

CHAPTER 1

A.C. Fundamentals

1.1 WHAT IS ALTERNATING CURRENT (A.C.)?

Alternating current is the current which constantly changes in amplitude, and which reverses direction at regular intervals. We know that direct current flows only in one direction, and that the amplitude of current is determined by the number of electrons flowing past a point in a circuit in one second. If, for example, a coulomb of electrons moves past a point in a wire in one second and all of the electrons are moving in the same direction, the amplitude of direct current in the wire is one ampere. Similarly, if half a coulomb of electrons moves in one direction past a point in the wire in half a second, then reverses direction and moves past the same point in the opposite direction during the next half-second, a total of one coulomb of electrons passes the point in one second. The amplitude of the alternating current is one ampere.

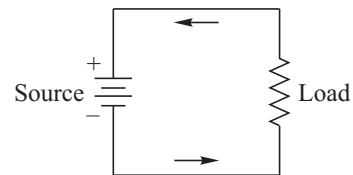


Fig. 1.1

1.2 ALTERNATING CURRENT VS. DIRECT CURRENT

The figure shown above shows the schematic diagram of a very basic D.C. circuit. It consists of nothing more than a source (a producer of electrical energy) and a load (whatever is to be powered by that electrical energy). The source can be any electrical source: a chemical battery, an electronic power supply, a mechanical generator, or any other possible continuous source of electrical energy. For simplicity, we represent the source in this figure as a battery.

At the same time, the load can be any electrical load: a light bulb, electronic clock or watch, electronic instrument, or anything else that must be driven by a continuous source of electricity. The figure here represents the load as a simple resistor.

Regardless of the specific source and load in this circuit, electrons leave the negative terminal of the source, travel through the circuit in the direction shown by the arrows, and eventually return to the positive terminal of the source. This action continues for as long as a complete electrical circuit exists.

Now consider the same circuit with a single change, as shown in the Fig. 1.2. This time, the energy source is constantly changing.

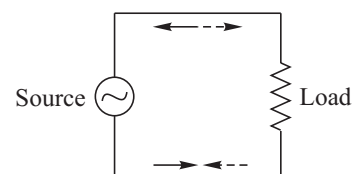


Fig. 1.2

It begins by building up a voltage which is positive on top and negative on the bottom, and therefore pushes electrons through the circuit in the direction shown by the solid arrows. However, then the source voltage starts to fall off, and eventually reverse polarity. Now current will still flow through the circuit, but this time in the direction shown by the dotted arrows. This cycle repeats itself endlessly, and as a result the current through the circuit reverses direction repeatedly. This is known as an alternating current.

1.3 DISADVANTAGES OF D.C. COMPARED TO A.C.

Disadvantage of the direct-current system becomes evident when the direct current (I) from the generating station must be transmitted a long distance over wires to the consumer. When this happens, a large amount of power is lost due to the resistance (R) of the wire. The power loss is equal to I^2R . However, this loss can be greatly reduced if the power is transmitted over the lines at a very high voltage level and a low current level. This is not a practical solution to the power loss in the d.c. system since the load would then have to be operated at a dangerously high voltage. Because of the disadvantages related to transmitting and using direct current, practically all modern commercial electric power companies generate and distribute alternating current. (a.c.).

Unlike direct voltages, alternating voltages can be stepped up or down in amplitude by a device called a transformer. Use of the transformer permits efficient transmission of electrical power over long-distance lines. At the electrical power station, the transformer output power is at high voltage and low current levels. At the consumer end of the transmission lines, the voltage is stepped down by a transformer to the value required by the load. Due to its inherent advantages and versatility, alternating current has replaced direct current in all but a few commercial power distribution systems.

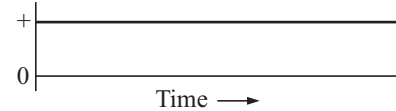


Fig. 1.3

1.4 PROPERTIES OF ALTERNATING CURRENT

A D.C. power source, such as a battery, outputs a constant voltage over time, as depicted in Fig. 1.4. Of course, once the chemicals in the battery have completed their reaction, the battery will be exhausted and cannot develop any output voltage. But until that happens, the output voltage to the right will remain essentially constant. The same is true for any other source of D.C. electricity: the output voltage remains constant over time.

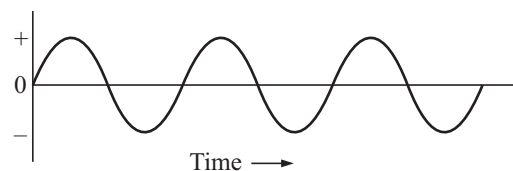


Fig. 1.4

By contrast, an A.C. source of electrical power changes constantly in amplitude and regularly here. Because the changes are so regular, alternating voltage and current have a number of properties associated with any such waveform. These basic properties include the following list:

Frequency: One of the most important properties of any regular waveform identifies the number of complete cycles it goes through in a fixed period of time. For standard measurements, the period of time is one second, so the frequency of the wave is commonly measured in cycles per second (cycles/sec) and, in normal usage, is expressed in units of Hertz (Hz). It is represented in mathematical equations by the letter ' f '.

Period: Sometimes we need to know the amount of time required to complete one cycle of the waveform, rather than the number of cycles per second of time. This is logically the reciprocal of frequency. Thus, period is the time duration of one cycle of the waveform, and is measured in seconds/cycle.

Wavelength: Because an A.C. wave moves physically as well as changing in time, sometimes we need to know how far it moves in one cycle of the wave, rather than how long that cycle takes to complete. This of course depends on how fast the wave is moving as well. The Greek letter (lambda) is used to represent wavelength in mathematical expressions. And, $\lambda = c/f$. As shown in the figure to

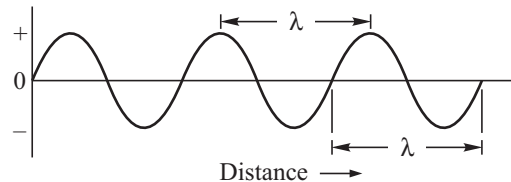


Fig. 1.5

the above, wavelength can be measured from any part of one cycle to the equivalent point in the next cycle. Wavelength is very similar to period as discussed above, except that wavelength is measured in distance per cycle while period is measured in time per cycle.

Amplitude: Mathematically, the amplitude of a sine wave is the value of that sine wave at its peak. This is the maximum value, positive or negative, that it can attain. However, when we speak of an A.C. power system, it is more useful to refer to the effective voltage or current.

1.5 THE SINE WAVE

In discussing alternating current and voltage, you will often find it necessary to express the current and voltage in terms of maximum or peak values, peak-to-peak values, effective values, average values, or instantaneous values. Each of these values has a different meaning and is used to describe a different amount of current or voltage.

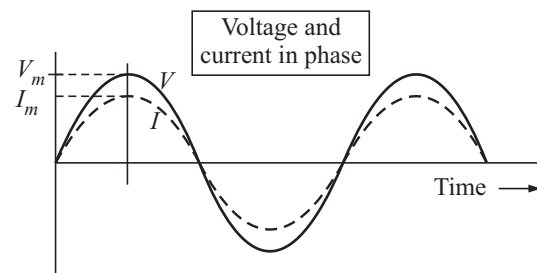


Fig. 1.6

Peak Value [I_p]

Refer to figure, it is the maximum value of voltage [V_p] or Current [I_p]. The peak value applies to both positive and negative values of the cycle.

Peak-Peak value [I_{p-p}]

During each complete cycle of ac there are always two maximum or peak values, one for the positive half-cycle and the other for the negative half-cycle. The difference between the peak positive value and the peak negative value is called the peak-to-peak value of the sine wave. This value is twice the maximum or peak value of the sine wave and is sometimes used for measurement of ac voltages.

Note the difference between peak and peak-to-peak values in figure below. Usually alternating voltage and current are expressed in effective values rather than in peak-to-peak values.

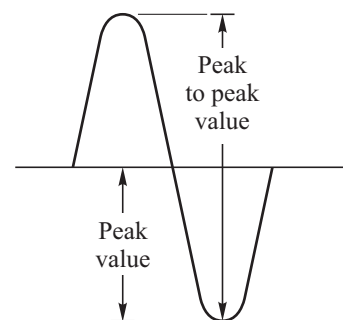


Fig. 1.7

Instantaneous Value

The instantaneous value of an alternating voltage or current is the value of voltage or current at one particular instant. The value may be zero if the particular instant is the time in the cycle at which the polarity of the voltage is changing. It may also be the same as the peak value, if the selected instant is the time in the cycle at which the voltage or current stops increasing and starts decreasing. There are actually an infinite number of instantaneous values between zero and the peak value.

1.6 AVERAGE VALUE

The average value of an alternating current or voltage is the average of all the instantaneous values during one alternation. Since the voltage increases from zero to peak value and decreases back to zero during one alternation, the average value must be some value between those two limits.

Derivation of Average Value of Current [I_{AV}]

Average value derivation: The average value of A.C. is the average over one complete cycle and is clearly zero, because there are alternately equal positive and negative half cycles.

Alternating current is represented as $I = I_0 \sin \omega t$

$$\begin{aligned} I_{\text{mean}} &= \frac{\int_0^{T/2} I_0 \sin \omega t \, dt}{\int_0^{T/2} dt} \\ &= \frac{I_0}{T/2} \cdot \frac{1}{\omega} [-\cos \omega t]_0^{T/2} \\ &= \frac{2I}{T} \cdot \frac{T}{2\pi} \left[\cos 0^\circ - \cos \frac{\omega T}{2} \right] \\ &= \frac{I_0}{\pi} \left[\cos 0^\circ - \cos \frac{\omega}{2} \cdot \frac{2\pi}{\omega} \right] \\ &= \frac{2I_0}{\pi} = \frac{2}{\pi} \times \text{Peak value of current} \end{aligned}$$

$$\text{Similarly, } E_{\text{mean}} = \frac{2E_0}{\pi} = \frac{2}{\pi} \times \text{Peak value of voltage}$$

Moving-coil meters respond to the average value of the current through them.

Root Mean Square Value

Circuit currents and voltage in A.C. circuits are generally stated as root-mean-square or rms values rather than by quoting the maximum values. The root-mean-square for a current is defined by

$$I_{\text{rms}} = \sqrt{(I^2)_{\text{avg}}}$$

That is, you take the square of the current and average it, then take the square root. When this process is carried out for a sinusoidal current

$$\left[I_m^2 \sin^2 \omega t \right]_{\text{avg}} = \frac{I_m^2}{2} \quad \text{so} \quad I_{\text{rms}} = \sqrt{(I^2)_{\text{avg}}} = \frac{I_m}{\sqrt{2}}$$

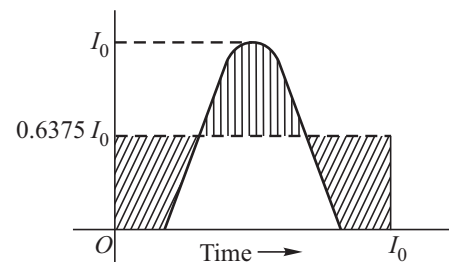


Fig. 1.8

Since the A.C. voltage is also sinusoidal, the form of the rms voltage is the same. These rms values are just the effective value needed in the expression for average power to put the A.C. power in the same form as the expression for D.C. power in a resistor. In a resistor where the power factor is equal to 1.

$$P_{\text{avg}} = \frac{V_m I_m}{2} \cos \phi = V_{\text{rms}} I_{\text{rms}} \cos \phi = V_{\text{rms}} I_{\text{rms}}, \text{ for a resistor } R$$

Form Factor = $\frac{V_{\text{rms}}}{V_{\text{ave}}}$ or $\frac{I_{\text{rms}}}{I_{\text{ave}}} = 1.11$ (approx.)

Derivation of RMS Value

RMS value of current is defined as $I_{\text{rms}} = \sqrt{I^{-2}}$. The average value of I^2 over one complete cycle is given by

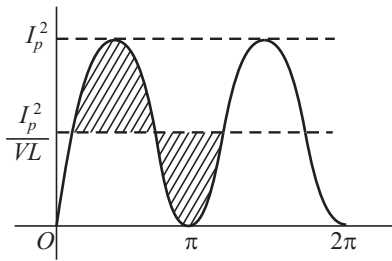


Fig. 1.9

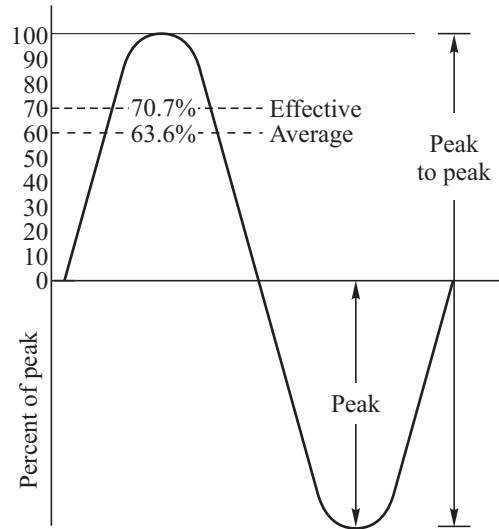


Fig. 1.10

$$I^{-2} = \frac{\int_0^T I^2 dt}{\int_0^T dt} = \frac{1}{T} \int_0^T I_0^2 \sin^2 \omega t dt$$

$$I^{-2} = \frac{I_0^2}{T} \int_0^T \left(\frac{1 - \cos 2\omega t}{2} \right) dt$$

$$= \frac{I_0^2}{2T} \left[t - \frac{\sin 2\omega t}{2\omega} \right]_0^T = \frac{I_0^2}{2T} [T - 0]$$

$$I^{-2} = \frac{I_0^2}{2}$$

Thus the root mean square value of an alternating current is $I_{\text{rms}} = \sqrt{I^{-2}} = \sqrt{\frac{I_0^2}{2}} = \frac{I_0}{\sqrt{2}}$

Similarly the RMS value of an a.c. voltage is

$$V_{\text{rms}} = \frac{V_0}{\sqrt{2}} \text{ or } \frac{E_0}{\sqrt{2}}.$$

The rms value is the effective value required in power calculations. The rms value of a sine-wave current produces the same heating effect in a resistor as an identical d.c. current.

1.7 SINE WAVES IN PHASE

When a sine wave of voltage is applied to a resistance, the resulting current is also a sine wave. This follows Ohm's law which states that current is directly proportional to the applied voltage. To be in phase, the two sine waves must go through their maximum and minimum points at the same time and in the same direction.

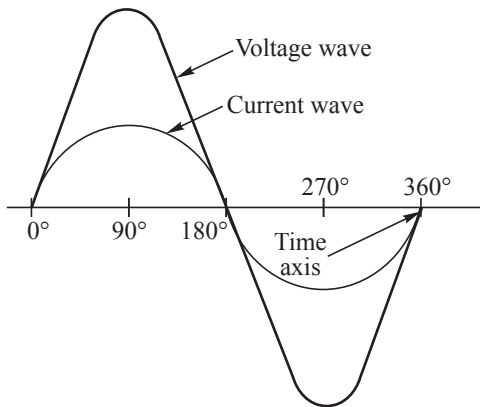


Fig. 1.11

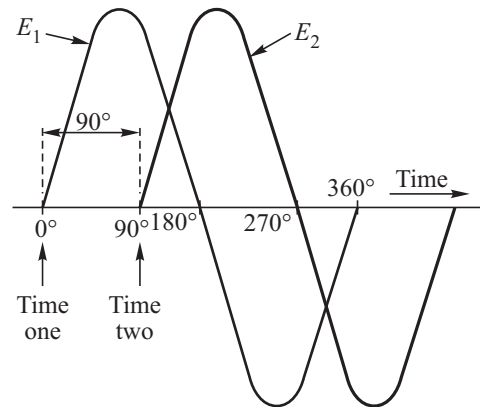


Fig. 1.12

Sine Waves Out of Phase

Figure below shows voltage wave E_1 which is considered to start at 0° (time one). As voltage wave E_1 reaches its positive peak, voltage wave E_2 starts its rise (time two). Since these voltage waves do not go through their maximum and minimum points at the same instant of time, a phase difference exists between the two waves. The two waves are said to be out of phase. For the two waves in figure, the phase difference is 90° .

1.8 THE OPERATOR J

Many problems in A.C. get simplified by the use of operator $j = \sqrt{-1}$, which is an imaginary quantity. Multiplication of a given vector quantity \vec{A} by j or $\sqrt{-1}$ denotes the rotation of the vector through 90° in the anticlockwise direction (*i.e.* from OX to OY in Fig. 1.13). Similarly a multiplication by $-j$ represents the rotation of the vector through 90° in the clockwise direction (*i.e.*, from OX to OY'). Multiplication by $j^2 = -1$ means rotation by 90° and a further rotation by 90° *i.e.* the application of j^2 to a vector rotates it through 180° (from OX to OX'). Thus

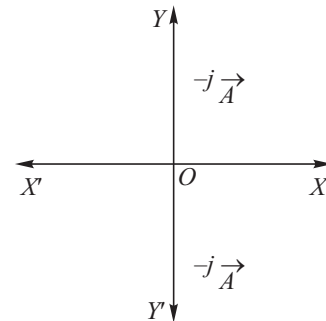


Fig. 1.13

$$j^2 A = -\bar{A}$$

Which is a vector equal in magnitude but opposite in direction to \bar{A} .

1.9 COMPLEX NUMBERS

The standard symbol for the set of all complex numbers is C .

For example, the equation $z = x + yj$ is to be understood as saying that the complex number z is the sum of the real number x and the real number y times i . In general, the x part of a complex number $z = x + yj$ is called the real part of z , while y is called the imaginary part of z . (Sometimes yj is called the imaginary part.)

When we use the xy -plane for the complex plane C , we'll call the x -axis by the name real axis, and the y -axis we'll call the imaginary axis.

Real numbers are to be considered as special cases of complex numbers; they're just the numbers $x + yj$ when y is 0, that is, they're the numbers on the real axis. For instance, the real number 2 is $2 + 0i$. The numbers on the imaginary axis are sometimes called purely imaginary numbers.

Two complex numbers are equal if and only if their real parts are equal and their imaginary parts are equal. *i.e.*, $a + bi = c + di$ if and only if $a = c$ and $b = d$.

Example 1.

$$2 - 5i$$

$$6 + 4i$$

$$0 + 2i = 2i$$

$$4 + 0i = 4.$$

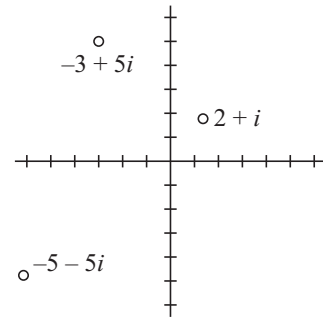


Fig. 1.14

1.10 COMPLEX ARITHMETIC

When a number system is extended the arithmetic operations must be defined for the new numbers, and the important properties of the operations should still hold. For example, addition of whole numbers is commutative. This means that we can change the order in which two whole numbers are added and the sum is the same: $3 + 5 = 8$ and $5 + 3 = 8$.

We need to define the four arithmetic operations on complex numbers.

Addition and Subtraction

To add or subtract two complex numbers, you add or subtract the real parts and the imaginary parts.

$$(a + jb) + (c + jd) = (a + c) + j(b + d)$$

$$(a + jb) - (c + jd) = (a - c) + j(b - d)$$

Example 2.

$$(3 - 5j) + (6 + 7j) = (3 + 6) + (-5 + 7)j = 9 + 2j$$

$$(3 - 5j) - (6 + 7j) = (3 - 6) + (-5 - 7)j = -3 - 12j.$$

Multiplication

The formula for multiplying two complex numbers is

$$(a + bj) * (c + dj) = (ac - bd) + (ad + bc)j.$$

You do not have to memorize this formula, because you can arrive at the same result by treating the complex numbers like expressions with a variable, multiply them as usual, then simplify. The only difference is that powers of i do simplify, while powers of x do not.

Example 3.

$$\begin{aligned}(2 + 3i)(4 + 7i) &= 2*4 + 2*7i + 4*3i + 3*7*i^2 \\ &= 8 + 14i + 12i + 21*(-1) \\ &= (8 - 21) + (14 + 12)i \\ &= -13 + 26i.\end{aligned}$$

Notice that in the second line of the example, the i^2 has been replaced by -1 .

Using the formula for multiplication, we would have gone directly to the third line.

EXERCISE

Perform the following operations:

(a) $(-3 + 4i) + (2 - 5i)$

(b) $3i - (2 - 4i)$

(c) $(2 - 7i)(3 - 4i)$

(d) $(1 + i) + (2 - 3i)$

Division

Definition: The conjugate (or complex conjugate) of the complex number $a + bi$ is $a - bi$. Conjugates are important because of the fact that a complex number times its conjugate is real; *i.e.*, its imaginary part is zero.

$$(a + bi)(a - bi) = (a^2 + b^2) + 0i = a^2 + b^2.$$

Example 4.

Number	Conjugate	Product
$2 + 3i$	$2 - 3i$	$4 + 9 = 13$
$3 - 5i$	$3 + 5i$	$9 + 25 = 34$
$4i$	$-4i$	16

Suppose we want to do the division problem $(3 + 2i) \div (2 + 5i)$. First, we want to rewrite this as a fractional expression $\frac{3 + 2i}{2 + 5i}$.

Even though we have not defined division, it must satisfy the properties of ordinary division. So, a number divided by itself will be 1, where 1 is the multiplicative identity; *i.e.*, 1 times any number is that number.

So, when we multiply $\frac{3 + 2i}{2 + 5i}$ by $\frac{3 - 5i}{3 - 5i}$, we are multiplying by 1 and the number is not changed.

Notice that the quotient on the right consists of the conjugate of the denominator over itself. This choice was made so that when we multiply the two denominators, the result is a real number. Here is the complete division problem, with the result written in standard form.

$$\begin{aligned}\frac{3+2i}{2+5i} &= \frac{3+2i}{2+5i} \times \frac{2-5i}{2-5i} \\ &= \frac{(3+2i)(2-5i)}{(2+5i)(2-5i)} \\ &= \frac{16-11i}{29} = \frac{16}{29} - \frac{11}{29}i.\end{aligned}$$

1.11 PHASORS

In an a.c. circuit, the e.m.f. or current vary sinusoidally with time and may be mathematically represented as

$$E = E_0 \sin \omega t$$

and

$$I = I_0 \sin (\omega t \pm \theta)$$

Where θ is the phase angle between alternating e.m.f. and current.

Displacement of S.H.M. also varies sinusoidally with time *i.e.*

$$Y = A \sin \omega t$$

And its instantaneous value is equal to the projection of the amplitude A on Y -axis. Therefore, instantaneous values of alternating e.m.f. (E) and current (I) may be considered as the projections of e.m.f. amplitude (E_0) and current amplitude (I_0) respectively. The quantities, such as alternating e.m.f. and alternating current are called phasor. Thus a phasor is a quantity which varies sinusoidally with time and represented as the projection of rotating vector.

1.12 PHASOR DIAGRAM

The generator at the power station which produces our A.C. mains rotates through 360 degrees to produce one cycle of the sine wave form which makes up the supply.

In the next diagram there are two sine waves.

They are out of phase because they do not start from zero at the same time.

To be in phase they must start at the same time.

The waveform A starts before B and is LEADING by 90 degrees.

Waveform B is LAGGING A by 90 degrees.

The next left hand diagram, known as a PHASOR DIAGRAM, shows this in another way.

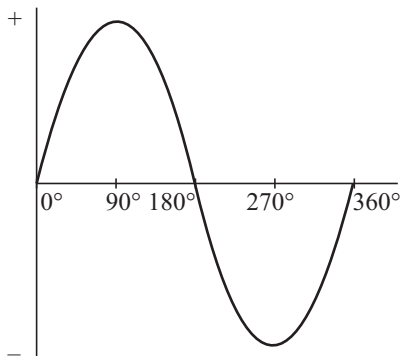


Fig. 1.15

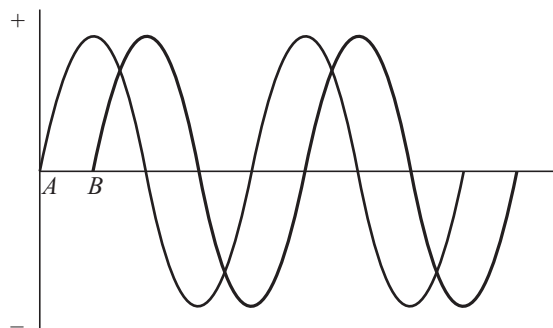


Fig. 1.16

The phasors are rotating anticlockwise as indicated by the arrowed circle.

A is leading B by 90 degrees.

The length of the phasors is determined by the amplitude of the voltages A and B.

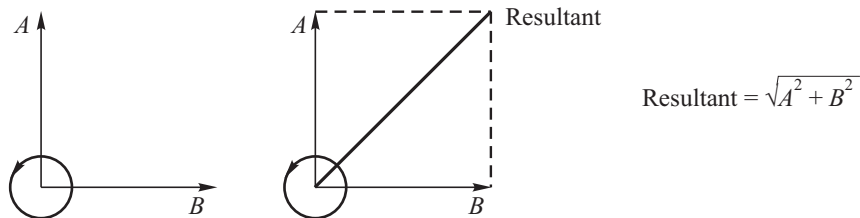


Fig. 1.17

Since the voltages are of the same value then their phasors are of the same length. If voltage A was half the voltage of B then its phasor would be half the length of B.

The voltages A and B cannot be added together directly to find the resulting voltage, because they are not in phase.

The result of the two voltages can be found by completing the phasor diagram as shown on the right.

The resulting voltage is slightly greater in amplitude than A or B and leads B by 45 degrees and lags A by 45 degrees.

Since the two voltages are 90 degrees apart, then the resultant can be found by using Pythagoras, as shown.

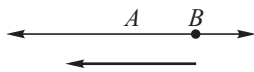


Fig. 1.18

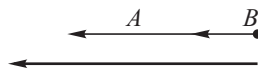


Fig. 1.19

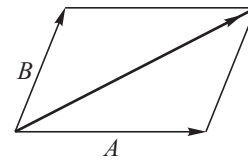


Fig. 1.20

It is sometimes helpful to treat the phase as if it defines a vector in a plane. The usual reference for zero phase is taken to be the positive x-axis and is associated with the resistor since the voltage and current associated with the resistor are in phase. The length of the phasor is proportional to the magnitude of the quantity represented, and its angle represents its phase relative to that of the current through the resistor. The phasor diagram for the RLC series circuit shows the main features.

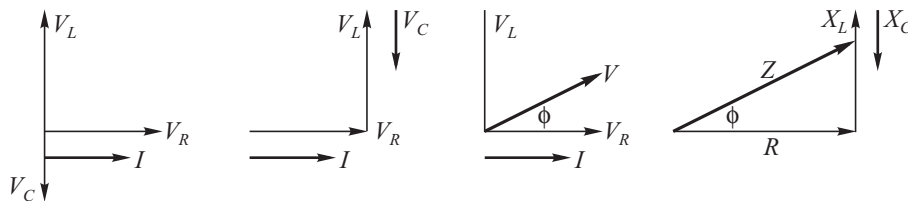


Fig. 1.21

$$V = \sqrt{V_R^2 + (V_L - V_C)^2} \qquad Z = \sqrt{R^2 + (X_L - X_C)^2}$$

$$\phi = \tan^{-1} \frac{V_L - V_C}{V_R} \qquad \phi = \tan^{-1} \frac{X_L - X_C}{R}$$

Note that the phase angle, the difference in phase between the voltage and the current in an A.C. circuit, is the phase angle associated with the impedance Z of the circuit.

1.13 PHASOR REPRESENTATION OF SINUSOIDAL CURRENTS AND VOLTAGES

A phase vector ("phasor") is a representation of a sine wave whose amplitude (A), phase (θ), and frequency (ω) are time-invariant. It is a subset of a more general concept called analytic representation. Phasors reduce the dependencies on these parameters to three independent factors, thereby simplifying certain kinds of calculations. In older texts, a phasor is also referred to as a sinor.

An alternating current or voltage is graphically represented by a line fixed at one end and rotating anticlockwise with a constant angular velocity ω radians/sec. The rotating line is called a phasor. The instantaneous value of the voltage or current is given by the length of the phasor and the instantaneous phase by the angle which the phasor makes with reference line. The phase angle is taken positive in anticlockwise direction and negative in clockwise direction.

The figure shown below shows the phasor representation of a sinusoidal voltage. Here OA represents phasor. Its projection on the vertical axis will give the instantaneous value of the e.m.f., *i.e.*,

$$OB = OA \sin \omega t$$

or

$$E = E_o \sin \omega t.$$

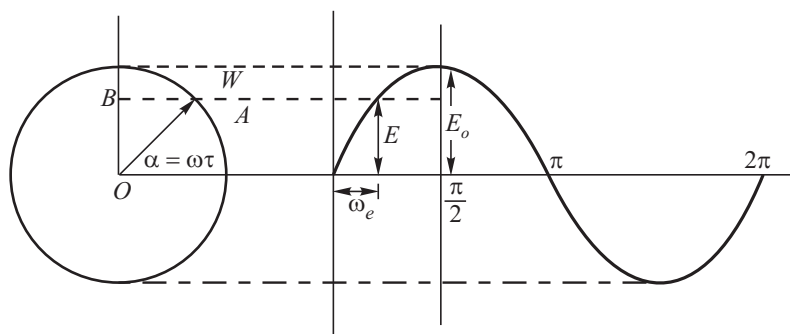


Fig. 1.22

The fraction of a cycle or time period that has elapsed since an alternating current or voltage last passed a given reference point, which is generally the starting point, is called its phase.

Phase of the alternating current or voltage may be expressed in time measured in seconds or fraction of a time period or the angle expressed in the degree or radians.

The phase difference between two alternating quantities is more important than their absolute phases. If two alternating current or voltages act simultaneously in the same circuit, they may do so in such a manner that their peak values do not occur at the same time. The time interval between two positive peak values of a.c. current or voltage is known as the phase difference. The quantity which reaches its maximum value earlier as compared to the other quantity is said to lead on the