1.1 Introduction

- A transmission line always has, series resistance, series inductive reactance and shunt capacitive reactance.
- The resistance is dependent upon the material from which the conductor is made.
- The inductance is formed as the conductor is surrounded by the magnetic lines of force.
- The capacitance of the line is formed as the conductor is carrying current acts as a capacitor with the earth which is always at lower potential then the conductor and the air between them forms a dielectric medium.
- Thus, the performance of transmission lines is dependent upon these three line constants. For instance, the voltage drop in the line depends upon the values of the above three line constants. Similarly, the resistance of the transmission line conductors is the most important cause of power loss in the line and determines the transmission efficiency.

1.2 Classification of overhead transmission lines

1. Short transmission lines: - upto 50 km – 80 km (<20 kV)
2. Medium transmission lines: - upto 80 km – 200 km (>20 kV - <100 kV)
3. Long transmission lines: - more than 150 km or 200 km (>100 kV)

1.3 Performance of transmission lines

While studying the performance of a transmission line, it is desirable to determine its voltage regulation and transmission efficiency.

1. Voltage regulation: - When a transmission line is carrying current, there is a voltage drop in the line due to resistance and reactance of the line. The result is that receiving end voltage V_R is generally less than the sending voltage V_S .

   The Voltage drop $(V_S - V_R)$ in the line is expressed as a percentage of receiving end voltage V_R is called voltage regulation.

   Mathematically

$$\%\text{Voltage Regulation} = \frac{V_S - V_R}{V_R} x100$$

   Obviously, it is desirable that the voltage regulation of transmission line should be low i.e., the increase in load current should make very little difference in the receiving end voltage.

2. Transmission efficiency: - The power obtained at receiving end of a transmission line is generally less than the sending end power due to losses in the line resistance.
   The ratio of receiving end power to the sending end power of a transmission line is known as the transmission efficiency of line i.e.

$$\%\text{Transmission Efficency}, \eta = \frac{\text{Receiving end power}}{\text{Sending end power}} x100$$

$$= \frac{V_R I_R \cos\phi_R}{V_S I_S \cos\phi_S} x100$$

$$= \frac{V_R I_R \cos\phi_R}{V_R I_R \cos\phi_R + \text{ losses}} x100$$

Where V_R, I_R and $\cos\phi_R$ are receiving end voltage, current and power factor while V_S, I_S and $\cos\phi_S$ are sending end voltage, current and power factor.

1.4 Performance of single phase short transmission lines

- The capacitance of short lines is negligible and usually not considered. Therefore, only resistance and inductance of the line are considered.
- The equivalent circuit of a single phase short transmission line is shown in the fig. 1.1(a) The vector diagram taking current as reference is shown in the fig 1.1(b).
- Here, the line resistance and inductance are shown as lumped or concentrated instead of being distributed.

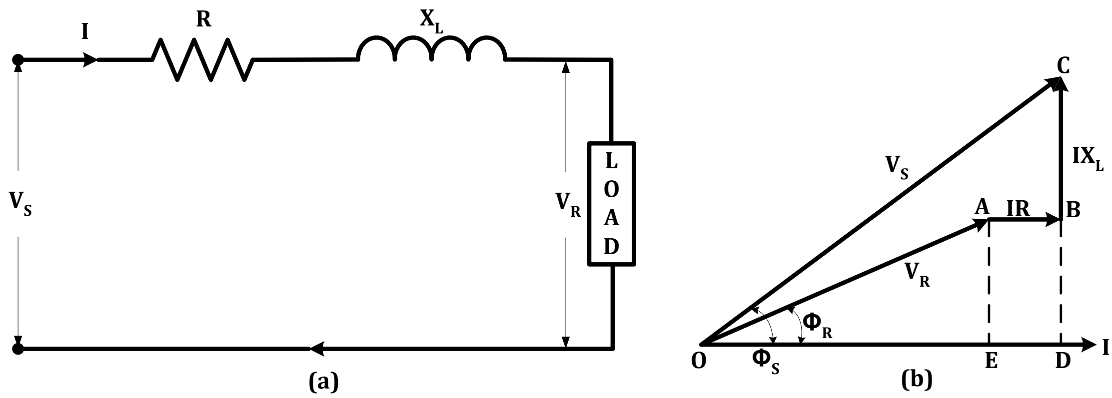


Figure 1.1 Short Transmission Line (a) Circuit and (b) Vector Diagram (Current as Reference)

$I =$ load current

$R =$ loop resistance i.e., resistance of both conductors

$X_L =$ loop reactance

$V_R =$ receiving end voltage

$\cos\phi_R =$ receiving end power factor (lagging)

$V_S =$ sending end voltage

$\cos\phi_S =$ sending end power factor

$$(OC)^2 = (OD)^2 + (DC)^2$$

$$= (OE + ED)^2 + (DB + BC)^2$$

$$= (V_R \cos\phi_R + IR)^2 + (V_R \sin\phi_R + IX_L)^2$$

$$V_S = \sqrt{(V_R \cos\phi_R + IR)^2 + (V_R \sin\phi_R + IX_L)^2}$$

$$\%\text{Voltage Regulation} = \frac{V_S - V_R}{V_R} x100$$

$$\text{Sending end p.f., } \cos\phi_S = \frac{\text{OD}}{\text{OC}}$$

$$= \frac{V_R \cos\phi_R + IR}{V_S}$$

$$\text{Power delivered} = V_R I \cos\phi_R$$

$$\%\text{ Transmission Efficiency} = \frac{V_R I \cos\phi_R}{V_S I \cos\phi_S} x100$$

$$= \frac{V_R I \cos\phi_R}{V_R I \cos\phi_R + \text{ losses}} x100$$

$$\text{Losses} = I^2 R$$

Solution under complex notation. It is often convenient to make the line calculation in complex notation.

Taking V_R as the reference phasor, the phasor diagram is shown in the fig. 1.2(b). It is clear that V_S the phasor sum of V_R and IZ

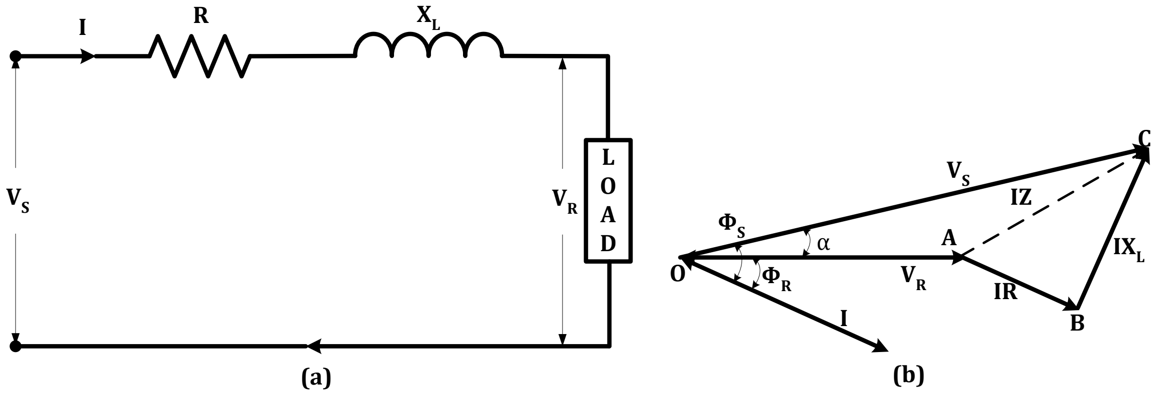


Figure 1.2 Short Transmission Line (a) Circuit and (b) Vector Diagram (Voltage as Reference)

$$V_R = V_R \angle 0$$
$$= V_R + \text{ j}0$$
$$I = I\angle -\phi_R \text{ (For lagging p.f.)}$$
$$= I(\cos\phi_R - j\sin\phi_R)$$
$$Z = R + jX_L$$
$$V_S = V_R + IZ$$
$$= (V_R + \text{ j}0) + I(\cos\phi_R - j\sin\phi_R)(R + jX_L)$$
$$V_S = V_S \angle \phi_S$$

$$\%\text{Voltage Regulation} = \frac{V_S - V_R}{V_R} x100$$

$$\text{Sending end p.f.= } \cos\phi_S$$

$$\text{Power delivered } = V_R I \cos\phi_R$$

$$\%\text{ Transmission Efficiency} = \frac{V_R I \cos\phi_R}{V_S I \cos\phi_S} x100$$

$$= \frac{V_R I \cos\phi_R}{V_R I \cos\phi_R + \text{ losses}} x100$$

$$\text{Losses } = I^2 R$$

(Note: Voltage regulation and power are scalar quantities)

1.5 Performance of medium transmission line

- In short transmission line calculations, the effect of the line capacitance is neglected because each line has smaller lengths and transmit power at relatively low voltages (<20kV).
- As the length (usually >80 km) and voltage (usually >20 kV) of the line increases, the capacitance gradually becomes of greater importance and cannot be neglected.
- The capacitance of the line is uniformly distributed over its entire length. However, to make the calculations simple, the capacitance of the system is assumed to be divided up in lumped or concentrated form of capacitors across the line at one or more points.
- The most common methods of representations of medium transmission lines are
  1. End condenser method
  2. Nominal T method
  3. Nominal π method

1.5.1 End condenser method

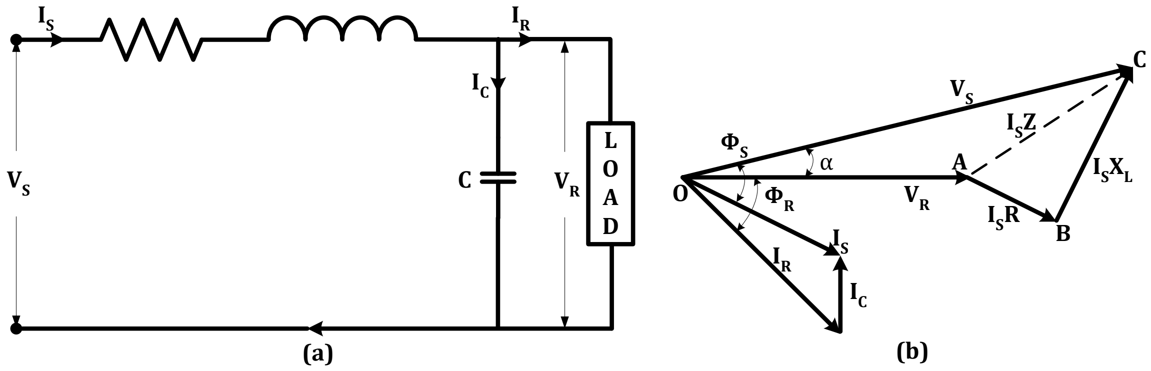


Figure 1.3 End Condenser Method (a) Circuit and (b) Vector Diagram

- In this method, the capacitance of the line is lumped or concentrated at the receiving end of the line as shown in fig. 1.3(a)
- This method of localizing the line capacitance at the load end overestimates the effect of capacitance.

Taking V_R as the reference phasor, the phasor diagram is shown in the fig. 1.3(b)

$I_R =$ load current per phase

$R =$ resistance per phase

$X_L =$ inductive reactance per phase

$C =$ capacitance per phase

$X_C =$ capactive reactance per phase

$I_C =$ capacitive current

$V_R =$ receiving end voltage

$\cos\phi_R =$ receiving end power factor (lagging)

$V_S =$ sending end voltage

$\cos\phi_S =$ sending end power factor

Taking receiving end voltage V_R as reference

$$\begin{aligned}
V_R &= V_R\angle 0 \\
&= V_R + j0 \\
I_R &= I_R\angle -\phi_R \\
&= I_R(\cos\phi_R - j\sin\phi_R) \\
I_C &= \frac{V_R}{X_C}\angle 90 \\
&= V_R\omega C\angle 90 \\
&= jV_R\omega C \\
&= jV_R 2\pi fC
\end{aligned}$$

The sending end current I_S is the vector summation of load current I_R and capacitive current I_C i.e.

$$\begin{aligned}
I_S &= I_R + I_C \\
&= I_R(\cos\phi_R - j\sin\phi_R) + j2\pi fCV_R \\
&= I_R\cos\phi_R + j(-I_R\sin\phi_R + 2\pi fCV_R)
\end{aligned}$$

$$\begin{aligned}
\text{Voltage drop/phase} &= I_S Z \\
&= I_S(R + jX_L) \\
V_S &= V_R + I_S Z \\
&= V_R + I_S(R + jX_L)
\end{aligned}$$

$$\%\text{Voltage Regulation} = \frac{V_S - V_R}{V_R} x100$$

$$\eta = \frac{\text{Power delivered / phase}}{\text{Power delivered / phase + losses / phase}} x100$$

$$= \frac{V_R I_R \cos\phi_R}{V_R I_R \cos\phi_R + I_S^2 R} x100$$

$$\text{Losses / phase} = I_S^2 R$$

$$\text{Total losses} = 3I_S^2 R$$

(Note: Voltage regulation and power are scalar quantities)

1.5.2 Nominal *T* method

- In this method, the whole line capacitance is assumed to be concentrated at the middle point of the line.
- Half of the line resistance and reactance are lumped on the either side as shown in fig. 1.4(a). In this arrangement, full charging current flows over half the line.

Taking V_R as the reference phasor, the phasor diagram is shown in the fig. 1.4(b)

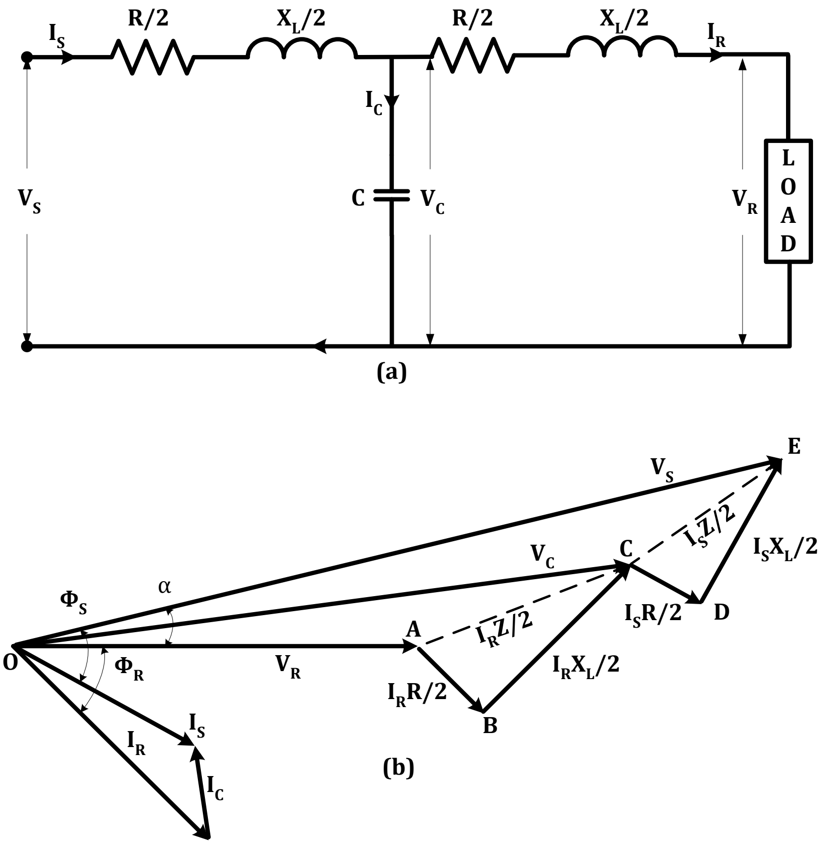


Figure 1.4 Nominal T method (a) Circuit and (b) Vector Diagram

$I_R =$ load current per phase
$R =$ resistance per phase
$X_L =$ inductive reactance per phase
$C =$ capacitance per phase
$X_C =$ capactive reactance per phase
$I_C =$ capacitive current
$V_R =$ receiving end voltage
$\cos\phi_R =$ receiving end power factor (lagging)
$V_C =$ Voltage across capacitor C
$V_S =$ sending end voltage
$\cos\phi_S =$ sending end power factor

Taking receiving end voltage V_R as reference

$$V_R = V_R\angle 0$$
$$= V_R + j0$$
$$I_R = I_R\angle -\phi_R$$
$$= I_R(\cos\phi_R - j\sin\phi_R)$$
$$\frac{Z}{2} = \frac{R}{2} + j\frac{X_L}{2}$$
$$\text{Voltage drop in half section} = I_R\frac{Z}{2}$$
$$V_C = V_R + I_R\frac{Z}{2}$$
$$= V_R + I_R(\cos\phi_R - j\sin\phi_R)\left(\frac{R}{2} + j\frac{X_L}{2}\right)$$

$$I_C = \frac{V_C}{X_C}\angle 90$$
$$= V_C\omega C\angle 90$$
$$= jV_C\omega C$$
$$= jV_C 2\pi fC$$
$$I_S = I_R + I_C$$
$$\text{Voltage drop other half section} = I_S\frac{Z}{2}$$
$$V_S = V_C + I_S\frac{Z}{2}$$
$$= V_C + I_S\left(\frac{R}{2} + j\frac{X_L}{2}\right)$$

$$\%\text{Voltage Regulation} = \frac{V_S - V_R}{V_R} x100$$

$$\eta = \frac{\text{Power delivered / phase}}{\text{Power delivered / phase + losses / phase}} x100$$

$$= \frac{V_R I_R \cos\phi_R}{V_R I_R \cos\phi_R + I_S^2 \frac{R}{2} + I_R^2 \frac{R}{2}} x100$$

$$\text{Losses / phase} = I_S^2 \frac{R}{2} + I_R^2 \frac{R}{2}$$

$$\text{Total losses} = 3\left(I_S^2 \frac{R}{2} + I_R^2 \frac{R}{2} \right)$$

1.5.3 Nominal π method

- In this method, the capacitance of each conductor (i.e. line to neutral) is divided into two halves; one half being lumped at the sending end and the other half at the receiving end as shown in the fig. 1.5(a).
- It is obvious that capacitance at the sending end has no effect on the line drop. However, its charging current must be added to the line current to obtain the total sending end current.

Taking V_R as the reference phasor, the phasor diagram is shown in the fig. 1.5(b).

$I_R =$ load current per phase
$R =$ resistance per phase
$X_L =$ inductive reactance per phase
$C =$ capacitance per phase
$X_C =$ capactive reactance per phase

I_{C1} and $I_{C2} =$ capacitive current
$I_L =$ line current
$V_R =$ receiving end voltage
$\cos\phi_R =$ receiving end power factor (lagging)
$V_S =$ sending end voltage
$\cos\phi_S =$ sending end power factor

Taking receiving end voltage V_R as reference

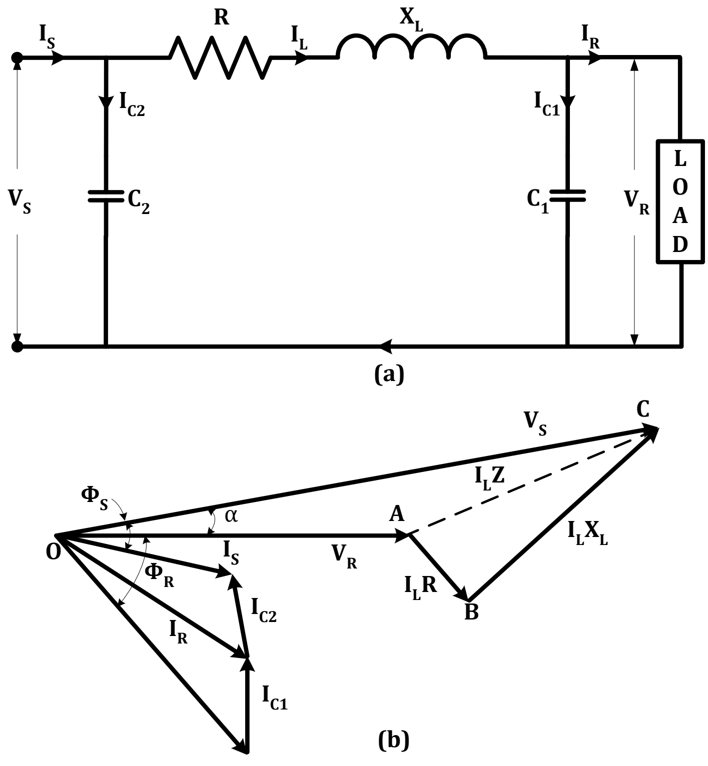


*Figure 1. 5 Nominal **π** Method (a) Circuit Diagram and (b) Vector Diagram*

$$V_R = V_R\angle 0$$
$$= V_R + j0$$
$$I_R = I_R\angle -\phi_R$$
$$= I_R(\cos\phi_R - j\sin\phi_R)$$
$$I_{C1} = V_R\omega\frac{C}{2}\angle 90$$
$$= jV_R\omega\frac{C}{2}$$
$$= jV_R\pi fC$$
$$I_L = I_R + I_{C1}$$
$$Z = R + jX_L$$
$$\text{Voltage drop in line} = I_L Z$$
$$V_S = V_R + I_L Z$$
$$= V_R + I_L\left(R + jX_L\right)$$
$$I_{C2} = V_S\omega\frac{C}{2}\angle 90$$
$$= jV_S\omega\frac{C}{2}$$
$$= jV_S\pi fC$$
$$I_S = I_L + I_{C2}$$

$$\%\text{Voltage Regulation} = \frac{V_S - V_R}{V_R} x100$$

$$\eta = \frac{\text{Power delivered / phase}}{\text{Power delivered / phase + losses / phase}} x100$$

$$= \frac{V_R I_R \cos\phi_R}{V_R I_R \cos\phi_R + I_L^2 R} x100$$

$$\text{Losses / phase} = I_L^2 R$$

$$\text{Total losses} = 3 I_L^2 R$$

1.6 Long transmission lines

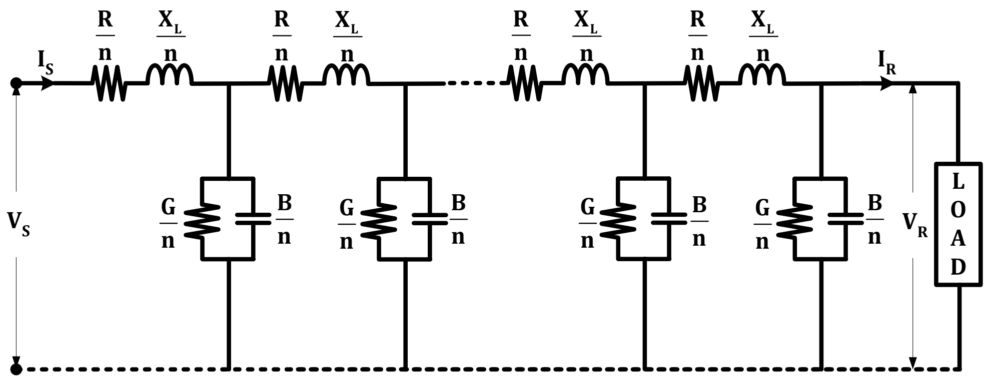


Figure 1.6 Equivalent Circuit of Long Transmission Line

- Fig. 1.6 shows the equivalent circuit of a 3-phase long transmission line on a phase-neutral basis. The whole line is divided into n sections, each section having line constants $\frac{1}{n}th$ of those for the whole line.
- The line constants are uniformly distributed over the entire length of line.
- The resistance and inductive reactance are series elements.
- The leakage susceptance (B) and leakage conductance (G) are shunt elements. The leakage susceptance is due to capacitance exist between line and neutral. The leakage conductance considers the energy losses occurring through leakage over the insulators or due to corona effect between conductors. Admittance = $\sqrt{G^2 + B^2}$
- The leakage current through shunt admittance is maximum at the sending end of the line and decreases continuously as the receiving end of the circuit is approached at which point its value is zero.

1.6.1 Performance of long transmission line

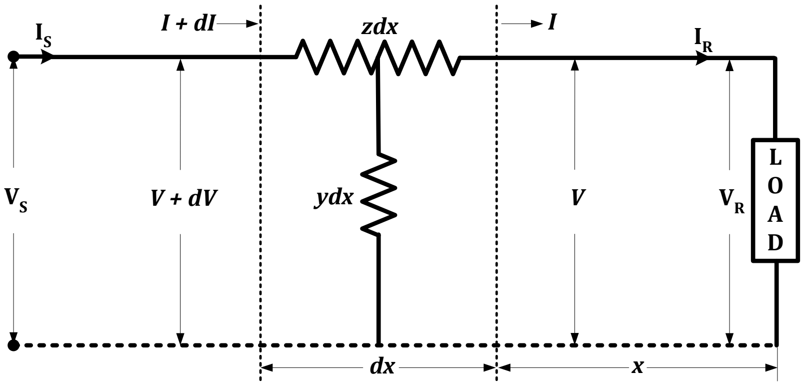


Figure 1.7 Small Element of a Long Transmission Line

- Consider a small element in the line of length dx situated at a distance x from the receiving end

$$z = \text{series impedance of the line per unit length}$$
$$y = \text{shunt admittance of the line per unit length}$$
$$V = \text{ voltage at end of the element towards receiving end}$$
$$V + dV = \text{voltage at the end of element towards sending end}$$
$$I + dI = \text{ current entering the element } dx$$
$$I = \text{current leaving the element } dx$$

- For the small element dx

$$zdx = \text{series impedance}$$
$$ydx = \text{shunt admittance}$$
$$dv = Izdx$$
$$\frac{dv}{dx} = Iz$$

- Similarly, current entering the element is $I + dI$ and current leaving is I. Therefore, current through the shunt element is dI

$$\therefore dI = Vydx$$
$$\therefore \frac{dI}{dx} = Vy$$

- Now differentiating above equation

$$\frac{d^2V}{dx} = \frac{dI}{dx}z$$
$$\frac{d^2V}{dx} = Vyz$$
$$\frac{d^2V}{dx} - yzV = 0$$

$$\therefore D^2-\left(\sqrt{yz}\right)^2=0 \quad \left(\begin{array}{l}\because y=\text{CF} \\ =C_1e^{m_1x}+C_2e^{m_2x} \text{ for ODE}\end{array}\right)$$

$$D=-\sqrt{yz} \text{ and } D=\sqrt{yz}$$

$$V=C_1e^{\sqrt{yz}\,x}+C_2e^{-\sqrt{yz}\,x}$$

$$\text{Now } \frac{dV}{dx}=C_1\sqrt{yz}e^{\sqrt{yz}\,x}-C_2\sqrt{yz}e^{-\sqrt{yz}\,x}=Iz$$

$$I=C_1\sqrt{\frac{y}{z}}e^{\sqrt{yz}\,x}-C_2\sqrt{\frac{y}{z}}e^{-\sqrt{yz}\,x}$$

At $x=0$, $V=V_R$ and $I=I_R$,

$$V_R=C_1+C_2$$

$$I_R=\sqrt{\frac{y}{z}}\left(C_1-C_2\right)$$

$$=\frac{1}{Z_C}\left(C_1-C_2\right)\left(\because \sqrt{\frac{z}{y}}=Z_C\right)$$

$$Z_CI_R=C_1-C_2$$

$$C_1=\frac{V_R+Z_CI_R}{2} \text{ and } C_2=\frac{V_R-Z_CI_R}{2}$$

$$V=\left(\frac{V_R+Z_CI_R}{2}\right)e^{\sqrt{yz}\,x}+\left(\frac{V_R-Z_CI_R}{2}\right)e^{-\sqrt{yz}\,x}$$

$$I=\left(\frac{\frac{V_R}{Z_C}+I_R}{2}\right)e^{\sqrt{yz}\,x}-\left(\frac{\frac{V_R}{Z_C}-I_R}{2}\right)e^{-\sqrt{yz}\,x}$$

$$V=V_R\left(\frac{e^{\sqrt{yz}\,x}+e^{-\sqrt{yz}\,x}}{2}\right)+Z_CI_R\left(\frac{e^{\sqrt{yz}\,x}-e^{-\sqrt{yz}\,x}}{2}\right)$$

$$I=\frac{V_R}{Z_C}\left(\frac{e^{\sqrt{yz}\,x}+e^{-\sqrt{yz}\,x}}{2}\right)+I_R\left(\frac{e^{\sqrt{yz}\,x}-e^{-\sqrt{yz}\,x}}{2}\right)$$

$$V=V_R\left(\frac{e^{\sqrt{yz}\,x}+e^{-\sqrt{yz}\,x}}{2}\right)+Z_CI_R\left(\frac{e^{\sqrt{yz}\,x}-e^{-\sqrt{yz}\,x}}{2}\right)$$

$$I=\frac{V_R}{Z_C}\left(\frac{e^{\sqrt{yz}\,x}+e^{-\sqrt{yz}\,x}}{2}\right)+I_R\left(\frac{e^{\sqrt{yz}\,x}-e^{-\sqrt{yz}\,x}}{2}\right)$$

$$\text{Now, } \sinh x = \frac{e^x - e^{-x}}{2} \text{ and } \cosh x = \frac{e^x + e^{-x}}{2}$$

$$V = V_R \cosh\sqrt{yz}x + Z_C I_R \sinh\sqrt{yz}x$$

$$I = \frac{V_R}{Z_C}\sinh\sqrt{yz}x + I_R \cosh\sqrt{yz}x$$

Now, if x=l

$$V_S = V_R \cosh\sqrt{YZ} + Z_C I_R \sinh\sqrt{YZ}$$

$$I_S = \frac{V_R}{Z_C}\sinh\sqrt{YZ} + I_R \cosh\sqrt{YZ}$$

$$\text{And } \cosh\sqrt{YZ} = \left(1 + \frac{ZY}{2} + \frac{Z^2Y^2}{24} + ...\right)$$

$$\sinh\sqrt{YZ} = \left(\sqrt{ZY} + \frac{(ZY)^{3/2}}{6} + ...\right)$$

$$\eta = \frac{\text{Power delivered / phase}}{\text{Power input / phase}} x100$$

$$= \frac{V_R I_R \cos\phi_R}{V_S I_S \cos\phi_S} x100$$

$$\text{Losses / phase} = V_S I_S \cos\phi_S - V_R I_R \cos\phi_R$$

$$\text{Total losses} = 3\text{xLosses / phase}$$

1.7 Generalized circuit constants of a transmission lines

- In any four terminal network, the input voltage and input current can be expressed in terms of output voltage and current.
- Therefore, the input voltage V_S and input current I_S of a 3-phase transmission line can be expressed as:

$$V_S = AV_R + BI_R$$
$$I_S = CV_R + DI_R$$

Where,

$V_S =$ sending end voltage per phase
$I_S =$ sending end current
$V_R =$ receiving end voltage per phase
$I_R =$ receiving end current

- A, B, C and D are the generalized circuit constants of the transmission line and are complex numbers.
- The constants A and D are dimensionless whereas the dimensions of B and C are ohms and siemens respectively. And for a given transmission line A=D and $AD - BC = 1$.

1.7.1 Short lines

- In short transmission lines, the effect of line capacitance is neglected. Fig. 1.8 shows the circuit of short transmission line (1 Phase).

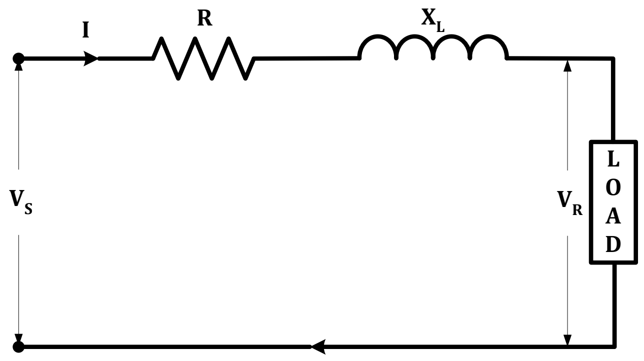


Figure 1.8 Circuit of Short Transmission Lines

Therefore,

$$I_S = I_R$$
$$V_S = V_R + I_R Z$$

Comparing these equation with basic equation we get

$$A = 1,\ B = Z,\ C = 0 \text{ and } D = 1$$

1.7.2 Medium lines – Nominal *T* method

- In this method, the whole line to neutral capacitance is assumed to be concentrated at the middle point of the line and half of the resistance and reactance are lumped on either side as shown in fig. 1.9.

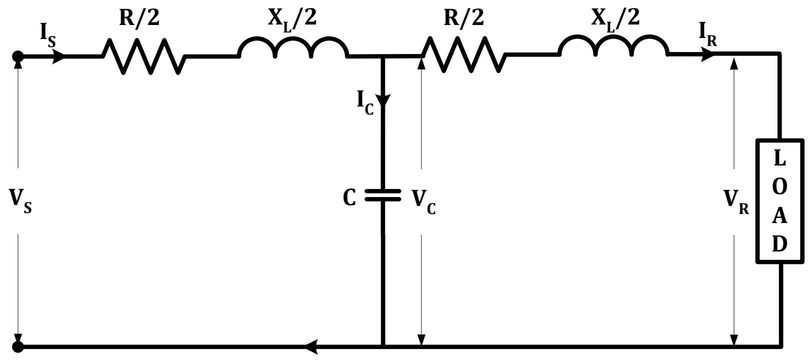


Figure 1.9 Circuit of Nominal T Method

$$V_C = V_R + I_R \frac{Z}{2}$$
$$V_S = V_C + I_S \frac{Z}{2}$$

$$I_C = \frac{V_C}{X_C}\angle 90$$
$$= V_C \omega C \angle 90$$
$$= jV_C \omega C$$
$$= V_C Y$$
$$I_S = I_R + I_C$$

$$V_S = V_C + (I_R + I_C)\frac{Z}{2}$$
$$= V_C + (I_R + V_C Y)\frac{Z}{2}$$
$$= V_C\left(1 + \frac{YZ}{2}\right) + I_R\frac{Z}{2}$$
$$= \left(V_R + I_R\frac{Z}{2}\right)\left(1 + \frac{YZ}{2}\right) + I_R\frac{Z}{2}$$
$$= \left(1 + \frac{YZ}{2}\right)V_R + \left(\frac{Z}{2} + \frac{YZ^2}{4} + \frac{Z}{2}\right)I_R$$
$$V_S = \left(1 + \frac{YZ}{2}\right)V_R + \left(Z + \frac{YZ^2}{4}\right)I_R$$

$$I_S = I_R + V_C Y$$
$$= I_R + \left(V_R + I_R\frac{Z}{2}\right)Y$$
$$I_S = V_R Y + I_R\left(1 + \frac{YZ}{2}\right)$$

$$A = \left(1 + \frac{YZ}{2}\right),\ B = \left(Z + \frac{YZ^2}{4}\right),\ C = Y \text{ and } D = \left(1 + \frac{YZ}{2}\right)$$

1.7.3 Medium lines – Nominal π method

- In this method, the line to neutral capacitance is divided into two halves; one half being concentrated at load end and other half at the sending end as shown in fig. 1.10.

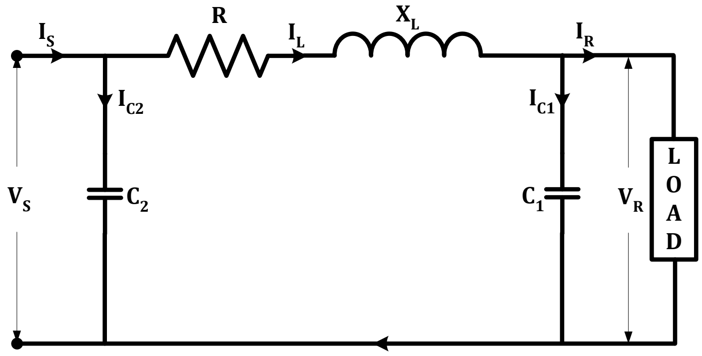


Figure 1.10 Circuit of Nominal π Method

$$I_L = I_R + I_{C1}$$

$$\begin{aligned}I_{C1} &= V_R \omega \frac{C}{2} \angle 90 \\ &= jV_R \omega \frac{C}{2} \\ &= V_R \frac{Y}{2}\end{aligned}$$

$$\begin{aligned}V_S &= V_R + I_L Z \\ &= V_R + (I_R + I_{C1}) Z \\ &= V_R + \left(I_R + V_R \frac{Y}{2}\right) Z\end{aligned}$$

$$V_S = \left(1 + \frac{YZ}{2}\right) V_R + Z I_R$$

$$\begin{aligned}I_{C2} &= V_S \omega \frac{C}{2} \angle 90 \\ &= jV_S \omega \frac{C}{2} \\ &= V_S \frac{Y}{2}\end{aligned}$$

$$\begin{aligned}I_S &= I_L + I_{C2} \\ &= I_R + V_R \frac{Y}{2} + \left[\left(1 + \frac{YZ}{2}\right) V_R + Z I_R\right] \frac{Y}{2}\end{aligned}$$

$$I_S = \left(Y + \frac{Y^2 Z}{2}\right) V_R + \left(1 + \frac{YZ}{2}\right) I_R$$

$$A = \left(1 + \frac{YZ}{2}\right),\ B = Z,\ C = \left(Y + \frac{Y^2 Z}{2}\right) \text{ and } D = \left(1 + \frac{YZ}{2}\right)$$

1.7.4 Long lines

- By rigorous method, the sending end voltage and current of a long transmission line are given by

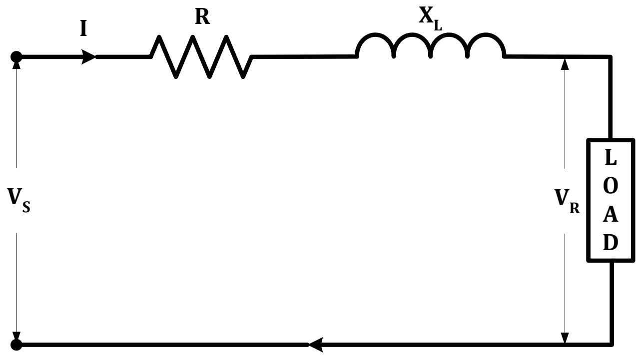
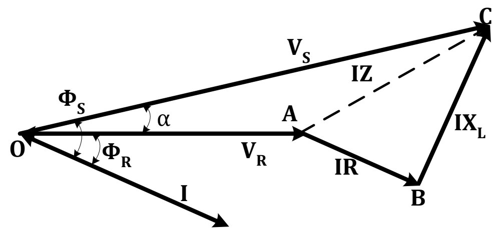
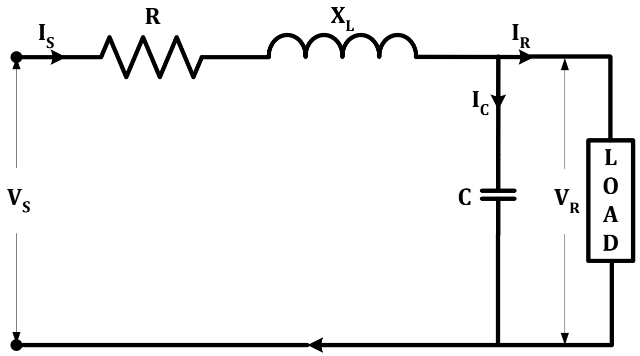
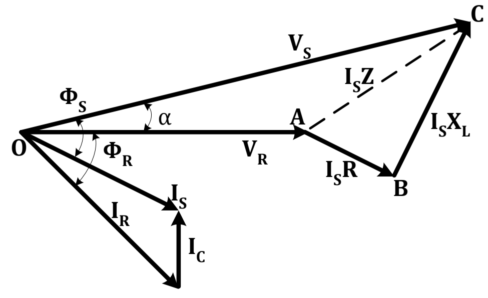
$$V_S = V_R \cosh\sqrt{YZ} + Z_C I_R \sinh\sqrt{YZ}$$
$$I_S = \frac{V_R}{Z_C}\sinh\sqrt{YZ} + I_R \cosh\sqrt{YZ}$$

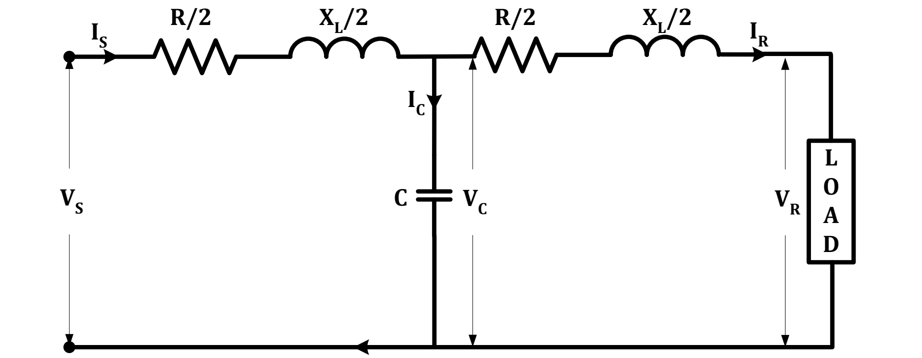
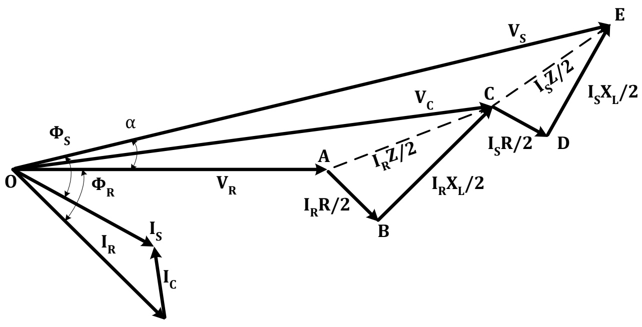
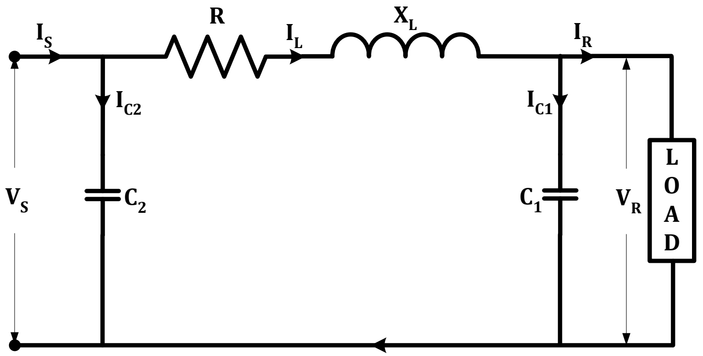
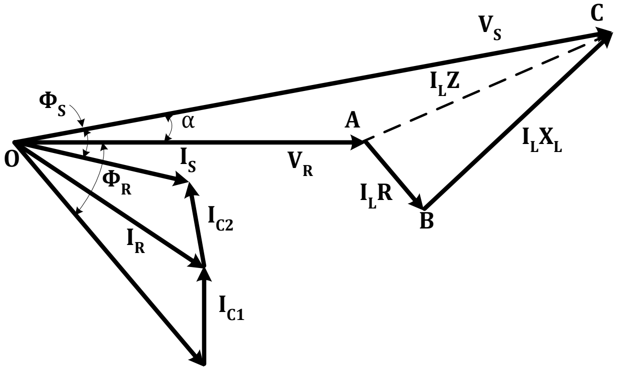
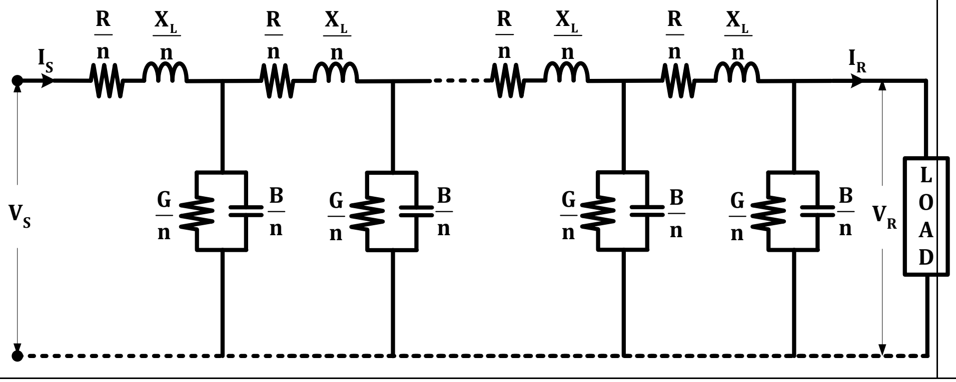
$$A = \cosh\sqrt{YZ},\ B = Z_C \sinh\sqrt{YZ},\ C = \frac{1}{Z_C}\sinh\sqrt{YZ} \text{ and } D = \cosh\sqrt{YZ}$$

1.8 Comparison between different methods for performance of transmission lines

(1. Short Lines, 2. End Condenser Method, 3. Nominal T Method, 4. Nominal π Method and 5. Rigorous Method)

Table 1.1 Comparison Between Different Methods

Sr. No.	Circuit Diagram	Vector Diagram	ABCD Parameters
1			$A=1$ $B=Z$ $C=0$ $D=1$
2			$A=1+YZ$ $B=Z$ $C=Y$ $D=1$

3			$A=\left(1+\frac{YZ}{2}\right)$ $B=\left(Z+\frac{YZ^2}{4}\right)$ $C=Y$ $D=\left(1+\frac{YZ}{2}\right)$
4			$A=\left(1+\frac{YZ}{2}\right)$ $B=Z$ $C=\left(Y+\frac{Y^2Z}{2}\right)$ $D=\left(1+\frac{YZ}{2}\right)$
5			$A=\cosh\sqrt{YZ}$, $B=Z_C\sinh\sqrt{YZ}$, $C=\frac{1}{Z_C}\sinh\sqrt{YZ}$, $D=\cosh\sqrt{YZ}$

1.9 Power flow through a transmission line

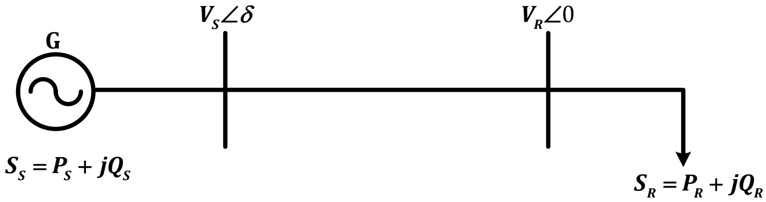


Figure 1.11 Transmission Line

$$V_R\angle 0 = \text{Receiving end voltage}$$
$$V_S\angle\delta = \text{Sending end voltage}$$
$$\delta = \text{Angle between } V_S \text{ and } V_R$$

Line Constants

$$A = A\angle\alpha$$
$$B = B\angle\beta$$
$$C = C\angle\gamma$$
$$D = D\angle\Delta$$

$$S_R = P_R + jQ_R$$
$$= V_R I_R^*$$

$$V_S = AV_R + BI_R$$
$$I_S = CV_R + DI_R$$

$$I_R = \frac{V_S - AV_R}{B}$$
$$= \frac{V_S\angle\delta - A\angle\alpha x V_R\angle 0}{B\angle\beta}$$
$$= \frac{V_S\angle(\delta-\beta)}{B} - \frac{AV_R\angle(\alpha-\beta)}{B}$$
$$I_R^* = \frac{V_S\angle(\beta-\delta)}{B} - \frac{AV_R\angle(\beta-\alpha)}{B}$$

$$S_R = V_R I_R^*$$
$$= \frac{V_S V_R\angle(\beta-\delta)}{B} - \frac{AV_R^2\angle(\beta-\alpha)}{B}$$
$$= P_R + jQ_R$$

$$\therefore P_R = \frac{V_S V_R}{B}\cos(\beta-\delta) - \frac{AV_R^2}{B}\cos(\beta-\alpha)$$
$$\therefore Q_R = \frac{V_S V_R}{B}\sin(\beta-\delta) - \frac{AV_R^2}{B}\sin(\beta-\alpha)$$

$$I_R = \frac{I_S - CV_R}{D}$$
$$V_S = AV_R + \frac{B}{D}(I_S - CV_R)$$
$$DV_S = ADV_R + BI_S - BCV_R$$
$$I_S = \frac{DV_S - V_R}{B} \quad (\because AD - BC = 1)$$
$$= \frac{D\angle\Delta V_S\angle\delta - V_R\angle 0}{B\angle\beta}$$
$$= \frac{DV_S\angle(\Delta+\delta-\beta)}{B} - \frac{V_R\angle(-\beta)}{B}$$

$$S_S = V_S I_S^*$$
$$I_S^* = \frac{DV_S\angle(\beta-\Delta-\delta)}{B} - \frac{V_R\angle\beta}{B}$$

$$S_S = P_S + jQ_S$$
$$= \frac{DV_s^2\angle(\beta-\Delta)}{B} - \frac{V_S V_R\angle(\delta+\beta)}{B}$$
$$\therefore P_S = \frac{DV_s^2}{B}\cos(\beta-\Delta) - \frac{V_S V_R}{B}\cos(\delta+\beta)$$
$$\therefore Q_S = \frac{DV_s^2}{B}\sin(\beta-\Delta) - \frac{V_S V_R}{B}\sin(\delta+\beta)$$

Equation of circle is

$$(x-h)^2 + (y-g)^2 = r^2$$

For receiving end circle diagram

$$P_R + \frac{AV_R^2}{B}\cos(\beta-\alpha) = \frac{V_S V_R}{B}\cos(\beta-\delta)$$
$$Q_R + \frac{AV_R^2}{B}\sin(\beta-\alpha) = \frac{V_S V_R}{B}\sin(\beta-\delta)$$

Squaring and adding equation we have

$$\left[P_R + \frac{AV_R^2}{B}\cos(\beta-\alpha)\right]^2 + \left[Q_R + \frac{AV_R^2}{B}\sin(\beta-\alpha)\right]^2 = \frac{V_S^2 V_R^2}{B^2}\left[\cos^2(\beta-\delta)+\sin^2(\beta-\delta)\right]$$

$$\text{x-coordinate of the center} = -\frac{AV_R^2}{B}\cos(\beta-\alpha)$$

$$\text{y-coordinate of the center} = -\frac{AV_R^2}{B}\sin(\beta-\alpha)$$

$$\text{Radius} = \frac{V_S V_R}{B}$$

For sending end circle diagram

$$P_S - \frac{DV_s^2}{B}\cos(\beta-\Delta) = -\frac{V_S V_R}{B}\cos(\delta+\beta)$$

$$Q_S - \frac{DV_s^2}{B}\sin(\beta-\Delta) = -\frac{V_S V_R}{B}\sin(\delta+\beta)$$

Squaring and adding equation we have

$$\left[P_S - \frac{DV_s^2}{B}\cos(\beta-\Delta)\right]^2 + \left[Q_S - \frac{DV_s^2}{B}\sin(\beta-\Delta)\right]^2 = \frac{V_S^2 V_R^2}{B^2}\left[\cos^2(\delta+\beta)+\sin^2(\delta+\beta)\right]$$

$$\text{x-coordinate of the center} = \frac{DV_s^2}{B}\cos(\beta-\Delta)$$

$$\text{y-coordinate of the center} = \frac{DV_s^2}{B}\sin(\beta-\Delta)$$

$$\text{Radius} = \frac{V_S V_R}{B}$$