3.1. Concept and Classification of Resistances and measurement methods

Concept

- The choice of a suitable method of measuring resistance depends on several different factors. The range of resistance to be measured, that is, whether low, medium, or high, is the most important factor.
- In any particular case the choice of method will be limited to one or two, depending on the equipment available.
- The different methods of resistance measurement can be divided into two classes, laboratory and workshop methods. This division again depends on the accuracy with which the measurement is to be made.
- Laboratory measurements are expected to be more accurate with a given method. On the other hand the workshop measurements are less accurate, but are simple and, if possible, direct reading.

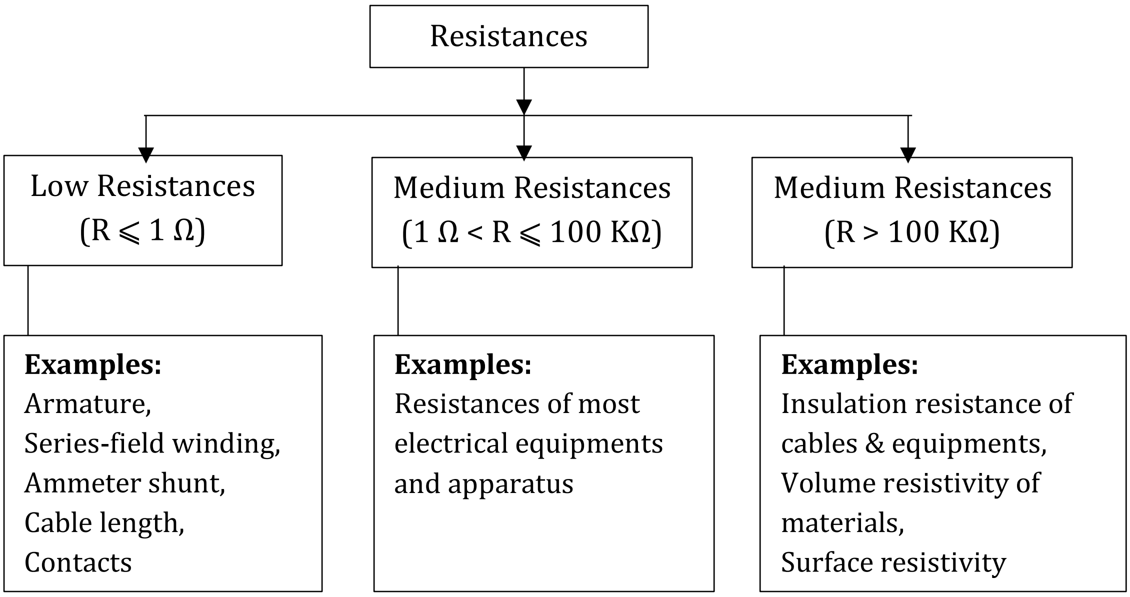


Figure 3. 1 Classification of resistances

- Low Resistances: Resistances of about 1 Ω and under are included in this class. Measurement of low resistances is required for determination of resistance of armatures, and series field windings of large machines, ammeter shunts, cable lengths, contactors etc.
- Medium Resistances: Resistances ranging from about 1 Ω to about 100 KΩ are included in this class. Most of the electrical apparatus employed are of medium resistance.
- High Resistances: Resistances of 100 KΩ and above are usually termed as high resistances. Measurement of high resistances is required for determination of Resistance of high resistance circuit elements, Insulation resistance of components and built-up electrical equipment of all types, Volume resistivity of a material and Surface resistivity.

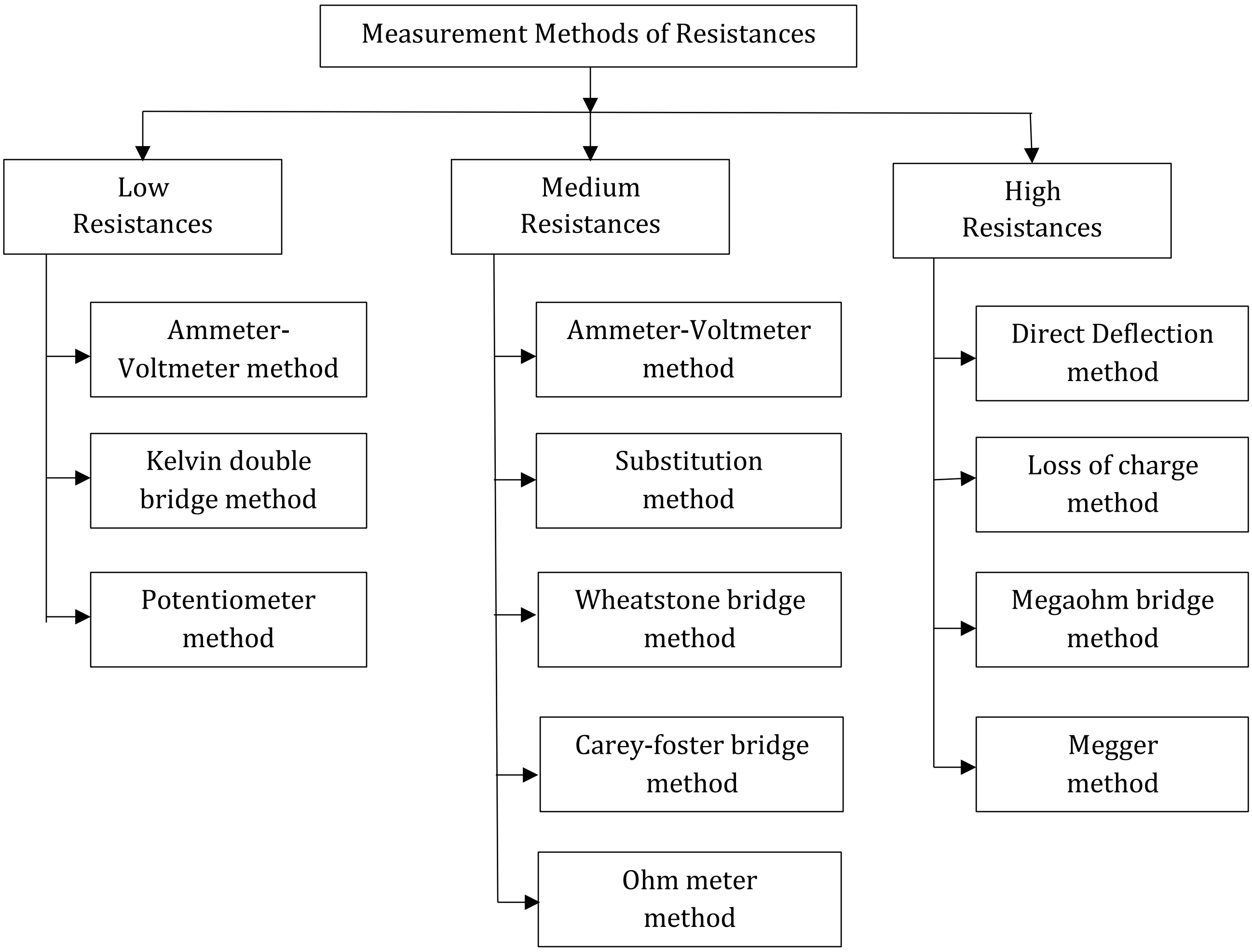


Figure 3. 2 Classification of resistance measurement methods

3.2. Measurement of High Resistances

Ammeter-Voltmeter method

- This method is very popular since the instruments required for this test are usually available in the laboratory.
- The two types of connections employed for ammeter-voltmeter method are shown in figure 3.3 (a) and (b). In both the cases, if readings of ammeter and voltmeter are taken, then the measured value of resistance is given by:

$$R_m = \frac{voltmeter\ reading}{ammeter\ reading} = \frac{V}{I}$$

- The measured value of resistance Rm, would be equal to the true value, R, if the ammeter resistance is zero and the voltmeter resistance is infinite, so that the conditions in the circuit are not disturbed. However, in practice this is not possible and hence both the methods give inaccurate results.
- **Consider circuit of figure 3.3 (a).** In this circuit the ammeter measures the true value of the current through the resistance but the voltmeter does not measure the true voltage across the resistance. The voltmeter indicates the sum of the voltages across the ammeter and the measured resistance.

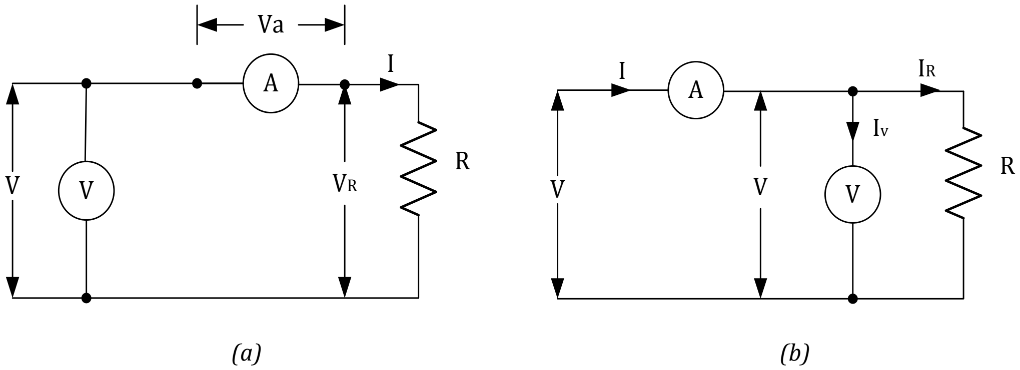


Figure 3. 3 (a) Case-1 (b) Case-2

- Let R_a be the resistance of the ammeter.

  ∴ Voltage across the ammeter, $V_a = IR_a$

  Now, measured value of resistance,

$$R_{m1} = \frac{V}{I} = \frac{V_R + V_a}{I} = \frac{IR + IR_a}{I} = R + R_a$$

  ∴ True value of resistance,

$$R = R_{m1} - R_a = R_{m1}\left(1 - \frac{R_a}{R_{m1}}\right)$$

  Thus the measured value of resistance is higher than the true value is equal to the measured only if the ammeter resistance, R_a, is zero.

$$Relative\ Error, \varepsilon_r = \frac{R_{m1} - R}{R} = \frac{R_a}{R} \quad \text{..........} (1)$$

- It is clear from above equation (1) that the error in measurement would be small if the value of resistance under measurement is large as compared to the internal resistance of the ammeter. Therefore the circuit of figure 3.3 (a) should be used when measuring high resistance value.
- **Consider circuit of figure 3.3 (b).** In this circuit the voltmeter measures the true value of the voltage but the ammeter measures the sum of currents through the resistance and the voltmeter.

  Let R_v be the resistance of the voltmeter.

$$\text{Current through the voltmeter}, I_v = \frac{V}{R_v}$$

  Measured value of resistance,

$$R_{m2} = \frac{V}{I} = \frac{V}{I_R + I_v} = \frac{V}{\frac{V}{R} + \frac{V}{R_v}} = \frac{R}{1 + \frac{R}{R_v}}$$

  True value of resistance,

$$R = \frac{R_{m2} R_v}{R_v - R_{m2}} = R_{m2}\left(\frac{1}{1 - \frac{R_{m2}}{R_v}}\right) \quad \text{..........} (2)$$

  From above equation (2), it is clear that the true value of resistance is equal to the measured value only if the resistance of voltmeter, R_v, is infinite.

Thus the measured value of resistance is smaller than the true value.

$$Relative\ error, \varepsilon_r = \frac{R_{m2} - R}{R} = \frac{{R_{m2}}^2}{R_v R}$$

The value of R_{m2} is approximately equal to R.

$$\therefore\ \varepsilon_r = -\frac{R}{R_v} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3)$$

It is clear from above equation (3) that the error in measurement would be small if the value of resistance under measurement is very small as compare to the resistance of the voltmeter. Hence, circuit of Figure 3.3 (b) should be used when measuring low resistance values.

- The relative errors for the two cases are equal when: $\frac{R_a}{R} = \frac{R}{R_v}$

Or when true value of resistance $R = \sqrt{R_a R_v}$ ……………………………… (4)

For resistances greater than the value given by above equation (4), the method of figure 3.3 (a) is used while for lower resistance method of figure 3.3 (b) is used.

Substitution method

- This method is very popular since the instruments required for this test are usually available in the laboratory.

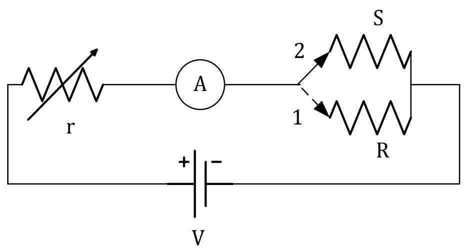


Figure 3. 4 Substitution method

- The circuit diagram for this method is shown in figure 3.4.
- R is the unknown resistance while S is a standard variable resistance. ‘A’ is an ammeter and ‘r’ is a regulating resistance. There is a switch for putting R and S into circuit alternately.
- The switch is put at position ‘1’ and resistance R is connected in the circuit. The regulating resistance r is adjusted till the ammeter pointer is at a chosen scale mark. Now, the switch is thrown to position ‘2’ putting the standard variable resistance S in the circuit. The value of S is varied till the same deflection as was obtained with R in the circuit is obtained.
- The settings of the dials of S are read. Since the substitution of one resistance for another has left the current unaltered, and provided that the emf of battery and the position of r are unaltered, the two resistances must be equal. Thus the value of unknown resistance R is equal to the dial settings of resistance S.
- The accuracy of this method is greatly affected if there is any change in the battery emf during the time the readings on the two settings are taken.
- This method is not widely used for simple resistance measurement and is used in a modified form for the measurement of high resistances. The substitution principle, however, is very

important and finds many applications in bridge methods and in high frequency a.c. measurements.

Wheatstone bridge method

- A wheatstone bridge has been in use longer than almost any electrical measuring instrument. It is still an accurate and reliable instrument and is extensively used in industry.
- The wheatstone bridge is an instrument for making **comparison measurements** and operates upon a **null indication** principle. This means the indication is independent of the calibration of the null indicating instrument or any of its characteristics. For this reason, very high degrees of accuracy can be achieved using wheatstone bridge.

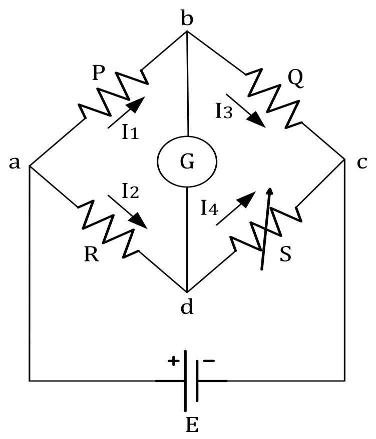


Figure 3. 5 Wheatstone bridge

- Figure 3.5 shows the basic circuit of a wheatstone bridge. It has four resistive arms, consisting of resistances P, Q, R and S together with a dc source of emf and a null detector, usually a galvanometer G. The current through the galvanometer depends on the potential difference between points c and d.
- The bridge is said to be balanced when there is no current through the galvanometer or when the potential difference across the galvanometer is zero. This occurs when the voltage from point 'b' to point 'd' equals the voltage from point 'd' to point 'b'; or by referring to the other battery terminal, when the voltage from point 'd' to point 'c' equals the voltage from point 'b' to point 'c'.
- For bridge balance, $I_1 P = I_2 R$ ………………………………………………………… (1)

  For the galvanometer current to be zero, the following conditions also exist:

  $$I_1 = I_3 = \frac{E}{P+Q} \ldots\ldots (2)$$

  $$\text{And} \quad I_2 = I_4 = \frac{E}{R+S} \ldots\ldots (3)$$

  Where E = emf of the battery.

  Combining above equations (1), (2) and (3), and simplifying,

  $$\therefore \frac{P}{P+Q} = \frac{R}{R+S}$$

From which $QR = PS$

If three of the resistances are known, the fourth may be determined from above equation and we obtain:

$$\therefore \boldsymbol{R} = \boldsymbol{S}\frac{\boldsymbol{P}}{\boldsymbol{Q}}$$

Where R is the unknown resistance, S is called the 'standard arm' of the bridge and P and Q are called the 'ratio arm'.

- The use of wheatstone bridge is limited to the measurements of resistances ranging from a few ohm to several mega-ohms.
- The upper limit is set by the reduction in sensitivity to unbalance caused by high resistance values.
- The lower limit for measurement is set by the resistance of the connecting leads and by contact resistance at the binding posts.

3.3. Measurement of Low Resistances

- The methods used for measurement of medium resistances are not suitable for measurement of low resistances i.e. resistances having a value under 1Ω. The reason is that the resistance of leads and contacts, though small, are appreciable in comparison in the case of low resistances.

Ammeter-Voltmeter method

- Low resistances are constructed with four terminals as shown in figure 3.6.

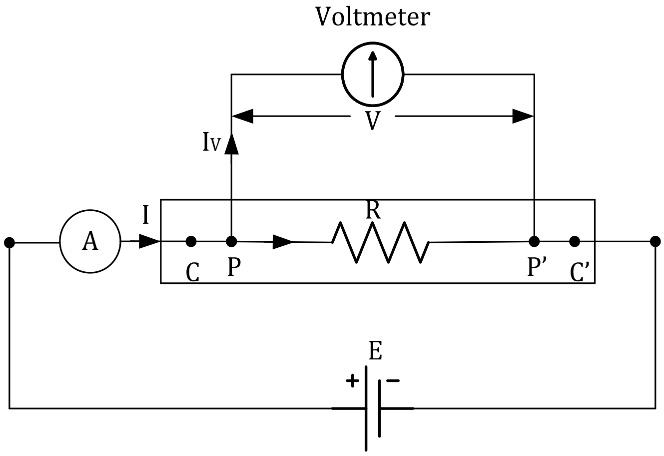


Figure 3. 6 Ammeter-voltmeter method for measuring 4 terminal resistances

- One pair of terminals CC' (called the current terminals) is used to lead current to and from the resistor. The voltage drop is measured between the other two terminals PP', called the potential terminals.
- The voltage V, indicated in figure 3.6, is thus I_R times the resistance R between terminals PP' and does not include any contact resistance drop that may be present at the current terminals CC'.

- Resistors of low values are thus measured in terms of resistance, between potential terminals, which becomes perfectly and precisely definite in value and is independent of the contact resistance drop at the current terminals.
- Contact resistance drop at the potential terminals need not be a source of error, as current crossing at these terminals is usually extremely small or even zero for null methods. Also this contact resistance now becomes a part of the potential circuit and is, therefore, a negligible part of the total resistance of the potential circuit since potential circuits have a high value of resistance.

Kelvin double bridge method

- The kelvin double bridge is a modification of the Wheatstone bridge and provides greatly increased accuracy in measurement of low value resistances.
- The kelvin double bridge incorporates the idea of a second set of ratio arms, hence the name double bridge and the use of four terminal resistors for low resistance arms.
- Figure 3.7 shows the schematic diagram of the kelvin bridge.

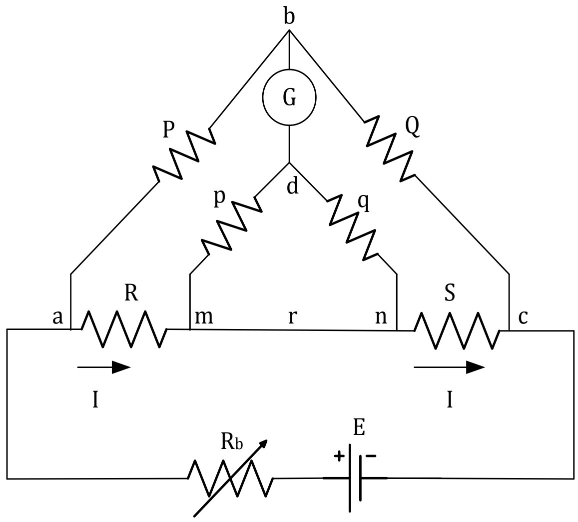


Figure 3. 7 Schematic diagram of Kelvin double bridge

- The first of ratio arms is P and Q. The second set of ratio arms, p and q is used to connect the galvanometer to a point d at the appropriate potential between points m and n to eliminate the effect of connecting lead of resistance r between the known resistance R, and the standard resistance S.
- The ratio $\frac{p}{q}$ is made equal to $\frac{P}{Q}$.
- Under balance conditions there is no current through the galvanometer, which means that the voltage drop between a and b, E_{ab} is equal to the voltage drop E_{amd} between a and c.

  Now $E_{ab} = \frac{P}{P+Q} E_{ac}$

  And $E_{ac} = I\left[R + S + \frac{(p+q)r}{p+q+r}\right]$ .... (1)

  And $E_{amd} = I\left[R + \frac{p}{p+q}\left\{\frac{(p+q)r}{p+q+r}\right\}\right] = I\left[R + \frac{pr}{p+q+r}\right]$ .... (2)

For zero galvanometer deflection,

$$E_{ab} = E_{amd}$$

$$\text{Or } \frac{P}{P+Q} I \left[R + S + \frac{(p+q)r}{p+q+r}\right] = I\left[R + \frac{pr}{p+q+r}\right]$$

$$\text{Or } R = \frac{P}{Q}S + \frac{qr}{p+q+r}\left[\frac{P}{Q} - \frac{p}{q}\right] \dots\dots\dots (3)$$

Now, if $\frac{P}{Q} = \frac{p}{q}$ equation (3) becomes,

$$\therefore \boldsymbol{R = \frac{P}{Q}S} \dots\dots\dots \textbf{(4)}$$

- Equation (4) is the usual working equation for the Kelvin Bridge. It indicates that the resistance of connecting lead, r, has no effect on the measurement, provides that the two sets of ratio arms have equal ratios.
- Equation (3) is useful, however, as it shows the error that is introduced in case the ratios are not exactly equal. It indicates that it is desirable to keep r as small as possible in order to minimize the errors in case there is a difference between ratios $\frac{P}{Q}$ *and* $\frac{p}{q}$.
- The effect of thermo-electric emfs can be eliminated by making another measurement with the battery connections reversed. The true value of R being the mean of the two readings.

Potentiometer method

- The potentiometer method of measurement of resistance is suitable for measurement of low resistances.

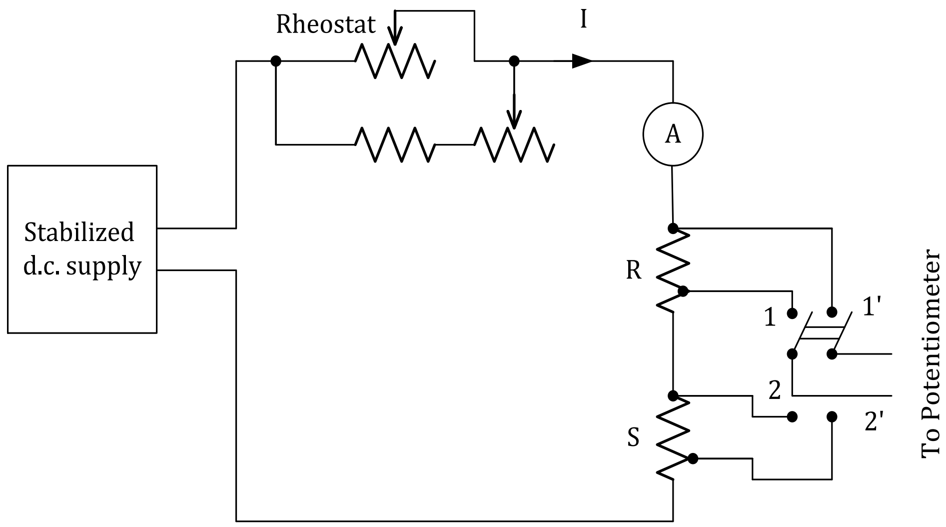


Figure 3. 8 Schematic diagram of potentiometer method for low resistance measurement

- The circuit of measurement of resistance with a potentiometer is shown in figure 3.8. The unknown resistance, R, is connected in series with a standard resistor S. The current through the circuit is controlled with the help of a rheostat. A two pole double throw switch is used. This switch, when put in position 1, 1’ connects the unknown resistance to the potentiometer. Suppose the reading of the potentiometer is V_R.

$$\therefore V_R = IR \dots\dots\dots (1)$$

Now the switch is thrown to position 2, 2’, this connects the standard resistor to the potentiometer. Suppose the reading of potentiometer is V_S.

$$\therefore \; V_S = IS \; \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \; (2)$$

From equations (1) and (2),

$$\therefore \boldsymbol{R = \frac{V_R}{V_S} S}$$

Since, the value of standard resistance S is accurately known, value of R can also be accurately known.

- The accuracy of this method depends upon the assumption that there is no change in the value of current when the two different measurements are taken. Therefore, a stable d.c. supply is absolutely necessary. The difficulty of ensuring this condition is the chief disadvantage of this method.
- The resistance of the standard resistor, S, which must be accurately known, should be of the same order as the resistance, R, under measurement. The ammeter inserted in the circuit is merely for indicating whether the current flowing through the circuit is within the capacity of the resistors or not otherwise the exact value of current flowing need not be known.
- It is desirable that the current flowing through the circuit be so adjusted that the value of voltage drop across each resistor is of the order of 1 volt.

3.4. Measurement of High Resistances

- The high resistances of the order of hundreds or thousands of Megaohm are often encountered in electrical equipment, and frequently must be measured.
- High accuracy is rarely required in such measurements, hence simple circuits are used. Since, the resistances under measurement have high values, very small currents are encountered in the measurement circuits. This aspect leads to several difficulties.

Direct deflection method

- The direct deflection method is basically that of figure 3.9.

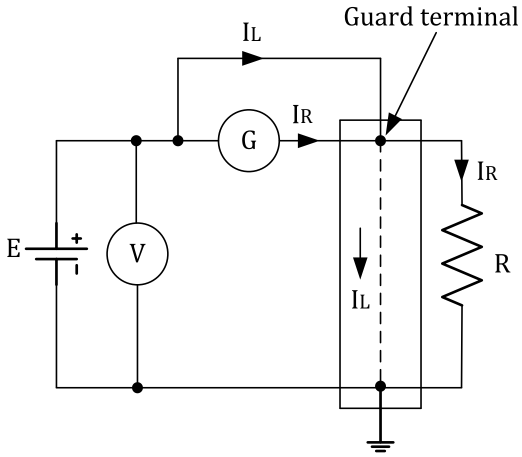
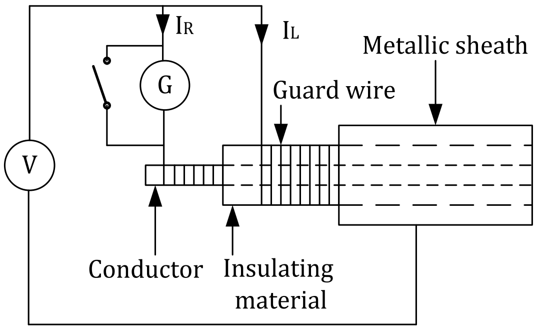


Figure 3. 9 Basic schematic diagram of direct deflection method for high resistance measurement

- For high resistances, such as insulation resistance of cables, a sensitive galvanometer of d'Arsonval type (usually having a current sensitivity of at 1000mm/μA at a scale distance of 1 metre) is used.
- In fact many sensitive type of galvanometers can detect currents from 0.1 – 1 nA. Therefore, with an applied voltage of 1 kV, resistances as high as 10^{12} to 10^{13} Ω can be measured.



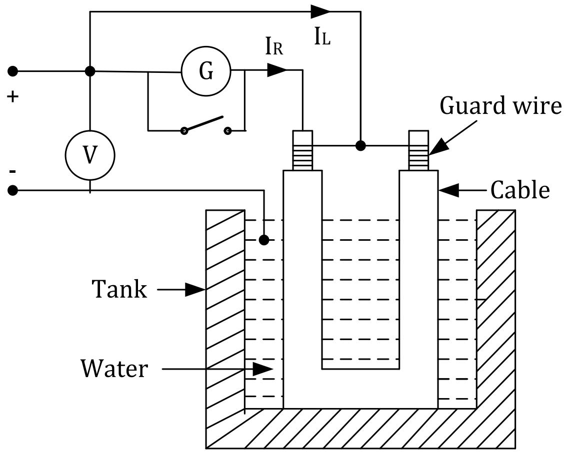
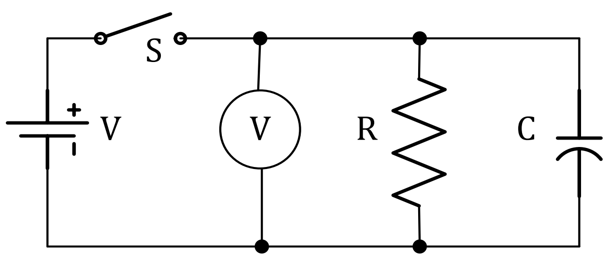


Figure 3. 10 (a) Measurement of insulation resistance of cable having sheath, (b) Measurement of insulation resistance of cable having no conducting sheath

- In figure 3.10 (a), the galvanometer G, measures the current I_R between the conductor and the metal sheath. The leakage current I_L over the insulating material is carried by the guard wire wound on the insulation and therefore does not flow through the galvanometer.
- From figure 3.10 (b), cables without metal sheaths can be tested in a similar way if cable, except the end or ends on which corrections are made, is immersed in water in a tank. The water and the tank then form the return path for the current. The cable is immersed in slightly saline water for about 24 hours and the temperature is kept constant and then the measurement is taken.
- The insulation resistance of the cable $R = \frac{V}{I_R}$.
- In some cases, the deflection of the galvanometer is observed and its scale is afterwards calibrated by replacing the insulation by a standard high resistance, the galvanometer shunt being varied, as required to give a deflection of the same order as before.

<u>Loss of charge method</u>

- The loss of charge method is basically that of figure 3.11.



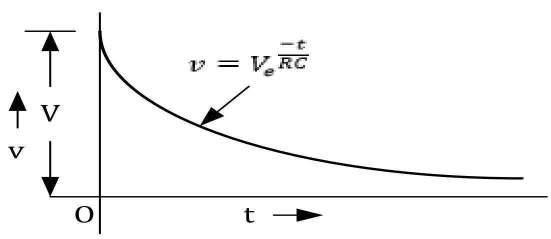


Figure 3. 11 Circuit diagram of loss of charge method   *Figure 3. 12 Variation of voltage with time for loss of charge method*

- In this method, the insulation resistance R to be measured is connected in parallel with a capacitor C and an electrostatic voltmeter.
- The capacitor is charged to some suitable voltage by means of a battery having voltage V and is then allowed to discharge through the resistance. The terminal voltage is observed over a considerable period of time during discharge.

- The voltage across the capacitor at any instant t after the application of voltage is

$$V = Ve^{\frac{-t}{RC}}$$

$$\text{Or} \frac{V}{v} = e^{\frac{-t}{RC}}$$

Or Insulation resistance

$$R = \frac{t}{C\, log_e \frac{V}{v}} = \frac{0.4343\, t}{C\, log_{10} \frac{V}{v}} \text{ .... } (1)$$

The variation of voltage v with time is shown in figure 3.12.

From equation (1) it follows that if V, v, C and t are known the value of R can be computed.

- If the resistance R is very large than the time for an appreciable fall in voltage is very large and thus process may become time-consuming. Also the voltage-time curve will thus be very flat and unless great care is taken in measuring voltages at the beginning and end of the time r, a serious error may be made in the ratio V/v causing a considerable corresponding error in the measured value of R. More accurate results may be obtained by change in the voltage V-v directly and calling this change as e, the expression for R becomes:

$$R = \frac{0.4343\, t}{C\, log_{10} \frac{V}{V-e}} \text{ .... } (2)$$

This change in voltage may be measured by a galvanometer.

Megaohm bridge method

- The figure 3.13 (a) shows a very high resistance R with its two main terminals A and B, and a guard terminal, which is put on the insulation. This high resistance may be diagrammatically represented as in figure 3.13 (b).
- The resistance R is between main terminals A and B and the leakage resistances R_{AG} *and* R_{BG} between the main terminals A and B of from a “Three terminal resistance”.

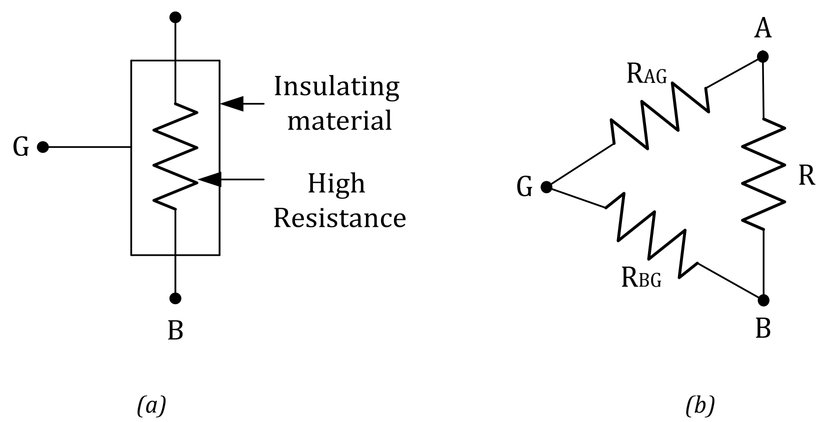


Figure 3. 13 Three terminal resistances

- The figure 3.14 shows the circuit of a completely self-containing Megaohm bridge which includes power supplies, bridge a members, amplifiers, and indicating instrument. It has a range from 0.1MΩ to 10^6 MΩ. The accuracy is within 3% for the lower part of the range to possible 10% above 10000MΩ.

- Sensitivity for balancing against high resistance is obtained by use of adjustable high voltage supplies of 500 V or 1000 V and the use of a sensitive null indicating arrangement such as a high gain amplifier with an electronic voltmeter or a C.R.O.
- The dial on Q is calibrated 1-10-100-1000 MΩ, with main decade 1-10 occupying greater part of the dial space. Since unknown resistance R = PS/Q the arm Q is made, tapered, so that the dial calibration is approximately logarithmic in the main decade, 1-10. Arm S gives five multipliers, 0.1, 1, 10, 100 and 1000.
- The junction of ratio arms P and Q is brought on the main panel and is designated as 'Guard' terminals.

Megger method (Insulation Resistance Tester)

- The Megger is one of the most important measuring device used by electrical engineers and is essentially used for measuring insulation resistance (IR) only.
- The insulation resistance quality of an electrical system degrades with time, environment condition i.e. temperature, humidity, moisture, and dust particles. It also get impacted negatively due to the presence of electrical and mechanical stress, so it is become very necessary to check the IR (insulation resistance) of equipment at a constant regular interval to avoid any measure fatal or electrical shock.
- **Types of Megger**
  - Manual (Analog) type (Hand operated)
  - Electronic (Digital) type (Battery operated)

Hand operated Megger (Analog)

Construction

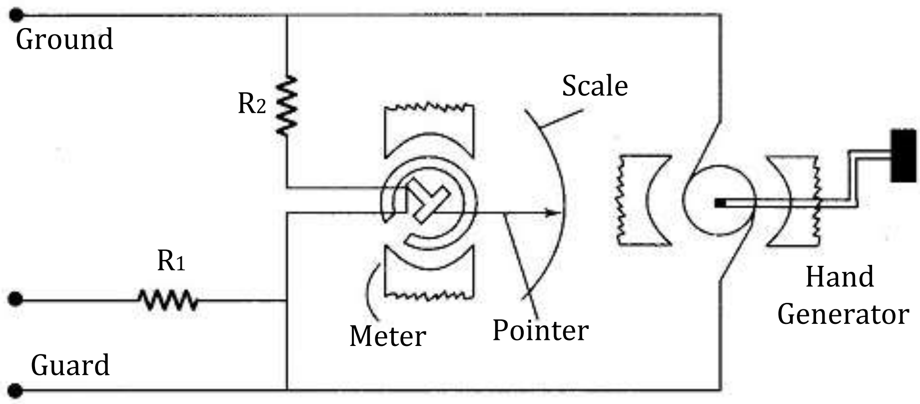


Figure 3. 14 Diagram of hand operated megger

- Deflection and control coil: Connected parallel to the generator, mounted at right angle to each other and maintain polarities in such a way to produced torque in opposite direction.
- Permanent magnets: Produce magnetic field to deflect pointer with North-South pole magnet.
- Pointer: One end of the pointer connected with coil another end deflects on scale from infinity to zero.
- Scale: A scale is provided in front-top of the megger from range 'zero' to 'infinity', enable us to read the value.

- D.C. generator or Battery connection: Testing voltage is produced by hand operated DC generator for manual operated megger.
- Pressure coil resistance and current coil resistance: Protect instrument from any damage because of low external electrical resistance under test.

Working

- Voltage for testing produced by hand operated megger by rotation of crank in case of hand operated type, a battery is used for digital tester.
- 500 V dc is sufficient for performing test on equipment range upto 440 volts. 1000 V to 5000 V is used for testing for high voltage electrical systems.
- Deflecting coil or current coil connected in series and allows flowing the electric current taken by the circuit being tested.
- In hand operated megger electromagnetic induction effect is used to produce the test voltage. Whereas in digital type megger battery are used to produce the testing voltage.
- As the voltage increases in external circuit the deflection of pointer increases with a increases of current.
- Hence, resultant torque is directly proportional to voltage and inversely proportional to current.

Advantages of Hand operated Megger

- Cheaper
- No external source required to operate

Disadvantages of Hand operated Megger

- Less accuracy
- Require very stable placement for operation
- Provides an analog display result
- Require very high care and safety during use

Electronic (Digital) type Megger

Construction

- Digital display: To show IR value in digital form
- Wire leads: For connecting megger with electrical external system to be tested
- Selection switches: Use to select electrical parameter ranges
- Indicators: To indicate various parameters status i.e. On-off, Power, Hold, Warning etc.

Advantages of Digital Megger

- Very high accuracy
- Easy to read
- One person can operate very easily
- Works perfectly even at very congested space
- Very handy
- Safe to use

Disadvantages of Digital Megger

- Require an external source (battery)
- Costlier as compared to analog megger

3.5. Measurement of Self-Inductance

- Alternating current methods are the best and most usual methods for the precise measurement of self and mutual inductance and capacitance, since it is generally more difficult to obtain accuracy with deflection methods.
- The measurement of self and mutual inductance by ac bridges is carried out in the same way as resistance is measured by dc bridge. AC bridges enable precise comparisons to be made, but adjustment is more complex, and great care is necessary to avoid error than when using dc bridge.

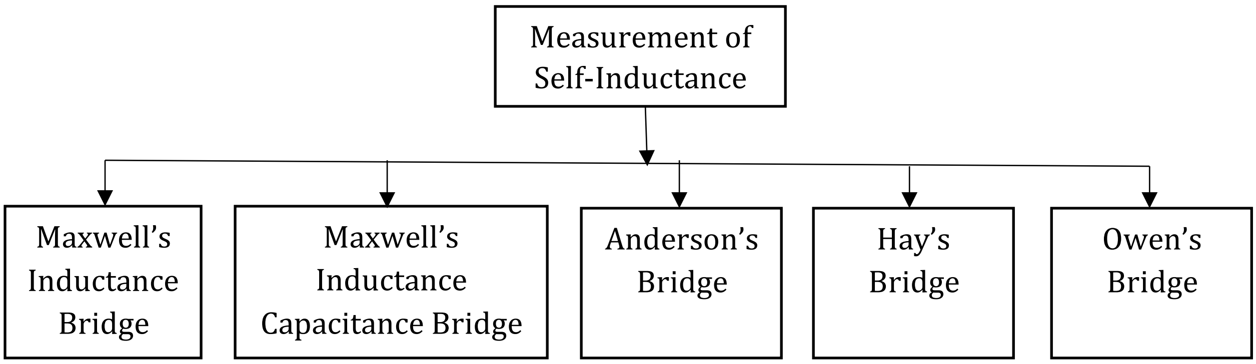


Figure 3. 15 Methods of Measurement of Self-Inductance

- The figure 3.15 shows the classification of self-inductance measurement methods. Among, Maxwell's inductance bridge and Anderson's bridge are widely used in laboratory practices.

<u>Maxwell's Inductance Bridge method</u>

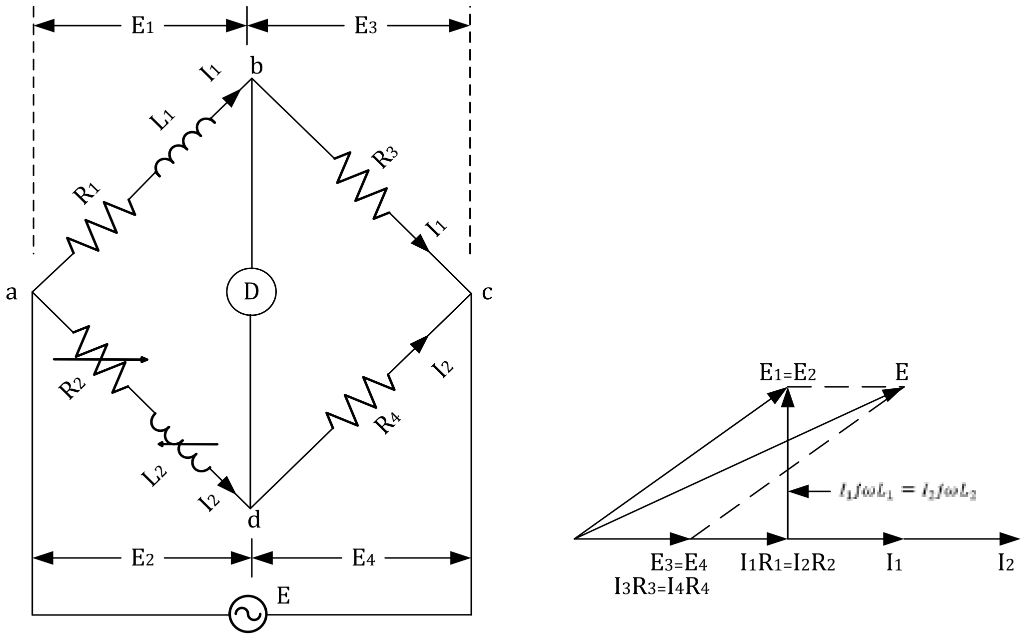


Figure 3. 16 Maxwell's Inductance Bridge (a) Circuit diagram (b) Phasor diagram

- This bridge circuit measures an inductance by comparison with a variable standard self-inductance. The connections and the phasor diagrams for balance conditions are shown in figure 3.16.

- Let,

  $L_1 = unknown\ inductance\ of\ resistance\ R_1$

  $L_2 = variable\ inductance\ of\ fixed\ resistance\ r_2$

  $R_2 = variable\ resistance\ connected\ in\ series\ with\ inductor\ L_2$

  $R_3, R_4 = known\ non - inductive\ resistances$

$$L_1 = \frac{R_3}{R_4} L_2 \ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

$$R_1 = \frac{R_3}{R_4}(R_2 + r_2) \ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

- Resistors R_3 and R_4 are normally a selection of values from 10, 100, 1000 and 10000 Ω. R_2 is a decade resistance box.
- In some cases, an additional known resistance may have to be inserted in series with unknown coil in order to obtain balance.

<u>Maxwell's Inductance Capacitance Bridge method</u>

- In this bridge, an inductance is measured by comparison with a standard variable capacitance. The connections and the phasor diagram at the balance conditions are given in figure 3.17.

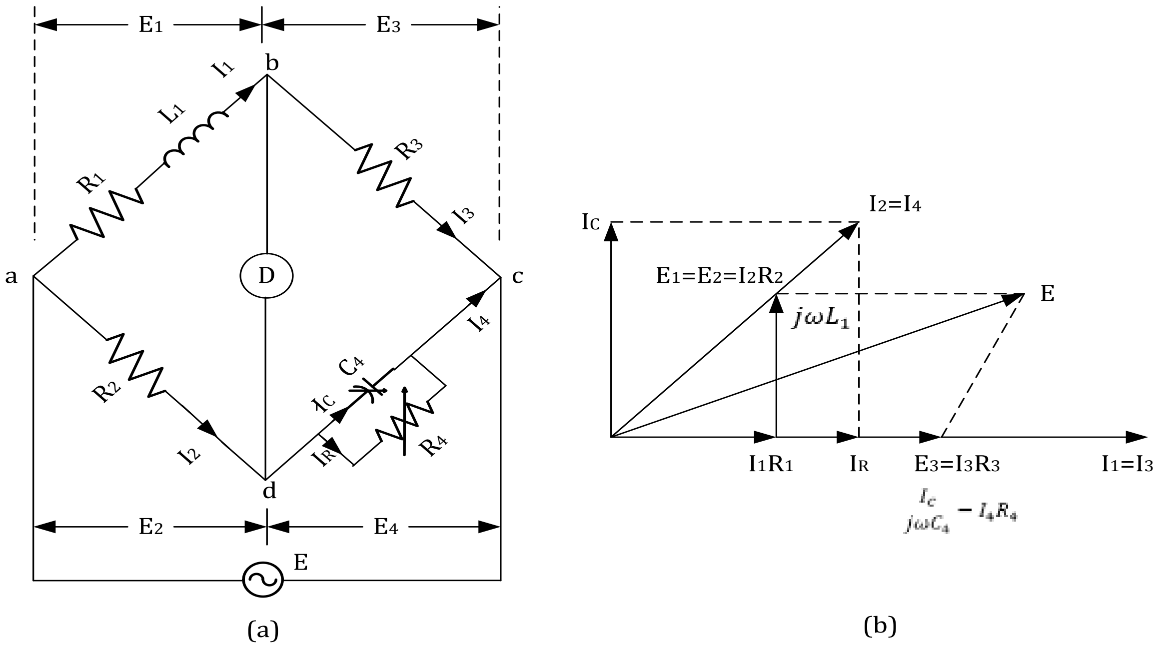


Figure 3. 17 Maxwell's Inductance Capacitance bridge (a) Circuit diagram (b) Phasor diagram

- Let,

  $L_1 = unknown\ inductance$

  $R_1 = effective\ resistance\ of\ inductor\ L_1$

  $C_4 = variable\ standard\ capacitor$

  $R_2, R_3, R_4 = known\ non - inductive\ resistances$

  Writing the equation for balance,

$$(R_1 + j\omega L_1)\left(\frac{R_4}{1 + j\omega C_4 R_4}\right) = R_2 R_3\ or\ R_1 R_4 + j\omega L_1 R_4$$

$$= R_2R_3 + j\omega R_2R_3C_4R_4$$

Separating the real and imaginary terms,

$$R_1 = \frac{R_2R_3}{R_4} \text{ ....} (1)$$

$$\text{And } L_1 = R_2R_3C_4 \text{ ....} (2)$$

Hence we have two variables R_4 and C_4 which appear in one of the two balance equations and hence the two equations are independent.

The expression for Q factor $Q = \omega\frac{L_1}{R_1} = \omega C_4R_4$ .... (3)

Advantages

- The two balance equations are independent if choose R4 and C4 as variable elements.
- The frequency does not appear in any of the two equations.
- This bridge yields simple expressions for unknowns L_1 and R_1 in terms of known bridge elements.
- This bridge is very useful for measurement of a wide range of inductance at power and audio frequencies.

Disadvantages

- This bridge requires a variable standard capacitor which may be very expensive if calibrated to a high degree of accuracy.
- The bridge is limited to measurement of low Q coils, (1<Q<10).

<u>Anderson's Bridge method</u>

- This bridge is a modification of the Maxwell's inductance capacitance bridge.
- In this method, the self-inductance is measured in terms of a standard capacitor.
- This method is applicable for precise measurement of self-inductance over a very wide range of values. The connections and the phasor diagram at the balance conditions are given in figure 3.18.
- Let,

  $L_1 = self - inductance\ to\ be\ measured$

  $R_1 = resistance\ of\ inductor\ L_1$

  $r_1 = resistance\ connected\ in\ series\ with\ L_1$

  $C = fixed\ standard\ capacitor$

  $r, R_2, R_3, R_4 = known\ non - inductive\ resistances$

  At balance,

$$I_1 = I_3\ and\ I_2 = I_c + I_4$$

$$Now, I_1R_3 = I_c \times \frac{1}{j\omega C}$$

$$\therefore I_c = I_1 j\omega CR_3$$

- Writing the other balance equations

$$I_1(r_1 + R_1 + j\omega L_1) = I_2R_2 + I_cr$$

$$\text{And } I_c = \left(r + \frac{1}{j\omega C}\right) = (I_2 - I_c)R_4$$

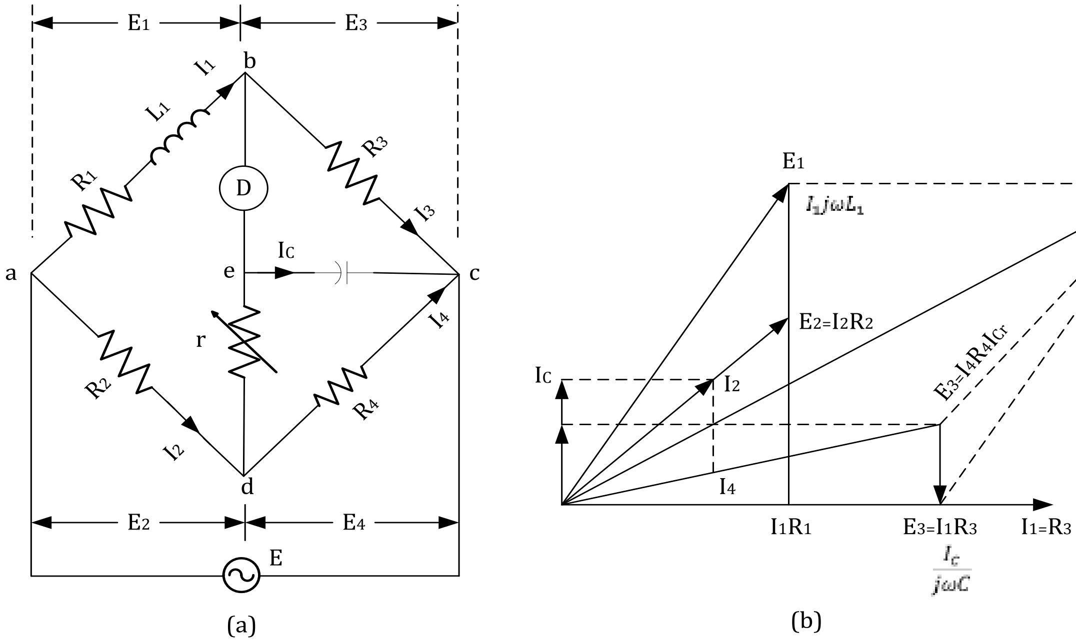


Figure 3. 18 Anderson's bridge (a) Circuit diagram (b) Phasor diagram

Substituting the value of I_c in the above equations, we have

$$I_1(r_1 + R_1 + j\omega L_1) = I_2R_2 + I_1 j\omega CR_3 r$$

$$\text{Or } I_1(r + R_1 + j\omega L_1 - j\omega CR_3 r) = I_2R_2 \text{ .... (1)}$$

$$\text{And } j\omega CR_3 I_1\left(r + \frac{1}{j\omega C}\right) = (I_2 - I_1 j\omega CR_3)R_4$$

$$\text{Or } I_1(j\omega CR_3 r + j\omega CR_3R_4 + R_3) = I_2R_4 \text{ .... (2)}$$

From equations (1) and (2),

$$I_1(r_1 + R_1 + j\omega L_1 - j\omega CR_3 r) = I_1\left(\frac{R_2R_3}{R_4} + \frac{j\omega CR_2R_3 r}{R_4} + j\omega CR_3R_2\right)$$

Equating the real and the imaginary parts,

$$R_1 = \frac{R_2R_3}{R_4} - r_1 \text{ .... (3)}$$

$$and\ L_1 = C\frac{R_3}{R_4}[r(R_4 + R_2) + R_2R_4] \text{ .... (4)}$$

An examination of balance equations reveals that to obtain easy convergence of balance, alternate adjustments of r_1 and r should be done as they appear in only one of the two balance equations.

Advantages

- It is much easier to obtain balance than in Maxwell's bridge for low Q-coils.
- A fixed capacitor can be used instead of a variable capacitor as in the case of Maxwell's bridge.
- This bridge may be used for accurate determination of capacitance in terms of inductance.

Disadvantages

- The Anderson's bridge is more complex than its prototype Maxwell's bridge.

- The Anderson's bridge has more parts and is more complicated to set up and manipulate. The balance equations are not simple and in fact are much more tedious.
- An additional junction point increases the difficulty of shielding the bridge.

Hay's Bridge method

- This bridge is a modification of the Maxwell's bridge.
- This bridge uses a resistance in series with the standard capacitor.
- The connection diagram and the phasor diagram shown given in figure 3.19.

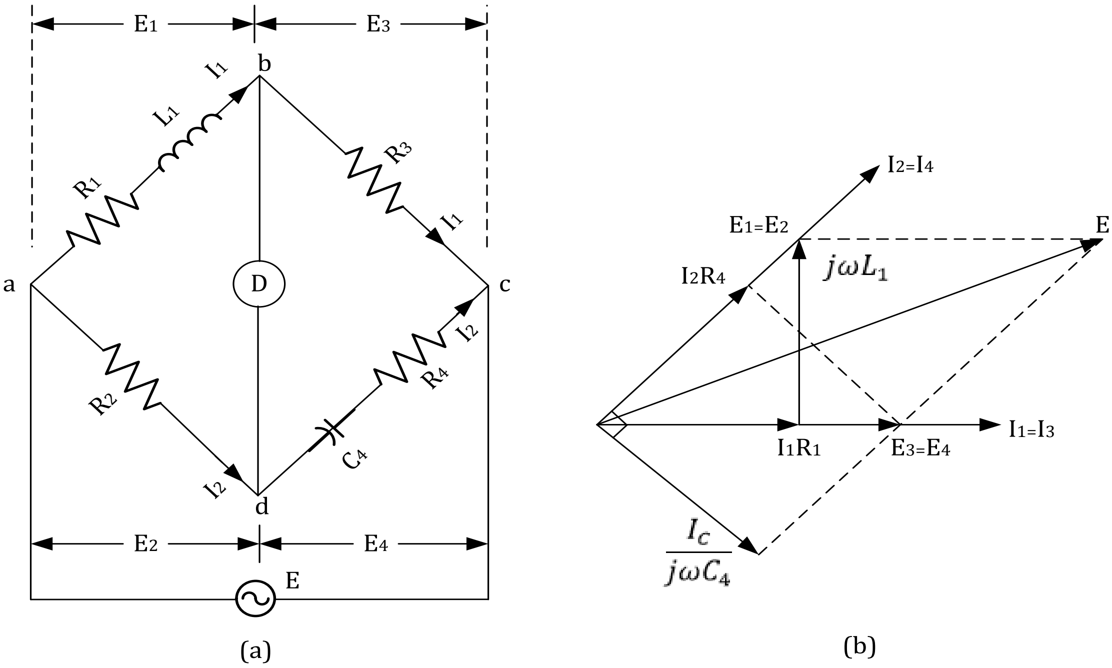


Figure 3. 19 Hay's bridge (a) Circuit diagram (b) Phasor diagram

- Let,

  $L_1 = unknown\ inductance\ having\ resistance\ R_1$

  $C_4 = standard\ capacitor$

  $C = fixed\ standard\ capacitor$

  $R_2, R_3, R_4 = known\ non-inductive\ resistances$

  At balance,

$$(R_1 + j\omega L_1)\left(R_4 - \frac{j}{\omega C_4}\right) = R_2R_3$$

$$or\ R_1R_4 + \frac{L_1}{C_4} + j\omega L_1R_4 - \frac{jR_1}{\omega C_4} = R_2R_3$$

  Separating the real and imaginary terms,

$$R_1R_4 + \frac{L_1}{C_4} = R_2R_3\ and\ L_1 = -\frac{R_1}{\omega^2R_4C_4}$$

  Solving the above two equations,

$$L_1 = \frac{R_2R_3C_4}{1+\omega^2{C_4}^2{R_4}^2}\ \ldots\ldots\ldots\ldots\ldots\ldots (1)$$

$$R_1 = \frac{\omega^2 R_2 R_3 R_4 {C_4}^2}{1+\omega^2 {C_4}^2 {R_4}^2} \ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ (2)$$

The Q factor of the coil is,

$$Q = \frac{\omega L_1}{R_1} = \frac{1}{\omega C_4 R_4} \ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ (3)$$

The expression for the unknown inductance and resistance contain the frequency term. Therefore it appears that the frequency of the source of supply to the bridge must be accurately known. This is not true for the inductance when a high Q coil is being measured, as is explained below.

$$L_1 = \frac{R_2 R_3 C_4}{1+\omega^2 {C_4}^2 {R_4}^2} \ but \ Q = \frac{1}{\omega C_4 R_4}$$

And therefore,

$$L_1 = \frac{R_2 R_3 C_4}{1+\left(\frac{1}{Q}\right)^2} \ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ (4)$$

For a value of Q > 10, the term $(1/Q)^2$ will be smaller than 1/100 and can be neglected.

Therefore equation (4) reduces to

$$L_1 = R_2 R_3 C_4 \ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ (5)$$

Which is the same as for a Maxwell's bridge.

Advantages

- This bridge gives very simple expression for unknown inductance for high Q coils, and is suitable for coils having Q>10.
- This bridge also gives a simple expression for Q factor.
- This bridge requires only a low value resistor for R_4 as compared to Maxwell's bridge.

Disadvantages

- The Hay's bridge is suited for the measurement of high Q inductors, especially those inductors having a Q>10. For inductors having Q<10, the term $(1/Q)^2$ in the expression for inductance L1 becomes rather important and thus cannot be neglected. Hence this bridge is not suited for measurement of coils having Q<10 and for these applications a Maxwell's bridge is more suited.

<u>Owen's Bridge method</u>

- This bridge may be used for measurement of an inductance in terms of capacitance.
- The connection diagram and the phasor diagram shown given in figure 3.20 under balance conditions.
- Let,

  $L_1 = unknown\ self - inductance\ having\ resistance\ R_1$

  $R_2 = variable\ non - inductive\ resistance$

  $R_3 = fixed\ non - inductive\ resistance$

  $C_2 = variable\ standard\ capacitor$

  $C_4 = fixed\ standard\ capacitor$

At balance,

$$(R_1 + j\omega L_1)\left(\frac{1}{j\omega C_4}\right) = \left(R_2 + \frac{1}{j\omega C_2}\right)R_3$$

$$or\ R_1R_4 + \frac{L_1}{C_4} + j\omega L_1 R_4 - \frac{jR_1}{\omega C_4} = R_2R_3$$

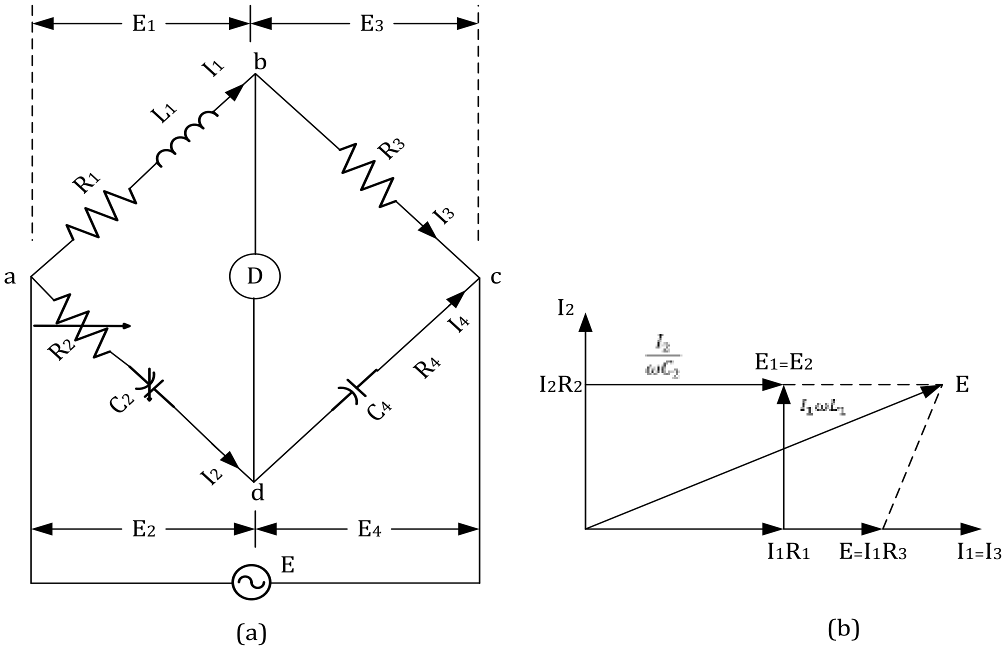


Figure 3. 20 Owen's bridge (a) Circuit diagram (b) Phasor diagram

Separating the real and imaginary terms,

$$L_1 = R_2R_3C_4 \text{ .... } (1)$$

$$R_1 = R_3\frac{C_4}{C_2} \text{ .... } (2)$$

Advantages

- Examining the equations for balance, we obtain two independent equations in case C_2 and R_2 are made variable. Since R_2 and C_2, the variable elements, are in the same arm, convergence to balance conditions is much easier.
- The balance equations are quite simple and do not contain any frequency component.
- The bridge can be used over a wide range of measurement of inductances.
- **Disadvantages**
- This bridge requires a variable capacitor which is an expensive item and also its accuracy is about 1%.
- The value of capacitance C2 tends to become rather large when measuring high Q coils.

3.6. Measurement of Capacitance

- Alternating current methods are the best and most usual methods for the precise measurement of self and mutual inductance and capacitance, since it is generally more difficult to obtain accuracy with deflection methods.

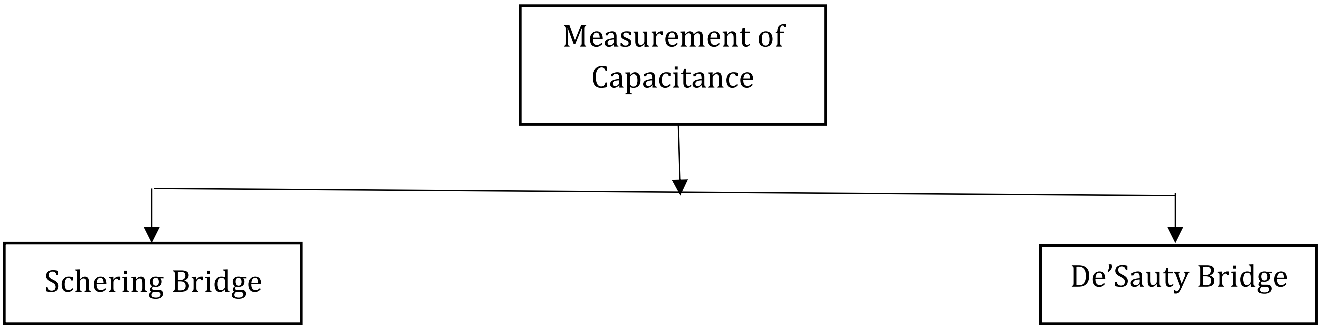


Figure 3. 21 Methods of measurement of capacitance

Schering Bridge method

- The connection and phasor diagram of the bridge under balance conditions are shown in figure 3.22.

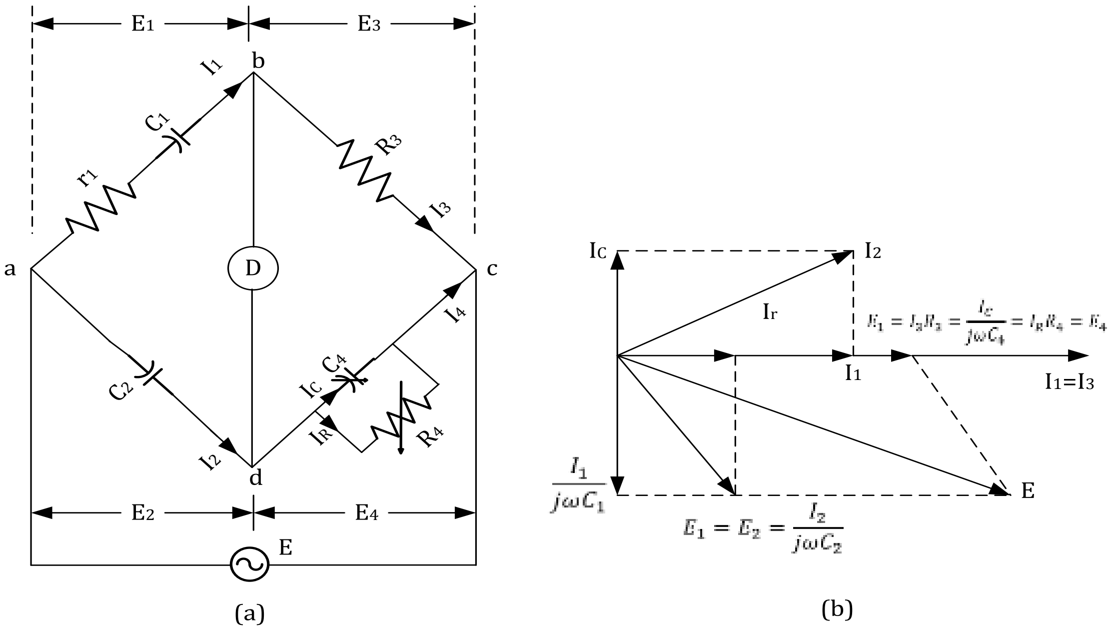


Figure 3. 22 Schering bridge (a) Circuit diagram (b) Phasor diagram

- Let,

  $C_1 = capacitor\ whose\ capacitance\ is\ to\ be\ determined$

  $r_1 = a\ series\ resistance\ representing\ the\ loss\ in\ the\ capacitor\ C_1$

  $C_2 = a\ standard\ capacitor$

  $R_3 = a\ non-inductive\ resistance$

  $C_4 = a\ variable\ capacitor$

  $R_4 = a\ variable\ non-inductive\ resistance\ in\ parallel\ with\ variable\ capacitor\ C_4$

- At balance,

$$\left(r_1 + \frac{1}{j\omega C_1}\right)\left(\frac{R_4}{1 + j\omega C_4 R_4}\right) = \frac{1}{j\omega C_2} R_3$$

$$\text{Or } \left(r_1 + \frac{1}{j\omega C_1}\right) R_4 = \frac{R_3}{j\omega C_2}(1 + j\omega C_4 R_4)$$

$$\text{Or } r_1 R_4 - \frac{jR_4}{\omega C_1} = -j\frac{R_3}{\omega C_2} + \frac{R_3 R_4 C_4}{C_2}$$

Equating the real and imaginary terms,

$$r_1 = \frac{R_3 C_4}{C_2} \ldots\ldots\ldots\ldots\ldots\ldots (1)$$

$$C_1 = C_2 \frac{R_4}{R_3} \ldots\ldots\ldots\ldots\ldots\ldots (2)$$

- Two independent balance equations are obtained if C_4 and R_4 are chosen as the variable elements.

  Dissipation factor,

$$D_1 = tan\delta = \omega C_1 r_1 = \omega\left(\frac{C_2 R_4}{R_3}\right) \times \left(\frac{R_3 C_4}{C_2}\right) = \omega C_4 R_4 \ldots\ldots\ldots\ldots\ldots\ldots (3)$$

  Therefore values of capacitance C1, and its dissipation factor are obtained from the values of bridge elements at balance.

- Since R_3 appears in both the balance equations and therefore there is some difficulty in obtaining balance but it has certain advantages.
- Since C_4 is a variable decade capacitance box, its setting in μF directly gives the value of the dissipation factor.

De Sauty's Bridge method

- The bridge is the simplest method of comparing two capacitances.
- The connections and phasor diagram of the bridge under balance conditions are shown in figure 3.23.

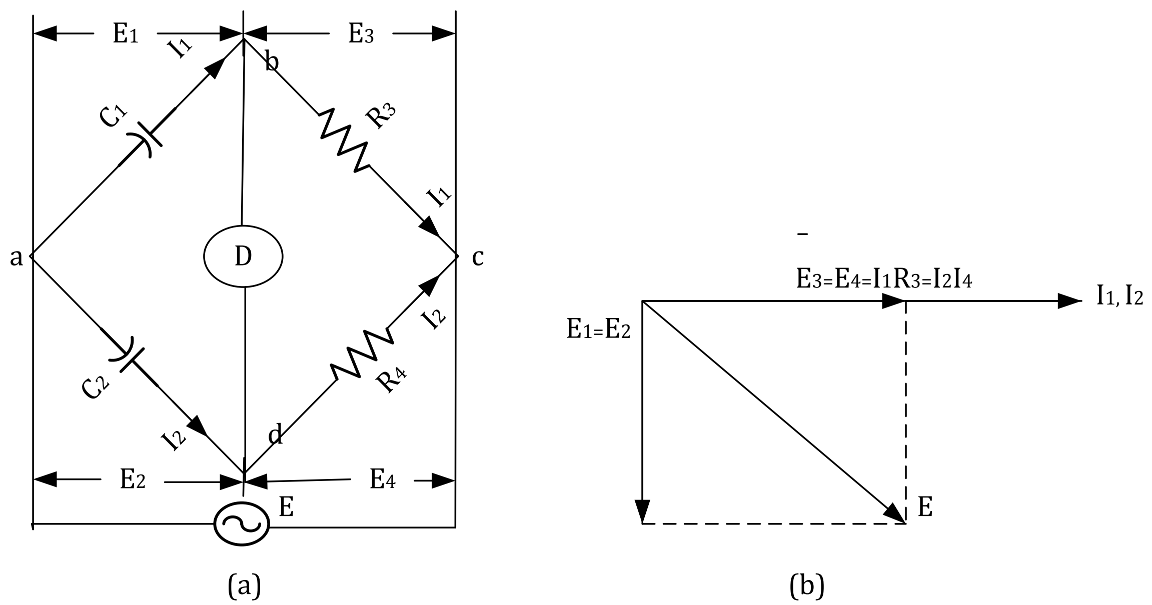


Figure 3. 23 Basic De Sauty's bridge (a) Circuit diagram (b) Phasor diagram

- Let,

  $C_1 = capacitor\ whose\ capacitance\ is\ to\ be\ determined$

  $C_2 = a\ standard\ capacitor$

  $R_3, R_4 = a\ non-inductive\ resistors$

- At balance,

$$\left(\frac{1}{j\omega C_1} R_4\right) = \left(\frac{1}{j\omega C_2}\right) R_3$$

$$\text{Or } C_1 = \frac{C_2 R_4}{R_3} \ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

- The balance can be obtained by varying either R_3 or R_4.
- The advantage of this bridge is its simplicity. But this advantage is nullified by the fact that it is impossible to obtain balance if both the capacitors are not free from dielectric loss. Thus with this method only loss-less capacitors like air capacitors can be compared.
- In order to make measurements on imperfect capacitors, the bridge is modified as shown in figure 3.24. This modification is due to Grover.
- Resistors R_1 and R_2 are connected in series with C_1 and C_2 respectively. R_1 and r_2 are resistances representing the loss component of the two capacitors.
- At balance,

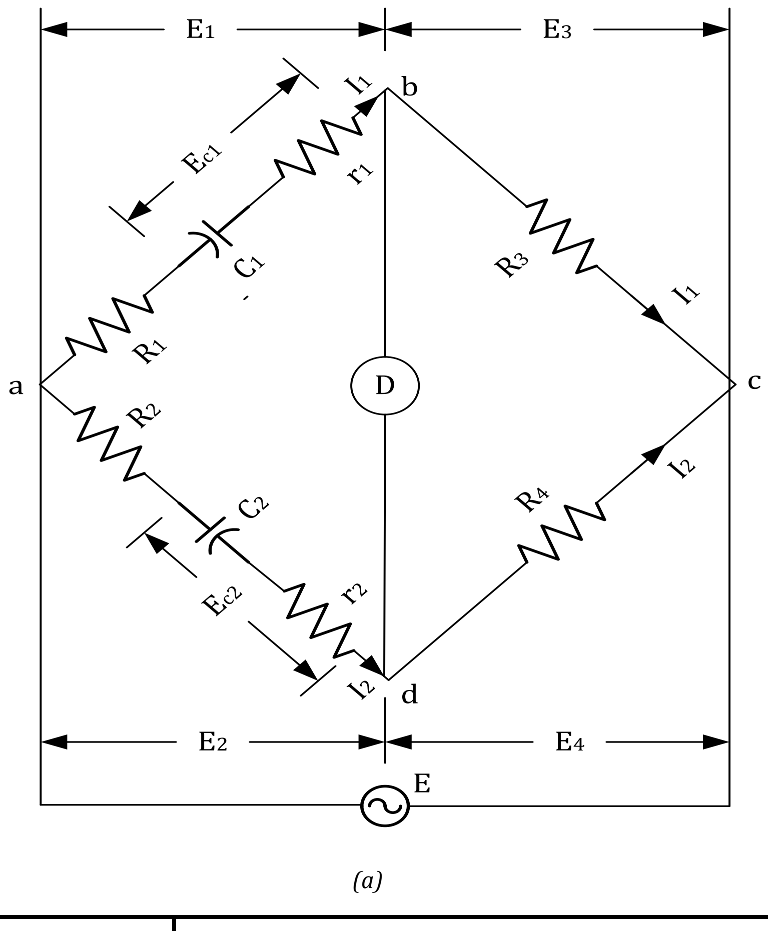
$$\left(R_1 + r_1 + \frac{1}{j\omega C_1}\right) R_4 = \left(R_2 + r_2 + \frac{1}{j\omega C_2}\right) R_3$$

From which,

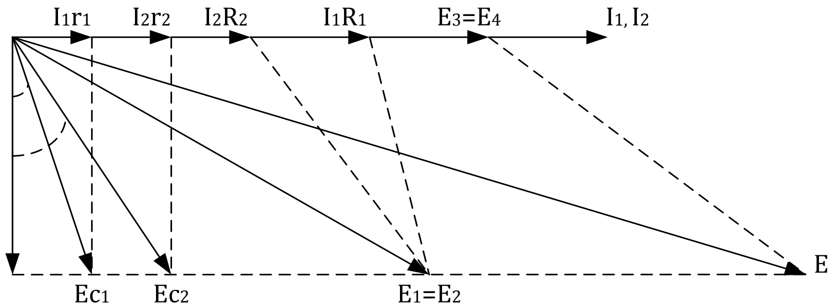
$$\frac{C_1}{C_2} = \frac{R_2 + r_2}{R_1 + r_1} = \frac{R_4}{R_3} \ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

The balance may be obtained by variation of resistances R_1, R_2, R_3, R_4.

Figure 3.24 shows the phasor diagram of the bridge under balance conditions. The angles δ_1 and δ_2 are the phase angles of capacitors C_1 and C_2 respectively.



(a)



(b)

Figure 3. 24 Modified De Sauty's bridge (a) Circuit diagram (b) Phasor diagram

Dissipation factors for the capacitors are:

$$D_1 = tan\delta_1 = \omega C_1 r_1 \ and \ D_2 = tan\delta_2 = \omega C_2 r_2$$

From equation (1),

$$\frac{C_1}{C_2} = \frac{R_2 + r_2}{R_1 + r_1}$$

$$\text{Or } C_2 r_2 - C_1 r_1 = C_1 R_1 - C_2 R_2$$

$$\text{Or } \omega C_2 r_2 - \omega C_1 r_1 = \omega(C_1 R_1 - C_2 R_2)$$

$$\therefore \ D_2 - D_1 = \omega(C_1 R_1 - C_2 R_2)$$

$$\text{But } \frac{C_1}{C_2} = \frac{R_4}{R_3}$$

$$\therefore \ C_1 = \frac{C_2 R_4}{R_3}$$

$$\text{Hence, } D_2 - D_1 = \omega C_2 \left(\frac{R_1 R_4}{R_3} - R_2\right) \ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

- Therefore, if the dissipation factor of one of the capacitors is known, the dissipation factor for the other can be determined.
- This method does not give accurate results for dissipation factor since its value depends on difference of quantities $\frac{R_1 R_4}{R_3}$ *and* R_2. These quantities are moderately large and their difference is very small and since this difference cannot be known with a high degree accuracy, the dissipation factor cannot be determined accurately.
