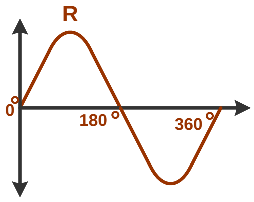
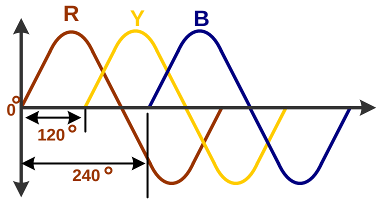
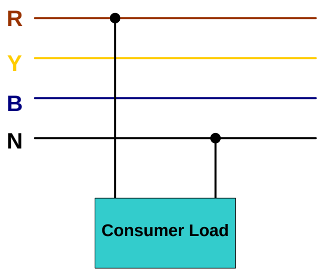
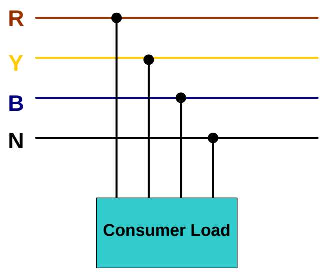
5.1 Comparison between single phase and three phase

Basis for Comparison	Single Phase	Three Phase
Definition	The power supply through one conductor.	The power supply through three conductors.
Wave Shape		
Number of wire	Require two wires for completing the circuit	Requires four wires for completing the circuit
Voltage	Carry 230V	Carry 415V
Phase Name	Split phase	No other name
Network	Simple	Complicated
Loss	Maximum	Minimum
Power Supply Connection		
Efficiency	Less	High
Economical	Less	More
Uses	For home appliances.	In large industries and for running heavy loads.

5.2 Generation of three phase EMF

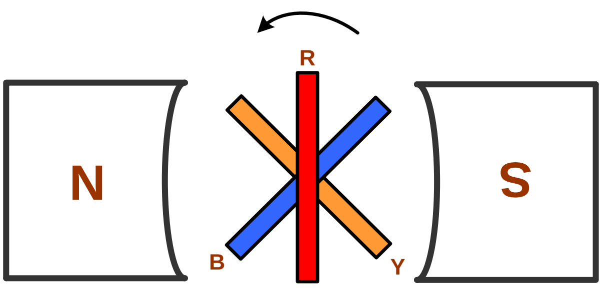


Figure 5.1 Generation of three phase emf

- According to Faraday's law of electromagnetic induction, we know that whenever a coil is rotated in a magnetic field, there is a sinusoidal emf induced in that coil.

- Now, we consider 3 coil C_1(R-phase), C_2(Y-phase) and C_3(B-phase), which are displaced 120^0 from each other on the same axis. This is shown in fig. 5.1.
- The coils are rotating in a uniform magnetic field produced by the N and S pols in the counter clockwise direction with constant angular velocity.
- According to Faraday's law, emf induced in three coils. The emf induced in these three coils will have phase difference of 120^0. i.e. if the induced emf of the coil C_1 has phase of 0^0, then induced emf in the coil C_2 lags that of C_1 by 120^0 and C_3 lags that of C_2 120^0.

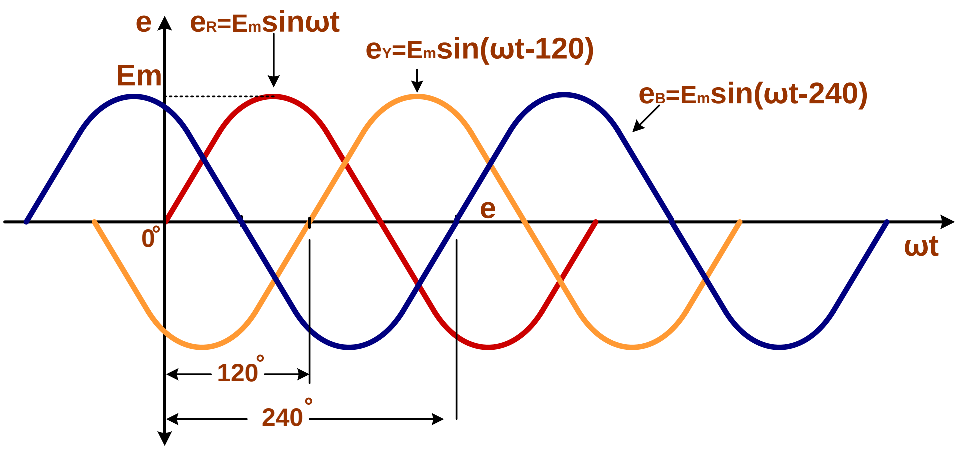


Figure 5.2 Waveform of Three Phase EMF

- Thus, we can write,

$$e_R = E_m \sin \omega t$$

$$e_Y = E_m \sin\left(\omega t - 120^0\right)$$

$$e_B = E_m \sin\left(\omega t - 240^0\right)$$

- The above equation can be represented by their phasor diagram as in the Fig. 5.3.

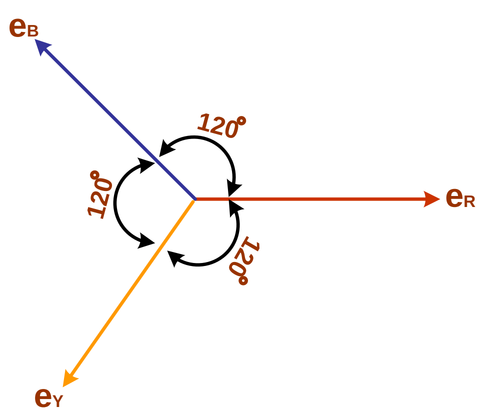


Figure 5.3 Phasor Diagram of Three Phase EMF

5.3 Important definitions

➢ Phase Voltage

It is defined as the voltage across either phase winding or load terminal. It is denoted by V_{ph}. Phase voltage V_{RN}, V_{YN} and V_{BN} are measured between R-N, Y-N, B-N for star connection and between R-Y, Y-B, B-R in delta connection.

- **Line voltage**

  It is defined as the voltage across any two-line terminal. It is denoted by V_L.

  Line voltage V_{RY}, V_{YB}, V_{BR} measure between R-Y, Y-B, B-R terminal for star and delta connection both.

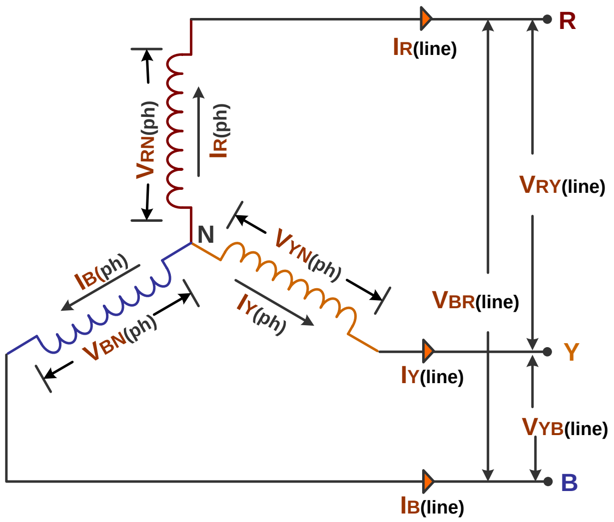


Figure 5.4 Three Phase Star Connection System

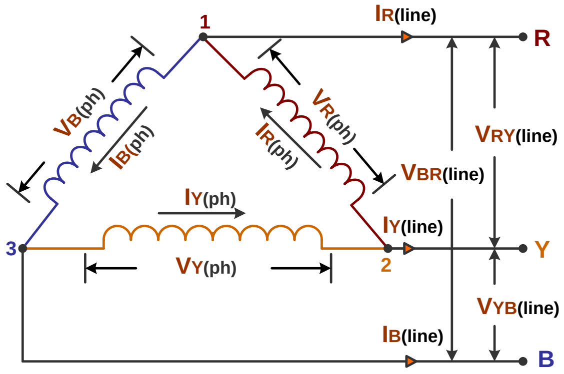


Figure 5.5 Three Phase Delta Connection System

- **Phase current**

  It is defined as the current flowing through each phase winding or load. It is denoted by I_{ph}.

  Phase current $I_{R(ph)}$, $I_{Y(ph)}$ and $I_{B(Ph)}$ measured in each phase of star and delta connection. respectively.

- **Line current**

  It is defined as the current flowing through each line conductor. It denoted by I_L.

  Line current $I_{R(line)}$, $I_{Y(line)}$, and $I_{B((line)}$ are measured in each line of star and delta connection.

- **Phase sequence**

  The order in which three coil emf or currents attain their peak values is called the phase sequence. It is customary to denoted the 3 phases by the three colours. i.e. red (R), yellow (Y), blue (B).

- **Balance System**

  A system is said to be balance if the voltages and currents in all phase are equal in magnitude and displaced from each other by equal angles.

- **Unbalance System**

  A system is said to be unbalance if the voltages and currents in all phase are unequal in magnitude and displaced from each other by unequal angles.

- **Balance load**

  In this type the load in all phase are equal in magnitude. It means that the load will have the same power factor equal currents in them.

- **Unbalance load**

  In this type the load in all phase have unequal power factor and currents.

5.4 Relation between line and phase values for voltage and current in case of balanced delta connection.

- **Delta (Δ) or Mesh connection**, starting end of one coil is connected to the finishing end of other phase coil and so on which giving a closed circuit.

Circuit Diagram

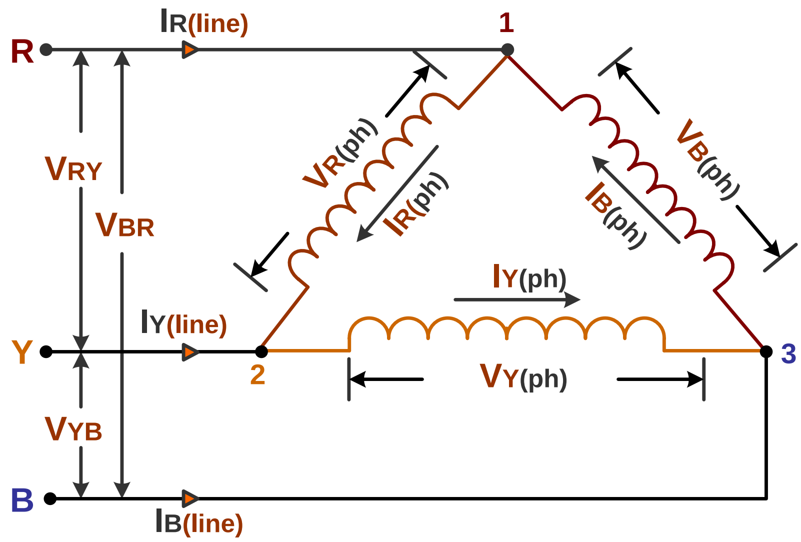


Figure 5.6 Three Phase Delta Connection

- Let,

  Line voltage, $V_{RY} = V_{YB} = V_{BR} = V_L$

  Phase voltage, $V_{R(ph)} = V_{Y(ph)} = V_{B(ph)} = V_{ph}$

  Line current, $I_{R(line)} = I_{Y(line)} = I_{B(line)} = I_{line}$

  Phase current, $I_{R(ph)} = I_{Y(ph)} = I_{B(ph)} = I_{ph}$

Relation between line and phase voltage

- For delta connection line voltage V_L and phase voltage V_{ph} both are same.

$$V_{RY} = V_{R(ph)}$$
$$V_{YB} = V_{Y(ph)}$$
$$V_{BR} = V_{B(ph)}$$
$$\therefore V_L = V_{ph}$$

Line voltage = Phase Voltage

Relation between line and phase current

- For delta connection,

$$I_{R(line)} = I_{R(ph)} - I_{B(ph)}$$
$$I_{Y(line)} = I_{Y(ph)} - I_{R(ph)}$$
$$I_{B(line)} = I_{B(ph)} - I_{Y(ph)}$$

- i.e. current in each line is vector difference of two of the phase currents.

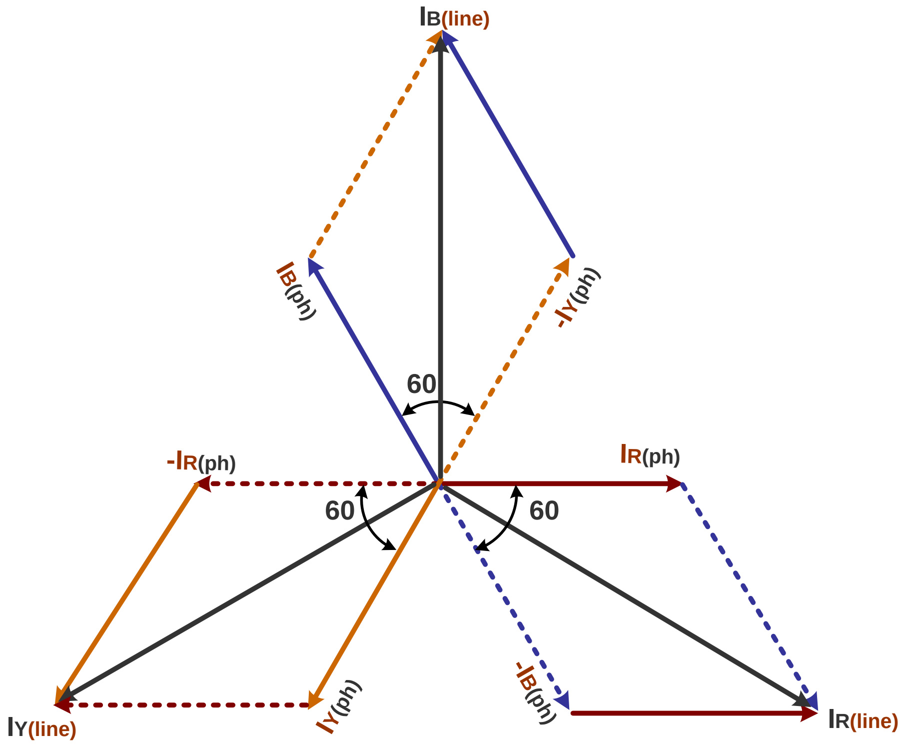


Figure 5.7 Phasor Diagram of Three Phase Delta Connection

- So, considering the parallelogram formed by I_R and I_B.

$$I_{R(line)}=\sqrt{I_{R(ph)}^{2}+I_{B(ph)}^{2}+2I_{R(ph)}I_{B(ph)}\cos\theta}$$

$$\therefore I_L=\sqrt{I_{ph}^{2}+I_{ph}^{2}+2I_{ph}I_{ph}\cos 60^\circ}$$

$$\therefore I_L=\sqrt{I_{ph}^{2}+I_{ph}^{2}+2I_{ph}^{2}\times\left(\frac{1}{2}\right)}$$

$$\therefore I_L=\sqrt{3I_{ph}^{2}}$$

$$\therefore I_L=\sqrt{3}I_{ph}$$

- Similarly, $I_{Y(line)}=I_{B(line)}=\sqrt{3}\,I_{ph}$
- Thus, in delta connection Line current = $\sqrt{3}$ Phase current

Power

$$P=V_{ph}I_{ph}\cos\phi+V_{ph}I_{ph}\cos\phi+V_{ph}I_{ph}\cos\phi$$

$$P=3V_{ph}I_{ph}\cos\phi$$

$$P=3V_L\left(\frac{I_L}{\sqrt{3}}\right)\cos\phi$$

$$\therefore P=\sqrt{3}V_LI_L\cos\phi$$

5.5 Relation between line and phase values for voltage and current in case of balanced star connection.

- In the **Star Connection**, the similar ends (either start or finish) of the three windings are connected to a common point called star or neutral point.

Circuit Diagram

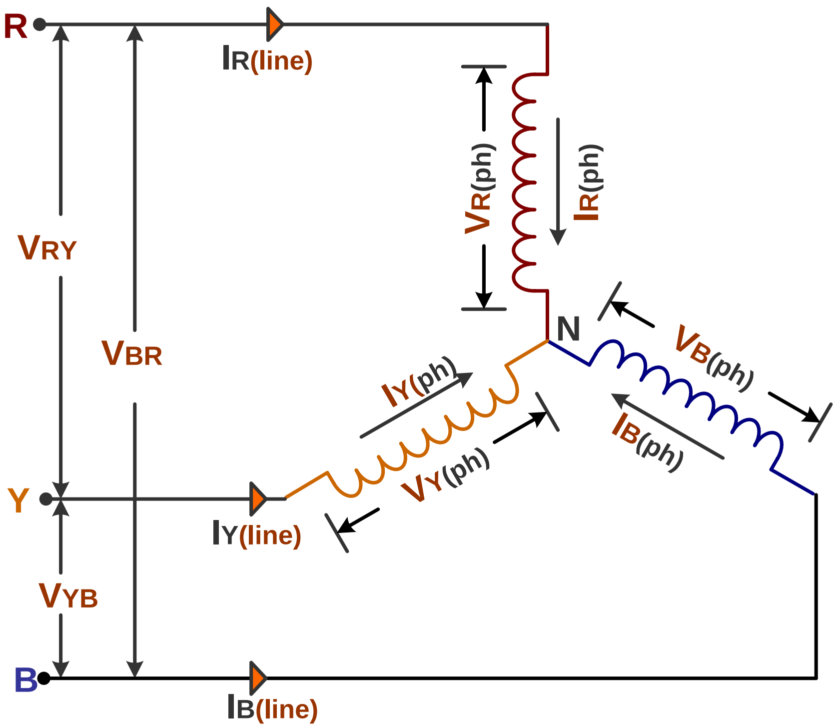


Figure 5.8 Circuit Diagram of Three Phase Star Connection

- Let,

  line voltage, $V_{RY} = V_{BY} = V_{BR} = V_L$

  phase voltage, $V_{R(ph)} = V_{Y(ph)} = V_{B(ph)} = V_{ph}$

  line current, $I_{R(line)} = I_{Y(line)} = I_{B(line)} = I_{line}$

  phase current, $I_{R(ph)} = I_{Y(ph)} = I_{B(ph)} = I_{ph}$

Relation between line and phase voltage

- For star connection, line current I_L and phase current I_{ph} both are same.

  $I_{R(line)} = I_{R(ph)}$

  $I_{Y(line)} = I_{Y(ph)}$

  $I_{B(line)} = I_{B(ph)}$

  $\therefore \quad I_L = I_{ph}$

  Line Current = Phase Current

Relation between line and phase voltage

- For delta connection,

$$V_{RY} = V_{R(ph)} - V_{Y(ph)}$$
$$V_{YB} = V_{Y(ph)} - V_{B(ph)}$$
$$V_{BR} = V_{B(ph)} - V_{R(ph)}$$

- i.e. line voltage is vector difference of two of the phase voltages. Hence,

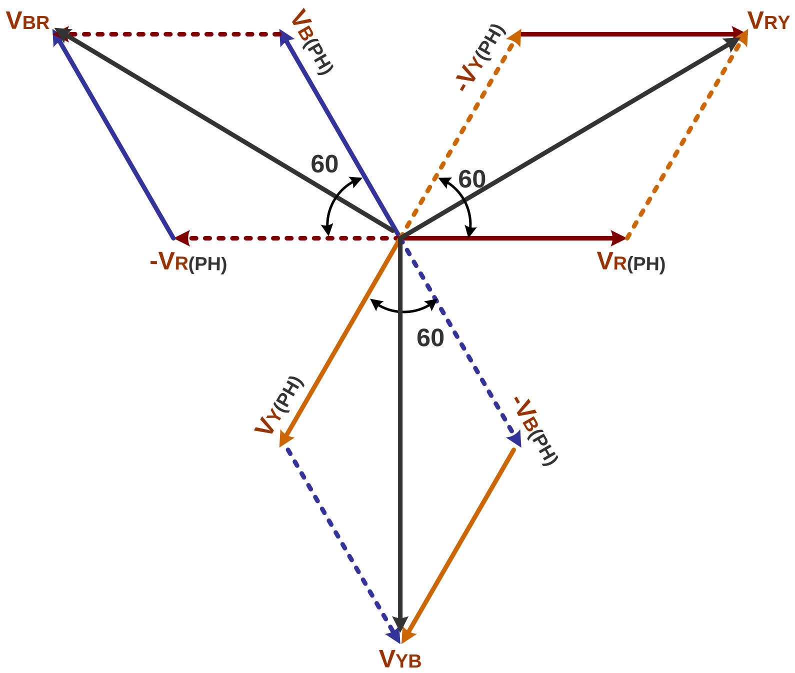


Figure 5.9 Phasor Diagram of Three Phase Star Connection

From parallelogram,

$$V_{RY} = \sqrt{V_{R(ph)}^{2} + V_{Y(ph)}^{2} + 2V_{R(ph)}V_{Y(ph)}\cos\theta}$$
$$\therefore V_L = \sqrt{V_{ph}^{2} + V_{ph}^{2} + 2V_{ph}V_{ph}\cos 60^\circ}$$
$$\therefore V_L = \sqrt{V_{ph}^{2} + V_{ph}^{2} + 2V_{ph}^{2} \times \left(1/2\right)}$$
$$\therefore V_L = \sqrt{3V_{ph}^{2}}$$
$$\therefore V_L = \sqrt{3}V_{ph}$$

- Similarly, $V_{YB} = V_{BR} = \sqrt{3}\ V_{ph}$
- Thus, in star connection Line voltage = $\sqrt{3}$ Phase voltage

Power

$$P = V_{ph}I_{ph}\cos\phi + V_{ph}I_{ph}\cos\phi + V_{ph}I_{ph}\cos\phi$$
$$P = 3V_{ph}I_{ph}\cos\phi$$
$$P = 3\left(\frac{V_L}{\sqrt{3}}\right)I_L\cos\phi$$
$$\therefore P = \sqrt{3}V_L I_L\cos\phi$$

5.6 Measurement of power in balanced 3-phase circuit by two-watt meter method

- This is the method for 3-phase power measurement in which sum of reading of two wattmeter gives total power of system.

Circuit Diagram

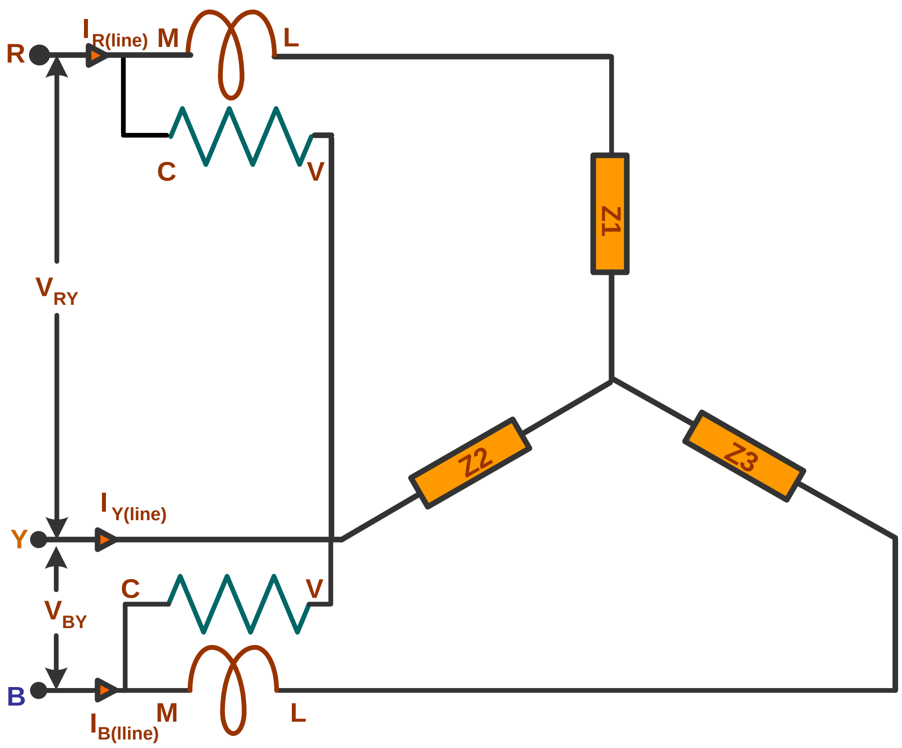


Figure 5.10 Circuit Diagram of Power Measurement by Two-Watt Meter in Three Phase Star Connection

- The load is considered as an inductive load and thus, the phasor diagram of the inductive load is drawn below in Fig. 5.11.

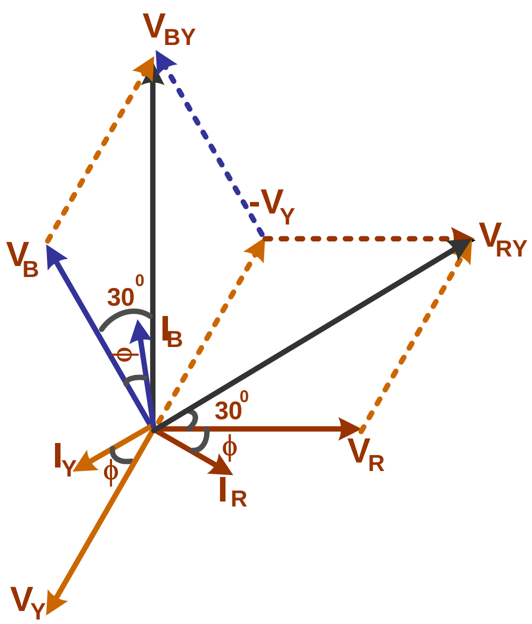


Figure 5.11 Phasor Diagram of Power Measurement by Two-Watt Meter in Three Phase Star Connection

- The three voltages V_{RN}, V_{YN} and V_{BN}, are displaced by an angle of 120^0 degree electrical as shown in the phasor diagram. The phase current lag behind their respective phase voltages by an angle ϕ. The power measured by the Wattmeter, W_1 and W_2.

Reading of wattmeter, $W_1 = V_{RY} I_R \cos\phi_1 = V_L I_L \cos(30+\phi)$

Reading of wattmeter, $W_2 = V_{BY} I_B \cos\phi_2 = V_L I_L \cos(30-\phi)$

Total power, P = W_1+W_2

$$\therefore P = V_L I_L \cos(30+\phi) + V_L I_L \cos(30-\phi)$$
$$= V_L I_L \left[\cos(30+\phi) + \cos(30-\phi)\right]$$
$$= V_L I_L \left[\cos 30 \cos\phi + \sin 30 \sin\phi + \cos 30 \cos\phi - \sin 30 \sin\phi\right]$$
$$= V_L I_L \left[2\cos 30 \cos\phi\right]$$
$$= V_L I_L \left[2\left(\frac{\sqrt{3}}{2}\right)\cos\phi\right]$$
$$= \sqrt{3} V_L I_L \cos\phi$$

- Thus, the sum of the readings of the two wattmeter is equal to the power absorbed in a 3-phase balanced system.

Determination of Power Factor from Wattmeter Readings

- As we know that

$$W_1 + W_2 = \sqrt{3} V_L I_L \cos\phi$$

Now,

$$W_1 - W_2 = V_L I_L \cos(30+\phi) - V_L I_L \cos(30-\phi)$$
$$= V_L I_L \left[\cos 30 \cos\phi + \sin 30 \sin\phi - \cos 30 \cos\phi + \sin 30 \sin\phi\right]$$
$$= V_L I_L \left[2\sin 30 \sin\phi\right]$$
$$= V_L I_L \left[2\left(\frac{1}{2}\right)\sin\phi\right] = V_L I_L \sin\phi$$

$$\therefore \frac{\sqrt{3}(W_1 - W_2)}{(W_1 + W_2)} = \frac{\sqrt{3} V_L I_L \sin\phi}{\sqrt{3} V_L I_L \cos\phi} = \tan\phi$$

$$\therefore \tan\phi = \frac{\sqrt{3}(W_1 - W_2)}{(W_1 + W_2)}$$

- Power factor of load given as,

$$\therefore \cos\phi = \cos\left(\tan^{-1}\frac{\sqrt{3}(W_1 - W_2)}{(W_1 + W_2)}\right)$$

Effect of power factor on wattmeter reading:

- From the Fig. 5.6, it is clear that for lagging power factor $\cos\phi$, the wattmeter readings are

$$W_1 = V_L I_L \cos(30+\phi)$$

$$W_2 = V_L I_L \cos(30-\phi)$$

- Thus, readings W_1 and W_2 will very depending upon the power factor angle ϕ.

p.f	ϕ	$W_1 = V_L I_L \cos(30+\phi)$	$W_2 = V_L I_L \cos(30-\phi)$	Remark
$\cos\phi = 1$	0^0	$\frac{\sqrt{3}}{2} V_L I_L$	$\frac{\sqrt{3}}{2} V_L I_L$	Both equal and +ve
$\cos\phi = 0.5$	60^0	0	$\frac{\sqrt{3}}{2} V_L I_L$	One zero and second total power
$\cos\phi = 0$	90^0	$-\frac{1}{2} V_L I_L$	$\frac{1}{2} V_L I_L$	Both equal but opposite