

LCM AND HCF PROBLEMS WITH SOLUTIONS

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LCM and HCF

LCM i.e. least common multiple is a number which is multiple of two or more than two numbers. For example: The common multiples of 3 and 4 are 12, 24 and so on. Therefore, l.c.m. is smallest positive number that is multiple of both. Here, l.c.m. is 12. HCF i.e. highest common factor are those integral values of number that can divide that number. LCM and HCF problems are very important part of all competitive exams.

Some important LCM and HCF tricks:

- 1) Product of two numbers = Their HCF \times Their LCM
- 2) HCF of given numbers always divides their LCM
- 3) HCF of given fractions = $\frac{\text{HCF of numerator}}{\text{LCM of denominator}}$
- 4) LCM of given fractions = $\frac{\text{LCM of numerator}}{\text{HCF of denominator}}$
- 5) If d is the HCF of two positive integer a and b , then there exist unique integer m and n , such that
$$d = am + bn$$
- 6) If p is prime and a, b are any integer then $\frac{p}{ab}$, This implies $\frac{p}{a}$ or $\frac{p}{b}$
- 7) HCF of a given number always divides its LCM

Most important points about LCM and HCF problems :

- 1) Largest number which divides x, y, z to leave same remainder = HCF of $y - x, z - y, z - x$.
- 2) Largest number which divides x, y, z to leave remainder R (i.e. same) = HCF of $x - r, y - r, z - r$.
- 3) Largest number which divides x, y, z to leave same remainder a, b, c = HCF of $x - a, y - b, z - c$
- 4) Least number which when divided by x, y, z and leaves a remainder R in each case = $(\text{LCM of } x, y, z) + R$

HCF and LCM questions:

Example 1: Least number which when divided by 35, 45, 55 and leaves remainder 18, 28, 38; is?

Solution: i) In this case we will evaluate LCM

ii) Here the difference between every divisor and remainder is same i.e. 17.

Therefore, required number = LCM of (35, 45, 55)-17 = (3465-17)= 3448.

Example 2: Least number which when divided by 5, 6, 7, 8 and leaves remainder 3, but when divided by 9, leaves no remainder?

Solution: l.c.m. of 5, 6, 7, 8 = 840

Required number = $840k + 3$

Least value of k for which $(840k + 3)$ is divided by 9 is 2

Therefore, required number = $840 \times 2 + 3$
= 1683

Example 3: Greater number of 4 digits which is divisible by each one of 12, 18, 21 and 28 is?

Solution: LCM of 12, 18, 21, 28 = 254

Therefore, required number must be divisible by 254

Greatest four digit number = 9999

On dividing 9999 by 252, remainder = 171

Therefore, $9999 - 171 = 9828$

Exercise

- The smallest number which when diminished by 7, is divisible by 12, 16, 18, 21 and 28 is
a) 1008 b) 1015 c) 1022
d) 1032 e) None of these
- The HCF of two numbers is 11 and their LCM is 7700. If one of the numbers is 275, then the other is
a) 279 b) 283 c) 308
d) 318 e) None of these
- The product of two numbers is 4107. If the HCF of those numbers is 37, then the greater number is
a) 101 b) 107 c) 109
d) 111 e) None of these
- The greatest possible length which can be used to measure exactly the length 7m, 3m, 85cm, 12m, 95 cm is
a) 15 cm b) 25 cm c) 35 cm
d) 42 cm e) None of these
- Find the greatest number that will divide 43, 91 and 183 so as to leave the same remained in each case.
a) 4 b) 7 c) 9

d) 13

e) None of these

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- 6) The G.C.D. of 1.08, 0.36 and 0.9 is
 a) 0.03 b) 0.9 c) 0.18
 d) 0.108 e) None of these
- 7) Three numbers are in the ratio $1 : 2 : 3$ and their HCF is 12. The numbers are
 a) 4, 8, 12 b) 5, 10, 15 c) 10, 20, 30
 d) 12, 24, 36 e) None of these
- 8) A rectangular courtyard 3.78 metres long and 5.25 metres wide is to be paved exactly with square tiles, all of the same size. What is the largest size of the tile which could be used for the purpose?
 a) 14cms b) 21 cms c) 42 cms
 d) Data inadequate e) None of these
- 9) The product of two numbers is 1320 and their HCF is 6. The LCM of the numbers is
 a) 220 b) 1314 c) 1326
 d) 7920 e) None of these
- 10) Product of two co-prime numbers is 117. Their LCM should be
 a) 1 b) 117 c) equal to their H.C.F.
 d) Cannot be calculated e) None of these
- 11) The maximum numbers of students among them 1001 pens and 910 pencils can be distributed in such a way that each student gets the same number of pens and same number of pencils is
 a) 91 b) 910 c) 1001
 d) 1911 e) None of these
- 12) 252 can be expressed as a product of prime as
 a) $2 \times 2 \times 3 \times 3 \times 7$ b) $2 \times 2 \times 2 \times 3 \times 7$ c) $3 \times 3 \times 3 \times 3 \times 7$
 d) $2 \times 3 \times 3 \times 3 \times 7$ e) None of these
- 13) Three different containers contain 496 litres, 403 litres and 713 litres of mixtures of milk and water respectively. What biggest measure can measure all the different quantities exactly?
 a) 1 litre b) 7 litre c) 31 litre
 d) 41 litre e) None of these
- 14) The HCF of two numbers is 8. Which one of the following can never be their LCM?
 a) 8 b) 12 c) 60
 d) 72 e) None of these
- 15) The LCM of two numbers is 495 and their HCF is 5. If the sum of the numbers is 10, then their difference is
 a) 10 b) 46 c) 70
 d) 90 e) None of these

- 16) Let N be the greatest numbers that will divide 43, 91 and 183 so as to leave the same remained in each case.
- a) 4
 - b) 7
 - c) 9
 - d) 13
 - e) None of these
- 17) Which of the following has most number of divisors?
- a) 99
 - b) 101
 - c) 176
 - d) 182
 - e) None of these
- 18) Which of the following is a pair of co-primes?
- a) (16, 62)
 - b) (18, 25)
 - c) (21, 35)
 - d) (23, 92)
 - e) None of these
- 19) The HCF of 1.75, 5.6 and 7 is :
- a) 0.07
 - b) 0.7
 - c) 3.5
 - d) 0.35
 - e) None of these
- 20) The product of two numbers is 4107. If the HCF of these numbers is 37, then the greater number is :
- a) 101
 - b) 107
 - c) 111
 - d) 185
 - e) None of these
- 21) The LCM of two numbers is 45 times their HCF. If one of the numbers is 125 and the sum of HCF and LCM is 1150, the other number is :
- a) 215
 - b) 220
 - c) 225
 - d) 235
 - e) None of these
- 22) The greatest number that exactly divides 105, 1001 and 2436 is :
- a) 3
 - b) 7
 - c) 11
 - d) 21
 - e) None of these
- 23) The smallest fraction, which each of $\frac{6}{7}$, $\frac{5}{14}$, $\frac{10}{21}$ will divide exactly is :
- a) $\frac{30}{7}$
 - b) $\frac{30}{98}$
 - c) $\frac{60}{147}$
 - d) $\frac{50}{294}$
 - e) None of these
- 24) The least number, which when divided by 48, 60, 72, 108 and 140 leaves 38, 50, 62, 98 and 130 as remainders respectively, is :
- a) 11115
 - b) 15110
 - c) 15120
 - d) 15210
 - e) None of these
- 25) The HCF of two numbers is 12 and their difference is 12. The numbers are :
- a) 66, 78
 - b) 70, 82
 - c) 94, 106
 - d) 84, 96
 - e) None of these

- 26) The HCF and LCM of two numbers are 11 and 385 respectively. If one number lies between 75 and 125, then that number is :
- a) 77 b) 88 c) 99
d) 110 e) None of these
- 27) The greatest number which can divide 1356, 1868 and 2764 leaving the same remainder 12 in each case, is :
- a) 64 b) 124 c) 156
d) 260 e) None of these
- 28) The least number which when increased by 5 is divisible by each one of 24, 32, 36 and 54 is :
- a) 427 b) 859 c) 869
d) 4320 e) None of these
- 29) Four different electronic devices make a beep after every 30 minutes, 1 hour, $1\frac{1}{2}$ hour and 1 hour 45 minutes respectively. All the devices beeped together at 12 noon. They will again beep together at :
- a) 12 midnight b) 3 am c) 6 am
d) 9 am e) None of these
- 30) Find the highest common factor of 36 and 84.
- a) 4 b) 6 c) 12
d) 18 e) None of these

Solutions

1. Option B

$$\begin{aligned} \text{Required numbers} &= (\text{LCM of } 12, 16, 18, 21, 28) + 7 \\ &= 1008 + 7 \\ &= 1015 \end{aligned}$$

2. Option C

$$\begin{aligned} \text{Other number} &= \left[11 \times \frac{7700}{275} \right] \\ &= 308 \end{aligned}$$

3. Option D

Let the numbers be $37a$ and $37b$.

$$\text{Then, } 37a \times 37b = 4107$$

$$ab = 3$$

Now, co-primes with product 3 are (1, 3)

So, the required numbers are $(37 \times 1, 37 \times 3)$

i.e., (1, 111)

Therefore greater number = 111

4. Option C

Required length = HCF of 700 cm, 385 cm and 1295 cm
= 35 cm

5. Option A

Required number
= HCF of (91 - 43), (183 - 91) and (183 - 43)
= HCF of 48, 92 and 140
= 4

6. Option C

Given numbers are 1.08, 0.36 and 0.90
HCF of 108, 36 and 90 is 18
HCF of given numbers = 0.18

7. Option D

Let the required numbers be x , $2x$ and $3x$
The, their HCF = x , So $x = 12$
The numbers are 12, 24, 36

8. Option B

Largest size of the tile.
HCF of 378 cm and 525 cm = 21 cms.

9. Option A

$$\begin{aligned} \text{LCM} &= \frac{\text{product of numbers}}{\text{HCF}} \\ &= \frac{1320}{6} \\ &= 220 \end{aligned}$$

10. Option B

HCF of co-prime numbers is 1
So, LCM = $\frac{117}{1}$
= 117

11. Option A

$$\begin{aligned}\text{Required number of students} &= \text{HCF of } 1001 \text{ and } 910 \\ &= 91\end{aligned}$$

12. Option A

$$\text{Clearly, } 252 = 2 \times 2 \times 3 \times 3 \times 7$$

13. Option C

$$\begin{aligned}\text{Required measurement} &= \text{HCF of } (496, 403, 713) \text{ litres} \\ &= 31 \text{ litres}\end{aligned}$$

14. Option C

HCF of two numbers divides their LCM exactly. Clearly, 8 is not a factor 60

15. Option A

Let the numbers be x and $(100 - x)$

$$\text{Then, } x(100 - x) = 5 \times 495$$

$$x^2 - 100x + 2475 = 0$$

$$(x - 55)(x - 45) = 0$$

$$x = 55 \text{ or } x = 45$$

Therefore, the numbers are 45 and 55

$$\text{Required difference} = (55 - 45) = 10$$

16. Option A

$$\begin{aligned}\text{Required number} &= \text{HCF of } (91 - 43), (183 - 91) \text{ and } (183 - 43) \\ &= \text{HCF of } 48, 92 \text{ and } 140 \\ &= 4\end{aligned}$$

17. Option C

$$99 = 1 \times 3 \times 3 \times 11;$$

$$101 = 1 \times 101$$

$$176 = 1 \times 2 \times 2 \times 2 \times 2 \times 11$$

$$182 = 1 \times 2 \times 7 \times 13$$

So, divisors of 99 are 1, 3, 9, 11, 33 and 99

Divisors of 101 are 1 and 1001

Divisors of 176 are 1, 2, 4, 8, 16, 22, 44, 88 and 176

Divisors of 182 are 1, 2, 7, 13, 14, 26, 91 and 182

Hence, 176 has the most number of divisors.

18. Option B

HCF of 18 and 25 is 1. So, they are co-primes.

19. Option D

Given numbers with two decimal places are : 1.75, 5.60 and 7.00. Without decimal places, these numbers are : 175, 560 and 700, whose HCF is 35
So, HCF of given numbers = 0.35

20. Option C

Let the numbers be $37a$ and $37b$. Then, $37a \times 37b = 4107$ $ab = 3$
Now, co-primes with product 3 are (1, 3)
So, the required numbers are $(37 \times 1, 37 \times 3)$ i.e., (1, 111)
So, greater number = 111

21. Option C

Let HCF be H and LCM be L . Then, $L = 45H$ and $L + H = 1150$
So, $45H + H = 1150$ or $H = 25$, So, $L = (1150 - 25) = 1125$
Hence, other number = $\left[\frac{25 \times 1125}{125} \right] = 225$

22. Option B

HCF of 2436 and 1001 is 7. Also, HCF of 105 and 7 is
So, HCF of 105, 1001 and 2436 is 7

23. Option A

Required fraction = LCM of $\frac{6}{7}, \frac{5}{14}, \frac{10}{21} = \frac{LCM \text{ of } 6, 5, 10}{HCF \text{ of } 7, 14, 21} = \frac{30}{7}$

24. Option B

Here $(48 - 38) = 10$, $(60 - 50) = 10$, $(72 - 62) = 10$, $(108 - 98) = 10$ & $(140 - 130) = 10$
So, required number = $(LCM \text{ of } 48, 60, 72, 108, 140) - 10 = 15120 - 10 = 15110$

25. Option D

Out of the given numbers, the two with HCF 12 and difference 12 are 84 and 96

26. Option A

Product of numbers = $11 \times 385 = 4235$
Let the numbers be $11a$ and $11b$. Then, $11a \times 11b = 4235$ $ab = 35$
Now, co-primes with product 35 are (1, 35) and (5, 7)
So, the numbers are $(11 \times 1, 11 \times 35)$ and $(11 \times 5, 11 \times 7)$
Since one number lies between 75 and 125, the suitable pair is (55, 77)
Hence, required number = 77

27. Option A

$$\begin{aligned}\text{Required number} &= \text{HCF of } (1356 - 12), (1868 - 12) \text{ and } (2764 - 12) \\ &= \text{HCF of } 1344, 1856 \text{ and } 2752 = 64\end{aligned}$$

28. Option B

$$\text{Required number} = (\text{LCM of } 24, 32, 36, 54) - 5 = 864 - 5 = 859$$

29. Option D

$$\begin{aligned}\text{Interval after which the devices will beep together} &= (\text{LCM of } 30, 60, 90, 105) \\ \text{min.} &= 1260 \text{ min.} = 21 \text{ hrs.}\end{aligned}$$

So, the devices will again beep together 21 hrs. after 12 noon i.e. at 9 am.

30. Option C

$$36 = 2^2 \times 3^2, 84 = 2^2 \times 3 \times 7$$