

a) 2 x 3 = 6.

b) 720 – 6 = 714 which is divisible by 7.

Ex [2] 14443 is not divisible by 7 because

a) 3 x 2 = 6.

b) 1444 – 6 = 1438.

c) 8 x 2 = 16.

d) 143 – 16 = 127 which is not divisible by 7.

Note: This method takes a lot of practice and is sometimes easier to just work it out individually.

Dividing By 8: A number is divisible by 8 if the last 3-digits are divisible by 8.

Ex [1] 34,168 is divisible by 8 because 168 is divisible by 8.

*To find the remainder of a number divided by 8 take the remainder of the last 3-digits. So if the last 3-digits are 013 then the number has a remainder of 5.

Dividing By 9: A number is divisible by 9 if the sum of the digits is divisible by 9.

Ex [1] 34,164 is divisible by 9 because 3+4+1+6+4 = 18 which is divisible by 9.

*To find the remainder of a number divided by 9, add the digits and find that remainder. So if the digits added together equal 13 then the number has a remainder of 4 since 13 divided by 9 has a remainder of 4.

Dividing By 10: A number is divisible by 10 if the last digit is a 0.

*To find the remainder of a number divided by 10 simply use the last digit.

Dividing By 11: A number is divisible by 11 if this is true:

1st Step: Starting from the one's digit add every other digit

2nd Step: Add the remaining digits together

3rd Step: Subtract 1st Step from the 2nd Step

*If this value is 0 then the number is divisible by 11. If it is not 0 then this is the remainder after dividing by 11 if it is positive. If the number is negative add 11 to it to get the remainder.

Ex [1] 6613585 is divisible by 11 since (5+5+1+6) – (8+3+6) = 0.

Dividing By 12: A number is divisible by 12 if it is divisible by 3 and by 4.

Ex [1] 34,164 is divisible by 12 because it is divisible by 3 and 4.

Double and Half Method:

A. The double and half method is used to make two numbers easier to multiply. The idea is you can double one number and half the other before multiplying.

Ex [1] 35 x 24 =_________.

a) 35 x 2 = 70.

b) 24 ÷ 2 = 12.

c) 70 x 12 = 840.

d) The answer is 840.

Ex [2] 57 x 22 =_________.

a) 57 x 2 = 114.

b) 22 ÷ 2 = 11.

c) 114 x 11 = 1254. See *Multiplying by 11*.

d) The answer is 1254.

B. This method can also be adapted to multiplying and dividing the numbers by an integer other than 2.

Ex [1] $6\,^{1}/_{4}$ x 96 =_________.

a) In this example you can multiply and divide by 4.

b) $6\,^{1}/_{4}$ x 4 = 25.

c) 96 ÷ 4 = 24.

d) 25 x 24 = 600. See *Multiplying by 25*.

e) The answer is 600.

Ex [2] 66 x 21 =_________.

a) In this example you can multiply and divide by 6.

b) 66 ÷ 6 = 11.

c) 21 x 6 = 126.

d) 126 x 11 = 1386. See *Multiplying by 11*.

e) The answer is 1386.

FOIL Method:

A. The FOIL method stands for – **F**irst, **O**utside, **I**nside, **L**ast. This method can be used to multiply any 2-digit numbers together. However, there are many cases where other strategies should be used since the FOIL method is somewhat time -consuming.

B. The FOIL method comes from algebra:

$$(10a + b)(10c + d) = 100ac + 10(bc + ad) + bd$$

C. Using numbers instead of variables we can get the following steps:

1. Multiply the ones digits together (or the last digits). 'L'
2. Multiply the inside numbers and the outside numbers and add them together. 'O' + 'I'.
3. Multiply the tens digits together (or the first digits). 'F'
4. Putting these steps together we get an answer in the form: 'F' ('O'+'I') 'L' or FOIL.
5. Keep track of any numbers you need to carry to add to any of these steps.

Ex [1] 12 x 34 =_________.

1 2 x 3 4 =

a) 2 x 4 = 8. Write down 8.

1 2 x 3 4 =

b) 2 x 3 + 1 x 4 = 6 + 4 = 10. Write down 0 and carry *1.

1 2 x 3 4 =

c) 1 x 3 = 3 + *1 = 4. Write down 4.

d) The answer is 408.

Ex [2] 45 x 24 =_________.

a) 5 x 4 = 20. Write 0, carry *2.

b) 5 x 2 + 4 x 4 = 10 + 16 = 26 + *2 = 28. Write 8, carry *2.

c) 4 x 2 = 8 + *2 = 10. Write 10.

d) The answer is 1080.

D. Sometimes the FOIL method can be adapted to multiply 3 digit numbers as well.

Ex [1] 114 x 42 =_________.

a) To multiply by 114 using the FOIL method, think of 'F' as being 11 in this case.

b) 4 x 2 = 8. Write 8.

c) 4 x 4 + 11 x 2 = 16 + 22 = 38. Write 8, carry *3.

d) 11 x 4 = 44 + *3 = 47. Write 47.

e) The answer is 4788.

Multiplying Mixed Numbers Whose Whole Numbers Are The Same And Whose Fractions Add To 1:

A. From algebra we learn:

$$a\frac{b}{c} + a\frac{c-b}{c} = a(a+1) + \frac{b}{c}\cdot\frac{c-b}{c}$$

B. Use the following rules:

1. Multiply the two fractions together. This is the fraction to the answer.
2. Multiply the whole number by that number plus 1. This is the whole number to the answer.

Ex [1] $4\frac{1}{4}$ x $4\frac{3}{4}$ = __________ (mixed number).

a) $^1/_4$ x $^3/_4$ = $^3/_{16}$. Write $^3/_{16}$.

b) 4 x (4 + 1) = 20. Write 20.

c) The answer is 20 $^3/_{16}$.

Ex [2] $9\frac{5}{8}$ x $9\frac{3}{8}$ = __________ (mixed number).

a) $^5/_8$ x $^3/_8$ = $^{15}/_{64}$. Write $^{15}/_{64}$.

b) 9 x (9 + 1) = 90. Write 90.

c) The answer is 90 $^{15}/_{16}$.

The following chart shows the relationship between common fractions, percents and decimals. This chart should be memorized. Look for obvious patterns to help you memorize these numbers. (For repeating decimals, … is used after a pattern has been established. On Number Sense tests, repeating decimals should never be used as an answer.)

Fractions	Percents	Decimals	Fractions	Percents	Decimals
$^1/_2$	50%	.5	$^1/_9$	11 $^1/_9$%	.111…
			$^2/_9$	22 $^2/_9$%	.222…
$^1/_3$	33 $^1/_3$%	.333…	$^4/_9$	44 $^4/_9$%	.444…
$^2/_3$	66 $^2/_3$%	.666…	$^5/_9$	55 $^5/_9$%	.555…
			$^7/_9$	77 $^7/_9$%	.777…
$^1/_4$	25%	.25	$^8/_9$	88 $^8/_9$%	.888…
$^3/_4$	75%	.75			
			$^1/_{10}$	10%	.1
$^1/_5$	20%	.2	$^3/_{10}$	30%	.3
$^2/_5$	40%	.4	$^7/_{10}$	70%	.7
$^3/_5$	60%	.6	$^9/_{10}$	90%	.9
$^4/_5$	80%	.8			
			$^1/_{11}$	9 $^1/_{11}$%	.0909…
$^1/_6$	16 $^2/_3$%	.1666…	$^2/_{11}$	18 $^2/_{11}$%	.1818…
$^5/_6$	83 $^1/_3$%	.8333…	$^3/_{11}$	27 $^3/_{11}$%	.2727…
			$^4/_{11}$	36 $^4/_{11}$%	.3636…
$^1/_7$	14 $^2/_7$%	.142857…	$^5/_{11}$	45 $^5/_{11}$%	.4545…
$^2/_7$	28 $^4/_7$%	.285714…	$^6/_{11}$	54 $^6/_{11}$%	.5454…
$^3/_7$	42 $^6/_7$%	.428571…	$^7/_{11}$	63 $^7/_{11}$%	.6363…
$^4/_7$	57 $^1/_7$%	.571428…	$^8/_{11}$	72 $^8/_{11}$%	.7272…
$^5/_7$	71 $^3/_7$%	.714285…	$^9/_{11}$	81 $^9/_{11}$%	.8181…
$^6/_7$	85 $^5/_7$%	.857142…	$^{10}/_{11}$	90 $^{10}/_{11}$%	.9090…
$^1/_8$	12 $^1/_2$%	.125	$^1/_{12}$	8 $^1/_3$%	.08333…
$^3/_8$	37 $^1/_2$%	.375	$^5/_{12}$	41 $^2/_3$%	.41666…
$^5/_8$	62 $^1/_2$%	.625	$^7/_{12}$	58 $^1/_3$%	.58333…
$^7/_8$	87 $^1/_2$%	.875	$^{11}/_{12}$	91 $^2/_3$%	.91666…
$^1/_{16}$	6 $^1/_4$%	.0625	$^3/_{16}$	18 $^3/_4$%	.1875
$^5/_{16}$	31 $^1/_4$%	.3125	$^7/_{16}$	43 $^3/_4$%	.4375
$^9/_{16}$	56 $^1/_4$%	.5625	$^{11}/_{16}$	68 $^3/_4$%	.6875
$^{13}/_{16}$	81 $^1/_4$%	.8125	$^{15}/_{16}$	93 $^3/_4$%	.9375

Finding The GCD (Greatest Common Denominator):

A. To find the GCD of 2 numbers, by definition, means to find the highest number that divides evenly into both numbers.

Ex [1] Find the GCD of 12 and 18.

a. In this example, the number 6 is the highest number that goes into both numbers. Notice that 3 and 2 also divide both numbers, but the GCD is 6.

B. There are some steps we can follow to make finding the GCD easier:

1. First, always look to see if the smaller number can divide into the larger number evenly. If it can then the smaller number is the GCD.
2. If not, then to make it easier, we can multiply the smaller number by a coefficient such that both numbers are relatively close.
3. Subtract the two numbers in step 2. This number will be the starting point. If this number can divide into both, then this is the GCD. If not, find a divisor of this number that can. If none can be found or if this number is 1, then the GCD is 1.

Ex [2] Find the GCD of 14 and 42.

a. Since 14 can divide into 42 evenly, 14 is the GCD.

b. The answer is 14.

Ex [3] Find the GCD of 12 and 74.

a. Since 12 cannot divide into 74 evenly, we need to find a number that when multiplied by 12 gets close to 74.

b. If we multiply 12 by 6, we get 72 which is close to 74.

c. 74 - 72 = 2. This is our starting point.

d. Since 2 divides into both numbers, 2 is the GCD.

e. The answer is 2.

Ex [4] Find the GCD of 13 and 56.

a. Since 13 cannot divide into 56 evenly, we need to find a number that when multiplied by 13 gets close to 56.

b. If we multiply 13 by 4, we get 52 which is close to 56.

c. 56 - 52 = 4. This is our starting point.

d. Since 4 does not go into 13 evenly, we need to find a factor of 4 that does. The factors of 4 are 1, 2, and 4. The only one that divides into both numbers is 1.

e. The answer is 1.

Finding The LCM (Least Common Multiple):

A. Finding the LCM of two numbers, by definition, means finding the smallest number that both numbers can divide into evenly.

Ex [1] Find the LCM of 12 and 8.

a. In this example both 12 and 8 can divide evenly into the number 24. Notice that they can also divide into 48. But the LCM is 24.

B. There are some steps we can follow to make finding the LCM easier:

1. First, look to see if the larger number is a multiple of the smaller number. If it is then this is the LCM.
2. If not, then the LCM can be found by the following:

$$\frac{a \times b}{GCD}$$

3. See *Finding The GCD*.
4. Both of these numbers can be divided by the GCD making the problem much easier.
5. Notice, if the GCD is 1, then the answer is simply the product of the two numbers.

Ex [2] Find the LCM of 8 and 24.

a. In this case 8 can divide into 24 evenly, so the LCM is 24.

b. The answer is 24.

Ex [3] Find the LCM of 25 and 48.

a. Since 25 does not divide into 48 evenly, we need to find the GCD.

b. The GCD is 1. So the LCM is the product of the 2 numbers.

c. 25 x 48 = 1200. See *Multiplying By 25*.

d. The answer is 1200.

Ex [4] Find the LCM of 3, 6, and 9.

a. In this example, we should take it 2 numbers at a time. So looking at the first 2 numbers, we can see that 6 is the LCM of 3 and 6 since 6 is a multiple of 3. So now we need to focus on the last two numbers.

b. Since 6 does not divide into 9, we need to find the GCD.

c. The GCD is 3. So the LCM is (6 x 9)/3 or 2 x 9 = 18.

d. The answer is 18.

Multiplying Two Numbers Less Than 100, But Close To 100:

A. From algebra we learn:

$$(100 - a)(100 - b) = 100((100 - a) - b) + ab$$

B. Using numbers instead of variables we get the following:

1. Find the difference between both of the numbers and 100.
2. Multiply these two values together and write it down. Make sure the answer takes up 2 place values.
3. Subtract the difference found in step 1 of one of the numbers with the remaining number. Write the result.

Ex [1] 98 x 97 =_________.

a) 100 – 98 = 2.

b) 100 – 97 = 3.

c) 2 x 3 = 6. Write 06 to take up 2 place values.

d) 98 – 3 = 95. Write 95. (You can also use 97 – 2 = 95)

e) The answer is 9506.

Ex [2] 88 x 93 =_________.

a) 100 – 88 = 12.

b) 100 – 93 = 7.

c) 12 x 7 = 84. Write 84.

d) 93 – 12 = 81. Write 81. (You can also use 88 – 7 = 81)

e) The answer is 8184.

Multiplying Two Numbers Greater Than 100, But Close To 100:

A. From algebra we learn:

$$(100 + a)(100 + b) = 100(100 + a + b) + ab$$

B. Using numbers instead of variables we get the following:

1. Multiply the one's digits together. Write this number down (make sure that it takes up 2 place values).
2. Add the one's digits together. Write the result (make sure that it takes up 2 place values).
3. Write 1.

Ex [1] 104 x 102 =_________.

a) 4 x 2 = 8. Write 08 to take up 2 place values.
b) 4 + 2 = 6. Write 06 to take up 2 place values.
c) Write 1.
d) The answer is 10608.

Ex [2] 106^2 =_________.

a) 6 x 6 = 36. Write 36.
b) 6 + 6 = 12. Write 12.
c) Write 1.
d) The answer is 11236.

Multiplying by 5:

A. This method will instruct you to write the answers from right to left.

1. When multiplying by 5, it is easier to divide by 2 and multiply by 10.
2. If the number you are multiplying is odd, then the last number will be a 5 (see Ex [2]), otherwise the last number is 0.

B. Examples:

Ex [1] 5 x 142 =_________.

a) 142 ÷ 2 = 71. Write 71.

b) Since 142 is even the last number is 0. Write 0.

c) The answer is 710.

Ex [2] 5 x 142857 =_______.

a) 142857 ÷ 2 = 71428 with a remainder of 1. Write 71428.

b) Since there is a remainder of 1, the last number is 5. Write 5.

c) The answer is 714285.

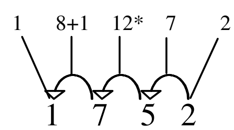
*Note: This trick works because 5 = 10 ÷ 2.

Multiplying By 11

A. When multiplying by 11 there are certain steps you need to follow:

1. Write down the last digit of the number.
2. Add the last digit to the number to its left. Write this number down, carry if necessary.
3. Moving left, keep adding the digit to the digit to its left. Write down the numbers, carrying if necessary
4. When you reach the first digit write this number down.

Ex [1] 1752 x 11 = __________.



a) Write down the 2.

b) 2 + 5 = 7. Write down 7.

c) 5 + 7 = 12. Write down 2 and carry the *1.

d) 7 + 1 = 8 + *1 = 9. Write down the 9.

e) Write down the 1 since there is nothing to carry.

f) The answer is 19272.

Ex [2] 35 x 11 = __________.

a) Write down 5.

b) Add 5 + 3 which equals 8. Write down 8.

c) Since there is nothing to carry write down 3.

d) The answer is 385.

Multiplying By 25:

A. Multiplying by 25 is one of the basic multiplication methods in number sense.

1. You can rewrite 25 to be 100 ÷ 4. So the first step is to divide by 4 and write this number down.
2. Depending on the remainder, referred to as (n MOD 4), the last numbers are the following:
   a) If (n MOD 4) = 0 then the last numbers are 00.
   b) If (n MOD 4) = 1 then the last numbers are 25.
   c) If (n MOD 4) = 2 then the last numbers are 50.
   d) If (n MOD 4) = 3 then the last numbers are 75.

B. Examples:

Ex [1] 25 x 84 =_________.

a) 84 ÷ 4 = 21. Write 21.
b) Since there is no remainder the last numbers are 00.
c) The answer is 2100.

Ex [2] 113 x 25 =_________.

a) 113 ÷ 4 = 18 with a remainder of 1. Write 18.
b) The last numbers are 25.
c) The answer is 1825.

Multiplying By 50:

A. Multiplying by 50 is the same as multiplying by 5, except if the number is even you write 00, and if the number is odd you write 50. (See *Multiplying by 5*)

1. When multiplying by 50, it is easier to divide by 2 and multiply by 100.
2. If the number you are multiplying is odd, then the last numbers will be a 50 (see Ex [2]), otherwise the last number is 00.

B. Examples:

Ex [1] 126 x 50 =_________.

a) 126 ÷ 2 = 63. Write 63.

b) Since 126 is even the last numbers are 00.

c) The answer is 6300.

Ex [2] 321 x 50 =_________.

a) 321 ÷ 2 = 160 with a remainder of 1. Write 160.

b) Since there is a remainder of 1 the last numbers are 50.

c) The answer is 16050.

Multiplying By 101:

A. When multiplying a 2 digit number by 101, simply write the number down twice.

Ex [1] 53 x 101 = 5353.

Ex [2] 49 x 101 = 4949.

B. Sometimes there are variations of 101 like 201, 203, 104, etc. In problems like these you can use the following:

Ex [1] 23 x 203 =_________.

a) Multiply 23 x 2 = 46.

b) Multiply 23 x 3 = 69.

c) The answer is 4669.

C. When multiplying a 3 digit number by 101 the following rules apply:

1. Write down the last 2 digits of the number.
2. Take the number in the hundred's digit and add back to the number.

Ex [1] 453 x 101 =_________.

a) Write 53.

b) 4 + 453 = 457. Write 457

c) The answer is 45753.

Ex [2] 998 x 101 =_________.

a) Write 98.

b) 9 + 998 = 1007. Write 1007.

c) The answer is 100798.

Adding a sequence in the form: 1 + 3 + ... + 2n-1:

A. A sequence in this form reduces to:

$$\sum_{i=1}^{n} 2i - 1 = 1 + 3 + 5 + \ldots + 2n - 1 = (\text{number of terms})^2$$

B. To find the number of terms easily just add 1 to the last number and divide by 2.

Ex [1] 1 + 3 + 5 + ... + 21 =_________.

a) Find the number of terms: (21 + 1) / 2 = 11.

b) $11^2 = 121$.

c) The answer is 121.

Ex [2] 1 + 3 + 5 + ... + 205 =_________.

a) Find the number of terms: (205 + 1) / 2 = 103.

b) $103^2 = 10909$. See *Multiplying Numbers Greater Than 100*.

c) The answer is 10909.

Multiplying Two Numbers Whose Ten's Digits Are The Same And Whose One's Digits Add To 10:

A. From algebra we learn:

$$(10a + b)(10a + (10-b)) = 100(a)(a + 1) + (b)(10 - b)$$

B. Using numbers instead of variables we get the following rules:

1. Multiply the one's digits together. Write this number down (make sure the number takes up 2 place values).
2. Multiply the number in the ten's digit by that number plus 1. Write the result.

Ex [1] 49 x 41 =_________.

a) 9 x 1 = 9. Write 09 to take up 2 place values.
b) 4 x (4 + 1) = 4 x 5 = 20. Write 20.
c) The answer is 2009.

Ex [2] 253 x 257 =_________.

a) 3 x 7 = 21. Write 21.
b) 25 x (25 + 1) = 25 x 26 = 650. Write 650. See *Multiplying by 25* or *Double and Half*.
c) The answer is 65021.

ORDER OF OPERATIONS -

When working with equations involving different operations, there are certain rules you must follow to get the correct answer. For example, you must multiply before you can add. These rules are called the *Order Of Operations*. A common way to remember the order of operations is by this sentence: Please Excuse My Dear Aunt Sally. Each letter represents an operation. P – parenthesis, E – exponents, M – multiplication, D – division, A – addition, S – subtraction. Each one must be followed in the order of the sentence. See below.

The order of operations follows:

Parentheses: ( ), [ ]: starting from the inside and working your way outside.
Exponents: starting from the inside and working your way outside.
Multiplication or Division: from left to right
Addition or Subtraction: from left to right

Look at the following examples:

Ex [1] 2 + 3 * 7 =__________

a) According to the order of operations, we must first multiply then add.
b) 3 * 7 = 21.
c) 2 + 21 = 23.
d) The answer is 23.

Ex [2] 10 – 5 – 3 – 1 =__________

a) According to the order of operations, we must subtract from left to right.
b) 10 – 5 – 3 – 1 = (10 – 5) – 3 – 1.
c) 10 – 5 – 3 – 1 = 5 – 3 – 1.
d) 10 – 5 – 3 – 1 = (5 – 3) – 1.
e) 10 – 5 – 3 – 1 = 2 – 1.
f) 10 – 5 – 3 – 1 = 1.
g) The answer is 1.

Ex [3] 2 x (8 – 3) =__________

a) According to the order of operations, we should do what is inside the () first.
b) 2 x (8 – 3) = 2 x 5.
c) 2 x (8 – 3) = 10.
d) The answer is 10.

Ex [4] $20 - [2 \times (5-3)^3] =$__________

a) According to the order of operations, we should do what is inside the () first.

b) $20 - (2 \times (5-3)^3) = 20 - (2 \times (2)^3)$.

c) $20 - (2 \times (5-3)^3) = 20 - (2 \times 8)$.

d) $20 - (2 \times (5-3)^3) = 20 - 16 = 4$.

e) The answer is 4.

Ex [5] $(3^2)^2 =$__________

a) According to the order of operations, we should calculate the exponents from the inside to the outside.

b) $(3^2)^2 = 9^2$

c) $(3^2)^2 = 81$.

d) The answer is 81.

Ex [6] $60 \div 4 \div 5 =$__________

a) According to the order of operations, we must divide from left to right.

b) $60 \div 4 \div 5 = (60 \div 4) \div 5$.

c) $60 \div 4 \div 5 = 15 \div 5$.

d) $60 \div 4 \div 5 = 3$.

e) The answer is 3.

Sometimes, we can change the order a little to help with calculating the answer. See below:

Ex [7] $32 \times 114 \div 8 =$__________

a) Since the order of operations says we can divide or multiply in the same step we can change this equation a little to make it easier.

b) Think of this as being $32 \div 8 \times 114$.

c) $32 \times 114 \div 8 = (32 \div 8) \times 114$.

d) $32 \times 114 \div 8 = 4 \times 114$.

e) $32 \times 114 \div 8 = 456$.

f) The answer is 456.

Ex [8] 6 x 4 x 8 ÷ 2 ÷ 3 =__________

a) To change this equation think of this as being 6 ÷ 3 x 4 ÷ 2 x 8. Now the problem is much easier.

b) 6 x 4 x 8 ÷ 2 ÷ 3 = (6 ÷ 3) x (4 ÷ 2) x 8.

c) 6 x 4 x 8 ÷ 2 ÷ 3 = 2 x 2 x 8.

d) 6 x 4 x 8 ÷ 2 ÷ 3 = 4 x 8.

e) 6 x 4 x 8 ÷ 2 ÷ 3 = 32.

Percents:

A. There are many ways of working with percents. Some of the ways are basic and can be solved just by knowing the information in the *fractions* section of the memorizations.

B. Most percents are in the form of small word problems, so just knowing what the words mean can be a HUGE help:

Of: Of means multiply.

Is: Is means equals.

What: What is the value you are looking for (i.e. the variable).

From: From means subtract.

Less Than: Less than means subtract

More Than: More than means addition

As: As means a ratio.

Into: Into means divide.

C. Using the words above, we can set up an equa tion that should be simple enough to solve.

Ex [1] 20% of 54 is 9% of ________.

a. Substituting we get - (.20 x 54) = .09 x n

b. Simplifying we get - (20 x 54)/9 = n. (Notice we can ignore the decimal places since there are the same number of decimal places in the denominator as in the numerator.)

c. (20 x 54) / 9 = 20 x 6 = 120.

d. The answer is 120.

Ex [2] If x% of 140 is 16.8, then x = __________.

a. Substituting we get - 140x = 16.8. (Ignore the percent for the time being. Just know that we will have to change the decimal to a percent in the end.)

b. Solving we get x = 16.8/140 or x = 168/1400 which simplifies to 24/200 or 12/100 which is .12. Changing this to a percent we get 12. (Notice that I first divided by 7, then divided by 2. Also, there is no need, since we are dealing with percents, to simplify it to its most basic form.)

c. The answer is 12.

Ex [3] 18% of 6 $^{2}/_{3}$ = __________.

a. Substituting we get - .18 x $^{20}/_{3}$.

b. Notice you need to change the mixed number to an improper fraction.

c. Simplifying we get - .06 x 20 or 1.2.

d. The answer is 1.2.

Ex [4] 5 more than 12% of 15 is __________.

a. Substituting from above we get 5 + .12 x 15 = ____.

b. Solving we get 5 + 1.8 = 6.8.

c. The answer is 6.8.

D. The following example is how to work percents in a different type of form. This problem is a little harder than just following the definitions above.

Ex [5] 14 is what percent more than 10? __________.

a. In this problem we have to find out what times 10 and added back to 10 = 14. In other words, 10n + 10 = 14.

b. Solving 10n + 10 = 14 we get 10n = 4 or n = .4 or 40%.

c. The answer is 40.

PRIME NUMBERS -

A prime number is a number that is only divisible by 1 and itself. Numbers that are not prime are called composite numbers. The smallest prime number, which happens to be the only even prime number, is 2. The prime numbers from 2 to 100 are listed below and should all be memorized.

List of Prime Numbers:

2	3	5	7
11	13	17	19
23	29		
31	37		
41	43	47	
53	59		
61	67		
71	73	79	
83	89		
97			

*Notice that there are only 25 primes between 0 and 100.

It is very important to learn prime numbers because many methods, like relatively prime, GCD, LCM, and others use prime numbers.

Ratios:

A. Number Sense uses ratios in a variety of ways. With ratios you simply write an expression and solve for the missing variable.

Ex [1] 5 is to 8 as 3 is to ________.

a. When working with ratios, it is easier to think in terms of fractions.

b. So $\frac{5}{8} = \frac{3}{x}$. Simplifying this equation we get 5x = 24.

c. Solving for x we get $^{24}/_{5}$.

d. The answer is $^{24}/_{5}$.

Ex [2] 25 is to 100 as what is to 16? _________.

a. Think of this as $\frac{25}{100} = \frac{x}{16}$.

b. Simplifying this expression we get (16)(25) = 100x.

c. This equals 400 = 100x. See *Multiplying by 25*.

d. x = 4.

e. The answer is 4.

Subtracting Reverses:

A. There are two forms of subtracting reverses and both forms use the same concept.

1. The first form is subtracting a 3-digit number from its reverse.

   Ex [1] 634 - 436 = __________.

2. The second form is subtracting a 4-digit number but sectioned in pairs, from its reverse.

   Ex [2] 2314 - 1423 = __________.

3. In both forms take the first number (or first pair) and subtract from the remaining first number (or second pair).

4. The answer follows this form:

   100n - n, where n is the result of step 3.

   Ex [1] 634 - 436 = __________.

   a. 6 - 4 = 2.

   b. 100(2) - 2 = 198.

   c. The answer is 198.

   Ex [2] 2314 - 1423 = __________.

   a. 23 - 14 = 9.

   b. 100(9) - 9 = 891.

   c. The answer is 891.

B. Be careful that the numbers you are subtracting are indeed reverses and also watch out to see if the answer should be negative or positive.

ROMAN NUMERALS -

Roman numerals use letters to represent values. Arabic numerals are the numbers *0, 1, 2, 3, 4, 5, 6, 7, 8, and 9* that we use today.

Students need to memorize the following chart of Roman numerals and the values they translate to in Arabic numerals.

Roman	Arabic
I	1
V	5
X	10
L	50
C	100
D	500
M	1000

To convert Roman numerals to Arabic numerals, students should learn the following rules:

1. If a Roman symbol is followed by a symbol that is equal or less than it, add the value of the two symbols together. For example, in the Roman numeral XV, the symbol X for 10 is followed by the symbol V for 5. Since 5 is less than 10, you add 10 + 5 = 15. So, XV = 15.

2. If a Roman symbol is followed by a symbol that is greater than it, subtract the value of the larger one minus the smaller one. For example, in the Roman numeral IX, the symbol I for 1 is followed by the symbol X for 10. Since 10 is greater than 1, subtract 10 - 1 = 9. So, *IX* = 9. (*Please note that you can only use the subtraction rule when dealing with the next place value only. For example, to write 45, you cannot write VL. You should correctly write XLV. The exception to this rule deals with nines: IX = 9, XC = 90, and CM = 900.)*

3. Always start on the left side of the numbers and work to the right.

4. There is only one correct way to write each part of a Roman numeral.

5. Never use more than three of the same Roman symbols in succession. If you think you need to use a fourth symbol, you probably need to use the subtraction rule.

Examples:

Ex [1] 17 = __________ (roman numerals)

a) 17 = 10 + 5 + 2

b) 17 = X + V + II

c) 17 = XVII in Roman numerals.

Ex [2] 19 = __________ (roman numerals)

If you use only addition on this number, you could get XVIIII. However, you cannot use more than three of the same symbols in succession. The number XVIIII has four I's, so we cannot write this number. We should use the subtraction method.

a) 19 = 10 + 9

b) 19 = 10 + (10 - 1)

c) 19 = X + X - I

d) 19 = XIX *Switch the last I and X to show subtraction

Ex [3] 273 = __________ (roman numerals)

a) 273 = 200 + 70 + 3

b) 273 = (100 + 100) + (50 + 10 + 10) + (1 + 1 + 1)

c) 273 = C C L X X I I I

d) 273 = CCLXXIII

Ex [4] XII = __________

Start from the left and break down the symbols. (Remember, if the first symbol is less than the next symbol, you need to subtract the values.)

a) XII = X + I + I

b) XII = 10 + 1 + 1

c) XII = 12

Ex [5] XXIX = __________

Working from the left to the right, we get XXIX = X + X + IX. Since I appears before an X, we subtract: X - I. So…

a) XXIX = X + X + IX
b) XXIX = X + X + (X - I)
c) XXIX = 10 + 10 + (10 - 1)
d) XXIX = 10 + 10 + 9
e) XXIX = 29

Ex [6] CCCXLIV = __________

There are two subtraction parts to this problem: XL = L - X = 50 - 10 = 40 and IV = V - I = 5 - 1 = 4.

a) CCCXLIV = CCC + XL + IV
b) CCCXLIV = 300 + 40 + 4
c) CCCXLIV = 344

Squaring A Number Ending In 5:

A. Squaring a number ending in 5 is very easy. The method comes from algebra:

$$(10a + 5)^2 = 100(a)(a + 1) + 25$$

B. Using numbers instead of variables we get the following:

1. Write down 25.
2. Multiply the number in the ten's digit by that number plus 1. Write this number down.

Ex [1] 35^2 =_________.

a) Write 25.

b) 3 x (3 + 1) = 3 x 4 = 12. Write 12.

c) The answer is 1225.

Ex [2] 115^2 =_________.

a) Think of 11 as being the number in the ten's digit.

b) Write 25.

c) 11 x (11 + 1) = 11 x 12 = 132. Write 132. See *Multiplying by 11*.

d) The answer is 13225.

Squaring A Number In The Range (40 – 49):

A. This method comes from algebra:

$$(50 - a)^2 = 100(25 - a) + a^2$$

B. Using numbers instead of variables we get the following:

1. Find the difference between the number and 50.
2. Square the result of step 1 and write it down (make sure it takes up 2 place values).
3. Subtract the result of step 1 from 25. Write this result.

Ex [1] 49^2 =_________.

a) The difference from 50 is 1.

b) $1^2 = 1$. Write 01 to take up 2 place values.

c) 25 – 1 = 24. Write 24.

d) The answer is 2401.

Ex [2] 42^2 =_________.

a) The difference from 50 is 8.

b) $8^2 = 64$. Write 64.

c) 25 – 8 = 17. Write 17.

d) The answer is 1764.

Squaring A Number In The Range (50 – 59):

A. This method comes from algebra:

$$(50 + a)^2 = 100(25 + a) + a^2$$

B. Using numbers instead of variables we get the following:

1. Square the one's digit and write it down (make sure it takes up 2 place values).
2. Add 25 to the one's digit. Write this result.

Ex [1] 53^2 =_________.

a) $3^2 = 9$. Write 09 to take up 2 place values.

b) 25 + 3 = 28. Write 28.

c) The answer is 2809.

Ex [2] 59^2 =_________.

a) $9^2 = 81$. Write 81.

b) 25 + 9 = 34. Write 34.

c) The answer is 3481.

Squaring A Number In The Range (90 – 99):

A. This method comes from algebra:

$$(100 - a)^2 = 100 \times [(100 - a) - a] + a^2$$

B. Using numbers instead of variables we get the following:

1. Find the difference between the number and 100.
2. Square the result of step 1 and write it down (make sure it takes up 2 place values).
3. Subtract the result of step 1 from the original number and write it down.

Ex [1] 97^2 =_________.

a) 100 – 97 = 3.

b) $3^2 = 9$. Write 09 to take up 2 place values.

c) 97 – 3 = 94. Write 94.

d) The answer is 9409.

Ex [2] 92^2 =_________.

a) 100 – 92 = 8.

b) $8^2 = 64$. Write 64.

c) 92 – 8 = 84. Write 84.

d) The answer is 8464.

Squaring A 2-Digit Number:

A. This method is similar to the *FOIL method* in that it is time-consuming and there are many other methods that are faster in certain situations. However, this method will work for any 2-digit number.

B. This method comes from algebra:

$$(10a + b)^2 = 100a^2 + 10(2ab) + b^2$$

C. Using numbers instead of variables we can get the following steps:

1. Square the one's digit. Write this number down, carry i f necessary.
2. Multiply the one's digit with the ten's digit and multiply by 2. Write this number down, carry if necessary.
3. Square the ten's digit. Write this number down.

Ex [1] 32^2 =_________.

a) $2^2 = 4$. Write 4.

b) 2 x 3 = 6. 6 x 2 = 12. Write 2, carry *1.

c) 3^2 = 9 + *1 = 10. Write 10.

d) The answer is 1024.

Ex [2] 78^2 =_________.

a) 8^2 = 64. Write 4, carry *6.

b) 7 x 8 = 56. 56 x 2 = 112 + *6 = 118. Write 8, carry *11.

c) 7^2 = 49 + *11 = 60. Write 60.

d) The answer is 6084.

D. This method can also be adapted for 3 digit numbers as well:

Ex [1] 123^2 =_________.

a) Think of 123 as (12)3 where 12 is the number in the ten's digit.

b) 3^2 = 9. Write 9.

c) 12 x 3 = 36. 36 x 2 = 72. Write 2, carry *7.

d) 12^2 = 144 + *7 = 151. Write 151.

e) The answer is 15129.

Working With Square Roots:

A. The trick to working with square roots is to know what range the square root is in. Since squaring numbers ending in 0 and *squaring numbers ending in 5* are both easy, we can get the answer within a range of 5.

B. If the number is a perfect square, then we can know what the ending digit will be of the answer by looking at the ending digit of the question.

If the number ends in a:

0	->	then the ending digit is a 0
1	->	then the ending digit is 1 or 9.
4	->	then the ending digit is 2 or 8.
5	->	then the ending digit is a 5.
6	->	then the ending digit is 4 or 6.
9	->	then the ending digit is 3 or 7.

C. After finding the last digit (or possibility between two digits) mentally chop off the last two digits and focus on the remaining digits.

D. Now, try to find a range of 5 that the number (in step c) is in. Once you do this you know the answer using step B. First, find a range of 10, then find out if the answer is in the upper-half of the range (i.e. ends in 5, 6, 7, 8, or 9) or if the answer is in the lower-half of the range (i.e. ends in 0, 1, 2, 3, or 4) by squaring the middle number (the number in the range that ends in 5).

Ex [1] $\sqrt{5329}$ = _________.

a. We know the last number must be a 3 or 7.

b. Chopping off the last two numbers we need to focus on the remaining numbers or 53.

c. We know that $7^2 = 49$ and $8^2 = 64$ so since 53 is between these two numbers, the answer is between 70 and 80.

d. Squaring 75 we get 5625. Since the original number is lower than this, we know the answer is in the lower-half.

e. The answer is 73.

Ex [2] $\sqrt{15876}$ = __________.

a. We know that the last number must be a 4 or 6.

b. Chopping off the last two numbers we are left with 158.

c. We know that $12^2 = 144$ and $13^2 = 169$ so the number is between 120 and 130 since 144 < 158 < 169.

d. Squaring 125, we get 15625. Since the original number is greater than this, we know the answer is in the upper-half.

e. The answer is 126.

Some people think that memorizing squares is a waste of time because there are easy methods to finding the squares of numbers. However, I believe that by memorizing the squares, at least through 25, you will have a distinct advantage over those who do not.

Therefore, I believe the following squares are essential:

$1^2 = 1$	$2^2 = 4$	$3^2 = 9$	$4^2 = 16$	$5^2 = 25$
$6^2 = 36$	$7^2 = 49$	$8^2 = 63$	$9^2 = 81$	$10^2 = 100$
$11^2 = 121$	$12^2 = 144$	$13^2 = 169$	$14^2 = 196$	$15^2 = 225$
$16^2 = 256$	$17^2 = 289$	$18^2 = 324$	$19^2 = 361$	$20^2 = 400$
$21^2 = 441$	$22^2 = 484$	$23^2 = 529$	$24^2 = 576$	$25^2 = 625$

*Note: I found it helpful to memorize through 30, but 25 should be efficient.

Shortcut - Multiplying by 11

This technique teaches you how to multiply any number by eleven, easily and quickly. We will take a few examples and from these you will see the pattern used and also how easy they are to do.

So, to begin let's try 12 time 11.

First things first you will ignore the 11 for the moment and concentrate on the 12.

Split the twelve apart, like so:

1

Add these two digits together 1 + 2 = 3

1+2 = 3

Place the answer, 3 in between the 12 to give 132

11 X12=132

Let's try another:

48 X 11

again, leave the 11 alone for a moment and work with the 48

4+8= 12

So now we have to put the 12 in between the 4 and 8 but don't do this:

4128 as that is wrong...

First, do this: Place the 2 from the twelve in between the 4 and 8 giving 428.

Now we need to input the 1 from the twelve into our answer also, and to do this just add the one from 12 to the 4 of 428 giving 528

Ok, one more

74 **X** 11

7+4 =11

7 (put the 1 from the right of 11 in) and 4 ,
then add the 1 from the left of 11 to the 7

74X11 =814

This is a really simple method and will save you so much time with your 11 times tables.

Shortcut - Multiplying by 12

So how does the 12's shortcut work?

Let's take a look.
12X7

the first thing is to always multiply the 1 of the twelve by the number we are multiplying by, in this case 7. So 1 X7=7.

Multiply this 7 by 10 giving 70.

Now multiply the 7 by the 2 of twelve giving 14. Add this to 70 giving 84.

Therefore7 X 12= 84

Let's try another:
17X 12

Remember, multiply the 17 by the 1 in 12 and multiply by 10

(Just add a **zero to the end):**

1 X 17 = 17, multiplied by 10 giving 170.

Multiply 17 by 2 giving 34.

Add 34 to 170 giving 204.

So 17X 12 =204

lets go one more
24X 12

Multiply 24 X 1 = 24. Multiply by 10 giving 240.

Multiply 24 by 2 48. Add to 240 giving us 288

24 X 12 288 (these are Seriously Simple Sums to do aren't they?!)

Shortcut - Is it divisible by four?

This little math trick will show you whether a number is divisible by four or not.
So, this is how it works.

Let's look at 1234

Does 4 divide evenly into 1234?

If it is an odd number, there is no way it will go in evenly.

So, for example, 4 will not go evenly into 1233 or 1235

now we know that for 4 to divide evenly into any number the number has to end with an even number.

Back to the question...
4 into 1234, the solution:

Take the last **number and add it to 2 times the second last number.** If 4 goes evenly into this number then you know that 4 will go evenly into the whole number.

So
4+(2X3) 10

4 goes into 10 two times with a remainder of 2 so it does not go in evenly.

Therefore 4 into 1234 does not go in completely.

Let's try 4 into 3436546

So, from our example, take the last number, 6 and add it to two times the penultimate number, 4

6+(2 X4)= 14

4 goes into 14 three times with two remainder.
So it doesn't go in evenly.

Let's try one more.
4 into 212334436

6+ (2X3) = 12

4 goes into 12 three times with 0 remainder.

Therefore 4 goes into 234436 evenly.

So what use is this trick to you?

You can also use it in working out whether the year you are calculating is a leap year or not.

Shortcut - Decimals Equivalents of Fractions

With a little practice, it's not hard to recall the decimal equivalents of fractions up to 10/11!

First, there are 3 you should know already:
1/2 = .5
1/3 = .333...
1/4 = .25

Starting with the thirds, of which you already know one:
1/3 = .333...
2/3 = .666...

You also know 2 of the 4ths, as well, so there's only one new one to learn:
1/4 = .25
2/4 = 1/2 = .5
3/4 .75

Fifths are very easy. Take the numerator (the number on top), double it, and stick a decimal in front of it.
1/5 = .2
2/5 = .4
3/5 = .6
4/5 = .8

There are only two new decimal equivalents to learn with the 6ths:

1/6 = .1666...
2/6 = 1/3 = .333...
3/6 = 1/2 = .5
4/6 = 2/3 = .666...
5/6 = .8333...

What about 7ths? We'll come back to them at the end. They're very unique.

8ths aren't that hard to learn, as they're just smaller steps than 4ths. If you have trouble with any of the 8ths, find the nearest 4th, and add .125 if needed:
1/8 = .125
2/8 = 1/4 = .25
3/8 = .375
4/8 = 1/2 = .5
5/8 = .625
6/8 = 3/4 = .75
7/8 = .875

9ths are almost too easy:
1/9 = .111...
2/9 = .222...
3/9 = .333...
4/9 = .444...
5/9 = .555...
6/9 = .666...
7/9 = .777...
8/9 = .888...

10ths are very easy, as well. Just put a decimal in front of the numerator:
1/10 = .1
2/10 = .2
3/10 = .3
4/10 = .4
5/10 = .5
6/10 = .6
7/10 = .7
8/10 = .8
9/10 = .9

Remember how easy 9ths were? 11th are easy in a same way, assuming you know multiples of 9:
1/11 = .090909...
2/11 = .181818...
3/11 = .272727...
4/11 = .3 63636...
5/11 = .454545...
6/11 = .545454...
7/11 = .636363...
8/11 = .727272...
9/11 = .818181...
10/11 =.909090...

As long as you can remember the pattern for each fraction, it is quite simple to work out the decimal place as far as you want or need to go!

One-seventh is an interesting number:
1/7 = .142857142857142857...

For now, just think of one-seventh as: .142857
See if you notice any pattern in the 7ths:
1/7 = .142857...
2/7 = .2857 14...
3/7 = .428571...
4/7 = .571428...
5/7 = .714285...
6/7 = .857142...

Notice that the 6 digits in the 7ths ALWAYS stay in the same order and the starting digit is the only thing that changes!

If you know your multiples of 14 up to 6, it isn't difficult to work out where to begin the decimal number.

Look at this:
For 1/7, think "1 X 14", giving us .14 as the starting point.
For 2/7, think "2 X 14", giving us .28 as the starting point.
For 3/7, think "3 X 14", giving us .42 as the starting point.
For 4/14, 5/14 and 6/14, you'll have to adjust upward by 1:
For 4/7, think "(4 X 14) + 1", giving us .57 as the starting point.
For 5/7, think "(5 X 14) + 1", giving us .71 as the starting point.
For 6/7, think "(6 X 14) + 1", giving us .85 as the starting point.
Practice these, and you'll have the decimal equivalents of everything from 1/2 to 10/11 at your finger tips!

Shortcut - Converting Kilos to pounds

In this section you will learn how to convert Kilos to Pounds, and Vice Versa.

Let's start off with looking at converting Kilos to pounds. 86 kilos into pounds:

Step one, multiply the kilos by TWO.

To do this, just double the kilos.

86x2= 172

Step two, divide the answer by ten.

To do this, just put a decimal point one place in from the right.

172/10= 17.2

Step three, add step two's answer to step one's answer.

172 + 17.2 = 189.2

86 Kilos = 189.2 pounds

Let's try:
50 Kilos to pounds:

Step one, multiply the kilos by **TWO.**
To do this, just double the kilos.
50x2= 100

Step Two, divide the answer by ten.
To do this, just put a decimal point one place in from the right.
100/10=10

Step three, add step two's answer to step one's answer.
100+10=110

50 Kilos = 110 pounds

Shortcut - Adding Time

Here is a nice simple way to add hours and minutes together:

Let’s add 1 hr and 35 minutes and 3 hr 55 minutes together.

What you do is this:

make the 1 hr 35 minutes into one number, which will give us 135 and do the same for the other number,

3 hours 55 minutes, giving us 355

Now you want to add these two numbers together:

135 + 355 = 490

So we now have a sub total of 490.

What you need to do to this and all sub totals is add the **time constant of 40.**

No matter what the hours and minutes are, just add the 40 time constant to the sub total.

490 + 40 = 530

so we can now see **our answer is** 5 **hrs and 30 minutes!**

Shortcut - Temperature Conversions

This is a shortcut to convert Fahrenheit to Celsius and vice versa.

The answer you will get will not be the exact answer but this will give you the closest idea of answer.

Fahrenheit to Celsius:
Take 30 away from the Fahrenheit, and then divide the answer by two. This is your answer in Celsius.

Example: -
74 Fahrenheit - 30 = 44. Then divide by two, 22 Celsius.

So 74 Fahrenheit = 22 Celsius.

Celsius to Fahrenheit just do the reverse:
Double it, and then add 30.
30 Celsius double it, is 60, then add 30 is 90

30 Celsius = 90 Fahrenheit

Remember, the answer is not exact but it gives you a rough idea.

Shortcut - Converting Kilometres to Miles

This is a useful method for when travelling between imperial and metric countries and need to know what kilometres to miles are.

The formula to convert kilometres to miles is number of (kilometres / 8) X 5

So lets try 80 kilometres into miles

80/8 = 10

multiplied by 5 is 50 miles!

Another example

40 kilometres

40/8=5

5X5=25 miles