MENSURATION PROBLEMS SOLUTIONS

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Mensuration

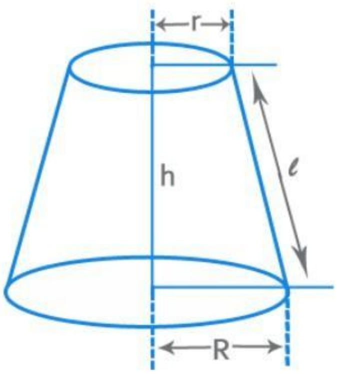
Solid Figures

	Volume	Total surface area	Lateral/curved surface area
Cube	$side^3$	$6 \times side^2$	$4 \times side^2$
Cuboid	L × B × H	2 (LB + LH + BH)	2 (LH + BH)
Cylinder	πr^2h	2πr (r + h)	2πrh
Cone	$(\frac{1}{3})\ \pi r^2$h	πr (r + L)	πrl $\left[\text{where L} = \sqrt{r^2 + h^2}\right]$
Sphere	$(\frac{4}{3})\ \pi r^3$	$4\ \pi r^2$	$4\ \pi r^2$
Hemisphere	$(\frac{2}{3})\ \pi r^3$	$3\ \pi r^2$	$2\ \pi r^2$

There are 4 body diagonals in a cube/cuboid of length ($\sqrt{3} \times$ side) and $\sqrt{l^2 + b^2 + h^2}$ respectively.

Frustum / Truncated Cone

It can be obtained by cutting a cone with a plane parallel to the circular base.

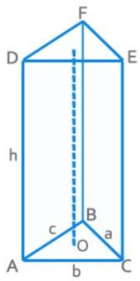


$$\text{Volume} = \frac{1}{3}\pi h\,(R^2 + r^2 + Rr)$$

Lateral Surface Area = π(R+r) L

Total Surface Area = $\pi(R+r)\,L + \pi\,(R^2 + r^2)$

Prism



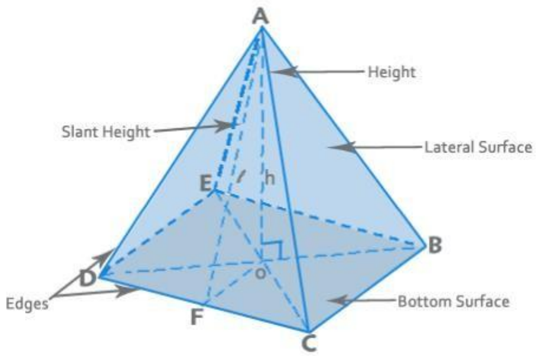
It is a solid with rectangular vertical faces and bases as congruent polygons (of n sides). It will have '2n' Vertices; 'n+2' Faces and '3n' Sides / Edges.

Lateral Surface Area = Perimeter × Height

Total Surface Area = Perimeter × Height + 2 ^{Area}Base

Volume = ^{Area}Base × Height

Pyramid



It is a figure in which the outer surfaces are triangular and converge at a point known as the apex, which is aligned directly above the centre of the base.

$$\text{Lateral Surface Area} = \frac{1}{2} \times \text{Perimeter} \times \text{Slant Height}$$

Total Surface Area = $\frac{1}{2}$ × Perimeter x Slant Height + $^{\text{Area}}$Base

Volume = $\frac{1}{3}$ × $^{\text{Area}}$Base × Height

Facts:

- If a sphere is inscribed in a cube of side a, the radius of the sphere will be $\frac{a}{2}$. If a sphere is circumscribed about a cube of side a, the radius of the sphere will be $\sqrt{3}\frac{a}{2}$.
- If a largest possible sphere is inscribed in a cylinder of radius 'a' and height h, its radius r will be
  r = $\frac{h}{2}$ {If 2a > h}, r = a {If 2a < h}
- If a largest possible sphere is inscribed in a cone of radius r and slant height equal to 2r, then the radius of sphere = $\frac{r}{\sqrt{3}}$
- If a cube is inscribed in a hemisphere of radius r, then the edge of the cube = r $\sqrt{\frac{2}{3}}$

Example 1:
The area of a rectanlgle is 460 square metres. If the length is 15% more than the breadth, what is the breadth of the rectangular field?

Solution:

Let breadth = x metres

Then length = $\frac{115x}{100}$ metres

= x × $\frac{115x}{100}$ = 460

x^2 = $\frac{460x \times 100}{115}$

x^2 = 400

x = 20

Example 2:
A rectangular field is to be fenced on three sides leaving a side of 20 feet uncovered. If the area of the field is 680 sq. ft., how many feet of fencing will be required?

Solution:
We are given with length and area, so we can find the breadth.
As length × breadth = Area
= > 20 × breadth = 680
= > breadth = 34 feet

Area to be fenced = 2B + L = 2 × 34 + 20 = 88 feet

Exercise

1) One side of a rectangular field is 15 m and one of its diagonals is 17 m. Find the area of the field.
a) 160 m^2 b) 120 m^2 c) 110 m^2
d) 130 m^2 e) None of these

2) The diagonals of two squares are in the ratio of 2 : 5. Find the ratio of their areas.
a) 25 : 4 b) 4 : 25 c) 10 : 1
d) 1 : 10 e) None of these

3) Find the area of a rhombus one side of which measures 20 cm and one diagonal 24 cm.
a) 375 cm^2 b) 378 cm^2 c) 390 cm^2
d) 384 cm^2 e) None of these

4) Find the area of right angled triangle whose base is 12 cm and hypotenuse 13 cm.
a) 30 cm^2 b) 60 cm^2 c) 70 cm^2
d) 80 cm^2 e) None of these

5) The length of a rectangle is 18 cm and its breadth is 10 cm. When the length is increased to 25 cm, what will be the breadth of the rectangle if the area remains the same?
a) 7 cm b) 7.1 cm c) 7.2 cm
d) 7.3 cm e) None of these

6) A rectangular parking space is marked out by painting three of its sides. If the length of the unpainted side is 9 feet, and the sum of the lengths of the painted sides is 37 feet, then what is the area of the parking space in square feet?
a) 46 b) 81 c) 126
d) 252 e) None of these

7) The length of a rectangular hall is 5 m more than its breadth. The area of the hall is 750 m^2. The length of the hall is :
a) 15 m b) 22.5 m c) 25 m
d) 30 m e) None of these

8) The cost of carpeting a room 18 m long with a carpet 75 cm wide at Rs.4.50 per metre is Rs.810. The breadth of the room is :
a) 7 m b) 7.5 m c) 8 m
d) 8.5 m e) None of these

9) The percentage increase in the area of a rectangle, if each of its sides is increased by 20%, is :
a) 40% b) 42% c) 44%
d) 46% e) None of these

10) 2 metres broad pathway is to be constructed around a rectangular plot on the inside. The area of the plot is 96 sq. m. The rate of construction is Rs.50 per square metre. Find the total cost of the construction.
a) Rs.2400 b) Rs.4000 c) Rs.4800
d) Data inadequate e) None of these

11) A sphere of 30 cm radius is dropped into a cylindrical vessel of 80 cmj diameter, which is partly filled wikth water, then its level rises by x cm. Find x:
a) 27.5 cm b) 22.5 cm c) 18.5 cm
d) Data inadequate e) None of these

12) Altitude and base of a right angle triangle are (x + 2) and (2x + 3) (in cm). If the area of the triangle be 60 cm^2, the length of the hypotenuse is :
a) 21 cm b) 13 cm c) 17 cm
d) 15 cm e) None of these

13) A rectangular lawn 60 m × 40 m has two road each 5 m wide running in the middle of it, one parallel to length and the other parallel to breadth. The cost of graveling the roads at 80 paise per sq. m is :
a) Rs.380 b) Rs.385 c) Rs.400
d) Data Inadequate e) None of these

14) What is the ratio of the area of larger square shaped plot to the area of the smaller square shaped plot?
a) 17 : 1 b) 25 : 9 c) 16 : 1
d) Data inadequate e) None of these

Solutions:

1. Option B

Other side = $\sqrt{17^2 - 15^2}$
= $\sqrt{289 - 225}$
= $\sqrt{64}$ = 8 m
So, area = (15 × 8) m^2 = 120 m^2

2. Option B

Let the diagonals of the squares be 2x and 5x respectively.
So, ratio of their areas = $\frac{1}{2} \times (2x^2) : \frac{1}{2} \times 5x^2 = 4x^2 : 25x^2$ = 4 : 25

3. Option D

Let other diagonal = 2x cm.
Since diagonals of a rhombus bisect each other at right angles, we have :

$(20)^2 = (12)^2 + x^2$
x = $\sqrt{20^2 - 12^2} = \sqrt{256}$ = 16 cm.
So, other diagonal = 32 cm.
So, area of rhombus = $\frac{1}{2}$ × (Product of diagonals) = $\left[\frac{1}{2} \times 24 \times 32\right] cm^2$ = 384 cm^2

4. Option A

Height of the triangle = $\sqrt{13^2 - 12^2}$ cm = $\sqrt{25}$ cm = 5 cm.
So, its area = $\frac{1}{2}$ × Base × Height = $\left[\frac{1}{2} \times 12 \times 5\right] cm^2$ = 30 cm^2

5. Option C

Let the breadth be b. Then, 25 × b = 18 × 10
b = $\left[\frac{18 \times 10}{25}\right]$ cm = 7.2 cm

6. Option C

Clearly we have L = 9 and L + 2b = 37 or b = 14
So, area = (L × b) = (9 × 14) sq. ft. = 126 sq. ft.

7. Option D

Let breadth = x metres. Then, length = $\left[x + 5\right]$ metres
Then, x (x + 5) = 750
x^2 + 5x - 750 = 0
(x + 30) (x - 25) = 0
x = 25
So, length = (x + 5) = 30 m

8. Option B
Length of the carpet = $\left[\frac{total\ cost}{Rate/m}\right] = \left[\frac{8100}{45}\right]$ m = 180 m.
Area of the room = Area of the carpet = $\left[180 \times \frac{75}{100}\right] m^2$ = 135 m^2
So, breadth of the room = $\left[\frac{Area}{length}\right] = \left[\frac{135}{18}\right]$ m = 7.5 m

9. Option C

Let original length = x metres and original breadth = y metres.
Original area = (xy) m^2
New length = $\left[\frac{120}{100} x\right]$ m = $\left[\frac{6}{5} x\right]$ m, New breadth = $\left[\frac{120}{100} y\right]$ m = $\left[\frac{6}{5} y\right]$ m

New area = $\left[\frac{6}{5}\ \text{x} \times \frac{6}{5}\ \text{y}\right]\ m^2 = \left[\frac{36}{25}\text{xy}\right]\ m^2$

So, increase % = $\left[\frac{11}{25}\ \text{xy} \times \frac{1}{xy} \times 100\right]$% = 44%

10. Option D

Lb = 96 (Given)

Area of pathway = [(L - 4) (b - 4) - Lb] = 16 - 4 (L + b), which cabe determined. So, data is inadequate.

11. Option B

Volume of water displaced = volume of sphere

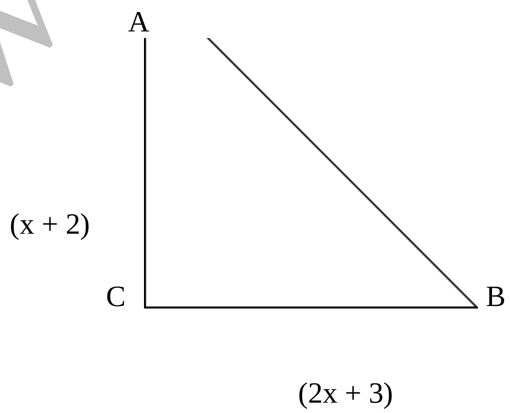
$\pi \times (40)^2 \times \text{h} = \frac{4}{3}\pi \times (30)^3$

h = $\frac{90}{4}$ = 22.5 cm

Thus, the level of water rises by 22.5 cm.

Note The volume of water will be calculated by considering it in the cylindrical shape since the water takes the shape of vessel in which it is filled.

12. Option C



Area of right angle triangle = $\frac{(x+\ 2)\ (2x+\ 3)}{2}$ = 60

$2x^2$ 7x + 6 = 120

$2x^2$ + 7x - 114 =n 0

Solving the avbove quadratic equation, we get x = 6

x + 2 = 8 cm

and 2x + 3 = 15 cm

So, Hypotenuse AB = $\sqrt{(8)^2 + (15)^2}$ = 17 cm

13. Option A

Area of path = (L + b - w) w

= (60 + 40 - 5) 5 = 475 m^2

Cost = Area × rate

= 475 × 0.8 = Rs.380

14. Option C