TRIGONOMETRY PROBLEMS SOLUTIONS

Bankers Point Explore The Intelligence

Trigonometry

Trigonometry is one of the most interesting chapters of Quantitative Aptitude section. Basically, it is a part of SSC and other bank exams syllabus. We will tell you the easy method to learn all the basics of trigonometry i.e. Trigonometric Ratios, facts and formulas.

Trigonometric Ratios

There are six trigonometric ratios. First three are the primary functions and last three are just the reciprocals of above three. Those are written as follows:

- Sin θ
- Cos θ
- Tan θ
- Cot θ
- Sec θ
- Cosec θ

Sin $\theta = \frac{Perpendicular}{Hypotenuse} = \frac{AB}{AC}$

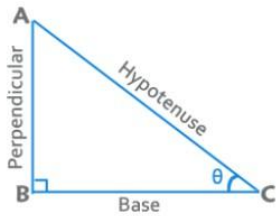
cos $\theta = \frac{Base}{Hypotenuse} = \frac{BC}{AC}$

tan $\theta = \frac{Perpendicular}{Base} = \frac{AB}{BC}$

cosec $\theta = \frac{1}{\sin\theta}$

sec $\theta = \frac{1}{\cos\theta}$

cot $\theta = \frac{1}{\tan\theta}$



Some basic identites

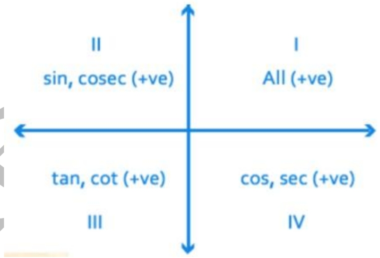
$sin^2\,\theta + cos^2\,\theta = 1$

$1 + tan^2\,\theta = sec^2\,\theta$

$1 + cot^2\,\theta = cosec^2\,\theta$

θ	0°	30°	45°	60°	90°
Sin θ	0	½	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
Cos θ	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
Tan θ	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	Not Defined
Cot θ	Not Defined	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0
Cosec θ	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	Not Defined
Sec θ	Not Defined	2	$\sqrt{2}$	$\frac{2}{\sqrt{2}}$	1

Signs of T-ratios in different quadrants:



Addition Formulae

Sin (A + B) = Sin A cos B + cos A sin B

Cos (A + B) = cos A cos B - sin A sin B

Tan (A + B) - $\frac{\tan A + \tan B}{1 - \tan A \tan B}$

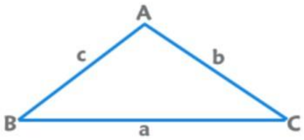
Subtraction Formulae

Sin (A - B) = sin A cos B - cos A sin B

cos (A - B) = cos A cos B + sin A sin B

tan (A - B) = $\frac{\tan A - \tan B}{1 + \tan A \tan B}$

Trigonometry Rules



Sine Rule : $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c} = \frac{1}{2R}$

Cosine Rule : Cos A = $\frac{b^2 + c^2 - a^2}{2bc}$

Cos B = $\frac{a^2 + c^2 - b^2}{2ac}$

Cos C = $\frac{a^2 + b^2 - c^2}{2ab}$

Examples with Solution

Example 1: Find the value of:

$$\frac{2\tan 30°}{1 + \tan 2\ 30°}$$

Solution: $\frac{2\tan 30°}{1 + \tan 2\ 30°} = \frac{(2/\sqrt(3)}{(1+(\frac{1}{\sqrt{3})})2)}$

= `(2/sqrt3)/(1+(1/sqrt3)^2)`

= $\frac{2/\sqrt{3}}{1+(1/\sqrt{3})^2}$

Solving above will give = $\sqrt{3}$

Example 2: If tan θ = $\frac{\sqrt{2}}{\sqrt{3}}$, then what will be the value of Cos θ?

Solution: tan θ = $\frac{P}{B}$

Therefore, P =√2 and B = √3

Using Pythagoras Theorem, $H^2 = P^2 + B^2$

$H^2 = 2 + 3 = 5$

⇒ H = √5

Therefore, Cos `theta` = B/H = `sqrt3/ sqrt5`

Exercise

1) One angle of a triangle is 54° and another angle is $\frac{\pi}{4}$ radian. Find the third angle in centesimal unit.

a) 90 b) 60 c) 80
d) 30 e) None of these

2) In a circle of diameter 60 m, the length of a chord is 30 m. Find the length of the minor arc on one side of the chord. What is the length of the major arc?

a) 157.1 m   b) 160 m   c) 179 m
d) 199 m   e) None of these

3) Two arcs of two different circles are of equal lengths. If these arcs subtends angles of 45° and 60° at the centres of the circles. Find the ratio of the radii of the two circles.

a) 3 : 4   b) 5 : 1   c) 4 : 3
d) 6 : 2   e) None of these

4) Sin $\frac{\pi}{14}$ sin $\frac{3\pi}{14}$ sin $\frac{5\pi}{14}$ sin $\frac{7\pi}{14}$ sin $\frac{9\pi}{14}$ sin $\frac{11\pi}{14}$ sin $\frac{13\pi}{14}$ =

a) 0   b) 1   c) $\frac{1}{16}$
d) $\frac{1}{64}$ e) $\frac{1}{128}$

5) The maximum value of sin [x + $\frac{\pi}{6}$] + cos [x + $\frac{\pi}{6}$] in the interval (0, $\frac{\pi}{2}$) is attained at x =

a) $\frac{\pi}{2}$ b) $\frac{\pi}{3}$ c) $\frac{\pi}{6}$
d) $\frac{\pi}{12}$ e) None of these

6) If the circumradius of an isoceless triangle ABC is equal to AB (= AC), then angle A is equal to

a) $\frac{\pi}{2}$ b) $\frac{\pi}{3}$ c) $\frac{\pi}{6}$
d) $\frac{2\pi}{3}$ e) None of these

7) In a ΔABC, if cos $\frac{A}{a}$ = cos $\frac{B}{b}$ = cos $\frac{C}{c}$ and the side a = 2, then area of the triangle is:

a) 1   b) 2   c) $\frac{\sqrt{3}}{2}$
d) $\sqrt{3}$ e) None of these

8) tan (cos^{-1} x) = ?

a) $\sqrt{1 - x^2}$ b) $\frac{\sqrt{1+x^2}}{x}$ c) $\frac{\sqrt{1-x^2}}{x}$
d) $\frac{x}{1+x^2}$ e) None of these

9) In ΔABC, B = $\frac{\pi}{3}$ and C = $\frac{\pi}{4}$. Let D divide BC internally in the ratio 1 : 3, then

$\frac{\sin(BAD)}{\sin(\angle CAD)}$=

a) $\frac{1}{3}$ b) $\frac{1}{\sqrt{6}}$ c) $\frac{1}{\sqrt{3}}$

d) $\sqrt{\frac{2}{3}}$ e) None of these

10) In any ΔABC, 2ac sin $\frac{A - B + C}{2}$ =

a) $a^2 + b^2 - c^2$ b) $c^2 + a^2 - b^2$ c) $b^2 - c^2 - a^2$
d) $c^2 - a^2 - b^2$ e) None of these

11) In a ΔABC, ∠C = $\frac{\pi}{2}$, then 2 (r + R) is equal to

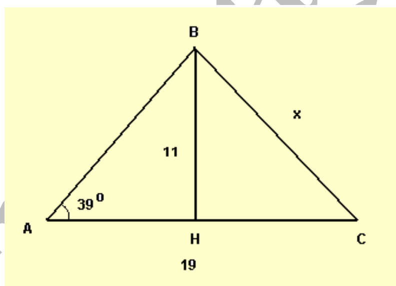
a) a + b b) b + c c) c + a
d) a + b + c e) None of these

12) If sin x + sin^2 x = 1, then cos^2 x + cos^4 x =

a) 0 b) 1 c) 1.5
d) 2 e) None of these

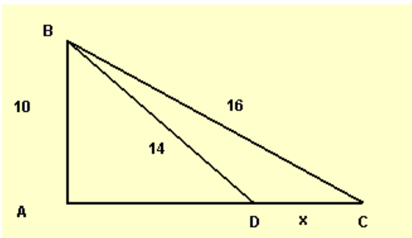
13) BH is perpendicular to AC. Find x the length of BC.

a) 12.3 b) 2.3 c) 3.2
d) 13.2 e) None of these



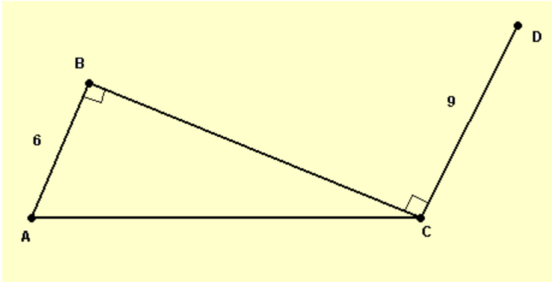
14) ABC is a right triangle with a right angle at A. Find x the length of DC?

a) 6.92 b) 9.26 c) 2.69
d) 9.62 e) None of these



15) In the figure below AB and CD are perpendicular to BC and the size of angle ACB is 31°. Find the length of segment BD.

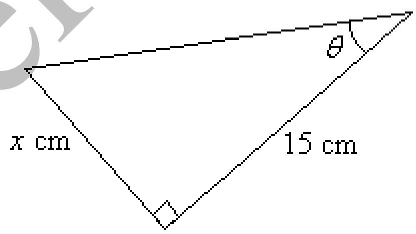
a) 14.3 b) 13.4 c) 14.1
d) 3.14 e) None of these



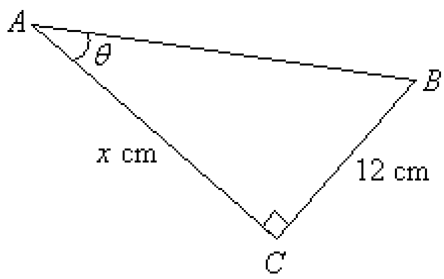
16) A rectangle has dimensions 10 cm by 5 cm. Determine the measures of the angles at the point where the diagonals intersect.
a) 53° b) 50° c) 65°
d) 60° e) None of these

17) The lengths of side AB and side BC of a scalene triangle ABC are 12 cm and 8 cm respectively. The size of angle C is 59°. Find the length of side AC.
a) 4.11 b) 11.14 c) 41.10
d) 14.0 e) None of these

18) Calculate the length of the side x, given that tan θ = 0.4
a) 10 cm b) 8 cm c) 9 cm
d) 6 cm e) None of these

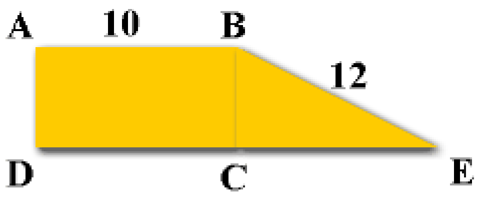


19) Calculate the length of the sidex, given than sin θ = 0.6
a) 16 cm b) 61 cm c) 32 cm
d) 64 cm e) None of these

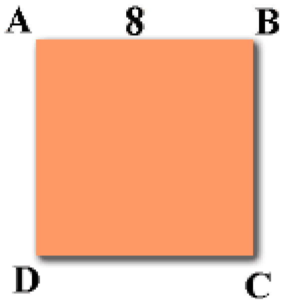


20) From a point on the ground 25 feet from the foot of a tree, the angle of elevation of the top of the tree is 32°. Find to the nearest foot, the height of the tree?
a) 48 feet b) 18 feet c) 22 feet
d) 16 feet e) None of these

21) In the figure below, ABCD is a rectangle whose perimeter is 30. The length of BE is 12. Find to the nearest degree, the measure of angle E?
a) 25 b) 32 c) 38
d) 41 e) None of these



22) In the figure ABCD is a square whose side is 8 units. Find the length of diagonal AC?
a) 11.9 b) 9.6 c) 9.9
d) 11.3 e) None of these



23) X is in quadrant 3, approximate sin (2 x) if cos (x) = - 0.2

a) 0.39 b) 0.65 c) 0.35

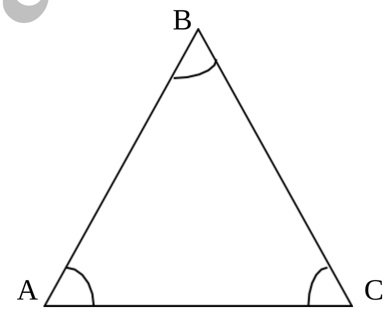
d) 1.25 e) None of these

24) X is an angle in quadrant 3 and sin (x) = $\frac{1}{3}$. Find sin (3x) and cos (3x)

a) $\frac{24}{30}$ b) $\frac{29}{35}$ c) $\frac{23}{27}$

d) $\frac{23}{29}$ e) None of these

Solutions:



1. Option A

Let ∠A = 54° and ∠B = $\frac{\pi}{4}$ rad

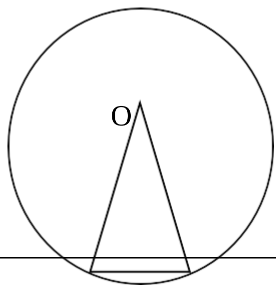
∴ B = 45°

Thus, ∠A + ∠B = 99°

∴ ∠C = 180 - 99 = 81° (∠A + ∠B + ∠C = 180°)

Now, since 90° 100 grade

81° = 81 × $\frac{100}{90}$ = 90 grade



A B
C

2. Option A

2r = 60 m r = 30 m
OA = OB = AB = 30 m
∴ Δ OAB is an equilateral triangle.
Thus, ∠AOB = 60° = $\frac{\pi}{3}$ rad
Now, since s = θ r = $\frac{\pi}{3}$ × 30
= 10 × 3.142
= 31.42 m (approx.)
Thus the length of minor arc ACB = 31.42 m and the length of major arc = (2 π r - minor arc)
= 2 π × 30 - 31.42 = 157.1 m (approx.)

3. Option C

45° = $\frac{\pi}{4}$ rad and 60° = $\frac{\pi}{3}$ rad
Let r_1 and r_2 be the radii of the two circles and s be the length of each arc.
∴ s = $r_1 \frac{\pi}{4} = r_2 \frac{\pi}{3}$ (s = rθ)
$\frac{r_1}{r_2} = \frac{4}{3}$
Hence the required ratio of radii = 4 : 3

4. Option D

Using sin (π - θ) = sin θ
sin $\left[\frac{9\pi}{14}\right]$ = sin $\left[\frac{5\pi}{14}\right]$
sin $\left[\frac{11\pi}{14}\right]$ = sin $\left[\frac{3\pi}{14}\right]$
sin $\left[\frac{13\pi}{14}\right]$ = sin $\left[\frac{\pi}{14}\right]$
Also, sin $\left[\frac{7\pi}{14}\right]$ = sin $\left[\frac{\pi}{2}\right]$ = 1
∴ Sin $\frac{\pi}{14}$ sin $\frac{3\pi}{14}$ sin $\frac{5\pi}{14}$ sin $\frac{7\pi}{14}$ sin $\frac{9\pi}{14}$ sin $\cancel{\frac{11\pi}{14}}$ sin $\cancel{\frac{13\pi}{14}}$
= [Sin $\frac{\pi}{14}$ sin $\frac{3\pi}{14}$ sin $\frac{5\pi}{14}$]
= $\left(\cos\frac{6\pi}{14}\cos\frac{4\pi}{14}\cos\frac{2\pi}{14}\right)^2$ [Using sin $\theta = \cos(\frac{\pi}{2}$ - θ)
= $\left(\cos\frac{\pi}{7}\cos\frac{2\pi}{7}\cos\frac{3\pi}{7}\right)^2$

$= -\left(\cos\frac{\pi}{7}\cos\frac{2\pi}{7}\cos\frac{4\pi}{7}\right)^2$ $\left[\cos\left[\frac{3\pi}{7}\right] = \cos\left[\pi - \frac{4\pi}{7}\right] = -\cos\left[\frac{4\pi}{7}\right]\right]$

$= \left(\frac{\sin\frac{2^3\pi}{7}}{2^3\sin\frac{\pi}{7}}\right)$

$= \left(\frac{\sin\frac{8\pi}{7}}{8\sin\frac{\pi}{7}}\right)^2$

$\frac{1}{64}$ $[\sin(8\pi/7) = \sin(\pi + \pi/7) = -\sin(\pi/7)]$

5. Option D

$\sin\left[x + \frac{\pi}{6}\right] + \cos\left[x + \frac{\pi}{6}\right] = \sqrt{2}$ $\left[\frac{1}{\sqrt{2}}\sin\left(x + \frac{\pi}{6}\right) + \frac{1}{\sqrt{2}}\cos\left(x + \frac{\pi}{6}\right)\right.$

$= \sqrt{2}\left[\sin\left(x + \frac{\pi}{6}\right)\cos\frac{\pi}{4} + \cos\left(x + \frac{\pi}{6}\right)\sin\frac{\pi}{4}\right]$

$= \sqrt{2}\left[\sin\left(x + \frac{5\pi}{12}\right)\right]$

This is maximum when $\sin\left(x + \frac{5\pi}{12}\right) = 1$

$x + \frac{5\pi}{12} = \frac{\pi}{2}$

$x = \frac{\pi}{12}$

6. Option D

$\text{Sin } B = \frac{b}{2R}$

$= \frac{AC}{2}$

$= \frac{R}{2R}$ $[\text{Given } AB = AC = R]$

$= \frac{1}{2}$

$B = \frac{\pi}{6}$ or $\frac{5\pi}{6}$

But, when $B = \frac{5\pi}{6}$, $C = \frac{5\pi}{6}$ $[AB = AC \Rightarrow B = C]$

$\Rightarrow B + C > \pi$

So, $B = \frac{5\pi}{6}$ not possible

$\therefore B = \frac{\pi}{6}$

$C = \frac{\pi}{6}$ $[AB = AC \Rightarrow B = C]$

$A = \pi - \left[\frac{\pi}{6} + \frac{\pi}{6}\right]$

$A = \frac{2\pi}{3}$

7. Option D

$\cos\frac{A}{a} = \cos\frac{B}{b} = \cos\frac{C}{c}$ [Given]

cos A/(2R sin A) = cos B/ (2R sin B) = cos C/ (2R sin C)

cot A = cot B = cot C

A = B = C

ΔABC is equilateral.

Area = $\frac{\sqrt{3}}{4}\ a^2 = \sqrt{3}$

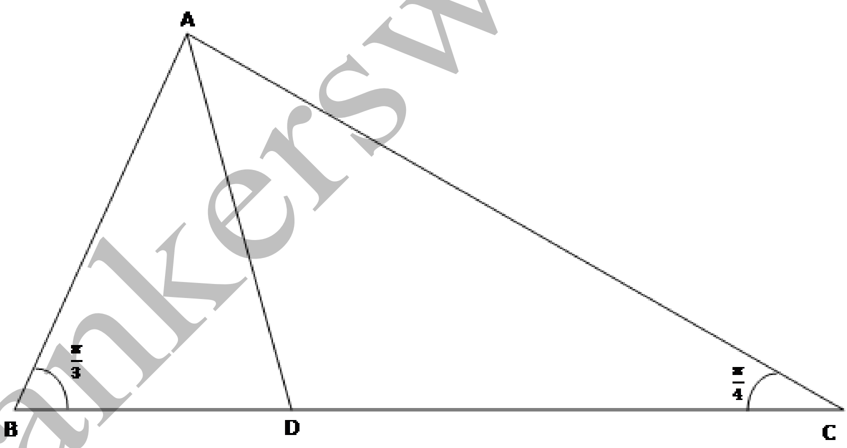
8. Option C

$\tan(cos^{-1}\ x) = \frac{\sin(cos^{-1}\ x)}{\cos(cos^{-1}\ x)}$

$= \frac{\sqrt{1-x^2}}{x}$

9. Option B

$\frac{BD}{DC} = \frac{1}{3}$ $\Big[$Given$\Big]$



From ΔABD,

BD/sin (∠BAD) = AD/sin (π/3) … (1)

From ΔACD,

DC/sin (∠CAD) = AD/sin (π/4) … (2)

Now, divide (1) by (2) and use BD/DC = 1/3

⇒ sin (∠BAD) / sin (∠CAD) = $\frac{1}{\sqrt{6}}$

10. Option B

$2\text{ ac}\sin\frac{A-B+C}{2} = 2\text{ ac}\sin\frac{A+B+C-2B}{2}$

= 2 ac sin $\frac{180° - 2B}{2}$
= 2 ac sin (90° - B)
= 2 ac cos B
= c^2 + a^2 - b^2

11. Option A

$\angle$C = $\frac{\pi}{2}$ ⇒ R = c/ (2 sin C) = c/2
Now, 2 (r + R) = 2 r+ 2 R
= 2 (s - c) tan (C/2) + 2 (c/2)
= 2s - 2c + c
= 2s - c
= a + b

12. Option B

sin x + sin^2 x = 1 [Given]
⇒ sin x = 1 - sin^2 x = cos^2 x
∴ cos^2 x + cos^4 x = sin x + sin^2 x = 1

13. Option A

BH perpendicular to AC means that triangles ABH and HBC are right triangles. Hence
tan (39°) = 11/AH or AH = 11/tan (39°)
HC = 19 - AH = 19 - 11/tan (39°)
Pythagora's theorem applied to right triangle HBC : 11^2 + HC^2 = x^2
Solve for x and substitute HC : x = sqrt [11^2 + $(19 - 11/\tan(39°)^2$]
= 12.3

14. Option C

Since angle A is right, both triangles ABC and ABD are right and therefore we can apply Pythagora's theorem.
14^2 = 10^2 + AD^2 , 16^2 = 10^2 + AC^2
Also x = AC - AD
= sqrt (16^2 - 10^2) - sqrt (14^2 - 10^2) = 2.69

15. Option B

Use right triangle ABC to write : tan (31°) = 6/BC, solve : BC = 6/tan (31°)
Use Pythagora's theorem in the right triangle BCD to write:
9^2 + BC^2 = BD^2
Solve above for BD and substitute BC : BD = sqrt [9 + $(6/\tan(31°)^2$]
= 13.4

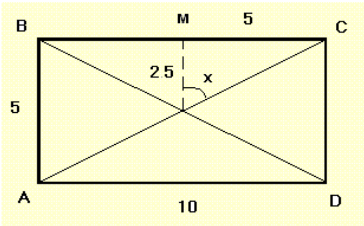
16. Option A

The diagram below shows the rectangle with the diagonals and half one of the angles with size x.

tan (x) = $\frac{5}{2.5}$ = 2, x = arctan (2)

larger angle made by diagonals 2x = 2 arctan (2) = 127°

smaller angle made by diagonals 180 2x = 53°



17. Option D

Let x be the length of side AC. Use the cosine law

12^2 = 8^2 + x^2 - 2 × 8 × x × cos (59°)

Solve the quadratic equation for x : x = 14.0 and x = - 5.7

x cannot be regative and therefore the solution is x = 14.0

18. Option D

tan θ = $\frac{opposite\ side}{adjacent\ side}$

0.4 = $\frac{x}{15}$

x = 0.4 × 15 = 6 cm

19. Option A

Sin θ = $\frac{opposite}{hypotenuse}$

0.6 = $\frac{12}{AB}$

AB = $\frac{12}{0.6}$

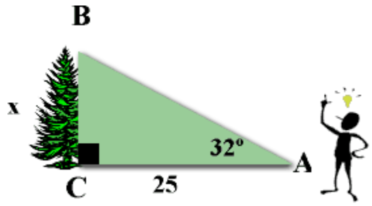
= 20 cm

Using Pythagoras theorem:

x = $\sqrt{20^2 - 12^2}$

= 16 cm

20. Option D

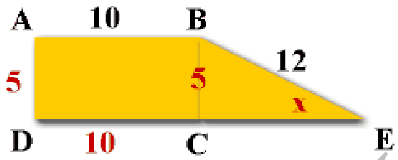


tan 32 = $\frac{x}{25}$

.6249 = $\frac{x}{25}$

x = 15.6225 = 16 feet

21. Option A



The opposite sides of a rectangle are equal. If the perimeter is 30, the sides are as shown above.

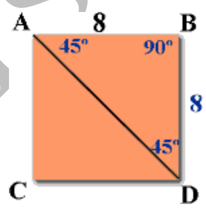
Use trigonometry to find x:

sin x = $\frac{5}{12}$

sin x = .4167

x = 24.6 = 25

22. Option D



The diagonal of a square creates two 45° - 45° - 90° triangles. You have options for a solution : the Pythagorean Theorem

sin 45° = $\frac{8}{x}$

x = 11.3

23. Option A

sin (2x) = 2 sin (x) cos (x)

sin x = - sqrt $\left[1 - (-0.2)^2\right]$

sin (2x) = 2 $\left[- \text{sqrt} \left[1 - (-0.2)^2\right]\right.$

= 0.39

24. Option C

sin (3x) = 3 sin x - 4 sin^3 x

$= 3\left[\frac{1}{3}\right] - 4\left[\frac{1}{3}\right]^3$

$= \frac{23}{27}$